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A lattice study of the exclusive $B \to K^*\gamma$ decay amplitude, using the Clover action at $\beta = 6.0$

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Abstract

We present the results of a numerical calculation of the $B \to K^*\gamma$ form factors. The results have been obtained by studying the relevant correlation functions at $\beta = 6.0$, on an $18^3 \times 64$ lattice, using the O(a)-improved fermion action, in the quenched approximation. From the study of the matrix element $\langle K^*|\bar{s}\sigma_{\mu\nu}b|B\rangle$ we have obtained the form factor $T_1(0)$ which controls the exclusive decay rate. The results are compared with the recent results from CLEO. We also discuss the compatibility between the scaling laws predicted by the Heavy Quark Effective Theory (HQET) and pole dominance, by studying the mass- and q^2-dependence of the form factors. From our analysis, it appears that the form factors follow a mass behaviour compatible with the predictions of the HQET and that the q^2-dependence of T_2 is weaker than would be predicted by pole dominance.

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1 Introduction

An important class of B-meson decays is given by the weak radiative decays $B \to X_s\gamma$, where X_s is a strange hadronic state and where the emitted photon is real. These decays are extensively studied because they provide, through loop effects, interesting information on the Standard Model parameters (e.g. the combination of CKM matrix elements $|V_{ts}V_{tb}^*|$). Short-distance physics puts in the foreground an effective magnetic interaction, $b \to s\gamma$, originating from the so-called penguin diagrams. In these diagrams, the top quark dominates, whereas the charm and up quark contributions are suppressed by powers of the quark masses.

Radiative decays are also sensitive to physics beyond the Standard Model, through possible extra particles (SUSY particles, extra charged Higgs bosons, ...) contributing to the loops. Should there be deviations between the expected decay rates and the measured values, these could be a manifestation of new physics.

Among radiative decays, the process $B \to K^*\gamma$ has received an increasing attention, because of the experimental measurement of the $B \to K^*\gamma$ branching ratio, performed by the CLEO collaboration [1]: $BR(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$. More recently, the same collaboration has also measured the inclusive rate, obtaining $BR(B \to X_s\gamma) = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}$ [2].

Several methods have been employed to predict the inclusive $B \to X_s\gamma$ and exclusive $B \to K^*\gamma$ decay rates: Heavy Quark Effective Theory (HQET)[3], QCD sum rules [4]–[8], and quark models [9]–[11]. For the exclusive decay, the theoretical uncertainty, which was originally of more than two orders of magnitude, has been greatly reduced by the more recent studies. Still, there is a large spread between the different results. Lattice QCD offers the possibility to investigate rare B decays from first principles. The feasibility of the lattice approach was first demonstrated in 1991, by the work of Bernard, Hsieh and Soni [12]. Further results have been obtained by the UKQCD collaboration [13] and by the LNAL collaboration [14].

In this paper, we present a lattice calculation of the form factors T_1 and T_2 relevant for exclusive $B \to K^*\gamma$ decays, and of the dependence of these form factors on the heavy-quark mass and on the squared momentum transfer q^2. Preliminary results of our study can be found in ref. [15]. The study of the dependence of the form factors on the heavy-quark mass provides a good test of the validity of the scaling laws predicted by HQET, in a region of masses around the charm quark mass. Particular attention has been devoted to the understanding of these scaling laws and their relations to the q^2-dependence. This point will be discussed in detail in section 4.

In the same context, there is a very interesting challenge: assuming HQET and $SU(3)$ symmetry, the hadronic matrix elements $\langle K^*(\eta,k)|\bar{s}\sigma^{\mu\nu}q_\nu b|B(p)\rangle$ are related to those relevant for B-meson semileptonic decays: $\langle \rho(\eta,k)|\bar{s}\gamma^\mu b|B(p)\rangle$ and $\langle \rho(\eta,k)|\bar{s}\gamma^\mu\gamma_5 b|B(p)\rangle$ [16]–[18]. This implies relationships between the semileptonic and radiative form factors. These relations can be proved, in the infinite mass limit, near the zero recoil point, i.e. $q^2 \sim q^2_{max}$. A complete test of these relationships on the lattice is a very interesting check of QCD dynamics.

With some extra assumptions one can extend the HQET relations, to the region of small q^2 [18]–[20]. As an example, consider the ratio

$$\frac{R(B \to K^*\gamma)}{\frac{d\Gamma(B\to\rho\ell\bar{\nu}_\ell)}{dq^2}|_{q^2=0}} = \frac{192\pi^3}{G_F^2}\frac{1}{|V_{ub}|^2}\frac{(m_B^2-m_{K^*}^2)^3}{(m_B^2-m_\rho^2)^3}\frac{m_b^3}{(m_b^2-m_s^2)^3}|\mathcal{I}|^2 \quad , \tag{1}$$

where

$$R(B \to K^*\gamma) = \frac{\Gamma(B \to K^*\gamma)}{\Gamma(B \to X_s\gamma)} \quad , \quad \mathcal{I} = \frac{(m_B+m_\rho)}{(m_B-m_{K^*})}\frac{2\,T_1(0)}{A_0^{B\to\rho}(0)} \tag{2}$$

and $A_0^{B\to\rho}(0)$ is one of the six semileptonic form factors (see for example [25]). The quantity $\mathcal{I}$ can be measured on the lattice. According to [18]–[20], $\mathcal{I}$ should be close to 1. Unfortunately, the quality of our numerical results does not allow a calculation of $\mathcal{I}$ at present. For this reason, we only focus here on $B \to K^*\gamma$ decay.

We present results obtained by assuming different scaling laws, in the heavy-meson mass, for the form factors at $q^2 = 0$. These scaling laws will be discussed in detail below. The two possibilities correspond to a pole-dominance behaviour in q^2 either for T_1 or for T_2 and lead to quite different results.

If we assume that T_1 follows the pole-dominance behaviour, we expect a scaling law of the form $T_1(q^2=0) = T_2(q^2=0) \sim m^{-1/2}$, where m is the mass of the heavy quark. In this case, following ref. [21], we obtain

$$\begin{aligned} T_1(q^2=0) &= 0.23 \pm 0.02 \pm 0.02\,, \\ R &= 0.31 \pm 0.12\,, \\ BR(B \to K^*\gamma) &= (7.4 \pm 1.4^{+2.4}_{-1.7}) \times 10^{-5}\,. \end{aligned} \tag{3}$$

where $R = \Gamma(B \to K^*\gamma)/\Gamma(B \to X_s\gamma)$. The formulae used to compute R and $BR(B \to K^*\gamma)$ can be found in section 2. If instead, we assume that T_2 follows the pole dominance behaviour, we expect a scaling law of the form $T_1(q^2=0) = T_2(q^2=0) \sim m^{-3/2}$. In this case we find

$$\begin{aligned} T_1(q^2=0) &= 0.09 \pm 0.01 \pm 0.01\,, \\ R &= 0.05 \pm 0.02\,, \\ BR(B \to K^*\gamma) &= (1.1 \pm 0.3^{+0.4}_{-0.3}) \times 10^{-5}\,. \end{aligned} \tag{4}$$

The values of $T_1(q^2=0)$ in eqs. (3) and (4) are in good agreement with the results of other similar studies [12]–[14]. In the first case, eqs. (3), the agreement of the lattice predictions with the experimental measurements is rather satisfactory. In the second case, eqs. (4), either there is a problem with the lattice calculations, or the difference may come from long-distance contributions to the exclusive decay rate [22]. Unfortunately, the present lattice technology cannot estimate these (and others) long-distance contributions. Our study of the mass- and q^2-dependence of the form factors favours solution (3), even though, given the statistical accuracy of our results and the systematic errors, we cannot draw any firm conclusion at this stage.

The systematic difference between results (3) and (4) originates from the extrapolation to large meson masses and small q^2, which is needed to obtain the physical form factors. This problem is intrinsic to the lattice approach at values of the lattice spacing currently accessible in numerical simulations. In this respect, at present, the lattice approach is not very different from quark-model calculations. The q^2- and mass-dependence of the form factors, including those relevant to semileptonic decays, remains a crucial challenge for lattice calculations.

2 Effective Hamiltonian and notation

In this section, we introduce the essential notation and express the matrix elements of the effective Hamiltonian and the exclusive decay rate in terms of the relevant form factors. The operator basis of the effective Hamiltonian density, for weak radiative B-meson decays, consists of local four-quark operators O_n ($n = 1, 2, \ldots, 6$) and magnetic-type operators O_n ($n = 7, 8$) [26]–[31]:

$$\mathcal{H}_{\rm eff} = -V_{tb}V_{ts}^* G_F/\sqrt{2}\sum_{n=1}^{8} C_n(\mu)O_n(\mu), \tag{5}$$

where C_n are the Wilson coefficients. The operator that controls the $b \to s\gamma$ transition is:

$$O_7 = \Big(\frac{e}{16\pi^2}\Big) m_b\,(\bar{s}\sigma^{\mu\nu}b)_R\,F_{\mu\nu}\,. \tag{6}$$

From eqs. (5) and (6), one finds the matrix element for the transition $B \to K^*\gamma$

$$\mathcal{M} = \frac{eG_F m_b}{8\sqrt{2}\pi^2} C_7(m_b)V_{tb}V_{ts}^*\epsilon^{\mu *}\langle K^*|J_\mu|B\rangle\,, \tag{7}$$

where

$$J^\mu \equiv J_V^\mu + J_A^\mu \;=\; \bar{s}\sigma^{\mu\nu}\frac{1\;+\;\gamma_5}{2}q_\nu b \tag{8}$$

and ϵ^μ and $q^\mu = p^\mu - k^\mu$ are the photon polarization and momentum transfer, respectively. We parametrize the hadronic matrix element in eq. (7) as

$$\langle K^*(\eta_r, k)|J^\mu|B(p)\rangle = C_1^\mu T_1(q^2) + iC_2^\mu T_2(q^2) + iC_3^\mu T_3(q^2), \tag{9}$$

where

$$\begin{aligned}
C_1^\mu &= 2\epsilon^{\mu\alpha\rho\sigma}\eta_r^*(k)_\alpha p_\rho k_\sigma,\\
C_2^\mu &= \eta_r^{\mu\,*}(k)(M_B^2 - M_{K^*}^2) - \Big(\eta_r^*(k).q\Big)(p+k)^\mu,\\
C_3^\mu &= \Big(\eta_r^*(k).q\Big)\Big(\,q^\mu - \frac{q^2}{M_B^2 - M_{K^*}^2}(p+k)^\mu\,\Big)\,;
\end{aligned} \tag{10}$$

T_1, T_2 and T_3 are real dimensionless Lorentz-invariant form factors, and η is the K^* polarization vector. The vector current J_V^μ contributes only to T_1, the axial current J_A^μ only to T_2 and T_3 :

$$\begin{aligned}\langle K^*(\eta_r,k)|J_V^\mu|B(p)\rangle &= C_1^\mu T_1(q^2) \\ \langle K^*(\eta_r,k)|J_A^\mu|B(p)\rangle &= iC_2^\mu T_2(q^2) + iC_3^\mu T_3(q^2)\,;\end{aligned} \tag{11}$$

T_3 does not contribute to the physical rate, because its coefficient vanishes for a transversely polarized photon. By using the relation $\sigma^{\mu\nu}\gamma^5 = \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$, one finds

$$T_1(0) = T_2(0) \tag{12}$$

at $q^2 = 0$. From the matrix element (7), using (12), one obtains the decay width

$$\Gamma(B \to K^*\gamma) = \frac{\alpha}{128\pi^4} m_b^2 G_F^2 M_B^3 \left(1 - \frac{M_{K^*}^2}{M_B^2}\right)^3 |V_{tb}V_{ts}^*|^2 C_7(m_b)^2 |T_1(0)|^2. \tag{13}$$

The physics of this decay is then described by one form factor only, T_1 at $q^2 = 0$.

It is not convenient to compare the theoretical width (13) with its experimental value, because many theoretical uncertainties enter in this quantity: the renormalization scale, the matching of the lattice to the continuum operators, the value of Λ_{QCD}, the possible presence of new physics beyond the standard model, uncomputed higher-order corrections, etc. [21]. Now that the inclusive measure is available [2], it is much more informative to compare instead the exclusive-to-inclusive ratio of rates R because in this ratio most of the above-mentioned uncertainties cancel out. In terms of T_1, the ratio R can be written as

$$R = \left[\frac{\Gamma(B \to K^*\gamma)}{\Gamma(B \to X_s\gamma)}\right]^{th} = \left(\frac{M_B}{m_b}\right)^3 \left(1 - \frac{M_{K^*}^2}{M_B^2}\right)^3 \times \frac{4}{1 + (\lambda_1 - 9\lambda_2)/(2m_b^2)} \times |T_1(0)|^2\,, \tag{14}$$

where we have divided the exclusive rate (13) by the theoretical inclusive rate, computed in the HQET parton model. In this formalism, the parameters λ_1 and λ_2 describe the leading non-perturbative corrections (at order $O(1/m_b^2)$) to the parton-model predictions for the inclusive rate[1] [32].

The branching ratio $BR(B \to K^*\gamma)$ is conveniently expressed through a chain of ratios [21]

$$BR(B \to K^*\gamma) \;=\; R \times \left[\frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to X l \nu_l)}\right]^{th} \times BR^{exp}(B \to X l \nu_l). \tag{15}$$

[1] They are related to the kinetic energy of the b-quark inside the B-meson and to the B–B^* mass splitting.

3 Correlation functions and extraction of the form factors

In lattice simulations, from the study of two- and three-point correlation functions, one extracts the current matrix elements for a given momentum transfer and for a given polarization η of the K^*. This is by now a well established technique. The reader can find more details in refs. [33] and [34]. We follow more closely ref. [33].

In general, the form factors are obtained at values of q^2 that are constrained by the quark masses and the values of particle momenta accessible on a given lattice volume. For this reason, it is necessary to extrapolate T_1 and T_2 to $q^2 = 0$, in order to get the physical result. In the following, the different steps of the procedure to extract the form factors are briefly described[2].

The matrix elements have been computed for an initial meson at rest and a final vector meson with momentum $\vec{p}_{K^*}$. We have taken $\vec{p}_{K^*} \equiv 2\pi/(La)\ (0,0,0)$, $2\pi/(La)(1,0,0)$, $2\pi/(La)(1,1,0)$, $2\pi/(La)(1,1,1)$, and $2\pi/(La)(2,0,0)$; where L is the spatial extension of the lattice, in our case $L = 18$, and a is the lattice spacing. Correlation functions, which are equivalent under the hypercubic symmetry, have been averaged.

The initial (final) meson was created (annihilated) by using the pseudoscalar ($J_B = \bar{b}\gamma_5 q$) and local vector ($J^\alpha_{K^*} = \bar{q}\gamma^\alpha s$) densities, inserted at times $t_B/a = 28$ and $t_{K^*} = 0$, respectively. We have varied the time position of J_μ in the interval $t_J/a = 10$–14.

In order to obtain the hadronic matrix element, the following procedure has been used:

1. We have computed masses and source matrix elements by fitting the two-point correlation functions at large time distances to the expressions given below. For the pseudoscalar meson, we have used:

$$C_B(t_B) \equiv \langle 0|J_B(t_B)J_B^\dagger(\vec{x}=0,t=0)|0\rangle = \frac{Z_B}{2M_B}e^{-M_B t_B}, \qquad (16)$$

where $J_B(t_B) = \int d^3x J_B(\vec{x},t_B)$ and $Z_B^{1/2} = \langle 0|J_B|B(\vec{p}_B = \vec{0})\rangle$. For the vector current, one has

$$\begin{aligned} C^{\alpha\beta}_{K^*}(\vec{p}_{K^*},t_{K^*}) &= \langle 0|J^\alpha_{K^*}(\vec{p}_{K^*},t_{K^*})J^{\dagger\beta}_{K^*}(\vec{x}=0,t=0)|0\rangle \\ &= \Big(g^{\alpha\beta} - \frac{p^\alpha_{K^*}p^\beta_{K^*}}{M^2_{K^*}}\Big)\frac{Z_{K^*}}{2E_{K^*}}e^{-E_{K^*}t_{K^*}}, \qquad (17) \end{aligned}$$

where

$$J^\alpha_{K^*}(\vec{p}_{K^*},t_{K^*}) = \int d^3x\ e^{-i\vec{p}_{K^*}\cdot\vec{x}}J^\alpha_{K^*}(\vec{x},t_{K^*}). \qquad (18)$$

[2]We adopt the convention that B and K^* denote the pseudoscalar heavy meson and the vector light meson, whenever no confusion arises.

In eq. (17), Z_{K^*} is defined through

$$\eta_r^{\beta *}\sqrt{Z_{K^*}} = \langle K^*(\eta_r, \vec{p}_{K^*})|J^\beta_{K^*}|0\rangle\,. \tag{19}$$

2. We have then extracted the matrix elements from the ratios

$$R_{\mu\alpha} = \frac{C_3^{\mu\alpha}(t_B, t_J, \vec{p}_{K^*})}{C_{K^*}^{\alpha\alpha}(\vec{p}_{K^*}, t_J)C_B(t_B - t_J)}\,, \tag{20}$$

with

$$\begin{aligned} C_3^{\mu\alpha}(t_B, t_J, \vec{p}_{K^*}) &= \langle 0|J^\alpha_{K^*}(\vec{p}_{K^*}, 0)J^\mu(-\vec{p}_{K^*}, t_J)J_B^\dagger(t_B)|0\rangle = \\ &\sum_r \eta_r^\alpha \frac{\sqrt{Z_{K^*}}}{2E_{K^*}}e^{-E_{K^*}t_J}\frac{\sqrt{Z_B}}{2M_B}e^{-M_B(t_B-t_J)} \\ \times\quad &\langle K^*(\eta_r, \vec{p}_{K^*})|J^\mu|B\rangle\,, \end{aligned} \tag{21}$$

where the magnetic operator J^μ, renormalized in the continuum, is given in terms of the corresponding lattice bare operator by $J^\mu = Z_\sigma J^\mu_{\rm latt}$, with $Z_\sigma = 0.98$ taken from perturbation theory [35]. To obtain the matrix elements, we have used two different procedures, denoted by "analytic" and "ratio" methods, which are discussed in detail in ref. [33]. In the "ratio" method, for each fixed-time distance, the three-point correlation function is divided by the two-point functions of the B and K^* mesons (with corresponding momentum) as in eq. (20), in order to cancel the exponential time-dependence. The "analytic" method differs from the previous one in that, instead of dividing by the numerical two-point correlation functions, we divide by the corresponding analytic expressions using the source matrix elements (Z_B and Z_{K^*}) and the meson energies obtained from the fit of the two-point functions at zero momentum. The energy of the K^* si computed from the meson mass, as explained in eqs. (23) and (25) below. When both the initial and final mesons are at rest, and at large time distances, the two methods are practically equivalent and lead to almost identical results. However, when the meson momenta are different from zero the two methods are expected to agree only up to $O(a)$ effects.

In refs. [14, 34, 36], it was found that discretization errors appear to be reduced, if one uses, for the two-point correlation functions, the "lattice" dispersion relation of a free boson

$$\bar{C}(t, \vec{p}) = \frac{Z}{2\sinh E}e^{-Et}, \tag{22}$$

where

$$E = \frac{2}{a}\mathrm{arcsinh}\left(\sqrt{\sinh^2\left(\frac{ma}{2}\right) + \sum_{i=1,3}\sin^2\left(\frac{p_i a}{2}\right)}\right) \tag{23}$$

and $\vec{p}$ is the momentum of the meson. The same is true in our case. We have verified this point by studying the ratio $\mathcal{R}(t) = C(t, \vec{p})/\bar{C}(t, \vec{p})$, where $C(t, \vec{p})$ is the two-point correlation function of a meson with momentum $\vec{p}$, as computed in our simulation, and $\bar{C}(t,\vec{p})$ is given by the expression in eq. (22), with Z and m taken from the fit of the zero-momentum correlation to $\bar{C}(t,\vec{p}=\vec{0})$. At large time distances, we find that $\mathcal{R}(t)$ is closer to 1 (typically 1.05 ± 0.01 instead of 1.10 ± 0.02), if we use eq. (22) instead of the standard expression

$$\hat{C}(t,\vec{p}) = \frac{Z}{2E} e^{-Et}, \tag{24}$$

with

$$E = \sqrt{m^2 + |\vec{p}|^2}. \tag{25}$$

For this reason, throughout our analysis, we have fitted the two-point functions to $\bar{C}(t,\vec{p})$, eq. (22), in addition to the "standard" form $\hat{C}(t,\vec{p})$, eq. (24).

Using $\bar{C}(t,\vec{p})$, the "ratio" and "analytic" methods yield only slightly different results (see section 5).

4 q^2-dependence of the form factors and scaling laws

In order to obtain the form factors at the physical point, we need to extrapolate in both mass and momentum. The final results depend critically on the assumptions on the q^2- and heavy mass-dependence, which deserve a detailed discussion.

At fixed $\vec{p}_{K^*}$, with $|\vec{p}_{K^*}| \ll M_B$ in the B-meson rest frame, the following scaling laws can be derived [16]:

$$\frac{T_1}{\sqrt{M_B}} = \gamma_1 \times \left(1 + \frac{\delta_1}{M_B} + \dots\right), \qquad T_2\sqrt{M_B} = \gamma_2 \times \left(1 + \frac{\delta_2}{M_B} + \dots\right); \tag{26}$$

these are valid up to logarithmic corrections. As mentioned in the introduction, "scaling" laws for the form factors at $q^2 = 0$ can only be found by using extra assumptions on the q^2-dependence of the form factors. Such a procedure is acceptable, provided the "scaling" laws derived in this way respect the exact condition $T_1(0) = T_2(0)$. This is a non-trivial constraint: the q^2-behaviour of T_1 and T_2 has to compensate the different mass dependence of the two form factors near the zero recoil point, see eq. (26). Thus, for example, the popular assumption of pole dominance for both T_1 and T_2 would give $T_1(0) \sim M_B^{-1/2}$ and $T_2(0) \sim M_B^{-3/2}$, which is inconsistent with eq. (12).

In all lattice simulations, which try to compute B-meson form factors by extrapolating from lower heavy-quark masses, we are forced to make assumptions on the corresponding q^2-dependence. This is a consequence of the fact that the extrapolation in the mass, at fixed light-meson momentum, pushes the values of q^2 towards q^2_{max}. This problem can only be avoided by going to much smaller values

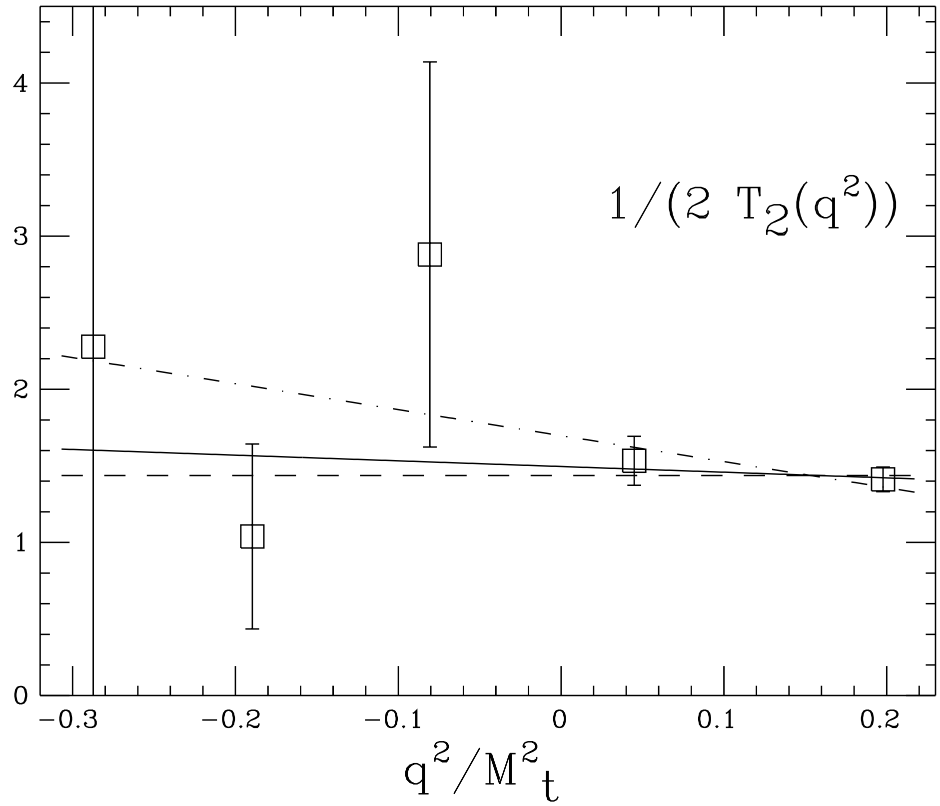

Figure 1: *The inverse form factor* $1/(2T_2(q^2))$ *is shown as a function of* q^2/M_t^2 *for* $K_H = 0.1200$*. The form factor has already been extrapolated in the mass of the light quark to the strange-quark mass. The dash-dotted line represents the pole-dominance behaviour, with the mass of the axial meson* M_t *taken from the fit of the two-point correlation function; the solid line corresponds to a pole-dominance behaviour, with* M_t *left as a free parameter; the dashed line is a fit with* $T_2(q^2)$ *constant in* q^2*.*

of the lattice spacing, thus allowing a large increase in the range of accessible masses and momenta. The assumptions on the q^2-dependence of the form factors can be tested, although in a small domain of momenta, directly on the numerical results.

In fig. 1, we show $1/(2T_2(q^2))$ as a function of the dimensionless variable q^2/M_t^2, where M_t is the appropriate mass for the axial meson exchanged in the t-channel, in the pole-dominance approximation. If pole dominance is valid, the form factor should have a linear behaviour in q^2, with a slope proportional to $1/M_t^2$. In the case of T_2, we found that the mass extracted from the slope is much larger (by a factor of order 2 in most of the cases) than the mass that we have obtained from the axial two-point correlation function. In other words,

$T_2(q^2)$ is flatter than predicted by pole dominance. In the case of T_1, the point at momentum $(0,0,0)$ cannot be computed. The absence of this point and the large statistical and systematic errors at large momenta make it difficult to test the validity of the pole-dominance hypothesis directly on our results. A consistency check will be provided is section 5. Although our data are not accurate enough to draw a definite conclusion, they suggest that assuming T_2 almost constant in q^2 and T_1 following pole dominance gives a good description of our results. This assumption is consistent with the "scaling" laws described above. We call this option $m^{-1/2}$-scaling. The $m^{-1/2}$-scaling is similar to what has been encountered in the QCD sum-rules calculation [37, 38] of the semileptonic $B \to \rho$ form factors V and A_1, which are related to T_1 and T_2 in the large-mass limit. In fig. 1, we also give the curves corresponding to a pole dependence for T_2 ($m^{-3/2}$-scaling). This possibility is also consistent with the "scaling" laws, if a dipole dominance behaviour is assumed for T_1, but it is disfavoured by our numerical results, as discussed above and shown in the figure. For these data, the uncorrelated χ^2, corresponding to the fit of T_2 to a constant, is $\chi^2 = 0.65$ to be compared to $\chi^2 = 1.15$, in the case of the fit of T_2 to a pole dominance behaviour. Similar results have been obtained at the other values of the heavy quark mass. It is clear, that an infinite number of possible q^2-dependence for $T_{1,2}$, which are compatible with the scaling laws, can be found. However it would be impossible to distinguish among them, given the statistical errors, the systematic uncertainty in the extraction of the form factors, the effects of $O(a)$, and the limited range in q^2 and M_B. Thus we take the two possibilities, $m^{-1/2}$-scaling and $m^{-3/2}$-scaling, as representative of a full class of "scaling" laws. In section 5, we will see that the two options lead to quite different results for the physical value of $T_1(0)$, which enters in the calculation of the $B \to K^*\gamma$ rate.

There remains to discuss the dependence on the light-quark masses (spectator and active). At fixed heavy-quark mass and light-meson momentum $\vec{p}_{K^*}$, the generic form factor F ($F = T_1, T_2$) has been extrapolated linearly in the mass of the active light quark

$$F = \alpha + \beta m_q \,, \tag{27}$$

to values corresponding to the physical strange meson K^*, assuming the form factors independent of the mass of the spectator quark. This is in agreement with the results of ref. [13], where it was shown that the dependence of the form factors on the mass of the spectator quark is very small. We believe that the extrapolation in the light-quark mass is unlikely to be a source of an important uncertainty within the present statistical accuracy.

5 Lattice setup and numerical results

The numerical simulation was performed on the 6.4 gigaflops APE machine, at $\beta = 6.0$, on an $18^3 \times 64$ lattice, using the SW-Clover action [39] in the quenched

Form factor	K_H	r-sinh	a-sinh	r-stand	a-stand
T_1	0.1330	0.35(3)	0.34(3)	0.33(3)	0.34(3)
T_2	0.1330	0.37(2)	0.37(2)	0.35(2)	0.35(2)
$\bar{T}_2$	0.1330	0.37(2)	0.36(2)	0.35(2)	0.35(2)
T_1	0.1250	0.34(4)	0.33(3)	0.31(3)	0.32(3)
T_2	0.1250	0.36(2)	0.36(2)	0.33(2)	0.33(2)
$\bar{T}_2$	0.1250	0.34(3)	0.33(3)	0.31(3)	0.32(3)
T_1	0.1200	0.33(4)	0.32(3)	0.30(3)	0.31(3)
T_2	0.1200	0.35(2)	0.35(2)	0.32(2)	0.32(2)
$\bar{T}_2$	0.1200	0.33(5)	0.32(4)	0.29(4)	0.31(4)
T_1	0.1150	0.33(4)	0.32(4)	0.29(3)	0.30(3)
T_2	0.1150	0.35(2)	0.35(2)	0.30(2)	0.30(2)
$\bar{T}_2$	0.1150	0.32(6)	0.31(6)	0.28(5)	0.30(6)
T_1	B	0.24(3)	0.23(3)	0.20(3)	0.21(3)
T_2	B	0.25(2)	0.25(2)	0.22(2)	0.22(2)
$\bar{T}_2$	B	0.22(5)	0.21(5)	0.19(5)	0.21(5)

Table 1: *T_1, T_2 and $\bar{T}_2$ at $q^2 = 0$ for different values of the heavy-quark masses, as extracted using the pole and constant behaviour respectively. We give in the second column the hopping parameter of the heavy quark (B denotes the extrapolation to the B-meson using the $m^{-1/2}$ scaling law); the light-quark mass has been extrapolated to the strange quark mass; the values of the form factors obtained with the ratio method from the sinh or standard fits (r-sinh and r-stand) or with the analytic method from the sinh or standard fits (a-sinh and a-stand) are shown from the third to the sixth columns.*

Form factor	K_H	r-sinh	a-sinh	r-stand	a-stand
T_1	0.1330	0.42(4)	0.40(4)	0.39(4)	0.40(4)
T_2	0.1330	0.36(2)	0.36(2)	0.34(2)	0.34(2)
T_1	0.1250	0.35(4)	0.34(3)	0.32(3)	0.33(3)
T_2	0.1250	0.32(2)	0.31(2)	0.29(2)	0.29(2)
T_1	0.1200	0.31(4)	0.31(3)	0.28(3)	0.29(3)
T_2	0.1200	0.29(2)	0.29(2)	0.26(2)	0.27(2)
T_1	0.1150	0.28(4)	0.28(3)	0.25(3)	0.26(3)
T_2	0.1150	0.27(2)	0.27(2)	0.24(2)	0.24(2)
T_1	B	0.10(1)	0.09(1)	0.08(1)	0.09(1)
T_2	B	0.09(1)	0.09(1)	0.08(1)	0.08(1)

Table 2: *T_1 and T_2 at $q^2 = 0$ for different values of the heavy-quark masses, as extracted using the dipole and pole behaviour respectively. We give in the second column the hopping parameter of the heavy quark (B denotes the extrapolation to the B-meson using the $m^{-3/2}$ scaling law); the light quark mass has been extrapolated to the strange-quark mass; the values of the form factors obtained with the ratio method from the sinh or standard fits (r-sinh and r-stand) or with the analytic method from the sinh or standard fits (a-sinh and a-stand) are shown from the third to the sixth column.*

$T(0) = T_1(q^2 = 0) = T_2(q^2 = 0)$				
Scaling law	r-sinh	a-sinh	r-stand	a-stand
$m^{-1/2}$	0.25(2)	0.25(2)	0.21(2)	0.22(2)
$m^{-3/2}$	0.09(1)	0.09(1)	0.08(1)	0.08(1)

Table 3: $T(0) = T_1(q^2 = 0) = T_2(q^2 = 0)$ *extrapolated to the B-meson using the $m^{-1/2}$ and the $m^{-3/2}$ scaling laws; the values of the form factors obtained with the ratio method from the sinh or standard fits (r-sinh and r-stand) or with the analytic method from the sinh or standard fits (a-sinh and a-stand) are shown from the second to the fifth columns.*

approximation. The results were obtained from a sample of 170 gauge configurations. The statistical errors have been estimated by a jacknife method, by decimating 10 configurations from the total set. For each configuration we have computed the quark propagators for 7 different values of the Wilson hopping parameter K_W, corresponding to "heavy" quarks, $K_H = 0.1150$, 0.1200, 0.1250, 0.1330, and "light" quarks, $K_L = 0.1425$, 0.1432 and 0.1440. In order to extract masses and source matrix elements, we have fitted the heavy and light meson two-point functions to eqs. (16) and (17) in the time interval $14 \leq t/a \leq 28$ and $10 \leq t/a \leq 28$ respectively. The relevant matrix elements of O_7 have been extracted, for different polarizations and momenta of the K^*, from the three-point function, see eq. (21), with the operator inserted at $10 \leq t_J/a \leq 14$. We found that the critical value of K_L, corresponding to the chiral limit, is $K_{cr} = 0.14545(1)$; the inverse lattice spacing, obtained from m_ρ, is $a^{-1} = 1.96(7)$ GeV; the value of the Wilson parameter for the strange quark, determined from the mass of the kaon, is $K_s = 0.1435(1)$. In the following, the labels "ratio" and "analytic" refer respectively to the ratio and analytic method as explained in section 3. The numbers quoted with "sinh" and "standard" are obtained respectively with the fits to eqs. (22) and (24) for the two-point functions (see section 3).

We now explain the analysis of the scaling behaviour of the form factors that, on the basis of the above discussion, has been done in combination with the study of their q^2-dependence. Unless otherwise stated, the form factors are those obtained after the extrapolation in the active light quark mass to the value of the hopping parameter corresponding to the strange quark.

1. According to HQET, when the mass of the meson is sufficiently large, T_2 at zero recoil should follow the behaviour given in eq. (26). A fit of our data for $T_2(q^2_{max})$ to $M_B^\alpha(a + b/M_B)$, with α, a and b as free parameters, gives $\alpha = -0.41\,(10)$, and $b/a = -350\,(70)$ MeV ($\alpha = -0.68\,(10)$, and $b/a = -480\,(50)$ MeV) with the sinh-fit (standard-fit). The exponent α is thus compatible with the value of $-1/2$ predicted by HQET, see eq. (26). To reduce the number of parameters, we have then extrapolated $T_2(q^2_{max})$ in $1/M_B$, with $\alpha = -1/2$, using eq. (26). In this case, we obtain $\gamma_2 = 20.2\,(1.6)$ MeV$^{1/2}$ and $\delta_2 = -430\,(50)$ MeV ($\gamma_2 = 16.8\,(1.4)$ MeV$^{1/2}$ and $\delta_2 = -320\,(50)$ MeV). These parameters correspond to the following values of $T_2(q^2_{max})$ for the B-meson

$$\begin{array}{lcc} & \text{sinh} & \text{standard} \\ T_2(q^2_{max}, B) = & 0.25(2) & 0.22(2)\,. \end{array} \qquad (28)$$

This result is also reported in table 1 as T_2 with the label B.

2. Let us assume the "pure-$m^{-1/2}$" behaviour for the form factors. By "pure-$m^{-1/2}$" behaviour, we mean that $T_2(q^2)$ is independent of q^2 (this is compatible with our results, cf. fig. 1) and that $T_1(q^2)$ follows a pole-dominance behaviour, with the mass of the vector meson as measured from the two-point correlation functions. In table 1, we compare, for all the values of the heavy-quark mass, $T_2 = T_2(q^2 = 0) = T_2(q^2_{max})$ with $T_1 = T_1(q^2 = 0) = T_1(q^2) \times (1 - q^2/M_t^2)$. The values of T_1 and T_2 are compatible within the statistical errors and the differences coming from different extrapolations (analytic, ratio, sinh, standard).

3. In order to check the stability of the results, we also performed a linear extrapolation in q^2 of $T_2(q^2)$ to $q^2 = 0$, at fixed heavy-quark mass, followed by a fit of $T_2(q^2 = 0)$ to eq. (26). The values obtained in this way are reported as $\bar{T}_2$ in table 1. Since we have not done any assumption on the M_B-dependence of the slope of the fit, this procedure is not a priori incompatible with the scaling laws discussed in the previous section. It should be noticed that, since we have an extra parameter in the fits, i.e. the linear slope in q^2, the extrapolated value $\bar{T}_2$ has a larger error.

4. Even though our results prefer a flat q^2-dependence for T_2, we have also fitted T_1 and T_2 by assuming a pure "$m^{-3/2}$" behaviour for the form factors. This means that we first fit the q^2-dependence of the form factors to $T_1(q^2) = T_1/(1 - q^2/M_t'^2)^2$ and $T_2(q^2) = T_2/(1 - q^2/M_t^2)$, where for each value of the heavy-quark mass, the value of M_t' and M_t are those computed from the corresponding two-point functions. The form factors extracted in this way are reported in tab. 2. We have then extrapolated in $1/M_B$ using the expressions $T_1\, M_B^{3/2} = \chi_1 \times (1 + \theta_1/M_B)$ and $T_2\, M_B^{3/2} = \chi_2 \times (1 + \theta_2/M_B)$.

5. We notice that, for the values of the heavy quark masses at which we have computed the form factors, the value of $T_1(q^2 = 0)$ ($T_2(q^2 = 0)$) extracted by assuming the pole (constant) q^2-behaviour differs at most by 20–25% from the value obtained by assuming the dipole- (pole-) q^2-depencence, as can be read in tables 1 and 2. The same would be true if we extrapolated the form factors using the two different scaling laws to the charm-quark mass. The values extrapolated to the B-meson instead, differ by about a factor of 2.

6. One should also take into account the distortion in the mass dependence of the form factors due to lattice artefacts at large quark masses. This effect has been measured non-perturbatively both by the APE collaboration at $\beta = 6.0$ [40] and the UKQCD collaboration at $\beta = 6.2$ [41]. Only with more accurate data, and exploring a much larger range of masses and momenta at larger values of β, will it be possible to decide the fundamental issue of the scaling behaviour.

7. We can exploit the information that the two form factors must be equal at $q^2 = 0$, by making a combined fit of T_1 and T_2, with the constraint $T(0) = T_1(q^2 = 0) = T_2(q^2 = 0)$. This can be done both in the $m^{-1/2}$ and $m^{-3/2}$ cases. The results are given in table 3.

8. As a consistency check of our results, we have also inverted the order of the fits, by fitting first $T_2(q^2_{\rm max})$ in $1/M_B$ and then assuming the q^2 behaviour of the form factor. The results are, within the errors, well compatible with those reported in tables 1 and 2.

From the results reported in tables 1–3, we quote

$$\begin{aligned} T_1(q^2 = 0) &= 0.23(2)(2) \qquad \text{scaling } m^{-1/2}\,, \\ T_1(q^2 = 0) &= 0.09(1)(1) \qquad \text{scaling } m^{-3/2}\,, \end{aligned} \tag{29}$$

from which we have derived the results of (3) and (4) in the introduction.

6 Conclusion

We have computed the form factor relevant for $B \to K^*\gamma$ decays. In order to extract the physical form factor from the lattice results, we have extrapolated in momentum transfer and in the mass of the heavy quark. The results, corresponding to different choices of the scaling law, can differ by about a factor of 2. Within large uncertainties, our study suggests that the scaling law "$m^{-1/2}$", by which $T_2(q^2)$ has a very small dependence on the momentum transfer, is preferred, in agreement with the results of ref. [14]. We cannot exclude, however, the "$m^{-3/2}$" scaling behaviour, or any intermediate solution. In order to improve the accuracy of the predictions, it is necessary to be able to work with heavier quark masses and to increase the range of q^2. This can only be achieved by going to larger lattices.

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