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Flipped $\mathbf{g}_\mu - \mathbf{2}$

John Ellis[a], **Jason L. Evans**[b], **Natsumi Nagata**[c],
Dimitri V. Nanopoulos[d] and **Keith A. Olive**[e]

[a] *Theoretical Particle Physics and Cosmology Group, Department of Physics, King's College London, London WC2R 2LS, United Kingdom; Theoretical Physics Department, CERN, CH-1211 Geneva 23, Switzerland; National Institute of Chemical Physics and Biophysics, Rävala 10, 10143 Tallinn, Estonia*

[b] *Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China*

[c] *Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo 113–0033, Japan*

[d] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics and Astronomy, Texas A&M University, College Station, TX 77843, USA; Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA; Academy of Athens, Division of Natural Sciences, Athens 10679, Greece*

[e] *William I. Fine Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA*

ABSTRACT

We analyze the possible magnitude of the supersymmetric contribution to $g_\mu - 2$ in a flipped SU(5) GUT model. Unlike other GUT models which are severely constrained by universality relations, in flipped SU(5) the U(1) gaugino mass and the soft supersymmetry-breaking masses of right-handed sleptons are unrelated to the other gaugino, slepton and squark masses. Consequently, the lightest neutralino and the right-handed smuon may be light enough to mitigate the discrepancy between the experimental measurement of $g_\mu - 2$ and the Standard Model calculation, in which case they may be detectable at the LHC and/or a 250 GeV e^+e^- collider, whereas the other gauginos and sfermions are heavy enough to escape detection at the LHC.

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1 Introduction

It is now 20 years since the first emergence of the discrepancy between the experimental value of $g_\mu - 2$ and the value calculated in the Standard Model [1]. The significance of this discrepancy has increased subsequently, with improved accuracy in the BNL measurements [2] and now the measurement by the Fermilab experiment [3], and the increased precision in the Standard Model calculation made possible, in particular, by improved determinations of the hadronic vacuum polarization and light-by-light contributions [4]. As soon as the first BNL result was announced, supersymmetric models were immediately proposed to explain the discrepancy [5, 6]. However, the popularity of the supersymmetric explanation has waned over the years, with the continuing lack of direct experimental evidence for supersymmetry, particularly at the LHC [7].

However, this dampening of supersymmetric enthusiasm is not entirely warranted. The absence at the LHC so far of squarks and gluinos does not bear directly on the possible masses of smuons and their sneutrino, the lighter chargino and the lightest neutralino, which would likely give the largest supersymmetric contributions to $g_\mu - 2$. However, in models that postulate universality relations at a high grand unification (GUT) scale, there are relations between the different gaugino masses and between the various soft supersymmetry-breaking sfermion masses. For example, in the constrained minimal supersymmetric Standard Model (CMSSM) [8], a universal gaugino mass, $m_{1/2}$, a scalar mass, m_0, and a trilinear term, A_0, are all defined at the GUT scale and, together with the ratio of Higgs vacuum expectation values (vev), $\tan\beta$, and the sign of the μ-term, define the sparticle spectrum at the weak scale when run down from the GUT scale. Prior to the LHC searches and the discovery of the Higgs boson, the CMSSM could easily account for the $g_\mu - 2$ discrepancy [5, 6], but the current experimental constraints exclude a significant supersymmetric contribution to $g_\mu - 2$ in this and similar models [9, 10]. However, if one treats the soft supersymmetry-breaking parameters as phenomenological quantities unconstrained by GUT-scale relations, the absence of sparticles at the LHC can be reconciled with a supersymmetric explanation of the $g_\mu - 2$ discrepancy [11, 12]. [1]

We show in this paper that a significant supersymmetric contribution is possible in one specific GUT model, namely flipped SU(5) (FSU(5)) [15]. [2] We recall that the difference in $a_\mu \equiv (g_\mu - 2)/2$ between the combination of the BNL and Fermilab data and the data-driven value recommended in [4] is $\Delta a_\mu = (251 \pm 59) \times 10^{-11}$, and that a recent lattice calculation [17] corresponds to $\Delta a_\mu = (107 \pm 69) \times 10^{-11}$. We find a region of the FSU(5) parameter space for which the supersymmetric contribution can reach $\Delta a_\mu|_{\rm FSU(5)} \gtrsim 140 \times 10^{-11}$, which would reduce the discrepancy with the data-driven calculation of a_μ to below 2 standard deviations, and remove entirely the discrepancy with the lattice calculation by the BMW collaboration [17].

[1] See [13, 14] for other supersymmetric interpretations of the $g_\mu - 2$ measurements.
[2] See [16] for a previous discussion of $g_\mu - 2$ in FSU(5).

2 Recap of the FSU(5) GUT

Specific GUT-motivated models can interpolate between the restrictive CMSSM and the relatively unconstrained phenomenological MSSM (pMSSM) [11, 18, 19]. In a minimal SU(5) GUT, while there is only a single universal gaugino mass, $m_{1/2} = M_5$, each generation of matter fields is split into $\mathbf{10}$ and $\bar{\mathbf{5}}$ representations, which may have separate soft scalar masses, m_{10} and $m_{\bar{5}}$, respectively [9]. Additionally, the Standard Model Higgs fields originate from a $\mathbf{5}$ and $\bar{\mathbf{5}}$ pair, which may also receive independent soft masses m_H and $m_{\bar{H}}$ as in an extension of the CMSSM with non-universal Higgs masses (NUHM) [20]. The common value of the gaugino masses at the GUT scale links the electroweak gaugino masses to the gluino mass, and the fact that both right- and left-handed (s)leptons find themselves in (super)multiplets containing (s)squarks links slepton masses to squark masses through renormalization-group running. Thus, despite its additional degrees of freedom beyond those in the CMSSM, the SU(5) model does not resolve the $g_\mu - 2$ discrepancy [9].

On the other hand, we recall that in FSU(5) there are two independent gauge group factors: in addition to the GUT SU(5) factor there is an 'external' U(1) factor. The masses of the usual SU(3), SU(2) and U(1) gauginos are related by SU(5) universality at the GUT scale, M_5, but the mass of the 'external' U(1) gaugino, M_{X1}, is in general independent. Liberated from the tyranny of GUT unification, this external U(1) gaugino could be much lighter than the other U(1) gaugino and the Higgsinos, enabling the lightest neutralino dark matter particle to be relatively light. We recall also that the right-handed sleptons are assigned to singlet representations of FSU(5), so their soft supersymmetry-breaking masses, m_1, are unrelated to those of the other sfermions, which have flipped assignments in $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations of SU(5). Therefore the mass of the right-handed smuon, $\tilde{\mu}_R$, is unrelated to the masses of the squarks and the left-handed smuon, $\tilde{\mu}_L$.

At one-loop order, there are contributions to $g_\mu - 2$ from a $\tilde{\mu}_R/\chi$ loop, a $\tilde{\mu}_L/\chi$ loop, and a diagram where the $\tilde{\mu}_R$ and $\tilde{\mu}_L$ mix (as well as chargino exchange diagrams). From the calculations in [21], we find that the neutralino exchange diagrams always dominates over the chargino exchange terms, and the dominant contribution comes from $\tilde{\mu}_R/\tilde{\mu}_L$ mixing, with the $\tilde{\mu}_R/\chi$ and $\tilde{\mu}_L/\chi$loop both sub-dominant. This is due in part to the relatively large values of μ and A_0 that contribute to left-right mixing. As we shall see, the $\tilde{\mu}_R$ might be sufficiently light, in combination with the lightest neutralino, χ, to reconcile the experimental measurement of $g_\mu - 2$ with the theoretical calculation of the Standard Model contribution.

More specifically, the assignments of representations and charges of each generation of particles in the matter sector of the theory are

$$\bar{f}_i(\bar{\mathbf{5}}, -3) = \{U_i^c, L_i\} \ , \quad F_i(\mathbf{10}, 1) = \{Q_i, D_i^c, N_i^c\} \ , \quad l_i(\mathbf{1}, 5) = E_i^c \ , \quad i = 1, 2, 3 \, , \tag{1}$$

where the charges are defined in the (SU(5), U(1)$_X$) basis. We note that there is an additional degree of freedom beyond the Standard Model contained in the $\mathbf{10}$, denoted by N^c, which can be interpreted as a right-handed neutrino. In order to generate the right-handed neutrino masses, the theory contains three or more SU(5) singlets ϕ_a.

In contrast to minimal SU(5), which is broken by an adjoint Higgs representation, FSU(5) is broken to the Standard Model gauge group by a pair of 10-dimensional Higgs representations:

$$H(\mathbf{10},1) = \{Q_H, D^c_H, N^c_H\}\ , \qquad \bar{H}(\bar{\mathbf{10}},-1) = \left\{\bar{Q}_H, \bar{D}^c_H, \bar{N}^c_H\right\}\ . \tag{2}$$

The MSSM Higgs bosons are embedded in another pair of Higgs representations:

$$h(\mathbf{5},-2) = \{T_{H_c}, H_d\}\ , \qquad \bar{h}(\bar{\mathbf{5}},2) = \left\{\bar{T}_{\bar{H}_c}, H_u\right\}\ , \tag{3}$$

where T_{H_c} and $T_{\bar{H}_c}$ denote color triplets, and H_d and H_u the MSSM Higgs doublets.

The conventional electroweak hypercharge is a linear combination of the $\mathrm{U}(1)_X$ gauge symmetry and the diagonal U(1) subgroup of SU(5), namely

$$\frac{Y}{2} = \frac{1}{\sqrt{15}} Y_{24} + \sqrt{\frac{8}{5}} Q_X\ , \tag{4}$$

where the Q_X charge is in units of $\frac{1}{\sqrt{40}}$ and

$$Y_{24} = \sqrt{\frac{3}{5}} \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}\right)\ . \tag{5}$$

The gauge bosons that get masses from the breaking of SU(5)×U(1)→ SU(3)×SU(2)×U(1) are $X(3,2)_{1/3}$, $\bar{X}(\bar{3},2)_{-1/3}$ and a singlet V_1, with masses

$$M_X = g_5 V\ , \qquad M_{V_1} = \sqrt{\frac{5}{2}}\left(\frac{24}{25} g_5^2 + \frac{1}{25} g_X^2\right)^{1/2} V\ , \tag{6}$$

where the vev $V = \langle N^c_{H_1}\rangle = \langle N^c_{H_2}\rangle$. The superpotential for this theory is

$$\begin{aligned} W = {} & \lambda_1^{ij} F_i F_j h + \lambda_2^{ij} F_i \bar{f}_j \bar{h} + \lambda_3^{ij} \bar{f}_i \ell^c_j h + \lambda_4 H H h + \lambda_5 \bar{H}\bar{H}\bar{h} \\ & + \lambda_6^{ia} F_i \bar{H} \phi_a + \lambda_7^a h \bar{h} \phi_a + \lambda_8^{abc} \phi_a \phi_b \phi_c + \mu_\phi^{ab} \phi_a \phi_b\ , \end{aligned} \tag{7}$$

where the indices i, j run over the three fermion families, the indices a, b, c have ranges ≥ 3, and for simplicity we have suppressed gauge group indices. We impose a $\mathbb{Z}_2$ symmetry $H \to -H$ to prevent the mixing of Standard Matter fields with Higgs colour triplets and elements of the Higgs decuplets. This symmetry also suppresses the supersymmetric mass term for H and $\bar{H}$, and thus suppresses dimension-five proton decay operators. The first three terms of the superpotential (7) provide the Standard Model Yukawa couplings. The splitting of the triplet and doublet masses in the Higgs 5-plets is accomplished naturally by the fourth and fifth terms in (7), as these terms yield masses only for the color triplets:

$$M_{H_C} = 4\lambda_4 V \qquad M_{\bar{H}_C} = 4\lambda_5 V\ . \tag{8}$$

The sixth term accounts for neutrino masses. The seventh term plays the role of the MSSM μ-term. The last two terms may play roles in cosmological inflation, along with λ_6, and also play roles in neutrino masses. GUT symmetry breaking, inflation, leptogenesis, and the generation of neutrino masses in this model have been discussed recently in [22–24].

The gauge and superpotential couplings of FSU(5) are matched to those of the MSSM at a renormalization scale, M_{GUT}, defined to be the scale where $g_2 = g_3$ [25]:

$$\begin{aligned} \alpha_2 = \alpha_3 = \alpha_5 \quad &, \quad 25\alpha_1^{-1} = 24\alpha_X^{-1} + \alpha_5^{-1} \, , \\ h_t = h_\nu = \lambda_2/\sqrt{2} \quad &, \quad h_b = 4\lambda_1 \, , \\ h_\tau = \lambda_3 \quad &, \end{aligned} \tag{9}$$

where $\alpha_1 \equiv (5/3)g_Y^2/(4\pi)$. Here we quote just the tree-level matching conditions, but our calculations include one-loop threshold corrections when the input universality scale, M_{in}, is above M_{GUT}, which will be discussed separately in a more general study [26]. We note that, unlike minimal SU(5), the neutrino Yukawa couplings are naturally fixed to be equal to the up-quark Yukawa couplings. This is a consequence of the flipping that puts the right-handed neutrinos into decuplets in FSU(5), instead of being singlets as in minimal SU(5), where their Yukawa couplings would be viewed as independent parameters.

The following GUT-scale parameters characterize the FSU(5) GUT model we study. As mentioned above, we include two independent gaugino masses, a common mass M_5 for the SU(5) gauginos $\tilde{g}, \tilde{W}$ and $\tilde{B}$, and an independent mass M_{X1} for the 'external' gaugino $\tilde{B}_X$. We also include three independent soft supersymmetry-breaking scalar masses, m_{10} for sfermions in the **10** representations of SU(5), m_5 for sfermions in the $\overline{\mathbf{5}}$ representations of SU(5), and m_1 for the right-handed sleptons in the singlet representations. All of these sfermion mass parameters are assumed to be generation-independent, and the trilinear soft supersymmetry-breaking parameters A_0 are assumed to be universal. As in the NUHM [20], we also assume independent soft supersymmetry-breaking for the **5** and $\overline{\mathbf{5}}$ Higgs representations, $m_{H_{1,2}}$, and treat the ratio of Higgs vevs, $\tan\beta$, as a free parameter. Finally, we assume that the Higgs mixing parameter $\mu > 0$, so as to obtain a supersymmetric contribution to $g_\mu - 2$ with the 'interesting' positive sign.

The matching conditions for the the soft supersymmetry-breaking terms at M_{GUT} are

$$\begin{aligned} M_2 = M_3 = M_5 \quad &, \quad 25M_1\alpha_1^{-1} = 24M_{X1}\alpha_X^{-1} + M_5\alpha_5^{-1} \, , \\ m_Q^2 = m_D^2 = m_N^2 = m_{10}^2 \quad &, \quad m_U^2 = m_L^2 = m_5^2 \, , \\ m_E^2 = m_1^2 \quad &, \\ m_{H_u}^2 = m_{h_2}^2 \quad &, \quad m_{H_d}^2 = m_{h_1}^2 \, , \\ A_t = A_\nu = A_b = A_\tau = A_0 \quad &. \end{aligned} \tag{10}$$

Once again, these are the tree-level matching conditions, though our calculations include the one-loop threshold corrections when $M_{in} > M_{GUT}$ and will be discussed separately in a more general study [26]. Full universality (as considered in [25]) would set $M_5 = M_{X1} = m_{1/2}$ and $m_{10} = m_5 = m_1 = m_{h_1} = m_{h_2} = m_0$.

Minimization of the Higgs potential determines μ and the B-term at the electroweak scale. This also determines the pseudoscalar Higgs mass, M_A, which we use as an input to `FeynHiggs 2.18.0` [27] to determine the masses of the remaining physical Higgs degrees of freedom. [3] Our FSU(5) model is therefore completely specified by the following set of

[3]Equivalently, as in [20], one can treat μ and M_A as input parameters and use the minimization conditions

parameters:

$$M_5,\, M_{X1},\, m_{10},\, m_5,\, m_1,\, \mu,\, M_A,\, A_0,\, \tan\beta\,. \tag{11}$$

If one were to assume universality at some high input scale, $M_{in} > M_{GUT}$, additional FSU(5) couplings such as λ_4, λ_5 and λ_6 would also need to be specified, and the relevant RGEs for flipped SU(5) were given in [25]. However, here it is assumed that $M_{in} = M_{GUT}$, so these parameters are unimportant for the results discussed here, except for the proton lifetime, which depends on $\lambda_{4,5}$. We need also to specify the mass of the heaviest left-handed neutrino, m_{ν_3}, which we take to be 0.05 eV. This and λ_6 fix the right-handed neutrino mass and μ_ϕ. However, our results are quite insensitive to these choices.

Maximization of the supersymmetric contribution to $g_\mu - 2$ requires, *a priori*, that either the $\tilde{\mu}_R$ and the lightest neutralino χ and/or the $\tilde{\nu}$ and the lighter chargino must be relatively light, i.e., $\lesssim 1$ TeV. The light $\chi/\tilde{\mu}_R$ option is favoured in FSU(5) by the fact that the U(1) gaugino mass and m_1 are independent of the other soft supersymmetry-breaking masses and relatively unconstrained, whereas the SU(2) gaugino mass is related by universality and the standard renormalization calculation to the gluino mass, which is strongly constrained by fruitless LHC searches [7], and the sneutrino mass is likewise constrained by lower limits on the right-handed up-squark mass. Therefore, we do not pursue the light chargino/$\tilde{\nu}$ option, but focus on the light $\chi/\tilde{\mu}_R$ option. Our computation of Δa_μ follows the analysis in [5], which is based on calculations in [21].

3 Results of FSU(5) Parameter Scan

We report now the results of a scan over the following ranges of the FSU(5) model parameters:

$$\begin{aligned}
M_5 &\in [1800, 5000]\ \text{GeV}, & M_1 &\in [100, 1000]\ \text{GeV}\,, && (12)\\
M_A &\in [1500, 3000]\ \text{GeV}, & \mu &\in [500, 5000]\ \text{GeV}\,, && (13)\\
m_{10} &\in [-1000, 4000]\ \text{GeV}, & m_{\overline{5}} &\in [-500, 1500]\ \text{GeV}\,, && (14)\\
m_1 &\in [-500, 1500]\ \text{GeV}, & A/M_5 &\in [0, 2]\,, && (15)\\
\tan\beta &\in [35, 40]\,, & & && (16)
\end{aligned}$$

including 2.2×10^6 points [4].

In making our scan, we implement the neutralino LSP requirement $m_{\ell_R} > m_\chi$. As mentioned above, we assume universality between the values of m_1 for the different singlet sleptons, so we consider the strongest available constraints across the ℓ_R of different generations, which are generally found for the $\tilde{e}_R$. LEP experiments established lower limits on $m_{\tilde{e}_R}$ that depend on other sparticle masses, in particular m_χ [32]. We assume a LEP

to solve for the two Higgs soft masses. This approach is taken here as it is more convenient when searching for parameter sets yielding a substantial contribution to $g_\mu - 2$.

[4] Negative values of soft supersymmetry-breaking scalar masses should be understood as $m^2/\sqrt{|m^2|}$. Such negative values are consistent with CMSSM-like phenomenology [28, 29] and with standard cosmology if the Standard Model vacuum is relatively long-lived when any charge- and/or colour-breaking minima occur [30, 31].

lower limit of 100 GeV in general, reducing to 73 GeV when $m_{\tilde{\mu}_R} - m_\chi \lesssim 2$ GeV. At the LHC, ATLAS has established the lower limit $m_{\tilde{\ell}_R} \gtrsim 450$ GeV when $m_\chi = 0$, where $\ell = e, \mu$, falling to $\gtrsim 200$ GeV when $m_\chi \simeq 180$ GeV [33], but these lower limits on the m_{ℓ_R} are absent for $m_\chi > 180$ GeV. An additional LHC constraint is present for compressed spectra when $m_{\mu_R} - m_\chi \lesssim 15$ GeV [34], which is maximized when $m_{\mu_R} - m_\chi \simeq 10$ GeV in which case it excludes $m_{\mu_R} \lesssim 150$ GeV. Therefore, in order to maximize the supersymmetric contribution to $g_\mu - 2$ we prioritize the region of parameter space where $m_\chi + 15\ \mathrm{GeV} < m_{\mu_R} \sim 100$ GeV, which constrains primarily m_1 and M_{X_1}. The other soft supersymmetry-breaking parameters are constrained primarily by unsuccessful LHC searches, and we also apply the constraint that m_h calculated using `FeynHiggs 2.18.0` [27] is within 3 GeV of the measured Higgs mass.

We do not use the relic neutralino density as a constraint, since the flipped SU(5) GUT model contains mechanisms for generating large amounts of entropy [23]. Nevertheless, in the regions of parameter space that provide the most sizeable contributions to Δa_μ, the lightest neutralino (typically mostly a bino) and the right-handed selectron and smuon are quite close in mass, $m_{\mu_R} - m_\chi \simeq 15 - 20$ GeV. In this case, the neutralino relic density is controlled by slepton coannihilation, which yields a relic density that is close to that needed to account for the cold dark matter density determined by recent microwave background analyses [35] (see also [14]).

The left panel of Fig. 1 shows a scatter plot of FSU(5) points in the $(m_{\tilde{\mu}_R}, m_\chi)$ plane color-coded according to the values of the supersymmetric contribution to a_μ that they yield, as indicated in the legend. The darker blue shading covers points with $m_{\tilde{\mu}_R} < m_\chi$, which are therefore excluded because the LSP is charged. The vertical red line represents the LEP constraint $m_{\tilde{e}_R} \gtrsim 100$ GeV [32], where we recall that $m_{\tilde{\mu}_R} = m_{\tilde{e}_R}$ within the approximations we use. Also visible at $m_{\tilde{\mu}_R} \lesssim 450$ GeV is the principal LHC Run 2 constraint on $\tilde{\ell}_R \to \ell\chi$ decay [33], where $\ell = e, \mu$ (blue line), and the additional constraint for $m_{\tilde{\mu}_R} < 150$ GeV and small $m_{\tilde{\mu}_R} - m_\chi$ [34] (red line). We see that points yielding $\Delta a_\mu > 50\,(100) \times 10^{-11}$, indicated by orange (yellow) boxes, are concentrated at $m_{\tilde{\mu}_R}, m_\chi \lesssim 500\,(250)$ GeV. We note that most of the points with supersymmetric contributions $\Delta a_\mu \gtrsim 100 \times 10^{-11}$ are allowed by the constraints mentioned above. In a dedicated study we found the largest value $\Delta a_\mu = 150 \times 10^{-11}$ for the point indicated by a black cross. [5]

The right panel of Fig. 1 displays stacked histograms of the numbers of points yielding values of $\tilde{a}_\mu \equiv \Delta a_\mu \times 10^{11}$ within the indicated ranges, binned according to the corresponding values of m_h calculated using `FeynHiggs 2.18.0`. We note that all the points with $\Delta a_\mu > 100 \times 10^{-11}$ correspond to $m_h < 123$ GeV. All points with $m_h > 122$ GeV are allowed if one adopts a conservative estimate of 3 GeV for the 2-σ uncertainty in the calculation of m_h. However, we note that the `FeynHiggs 2.18.0` code [27] returns a 1-σ uncertainty in m_h that is below 1 GeV for the points of greatest interest for $g_\mu - 2$. We find for scan points with $m_h > 123\,(124)$ GeV the following maximum values $\Delta a_\mu = 71\,(25) \times 10^{-11}$.

The left panel of Fig. 2 shows a scatter plot of FSU(5) points in the $(m_h, m_{\tilde{\mu}_R})$ plane and color-coded as in Fig. 1. The horizontal line represents the LEP lower limit on the slepton

[5] As mentioned above, the limit $m_{\tilde{\mu}_R} > 100$ GeV is relaxed to $m_{\tilde{\mu}_R} \gtrsim 73$ GeV when $m_{\tilde{\mu}_R} - m_\chi \lesssim 2$ GeV [34]. In a dedicated study of this exceptional region we found points with values of $\Delta a_\mu \gtrsim 220 \times 10^{-11}$.

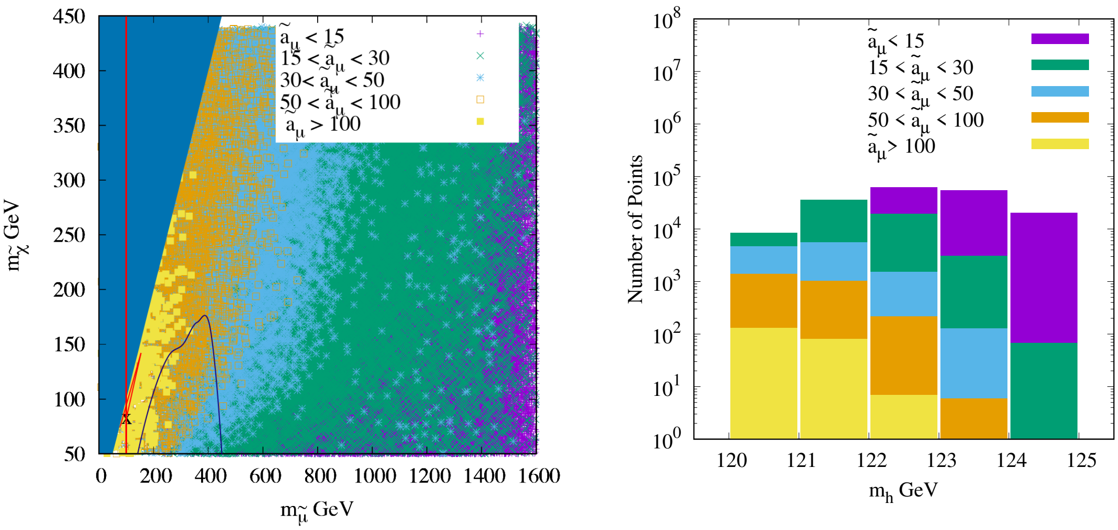

Figure 1: *Left panel: Scatter plot of flipped SU(5) points in the* $(m_{\tilde{\mu}_R}, m_\chi)$ *plane, color-coded according to the values of the supersymmetric contribution to* a_μ, $\tilde{a}_\mu \equiv \Delta a_\mu \times 10^{-11}$, *that they yield, as indicated in the legend. The diagonal line represents the constraint that the LSP is not charged, and the vertical line represents the LEP lower limit on the slepton mass [32]. Also visible at small masses are the LHC constraints on* $\tilde{\ell}_R \to \ell\chi$ *where* $\ell = e, \mu$ *[33]. The point with the largest value of* $\Delta a_\mu = 150 \times 10^{-11}$ *is indicated with a cross. Right panel: Stacked histograms of the numbers of points with* $\tilde{a}_\mu$ *and* m_h *in the indicated ranges.*

mass of 100 GeV [32]. We see that the the values of Δa_μ tend to decrease with increasing $m_{\tilde{\mu}_R}$ and m_h. The trend with $m_{\tilde{\mu}_R}$ was seen already in the left panel of Fig. 1, and the trend with m_h reflects the fact that larger values of m_h correspond in general to larger sparticle masses, in particular $\tilde{\mu}_L$. This suppresses $\tilde{\mu}_L/\tilde{\mu}_R$ mixing and hence the corresponding contribution to Δa_μ. The right panel of Fig. 2 shows a scatter plot of flipped SU(5) points in the $(\mu, m_{\tilde{\mu}_R})$ plane, where we see that the points yielding $\Delta a_\mu \gtrsim 50 \times 10^{-11}$ correspond to relatively large values of $\mu > 2500$ GeV, where the $\tilde{\mu}_R/\tilde{\mu}_L$ mixing contribution is enhanced.

Fig. 3 compares the ranges of the discrepancy, Δa_μ between the combination of the BNL and Fermilab measurements and the data-driven estimate of a_μ taken from the Theory Initiative [4] (green line) and the BMW lattice calculation [17] (black line), together with the range of the supersymmetric contribution to Δa_μ found in our general scan of the flipped SU(5) parameter space (red line). We see that the flipped SU(5) model could resolve completely the residual 1.5-σ discrepancy between the BMW lattice calculation [17] and the experimental measurements. It also reduces the discrepancy between the data-driven Standard Model estimate and the measurements to less than 2 standard deviations. [6] Also shown is the 2-σ range of Δa_μ found in a global analysis of the CMSSM that includes all relevant constraints from LHC Run 2, previous experiments and constraints on dark matter [9] (blue

[6]The red dashed line shows the additional range of Δa_μ that is found in the exceptional region where $m_{\tilde{\mu}_R} - m_\chi \lesssim 2$ GeV and $m_{\tilde{\mu}_R} \gtrsim 73$ GeV.

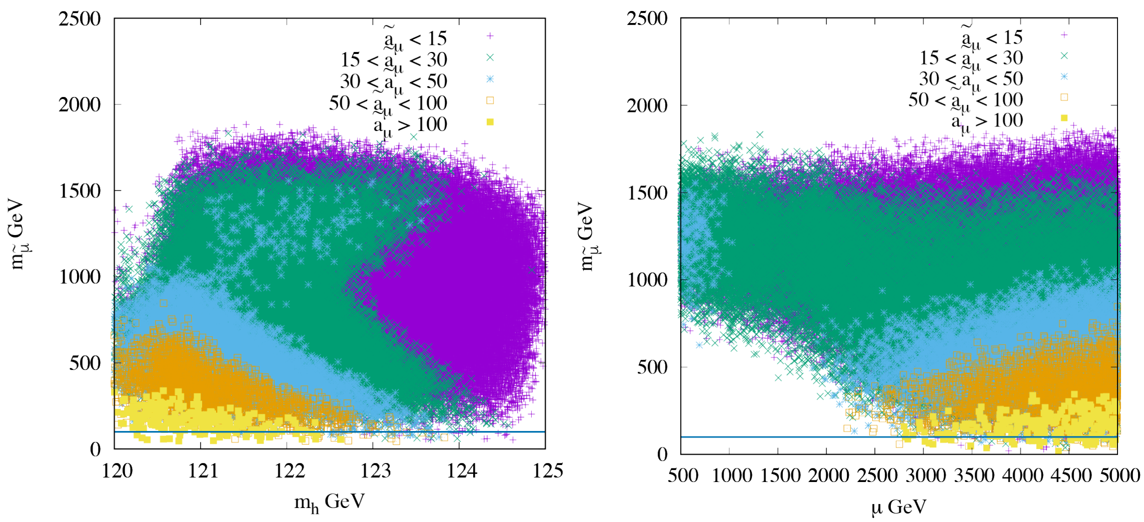

Figure 2: *Scatter plots of flipped SU(5) points in (left panel) the $(m_h, m_{\tilde{\mu}_R})$ plane and (right panel) the $(\mu, m_{\tilde{\mu}_R})$ plane, color-coded according to the values of the supersymmetric contribution to a_μ that they yield, as indicated in the legend. The horizontal lines represent the LEP lower limit on the slepton mass [32].*

line). We see that the supersymmetric contribution to Δa_μ in the CMSSM is ~ 30 times smaller than in flipped SU(5), and is negligible compared to the experimental discrepancies with the Standard Model calculations.

As has been mentioned above, the generic FSU(5) point that makes the largest contribution to a_μ yields $\Delta a_\mu = 150 \times 10^{-11}$. Table 1 shows the input parameters for this point, including those pertaining to the specification of the GUT model [7] and those pertaining to the supersymmetry scales. We also list in Table 1 the output MSSM particle masses and other observables. We observe that, apart from the lightest neutralino LSP and the $\tilde{\mu}_R$ (and the near-degenerate $\tilde{e}_R$), [8] the squarks and gluinos are in general far beyond the current reach of the LHC [7] and even the prospective reach of the HL-LHC [36], though within reach of FCC-hh [37] or SppC [38]. This is a general feature of points that yield interesting values of Δa_μ and $m_h > 122$ GeV. The optimal point is also compatible with the LHC Run 2 limits in the $(M_A, \tan\beta)$ plane [39]. The $\tilde{\mu}_R$ and $\tilde{e}_R$ might be within reach of future LHC searches via conventional missing-energy signatures [33] and/or dedicated searches in the compressed spectrum region [34], possibly using the LHC as a photon collider [40]. They could also be within the reach of an e^+e^- collider operating at 250 GeV in the center of mass, such as the ILC [41], FCC-ee [42] or CEPC [43].

Finally, we also show in Table 1 the values of some other observables for this point. The relic LSP density $\Omega_\chi h^2$ calculated assuming adiabatic cosmological evolution happens

[7] The GUT mass scales are largely determined by extrapolation from low-energy data, and are insensitive to the values of $\lambda_{4,5,6}$. Our results are also insensitive to m_{ν_3} within the range allowed by cosmological data.

[8] Note that the $\tilde{\tau}_R$ is much heavier than the $\tilde{\mu}_R$ and $\tilde{e}_R$, because $m^2_{H_d}$ has large negative values, which increase $m_{\tilde{\tau}_R}$ at low energies.

Input GUT parameters (masses in units of 10^{16} GeV)		
$M_{GUT} = 1.00$	$M_X = 0.79$	$V = 1.13$
$\lambda_4 = 0.1$	$\lambda_5 = 0.3$	$\lambda_6 = 0.001$
$g_5 = 0.70$	$g_X = 0.70$	$m_{\nu_3} = 0.05$ eV
Input supersymmetry parameters (masses in GeV units)		
$M_5 = 2460$	$M_1 = 240$	$\mu = 4770$
$m_{10} = 930$	$m_{\overline{5}} = 450$	$m_1 = 0$
$M_A = 2100$	$A_0/M_5 = 0.67$	$\tan\beta = 35$
MSSM particle masses (in GeV units)		
$m_\chi = 84$	$m_{\tilde{t}_1} = 4030$	$m_{\tilde{g}} = 5090$
$m_{\chi_2} = 2160$	$m_{\chi_3} = 5080$	$m_{\chi_4} = 5080$
$m_{\tilde{\mu}_R} = 101$	$m_{\tilde{\mu}_L} = 1600$	$m_{\tilde{\tau}_1} = 1010$
$m_{\tilde{q}_L} = 4470$	$m_{\tilde{d}_R} = 4250$	$m_{\tilde{u}_R} = 4170$
$m_{\tilde{t}_2} = 4410$	$m_{\tilde{b}_1} = 4170$	$m_{\tilde{b}_2} = 4400$
$m_{\chi^\pm} = 2160$	$m_{H,A} = 2100$	$m_{H^\pm} = 2100$
Other observables		
$\Delta a_\mu = 150 \times 10^{-11}$	$\Omega_\chi h^2 = 0.13$	$m_h = 122$ GeV
Normal-ordered ν masses:	$\tau_{p\to e^+\pi^0}\vert_{\rm NO} = 4.6 \times 10^{35}$ yrs	$\tau_{p\to \mu^+\pi^0}\vert_{\rm NO} = 4.7 \times 10^{36}$ yrs
Inverse-ordered ν masses:	$\tau_{p\to e^+\pi^0}\vert_{\rm IO} = 1.4 \times 10^{37}$ yrs	$\tau_{p\to \mu^+\pi^0}\vert_{\rm IO} = 9.8 \times 10^{35}$ yrs

Table 1: *Parameters and predictions of an FSU(5) point that yields* $\Delta a_\mu = 150 \times 10^{-11}$.

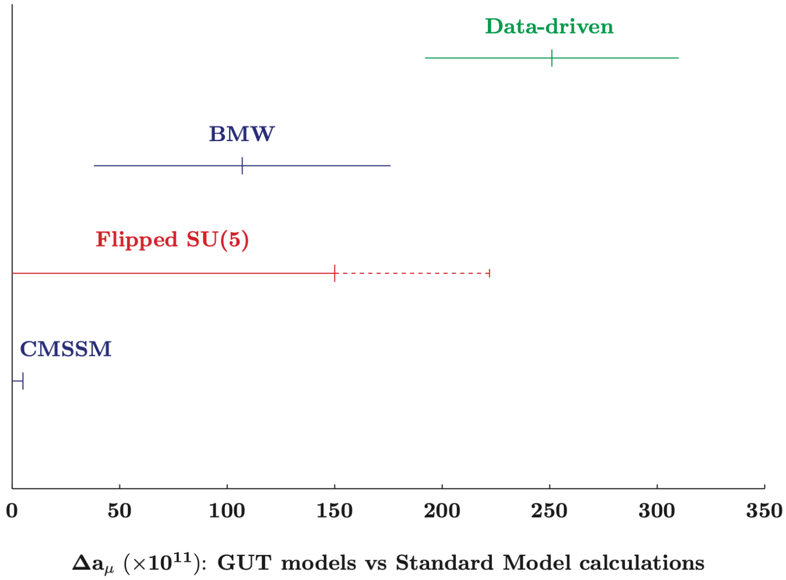

Figure 3: *Comparison of the ranges of the discrepancy in a_μ between the combination of the BNL and Fermilab measurements with the data-driven estimate taken from the Theory Initiative [4] (green line), from the BMW lattice calculation [17] (black range), and the ranges found in flipped SU(5) in this paper (red range, general region shown as solid line, extension in exceptional region shown dashed) and in the CMSSM [9] (blue range).*

to fall quite close to the range of cold dark matter density favoured by Planck [35] and other measurements, though this was not imposed *a priori*. This is because smaller values of $\Omega_\chi h^2$ are allowed if there is another source of cold dark matter, while a complete FSU(5) model of cosmology favours a large amount of entropy generation that would dilute even a quite substantial potential overdensity of LSPs [23]. However, for the point whose parameters are given in Table 1 and other, similar points, LSP coannihilations with the $\tilde{\mu}_R$ and $\tilde{e}_R$ naturally bring $\Omega_\chi h^2$ close to or within the range preferred by Planck even before any such entropy generation [14]. We also show in Table 1 predictions for the partial lifetimes for $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ in variants of the FSU(5) in which the light neutrino masses are ordered either normally (NO) or inversely (IO). We see that in all cases these partial lifetimes are well beyond the present experimental limits and the prospective reach of planned experiments such as Hyper-Kamiokande.

4 Summary

We have explored in this paper the range of possible values of the supersymmetric contribution to a_μ in the flipped SU(5) GUT model. This model has more parameters than the familiar CMSSM, or even a standard SU(5) GUT. Specifically, there are two independent gaugino mass parameters in flipped SU(5), one for the SU(5) adjoint gauginos, M_5, and another, M_{X1}, for the gaugino corresponding to the external U(1) factor. This decouples the mass of the lightest neutralino LSP from those of the gluino and the SU(2) gauginos. Also, flipped SU(5) has three independent soft supersymmetry-breaking scalar masses per generation, for the $\mathbf{10}, \overline{\mathbf{5}}$ and singlet representations of SU(5), $m_{10}, m_{\overline{5}}$ and m_1, compared to two parameters in standard SU(5) or just one mass parameter in the CMSSM. Moreover, since the supersymmetric partner of the right-handed muon is in a singlet representation of flipped SU(5), its mass avoids the constraints imposed on squarks and left-handed sleptons. The freedom in the choice of M_{X1} and m_1 allows the LSP and the $\tilde{\mu}_R$ to be much lighter than the other sparticles, opening up the possibility of a much larger contribution to a_μ than in the CMSSM, for example.

Indeed, we have found that the flipped SU(5) contribution to a_μ could be as large as $\sim 150 \times 10^{-11}$, even after taking the available LEP and LHC constraints into account, whereas these constraints favour values $\lesssim 5 \times 10^{-11}$ in the CMSSM. The potential flipped SU(5) contribution to a_μ would reduce the discrepancy between experiment and the data-driven calculation of the Standard Model contribution to below 2 standard deviations, and be completely consistent with the central value of the BMW lattice calculation. Flipped SU(5) is therefore an example of a GUT-based supersymmetric model that may bridge the gap between experiment and the Standard Model.

We have also discussed in this paper some other possible experimental signatures of this flipped SU(5) scenario for $g_\mu - 2$. The lightest supersymmetric particles, namely the lightest neutralino, the $\tilde{e}_R$ and the $\tilde{\mu}_R$ may all be detectable in dedicated searches at the LHC, or in experiments at a 250-GeV e^+e^- collider such as the ILC, FCC-ee or CEPC. On the other hand, the heavier supersymmetric particles would be beyond the reach of the LHC, and their detection would have to wait for FCC-hh or SppC. Suitable neutrino masses can be incorporated, with either normal or inverse mass ordering. In both cases, the flipped SU(5) model predicts a proton lifetime well beyond the current constraints and also beyond the reach of planned experiments. We note also that the cross section for spin-dependent dark matter scattering is far below the current experimental limit.

We will return soon to these and other issues in a more detailed study of the phenomenology of flipped SU(5) [26].

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