Excited Ξ_c^0 baryons within the QCD rum rule approach

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(Received 7 April 2020; accepted 13 November 2020; published 7 December 2020)

We systematically study mass spectra and decay properties of P-wave $\Xi_c^\prime$ baryons of the $SU(3)$ flavor $\mathbf{6}_F$, using the methods of QCD sum rules and light-cone sum rules within the framework of heavy quark effective theory. Our results suggest that the three excited Ξ_c^0 baryons recently observed by LHCb can be well explained as P-wave $\Xi_c^\prime$ baryons: the $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ are partner states of $J^P = 1/2^-$ and $3/2^-$, respectively, both of which contain one λ-mode orbital excitation; the $\Xi_c(2965)^0$ has $J^P = 3/2^-$ and also contains one λ-mode orbital excitation. We propose to search for another P-wave $\Xi_c^\prime$ state of $J^P = 5/2^-$ in the $\Lambda_c K/\Xi_c \pi$ mass spectral in future experiments. Its mass is about 56^{+30}_{-35} MeV larger than the $\Xi_c(2965)^0$, and its width is about $18.1^{+19.7}_{-8.3}$ MeV.

DOI: 10.1103/PhysRevD.102.114009

I. INTRODUCTION

The light quarks and gluons circle around the nearly static heavy quark inside the heavy baryon. This system is the QCD analogue of the hydrogen, but bounded by the strong interaction [1–3]. In three recent LHCb experiments [4–6], its spectra are found to have beautiful fine structures: five excited Ω_c^0 baryons were observed in the experiment [4]; four excited Ω_b^- baryons were observed in the experiment [5]; in the very recent experiment [6], three excited Ξ_c^0 baryons were observed simultaneously in the $\Lambda_c^+ K^-$ mass spectrum, whose parameters were measured to be as follows:

$$\Xi_c(2923)^0\colon\ M = 2923.04 \pm 0.25 \pm 0.20 \pm 0.14\ \text{MeV},\quad \Gamma = 7.1 \pm 0.8 \pm 1.8\ \text{MeV}, \tag{1}$$

$$\Xi_c(2939)^0\colon\ M = 2938.55 \pm 0.21 \pm 0.17 \pm 0.14\ \text{MeV},\quad \Gamma = 10.2 \pm 0.8 \pm 1.1\ \text{MeV}, \tag{2}$$

$$\Xi_c(2965)^0\colon\ M = 2964.88 \pm 0.26 \pm 0.14 \pm 0.14\ \text{MeV},\quad \Gamma = 14.1 \pm 0.9 \pm 1.3\ \text{MeV}. \tag{3}$$

These excited $\Omega_c^0/\Omega_b^-/\Xi_c^0$ baryons are good candidates of P-wave charmed and bottom baryons, whose observations have proved the rich internal structure of (heavy) hadrons [7–9].

The LHCb Collaboration [6] further pointed out that the $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ baryons are probably the substructures of $\Xi_c(2930)^0$ [10,11], while the $\Xi_c(2965)^0$ and $\Xi_c(2970)^0$ [12] might be different states. Various phenomenological methods and models have been applied to study these baryons. In Ref. [13], the author uses the constituent quark model to interpret the $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ as the λ-mode P-wave $\Xi_c^\prime$ baryons of $J^P = 3/2^-$ and $5/2^-$, and the $\Xi_c(2965)^0$ as the $J^P = 1/2^+$ $\Xi_c^\prime(2S)$ state. In Ref. [14], the authors use the chiral quark model to interpret them as the λ-mode P-wave $\Xi_c^\prime$ baryons of $J^P = 3/2^-$, $3/2^-$, and $5/2^-$, respectively. In Ref. [15], the authors use the method of QCD sum rules to interpret the $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ as P-wave $\Xi_c^\prime$ baryons of $J^P = 1/2^-$ and $3/2^-$, and the $\Xi_c(2965)^0$ as the $J^P = 1/2^+$ $\Xi_c^\prime(2S)$ or $\Xi_c(2S)$ state. In Ref. [16], the authors use the molecular picture to interpret the $\Xi_c(2923)^0$ as a $D\bar{\Lambda} - D\bar{\Sigma}$ molecule.

Besides, many phenomenological methods and models have been applied to understand the $\Xi_c(2930)^0$ and $\Xi_c(2970)^0$ previously observed by *BABAR* [10] and Belle [12], such as various quark models [17–29], various molecular explanations [30–34], the chiral perturbation theory [35,36], lattice QCD [37–39], QCD sum rules [40–46], etc. We refer to the reviews [9,47–49] and references therein for detailed discussions.

We have systematically studied mass spectra and decay properties of P-wave heavy baryons in Refs. [50–53] using the methods of QCD sum rules [54,55] and light-cone sum rules [56–60] within the framework of heavy quark effective theory (HQET) [61–63]. The results

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were combined in Ref. [53] so that a rather complete study within HQET was performed on both mass spectra and decay properties of P-wave bottom baryons. There we predicted four Ξ_b' baryons, three of which have finite and limited widths, while the rest one has a (nearly) zero width,

$$[\Xi_b'(1/2^-),1,1,\lambda]\colon\ M = 6.21 \pm 0.11\ \text{GeV},\quad \Gamma = 4.7^{+5.8}_{-3.3}\ \text{MeV}, \tag{4}$$

$$[\Xi_b'(3/2^-),1,1,\lambda]\colon\ M = 6.22 \pm 0.11\ \text{MeV},\quad \Gamma = 1.8^{+1.1}_{-1.0}\ \text{MeV}, \tag{5}$$

$$[\Xi_b'(3/2^-),2,1,\lambda]\colon\ M = 6.23 \pm 0.15\ \text{GeV},\quad \Gamma = 27.3^{+28.5}_{-14.2}\ \text{MeV}, \tag{6}$$

$$[\Xi_b'(5/2^-),2,1,\lambda]\colon\ M = 6.24 \pm 0.14\ \text{MeV},\quad \Gamma = 12.7^{+12.4}_{-6.1}\ \text{MeV}. \tag{7}$$

Their mass splittings are

$$M_{[\Xi_b'(3/2^-),1,1,\lambda]} - M_{[\Xi_b'(1/2^-),1,1,\lambda]} = 7 \pm 2\ \text{MeV},$$
$$M_{[\Xi_b'(5/2^-),2,1,\lambda]} - M_{[\Xi_b'(3/2^-),2,1,\lambda]} = 11 \pm 5\ \text{MeV}. \tag{8}$$

The above notations will be explained later, and we refer to Ref. [53] for their detailed decay channels. From our previous results [53], we guess that the $\Xi_c(2923)^0$, $\Xi_c(2939)^0$, and $\Xi_c(2965)^0$ are just the charmed partners of the $[\Xi_b'(1/2^-),1,1,\lambda]$, $[\Xi_b'(3/2^-),1,1,\lambda]$, and $[\Xi_b'(3/2^-),2,1,\lambda]$, respectively. To verify this, in this paper, we follow the same approach used in Refs. [50–53] to study the above excited Ξ_c^0 baryons recently observed by LHCb [5]. We shall find that all of them can be interpreted as P-wave Ξ_c' baryons of the $SU(3)$ flavor $\mathbf{6}_F$, so that both their mass spectra and decay properties can be well explained.

This paper is organized as follows. In Sec. II, we introduce our notations for P-wave Ξ_c' baryons and categorize them into four charmed baryon multiplets $[\Xi_c',1,0,\rho]$, $[\Xi_c',0,1,\lambda]$, $[\Xi_c',1,1,\lambda]$, and $[\Xi_c',2,1,\lambda]$. We use them to perform QCD sum rule analyses within the framework of heavy quark effective theory and calculate their masses. Then, in Sec. III, we study their decay properties, including their S-wave and D-wave decays into ground-state charmed baryons and pseudoscalar mesons (π or K) as well as their S-wave decays into ground-state charmed baryons and vector mesons (ρ or K^*). In Sec. IV, we discuss the results and conclude this paper.

II. MASS SPECTRA FROM QCD SUM RULES

We follow Ref. [18] and use the same notations to describe P-wave Ξ_c' baryons of the $SU(3)$ flavor $\mathbf{6}_F$. Each baryon consists of one charm quark and two light quarks, and contains one orbital excitation, which can be either between the two light quarks ($l_\rho = 1$) or between the charm quark and the two-light-quark system ($l_\lambda = 1$). Hence, there are ρ-mode excited Ξ_c' baryons ($l_\rho = 1$ and $l_\lambda = 0$) and λ-mode ones ($l_\rho = 0$ and $l_\lambda = 1$). Together with the color, flavor, and spin degrees of freedom, its internal structures are as follows:

(i) Color structure of the two light quarks is antisymmetric, that is, the color $\bar{\mathbf{3}}_C$.
(ii) Flavor structure of the two light quarks is symmetric, that is, the $SU(3)$ flavor $\mathbf{6}_F$.
(iii) Spin structure of the two light quarks can be either antisymmetric ($s_l \equiv s_{qq} = 0$) or symmetric ($s_l = 1$).
(iv) Orbital structure of the two light quarks can be either antisymmetric ($l_\rho = 1$) or symmetric ($l_\rho = 0$).

Considering that the total structure of the two light quarks is antisymmetric due to the Pauli principle, we can categorize P-wave Ξ_c' baryons into four multiplets, denoted as $[\mathbf{6}_F, j_l, s_l, \rho/\lambda]$. We show them in Fig. 1, where j_l denotes the total angular momentum of the light components ($j_l = s_l \otimes l_\rho \otimes l_\lambda$). Every multiplet contains one or two Ξ_c' baryons, whose total angular momenta are $j = j_l \otimes s_c = |j_l \pm 1/2|$, with s_c the charm quark spin.

To avoid confusions, we add a note here that the symbol Ξ_c but not Ξ_c' was used in the previous section to describe the baryon states observed in the LHCb experiment [6]. This is because the flavor symmetry of the two light quarks cannot be differentiated so that Ξ_c and Ξ_c' cannot be differentiated neither in the experiment. However, theoretically this can be done, and we use Ξ_c and Ξ_c' to denote baryons belonging to the $SU(3)$ flavor $\bar{\mathbf{3}}_F$ and $\mathbf{6}_F$, respectively.

We have systematically studied mass spectra of P-wave charmed baryons in Ref. [50] using QCD sum rules within HQET. In this method, we calculate the baryon mass through

$$m_{\Xi_c'(j^P),j_l,s_l,\rho/\lambda} = m_c + \bar{\Lambda}_{\Xi_c',j_l,s_l,\rho/\lambda} + \delta m_{\Xi_c'(j^P),j_l,s_l,\rho/\lambda}, \tag{9}$$

where m_c is the charm quark mass, $\bar{\Lambda}_{\Xi_c',j_l,s_l,\rho/\lambda} = \bar{\Lambda}_{\Xi_c'(|j_l-1/2|),j_l,s_l,\rho/\lambda} = \bar{\Lambda}_{\Xi_c'(j_l+1/2),j_l,s_l,\rho/\lambda}$ is extracted from the mass sum rules at the leading order, and $\delta m_{\Xi_c'(j^P),j_l,s_l,\rho/\lambda}$ is extracted from the mass sum rules at the $\mathcal{O}(1/m_c)$ order. We refer to Ref. [50] for their explicit expressions.

Equation (9) tells that the Ξ_c' mass depends significantly on the charm quark mass. Hence, there exists considerable (theoretical) uncertainty in our results for absolute values of baryon masses, and we cannot distinguish the three excited Ξ_c^0 baryons observed by LHCb [6] only by using their mass spectra. However, the mass splittings within the same multiplets are produced at the $\mathcal{O}(1/m_c)$ order with much less uncertainty, giving more useful information.

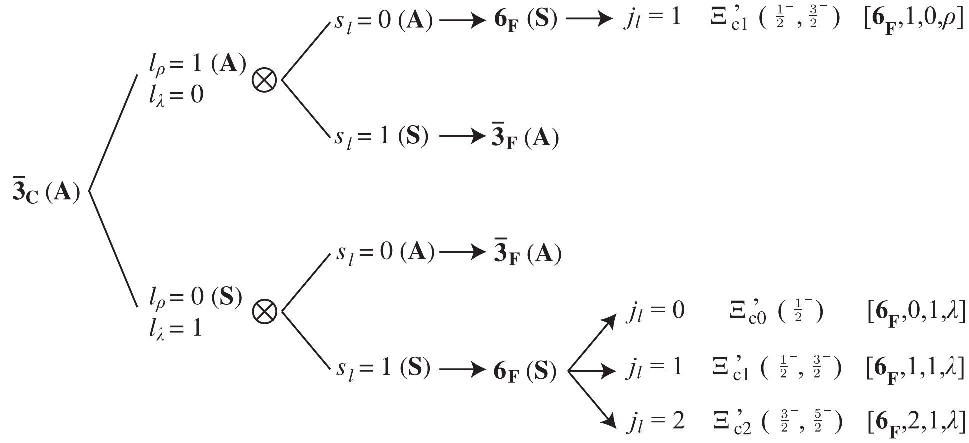

FIG. 1. Categorization of P-wave Ξ'_c baryons.

We can extract even more valuable information from decay properties of P-wave Ξ'_c baryons. Before doing this, we fine-tune one of the two free parameters in mass sum rules, the threshold value ω_c, to get a better description of the LHCb experiment [6]. The other free parameter, the Borel mass T, can be determined by using two criteria: the first is to require the high-order power corrections to be less than 30%, and the second is to require the pole contribution of P-wave Ξ'_c baryons to be larger than 20%. Note that there may exist some lower state, making it not easy to get an idea pole contribution at 50%. This is quite similar to the QCD sum rule study on excited heavy mesons [64]. Besides, the small pole contribution is mathematically due to the large powers of s in the spectral function, which makes the suppression of the Borel transformation on the continuum not so effective. For example, see Ref. [65] where the pole contribution of the $d^*(2380)$ is only about 0.0002 due to the large power of s in its spectral function. As a compensation, we need the third criterion, which requires the mass dependence on the threshold value ω_c to be weak.

After fixing these two parameters, all the other parameters can be calculated using the method of QCD sum rules with HQET. We summarize the obtained results in Table I, together with the parameters that are necessary to calculate decay widths through light-cone sum rules. Note that they are slightly different from those parameters for excited bottom baryons [53]. Their uncertainties are due to various QCD sum rule parameters [7,66–72] which are as follows:

$$\begin{aligned} m_s(1\ \text{GeV}) &= (137 \pm 27)\ \text{MeV}, \\ \langle g_s^2 GG \rangle &= (0.48 \pm 0.14)\ \text{GeV}^4, \\ \langle \bar{q}q \rangle &= -(0.24 \pm 0.01)^3\ \text{GeV}^3, \\ \langle \bar{s}s \rangle &= (0.8 \pm 0.1) \times \langle \bar{q}q \rangle, \\ \langle g_s \bar{q}\sigma G q \rangle &= -M_0^2 \times \langle \bar{q}q \rangle, \\ \langle g_s \bar{s}\sigma G s \rangle &= -M_0^2 \times \langle \bar{s}s \rangle, \\ M_0^2 &= (0.8 \pm 0.2)\ \text{GeV}^2. \end{aligned} \tag{10}$$

Our results suggest that the ρ-mode excitation is lower than the λ-mode, a behavior which is consistent with our previous results for their corresponding $SU(3)$ flavor $\bar{\mathbf{3}}_F$ multiplets [50], but in contrast to the quark model expectation [26]. However, this is possible simply because that the mass differences between different multiplets have considerable uncertainties in our QCD sum rule calculations, similar to the absolute values of baryon masses, but different from the mass differences within the same multiplet.

TABLE I. Mass spectra of P-wave Ξ'_c baryons belonging to the $SU(3)$ flavor $\mathbf{6}_F$ representation, evaluated using QCD sum rules within HQET. Here we also list the parameters that are necessary to calculate their decay widths through light-cone sum rules.

Multiplets	ω_c (GeV)	Working region (GeV)	$\bar{\Lambda}$ (GeV)	Baryon (j^P)	Mass (GeV)	Difference (MeV)	f (GeV^4)
$[\mathbf{6}_F(\Xi'_c), 1, 0, \rho]$	1.87	$0.26 < T < 0.34$	$1.36^{+0.12}_{-0.08}$	$\Xi'_c(1/2^-)$	$2.88^{+0.15}_{-0.13}$	13^{+6}_{-5}	$0.059^{+0.017}_{-0.011}$
				$\Xi'_c(3/2^-)$	$2.89^{+0.15}_{-0.13}$		$0.028^{+0.008}_{-0.005}$
$[\mathbf{6}_F(\Xi'_c), 0, 1, \lambda]$	1.57	$0.27 < T < 0.29$	$1.22^{+0.08}_{-0.07}$	$\Xi'_c(1/2^-)$	$2.90^{+0.13}_{-0.12}$	…	$0.041^{+0.010}_{-0.008}$
$[\mathbf{6}_F(\Xi'_c), 1, 1, \lambda]$	1.72	$T = 0.34$	$1.14^{+0.09}_{-0.08}$	$\Xi'_c(1/2^-)$	$2.91^{+0.13}_{-0.12}$	38^{+15}_{-13}	$0.041^{+0.008}_{-0.007}$
				$\Xi'_c(3/2^-)$	$2.95^{+0.12}_{-0.11}$		$0.019^{+0.004}_{-0.003}$
$[\mathbf{6}_F(\Xi'_c), 2, 1, \lambda]$	1.72	$0.27 < T < 0.32$	$1.24^{+0.15}_{-0.09}$	$\Xi'_c(3/2^-)$	$2.96^{+0.24}_{-0.15}$	66^{+29}_{-25}	$0.057^{+0.020}_{-0.012}$
				$\Xi'_c(5/2^-)$	$3.02^{+0.23}_{-0.14}$		$0.034^{+0.012}_{-0.007}$

III. WIDTHS FROM LIGHT-CONE SUM RULES

We have systematically studied decay properties of P-wave heavy baryons in Refs. [51–53] using light-cone sum rules within HQET, and the results are combined in Ref. [53] to study P-wave bottom baryons. In the present study, we replace the bottom quark by the charm quark, and redo all the calculations. We summarized all the sum rule equations in Appendix B. Their extracted results are given in Table II, where we have investigated all the possible S-wave and D-wave decays of P-wave Ξ'_c baryons into ground-state charmed baryons and light pseudoscalar mesons.

Their uncertainties are due to the parameters given in Table I as well as various light-cone sum rule parameters [59,60]. Because there are many input parameters with uncertainties (some of them are given in Appendix A), the uncertainties of our results are not so small, that is, they can be as large as $X^{+200\%}_{-67\%}$. Especially, the parameter $a_2^{\pi/K}$ of $\phi_{\pi/K;2}$ is 0.25 ± 0.15 [59,60], which causes the major uncertainties.

During the calculations, we have used the following mass values:

(i) For the $[\mathbf{6}_F(\Xi'_c), 1, 0, \rho]$ doublet, we use the following mass values taken from their mass sum rules:

$$M_{[\Xi'_c(1/2^-),1,0,\rho]} = 2.88^{+0.15}_{-0.13}\ \text{GeV},$$
$$M_{[\Xi'_c(3/2^-),1,0,\rho]} = 2.89^{+0.15}_{-0.13}\ \text{GeV}. \tag{11}$$

(ii) For the $[\mathbf{6}_F(\Xi'_c), 0, 1, \lambda]$ singlet, we use the following mass value taken from its mass sum rules:

$$M_{[\Xi'_c(1/2^-),0,1,\lambda]} = 2.90^{+0.13}_{-0.12}\ \text{GeV}. \tag{12}$$

(iii) For the $[\mathbf{6}_F(\Xi'_c), 1, 1, \lambda]$ doublet, we use the masses of $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ measured by LHCb [6],

$$M_{[\Xi'_c(1/2^-),1,1,\lambda]} = M_{\Xi_c(2923)^0} = 2923.04\ \text{GeV},$$
$$M_{[\Xi'_c(3/2^-),1,1,\lambda]} = M_{\Xi_c(2939)^0} = 2938.55\ \text{GeV}. \tag{13}$$

(iv) For the $[\mathbf{6}_F(\Xi'_c), 2, 1, \lambda]$ doublet, we use the mass of $\Xi_c(2965)^0$ as well as their mass sum rules,

$$M_{[\Xi'_c(3/2^-),2,1,\lambda]} = M_{\Xi_c(2965)^0} = 2964.88\ \text{MeV},$$
$$M_{[\Xi'_c(5/2^-),2,1,\lambda]} = M_{[\Xi'_c(3/2^-),2,1,\lambda]} + 56\ \text{MeV}. \tag{14}$$

From Table II, we quickly find that the $\Xi_c(2923)^0$, $\Xi_c(2939)^0$, and $\Xi_c(2965)^0$ may be interpreted as the P-wave Ξ'_c baryons $[\Xi'_c(1/2^-), 1, 1, \lambda]$, $[\Xi'_c(3/2^-), 1, 1, \lambda]$, and $[\Xi'_c(3/2^-), 2, 1, \lambda]$, respectively. However, there exist three discrepancies between our theoretical results and the LHCb measurements [6]: (i) the missing of the $\Lambda_c K$ decay channel for the former two baryons, (ii) the mass splitting between the former two baryons, and (iii) the total widths of the latter two baryons.

These discrepancies are possible and reasonable because the HQET is an effective theory, which works well for bottom baryons but not so well for charmed baryons. Hence, the three $J = 1/2^-$ Ξ'_c baryons can mix together and the three $J = 3/2^-$ Ξ'_c baryons can also mix together. Especially, a tiny mixing angle $\theta_1 \approx 0°$ is enough to make it possible to observe all of them in the $\Lambda_c K$ decay channel.

Since the two $\Xi'_c(3/2^-)$ baryons belonging to the $[\mathbf{6}_F, 1, 1, \lambda]$ and $[\mathbf{6}_F, 2, 1, \lambda]$ doublets are very close to each other, in this paper we consider their mixing explicitly,

$$\begin{pmatrix} |\Xi'_c(3/2^-)\rangle_1 \\ |\Xi'_c(3/2^-)\rangle_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix} \times \begin{pmatrix} |\Xi'_c(3/2^-), 1, 1, \lambda\rangle \\ |\Xi'_c(3/2^-), 2, 1, \lambda\rangle \end{pmatrix}, \tag{15}$$

where θ_2 is the mixing angle. Fine-tuning it to be $\theta_2 = 37 \pm 5°$, we show the obtained results in Table III. Very quickly, we find that this mixing mediates the widths of $[\Xi'_c(3/2^-), 1, 1, \lambda]$ and $[\Xi'_c(3/2^-), 2, 1, \lambda]$ and decreases the mass splitting within the $[\mathbf{6}_F, 1, 1, \lambda]$ doublet,

$$\begin{aligned}
M_{[\Xi'_c(3/2^-),1,1,\lambda]} &: 2.95^{+0.12}_{-0.11}\ \text{GeV} \to 2.94^{+0.12}_{-0.11}\ \text{GeV},\\
\Gamma_{[\Xi'_c(3/2^-),1,1,\lambda]} &: 4.4^{+4.5}_{-2.3}\ \text{MeV} \to 11.8^{+9.8}_{-4.2}\ \text{MeV},\\
M_{[\Xi'_c(3/2^-),2,1,\lambda]} &: 2.96^{+0.24}_{-0.15}\ \text{GeV} \to 2.97^{+0.24}_{-0.15}\ \text{GeV},\\
\Gamma_{[\Xi'_c(3/2^-),2,1,\lambda]} &: 30.7^{+35.0}_{-14.2}\ \text{MeV} \to 19.4^{+22.5}_{-9.1}\ \text{MeV},\\
\Delta M_{[\Xi'_c,1,1,\lambda]} &: 38^{+15}_{-13}\ \text{MeV} \to 27^{+16}_{-27}\ \text{MeV},\\
\Delta M_{[\Xi'_c,2,1,\lambda]} &: 66^{+29}_{-25}\ \text{MeV} \to 56^{+30}_{-35}\ \text{MeV}.
\end{aligned}$$

Now the $\Xi_c(2939)^0$ and $\Xi_c(2965)^0$ can be well explained by using the two $J^P = 3/2^-$ baryons $|\Xi'_c(3/2^-)\rangle_1$ and $|\Xi'_c(3/2^-)\rangle_2$, respectively.

Our QCD sum rule results are similar to the quark model calculations [14]. Besides, it is interesting to compare our results with the $SU(3)$ flavor symmetry. Take $[\Xi'_c(3/2^-), 2, 1, \lambda]$ as an example, its partial decay widths to the $\Lambda_c K$ and $\Xi_c\pi$ final states are evaluated to be $9.8^{+17.9}_{-7.2}$ and $17.0^{+29.7}_{-12.0}$ MeV, respectively. Their ratio is about 0.58, similar to following factor derived from the $SU(3)$ flavor symmetry:

$$\begin{aligned}
\mathcal{R} \equiv \frac{\Gamma(\Xi'_c \to \Lambda_c K)}{\Gamma(\Xi'_c \to \Xi_c \pi)} &\approx \frac{\Gamma(\Xi'^0_c \to \Lambda^+_c K^-)}{1.5 \times \Gamma(\Xi'^0_c \to \Xi^+_c \pi^-)}\\
&\sim \frac{g^2_{\Xi'^0_c \to \Lambda^+_c K^-}}{1.5 \times g^2_{\Xi'^0_c \to \Xi^+_c \pi^-}}\\
&= 0.67.
\end{aligned} \tag{16}$$

If considering that $M_{\Lambda_c} + M_K = 2782$ MeV is larger than $M_{\Xi_c} + M_\pi = 2607$ MeV, we can understand the above diversity even better.

TABLE II. Decay properties of P-wave Ξ_c' baryons belonging to the $SU(3)$ flavor $\mathbf{6}_F$ representation. Here the baryons are categorized according to the HQET. In the fifth column, Γ_S and Γ_D denote the relevant decay channel to be S-wave and D-wave, respectively. Their possible experimental candidates are given in the last column for comparisons. Note that there exists considerable uncertainty in our results for absolute values of baryon masses (the third column), but the mass splittings within the same doublets (the fourth column) are produced quite well with much less uncertainty.

Multiplet	Baryon (j^P)	Mass (GeV)	Difference (MeV)	Decay channel	Total width (MeV)	Candidate
$[\mathbf{6}_F, 1, 0, \rho]$	$\Xi_c'(1/2^-)$	$2.88^{+0.15}_{-0.13}$	13^{+6}_{-5}	$\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c'\pi) = 110^{+170}_{-80}$ MeV $\Gamma_D(\Xi_c'(1/2^-) \to \Xi_c^*\pi) = 0.15^{+0.23}_{-0.11}$ MeV $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c\rho \to \Xi_c\pi\pi) = 2\times10^{-4}$ MeV $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c'\rho \to \Xi_c'\pi\pi) = 5\times10^{-9}$ MeV	110^{+170}_{-80}	⋯
	$\Xi_c'(3/2^-)$	$2.89^{+0.15}_{-0.13}$		$\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c'\pi) = 0.63^{+0.99}_{-0.45}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c^*\pi) = 58^{+88}_{-39}$ MeV $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c^*\pi) = 0.03^{+0.05}_{-0.02}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c\rho \to \Xi_c\pi\pi) = 5\times10^{-4}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c'\rho \to \Xi_c'\pi\pi) = 2\times10^{-9}$ MeV	$58.5^{+88.4}_{-39.4}$	⋯
$[\mathbf{6}_F, 0, 1, \lambda]$	$\Xi_c'(1/2^-)$	$2.90^{+0.13}_{-0.12}$	⋯	$\Gamma_S(\Xi_c'(1/2^-) \to \Lambda_c K) = 400^{+610}_{-270}$ MeV $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c\pi) = 360^{+550}_{-250}$ MeV $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c'\rho \to \Xi_c'\pi\pi) = 0.03$ MeV	760^{+820}_{-370}	⋯
$[\mathbf{6}_F, 1, 1, \lambda]$	$\Xi_c'(1/2^-)$	$2.91^{+0.13}_{-0.12}$	38^{+15}_{-13}	$\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c'\pi) = 11.7^{+15.0}_{-8.0}$ MeV $\Gamma_D(\Xi_c'(1/2^-) \to \Xi_c^*\pi) = 0.12^{+0.22}_{-0.10}$ MeV $\Gamma_S(\Xi_c'(1/2^-) \to \Lambda_c K^* \to \Lambda_c K\pi) = 4\times10^{-8}$ MeV $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c\rho \to \Xi_c\pi\pi) = 1.7^{+7.6}_{-1.7}$ MeV $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c'\rho \to \Xi_c'\pi\pi) = 0.38^{+0.54}_{-0.30}$ MeV $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c^*\rho \to \Xi_c^*\pi\pi) = 2\times10^{-7}$ MeV	$13.9^{+16.8}_{-8.2}$	$\Xi_c(2923)^0$
	$\Xi_c'(3/2^-)$	$2.95^{+0.12}_{-0.11}$		$\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c'\pi) = 0.67^{+1.06}_{-0.52}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c^*\pi) = 3.3^{+4.3}_{-2.3}$ MeV $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c^*\pi) = 0.05^{+0.08}_{-0.04}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Lambda_c K^* \to \Lambda_c K\pi) = 2\times10^{-4}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c\rho \to \Xi_c\pi\pi) = 0.21^{+0.60}_{-0.20}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c'\rho \to \Xi_c'\pi\pi) = 0.12^{+0.19}_{-0.10}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c^*\rho \to \Xi_c^*\pi\pi) = 1\times10^{-3}$ MeV	$4.4^{+4.5}_{-2.3}$	$\Xi_c(2939)^0$ and $\Xi_c(2965)^0$
$[\mathbf{6}_F, 2, 1, \lambda]$	$\Xi_c'(3/2^-)$	$2.96^{+0.24}_{-0.15}$	66^{+29}_{-25}	$\Gamma_D(\Xi_c'(3/2^-) \to \Lambda_c K) = 9.8^{+17.9}_{-7.2}$ MeV $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c\pi) = 17.0^{+29.7}_{-12.0}$ MeV $\Gamma_D(\Xi_c'(3/2^-) \to \Sigma_c K) = 0.003^{+0.015}_{-0.003}$ MeV $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c'\pi) = 2.3^{+4.0}_{-1.7}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c^*\pi) = 2\times10^{-4}$ MeV $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c^*\pi) = 0.19^{+0.33}_{-0.14}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c'\rho \to \Xi_c'\pi\pi) = 1.4^{+2.2}_{-1.0}$ MeV $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c^*\rho \to \Xi_c^*\pi\pi) = 1\times10^{-3}$ MeV	$30.7^{+35.0}_{-14.2}$	
	$\Xi_c'(5/2^-)$	$3.02^{+0.23}_{-0.14}$		$\Gamma_D(\Xi_c'(5/3^-) \to \Lambda_c K) = 6.3^{+11.4}_{-4.6}$ MeV $\Gamma_D(\Xi_c'(5/2^-) \to \Xi_c\pi) = 9.6^{+15.8}_{-6.8}$ MeV $\Gamma_D(\Xi_c'(5/2^-) \to \Sigma_c K) = 0.02^{+0.09}_{-0.02}$ MeV $\Gamma_D(\Xi_c'(5/2^-) \to \Xi_c'\pi) = 0.70^{+1.30}_{-0.54}$ MeV $\Gamma_D(\Xi_c'(5/2^-) \to \Sigma_c^*\pi) = 4\times10^{-3}$ MeV $\Gamma_D(\Xi_c'(5/2^-) \to \Xi_c^*\pi) = 1.5^{+2.6}_{-1.1}$ MeV $\Gamma_S(\Xi_c'(5/2^-) \to \Xi_c^*\rho \to \Xi_c^*\pi\pi) = 0.02$ MeV	$18.1^{+19.7}_{-8.3}$	⋯

Our results are obtained using light-cone sum rules within HQET. It is also interesting to compare them with the results of Ref. [15], which are obtained using full QCD light-cone sum rule method. There the widths of the $1P$ Ξ_c' baryons with $J^P = 1/2^-$ and $3/2^-$ are calculated to be 7.2 ± 1.4 and 10.1 ± 2.1 MeV, respectively, supporting

TABLE III. Decay properties of P-wave Ξ_c' baryons of the $SU(3)$ flavor $\mathbf{6}_F$. The first column lists the baryons categorized according to the HQET, and the third column lists the baryons after considering the mixing. The possible experimental candidates are given in the last column for comparisons.

HQET state	Mixing	Mixed state	Mass (GeV)	Difference (MeV)	Decay channel (MeV)	Width (MeV)	Candidate
$[\Xi_c'(\frac{1}{2}^-), 0, 1, \lambda]$		$[\Xi_c'(\frac{1}{2}^-), 0, 1, \lambda]$	$2.90^{+0.13}_{-0.12}$	$\cdots$	$\Gamma_S(\Xi_c'(1/2^-) \to \Lambda_c K) = 400^{+610}_{-270}$ $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c \pi) = 360^{+550}_{-250}$ $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c' \rho \to \Xi_c' \pi\pi) = 0.03$	760^{+820}_{-370}	$\cdots$
$[\Xi_c'(\frac{1}{2}^-), 1, 1, \lambda]$	$\theta_1 \approx 0°$	$[\Xi_c'(\frac{1}{2}^-), 1, 1, \lambda]$	$2.91^{+0.13}_{-0.12}$		$\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c' \pi) = 11.7^{+15.0}_{-8.0}$ $\Gamma_D(\Xi_c'(1/2^-) \to \Xi_c^* \pi) = 0.12^{+0.22}_{-0.10}$ $\Gamma_S(\Xi_c'(1/2^-) \to \Lambda_c K^* \to \Lambda_c K \pi) = 4 \times 10^{-8}$ $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c \rho \to \Xi_c \pi\pi) = 1.7^{+7.6}_{-1.7}$ $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c' \rho \to \Xi_c' \pi\pi) = 0.38^{+0.54}_{-0.30}$ $\Gamma_S(\Xi_c'(1/2^-) \to \Xi_c^* \rho \to \Xi_c^* \pi\pi) = 2 \times 10^{-7}$	$13.9^{+16.8}_{-8.2}$	$\Xi_c(2923)^0$
$[\Xi_c'(\frac{3}{2}^-), 1, 1, \lambda]$		$\Xi_c'(\frac{3}{2}^-)_1$	$2.94^{+0.12}_{-0.11}$	27^{+16}_{-27}	$\Gamma_D(\Xi_c'(3/2^-) \to \Lambda_c K) = 2.3^{+4.3}_{-1.7}$ $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c \pi) = 4.6^{+8.1}_{-3.3}$ $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c' \pi) = 2.0^{+2.2}_{-1.2}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c^* \pi) = 2.1^{+2.6}_{-1.5}$ $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c^* \pi) = 0.14^{+0.16}_{-0.08}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Lambda_c K^* \to \Lambda_c K \pi) = 2 \times 10^{-6}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c \rho \to \Xi_c \pi\pi) = 0.13^{+0.39}_{-0.13}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c' \rho \to \Xi_c' \pi\pi) = 0.5^{+0.6}_{-0.3}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c^* \rho \to \Xi_c^* \pi\pi) = 1 \times 10^{-3}$	$11.8^{+9.8}_{-4.2}$	$\Xi_c(2939)^0$
$[\Xi_c'(\frac{3}{2}^-), 2, 1, \lambda]$	$\theta_2 = 37 \pm 5°$	$\Xi_c'(\frac{3}{2}^-)_2$	$2.97^{+0.24}_{-0.15}$	56^{+30}_{-35}	$\Gamma_D(\Xi_c'(3/2^-) \to \Lambda_c K) = 6.3^{+11.6}_{-4.7}$ $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c \pi) = 10.9^{+19.1}_{-7.8}$ $\Gamma_D(\Xi_c'(3/2^-) \to \Sigma_c K) = 4 \times 10^{-4}$ $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c' \pi) = 0.37^{+1.89}_{-0.36}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c^* \pi) = 1.3^{+1.80}_{-0.94}$ $\Gamma_D(\Xi_c'(3/2^-) \to \Xi_c^* \pi) = 0.03^{+0.16}_{-0.03}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Lambda_c K^* \to \Lambda_c K \pi) = 2 \times 10^{-5}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c \rho \to \Xi_c \pi\pi) = 0.12^{+0.36}_{-0.12}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c' \rho \to \Xi_c' \pi\pi) = 0.37^{+1.15}_{-0.36}$ $\Gamma_S(\Xi_c'(3/2^-) \to \Xi_c^* \rho \to \Xi_c^* \pi\pi) = 5 \times 10^{-3}$	$19.4^{+22.5}_{-9.1}$	$\Xi_c(2965)^0$
$[\Xi_c'(\frac{5}{2}^-), 2, 1, \lambda]$	$\cdots$	$[\Xi_c'(\frac{5}{2}^-), 2, 1, \lambda]$	$3.02^{+0.23}_{-0.14}$		$\Gamma_D(\Xi_v'(5/3^-) \to \Lambda_c K) = 6.3^{+11.4}_{-4.6}$ $\Gamma_D(\Xi_c'(5/2^-) \to \Xi_c \pi) = 9.6^{+15.8}_{-6.8}$ $\Gamma_D(\Xi_c'(5/2^-) \to \Sigma_c K) = 0.02^{+0.09}_{-0.02}$ $\Gamma_D(\Xi_c'(5/2^-) \to \Xi_c' \pi) = 0.70^{+1.30}_{-0.54}$ $\Gamma_D(\Xi_c'(5/2^-) \to \Sigma_c^* \pi) = 4 \times 10^{-3}$ $\Gamma_D(\Xi_c'(5/2^-) \to \Xi_c^* \pi) = 1.5^{+2.6}_{-1.1}$ $\Gamma_S(\Xi_c'(5/2^-) \to \Xi_c^* \rho \to \Xi_c^* \pi\pi) = 0.02$	$18.1^{+19.7}_{-8.3}$	$\cdots$

the interpretation of $\Xi_c(2939)^0$ and $\Xi_c(2965)^0$ as such states. These two values are consistent with our results that $\Gamma_{[\Xi_c'(1/2^-),1,1,\lambda]} = 13.9^{+16.8}_{-8.2}$ MeV and $\Gamma_{[\Xi_c'(3/2^-),1,1,\lambda]} = 11.8^{+9.8}_{-4.2}$ MeV, and our interpretations are the same for the $\Xi_c(2939)^0$ and $\Xi_c(2965)^0$.

IV. SUMMARY AND DISCUSSIONS

In the present study, we have investigated P-wave Ξ_c' baryons of the $SU(3)$ flavor $\mathbf{6}_F$ by systematically studying their mass spectra and decay properties using the methods of QCD sum rules and light-cone sum rules within the framework of heavy quark effective theory. The obtained results are summarized in Tables II and III, from which we can well understand the three excited Ξ_c^0 baryons recently observed by LHCb [6] as P-wave Ξ_c' baryons of the $SU(3)$ flavor $\mathbf{6}_F$.

There can be as many as seven P-wave Ξ_c' baryons, belonging to the following four multiplets:

$$\Xi_c'(1/2^-), \Xi_c'(3/2^-) \in [\mathbf{6}_F, 1, 0, \rho],$$
$$\Xi_c'(1/2^-) \in [\mathbf{6}_F, 0, 1, \lambda],$$
$$\Xi_c'(1/2^-), \Xi_c'(3/2^-) \in [\mathbf{6}_F, 1, 1, \lambda],$$
$$\Xi_c'(3/2^-), \Xi_c'(5/2^-) \in [\mathbf{6}_F, 2, 1, \lambda].$$

Our results suggest the following:

(i) The $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ can be interpreted as the P-wave Ξ_c' baryons of $J^P = 1/2^-$ and $3/2^-$, respectively, both of which belong to the $[\mathbf{6}_F(\Xi_c'), 1, 1, \lambda]$ doublet. The $\Xi_c(2965)^0$ can be interpreted as the P-wave Ξ_c' baryon of $J^P = 3/2^-$, belonging to the $[\mathbf{6}_F(\Xi_c'), 2, 1, \lambda]$ doublet. It has a partner state, Ξ_c' of $J^P = 5/2^-$, whose mass is about 56^{+30}_{-35} MeV larger and width about $18.1^{+19.7}_{-8.3}$ MeV. We propose to search for it in the $\Lambda_c K/\Xi_c \pi$ mass spectral in future experiments.

(ii) The HQET is an effective theory, which works well for bottom baryons but not so well for charmed baryons. This suggests that the three $J = 1/2^-$ Ξ_c' baryons can mix together and the three $J = 3/2^-$ ones can also mix together, making it possible to observe all of them in the $\Lambda_c K$ invariant mass spectrum. Especially, in this paper, we have explicitly considered the mixing between the two $\Xi_c'(3/2^-)$ baryons belonging to the $[\mathbf{6}_F, 1, 1, \lambda]$ and $[\mathbf{6}_F, 2, 1, \lambda]$ doublets, which mediates their widths as well as decreases the mass splitting within the $[\mathbf{6}_F, 1, 1, \lambda]$ doublet. The obtained results can be used to better describe the LHCb experiment [6].

(iii) The width of $[\Xi_c^0(1/2^-), 0, 1, \lambda]$ is too large for it to be observed in experiments. The widths of $\Xi_c'(1/2^-)$ and $\Xi_c'(3/2^-)$ belonging to the $[\mathbf{6}_F(\Xi_c'), 1, 0, \rho]$ doublet are evaluated to be about 110 and 59 MeV, respectively, making them not so easy to be observed. We notice that there is "an additional component" observed by LHCb in the energy region around 2900 MeV [6], which may be due to these two states.

The above conclusions are obtained by combining our systematical studies on mass spectra, mass splittings within the same multiplets, and decay properties of P-wave Ξ_c' baryons. Moreover, we have taken into account the five excited Ω_c^0 and four excited Ω_b^- baryons observed by LHCb [4,5], whose correspondences may be as follows [73]

$$[\mathbf{6}_F(?/2^-), 1, 0, \rho] : \Xi_c'(?/2^-) \sim \Omega_c^0(3000) \sim \Omega_b^-(6316),$$
$$[\mathbf{6}_F(1/2^-), 1, 1, \lambda] : \Xi_c^0(2923) \sim \Omega_c^0(3050) \sim \Omega_b^-(6330),$$
$$[\mathbf{6}_F(3/2^-), 1, 1, \lambda] : \Xi_c^0(2939) \sim \Omega_c^0(3066) \sim \Omega_b^-(6340),$$
$$[\mathbf{6}_F(3/2^-), 2, 1, \lambda] : \Xi_c^0(2965) \sim \Omega_c^0(3090) \sim \Omega_b^-(6350),$$
$$[\mathbf{6}_F(5/2^-), 2, 1, \lambda] : \Xi_c'(5/2^-) \sim \Omega_c^0(3119) \sim \Omega_b^-(5/2^-).$$

We shall discuss this in detail in our future work [74].

In the present study, we have investigated P-wave Ξ_c' baryons within the heavy quark effective theory. Because the finite charm quark mass breaks this symmetry explicitly, we have also considered the mixing effect between baryons having the same spin-parity quantum number, such as the mixing between $[\Xi_c'(\frac{3}{2}^-), 1, 1, \lambda]$ and $[\Xi_c'(\frac{3}{2}^-), 2, 1, \lambda]$. One can study them within some other schemes. For example, in Refs. [14,75], the authors studied excited Ξ_c' and Ω_b baryons in the j-j coupling scheme based on the chiral quark model. However, there they also considered the mixing effect, so the obtained "physical" basis can be eventually the same.

To end this paper, we note that the conclusions of the present study are just possible explanations, and there exist some other possibilities for the three excited Ξ_c^0 baryons observed by LHCb [6]. Further experimental and theoretical studies are still demanded to fully understand them, since the beautiful fine structures of the excited singly heavy baryons observed in the three LHCb experiments [4–6] have proved the rich internal structure of (heavy) hadrons, and their relevant studies are significantly improving our knowledge of the strong interaction.

ACKNOWLEDGMENTS

We thank Qi-Fang Lü for useful discussions. This project was supported by the National Natural Science Foundation of China under Grants No. 11722540 and No. 12075019, the Fundamental Research Funds for the Central Universities, and the Foundation for Young Talents in College of Anhui Province (Grant No. gxyq2018103).

APPENDIX A: INPUT PARAMETERS

We list masses of ground-state charmed baryons used in this paper [7] which are as follows:

$$\Lambda_c(1/2^+) : m = 2286.46 \text{ MeV},$$
$$\Xi_c(1/2^+) : m = 2469.34 \text{ MeV},$$
$$\Sigma_c(1/2^+) : m = 2453.54 \text{ MeV},$$
$$\Sigma_c^*(3/2^+) : m = 2518.1 \text{ MeV},$$
$$\Xi_c'(1/2^+) : m = 2576.8 \text{ MeV},$$
$$\Xi_b^*(3/2^+) : m = 2645.9 \text{ MeV},$$
$$\Omega_b(1/2^+) : m = 2695.2 \text{ MeV},$$
$$\Omega_b^*(3/2^+) : m = 2765.9 \text{ MeV}.$$

Their sum rule parameters can be found in Refs. [51,76].

We list masses and decay widths of pseudoscalar and vector mesons used in this paper [7] which are as follows:

$$
\begin{aligned}
\pi(0^-)&:\ m = 138.04\ \text{MeV},\\
K(0^-)&:\ m = 495.65\ \text{MeV},\\
\rho(1^-)&:\ m = 775.21\ \text{MeV},\\
&\quad\ \Gamma = 148.2\ \text{MeV},\quad g_{\rho\pi\pi} = 5.94,\\
K^*(1^-)&:\ m = 893.57\ \text{MeV},
\end{aligned}
\tag{A1}
$$

$$
\Gamma = 49.1\ \text{MeV},\quad g_{K^*K\pi} = 3.20,
\tag{A2}
$$

where the two coupling constants $g_{\rho\pi\pi}$ and $g_{K^*K\pi}$ are evaluated using the experimental widths of the ρ and K^* [7] through the Lagrangians

$$
\begin{aligned}
\mathcal{L}_{\rho\pi\pi} &= g_{\rho\pi\pi} \times (\rho^0_\mu \pi^+ \partial^\mu \pi^- - \rho^0_\mu \pi^- \partial^\mu \pi^+) + \cdots,\\
\mathcal{L}_{K^*K\pi} &= g_{K^*K\pi} K^{*+}_\mu \times (K^- \partial^\mu \pi^0 - \partial^\mu K^- \pi^0) + \cdots.
\end{aligned}
\tag{A3}
$$

We also list the light-cone distribution amplitudes of the K meson, which are taken from Ref. [60]. We refer to Refs. [59,60] for detailed discussions. The light-cone distribution amplitudes of the K meson used in this paper are

$$
\begin{aligned}
\langle 0|\bar q(z)\gamma_\mu\gamma_5 s(-z)|K(q)\rangle &= i f_K q_\mu \int_0^1 du e^{i(2u-1)q\cdot z}\left(\phi_{2;K}(u) + \frac{1}{4} z^2 \phi_{4;K}(u)\right)\\
&\quad + \frac{i}{2} f_K \frac{1}{q\cdot z} z_\mu \int_0^1 du e^{i(2u-1)q\cdot z}\psi_{4;K}(u),
\end{aligned}
\tag{A4}
$$

$$
\langle 0|\bar q(z) i\gamma_5 s(-z)|K(q)\rangle = \frac{f_K m_K^2}{m_s + m_q}\int_0^1 du e^{i(2u-1)q\cdot z}\phi^p_{3;K}(u),
\tag{A5}
$$

$$
\langle 0|\bar q(z)\sigma_{\alpha\beta}\gamma_5 s(-z)|K(q)\rangle = -\frac{i}{3}\frac{f_K m_K^2}{m_s + m_q}(q_\alpha z_\beta - q_\beta z_\alpha)\int_0^1 du e^{i(2u-1)q\cdot z}\phi^\sigma_{3;K}(u),
\tag{A6}
$$

$$
\begin{aligned}
\langle 0|\bar q(z)\gamma_\mu\gamma_5 g G_{\alpha\beta}(vz) s(-z)|K(q)\rangle &= q_\mu (q_\alpha z_\beta - q_\beta z_\alpha)\frac{1}{q\cdot z} f_K \Phi_{4;K}(v, q\cdot z)\\
&\quad + (q_\beta g^\perp_{\alpha\mu} - q_\alpha g^\perp_{\beta\mu}) f_K \Psi_{4;K}(v, q\cdot z),
\end{aligned}
\tag{A7}
$$

$$
\begin{aligned}
\langle 0|\bar q(z)\gamma_\mu i g \tilde G_{\alpha\beta}(vz) s(-z)|K(q)\rangle &= q_\mu (q_\alpha z_\beta - q_\beta z_\alpha)\frac{1}{q\cdot z} f_K \tilde\Phi_{4;K}(v, q\cdot z)\\
&\quad + (q_\beta g^\perp_{\alpha\mu} - q_\alpha g^\perp_{\beta\mu}) f_K \tilde\Psi_{4;K}(v, q\cdot z),
\end{aligned}
\tag{A8}
$$

$$
\begin{aligned}
\langle 0|\bar q(z)\sigma_{\mu\nu}\gamma_5 g G_{\alpha\beta}(vz) s(-z)|K(q)\rangle &= i f_{3K}(q_\alpha q_\mu g^\perp_{\nu\beta} - q_\alpha q_\nu g^\perp_{\mu\beta} - (\alpha \leftrightarrow \beta))\\
&\quad \times \int \mathcal{D}\underline{\alpha} e^{-iq\cdot z(\alpha_2 - \alpha_1 + v\alpha_3)}\Phi_{3;K}(\alpha_1, \alpha_2, \alpha_3),
\end{aligned}
\tag{A9}
$$

where $\tilde G_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$.

APPENDIX B: SUM RULE EQUATIONS

In this appendix, we give the sum rule equations used to study S-wave and D-wave decays of P-wave Ξ_c' baryons into ground-state charmed baryons and pseudoscalar mesons.

The sum rule equations for the $\Xi_b'^-[\frac{1}{2}^-]$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ are

$$
\begin{aligned}
G^D_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c'^0[\frac{1}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{f_\pi m_s u}{128\pi^2}\phi_{4;\pi}(u) + \frac{f_\pi u}{24}\langle\bar s s\rangle \phi_{2;\pi}(u) \\
&\quad + \frac{f_\pi m_s u}{8\pi^2 t^2}\phi_{2;\pi}(u) + \frac{f_\pi m_\pi^2 u}{24(m_u+m_d)\pi^2 t^2}\phi^\sigma_{3;\pi}(u) + \frac{f_\pi m_s u t^2}{576(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u) + \frac{f_\pi u t^2}{384}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad + \frac{f_\pi u t^2}{384}\langle g_s\bar s\sigma G s\rangle\phi_{2;\pi}(u) + \frac{f_\pi u t^4}{6144}\langle g_s\bar s\sigma G s\rangle\phi_{4;\pi}(u)\bigg) \\
&\quad - \int_0^\infty dt\int_0^1 du\int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \\
&\quad \times \bigg(-\frac{i f_{3\pi} u v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha) + \frac{i f_{3\pi}\alpha_2 u v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha) + \frac{i f_{3\pi}\alpha_3 u v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha) - \frac{i f_{3\pi} u v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha) \\
&\quad + \frac{i f_{3\pi}\alpha_2 v\cdot q}{4\pi^2 t}\Phi_{3\pi}(\underline\alpha) - \frac{i f_{3\pi} v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha) + \frac{f_{3\pi} u}{4\pi^2 t^2}\Phi_{3;\pi}(\underline\alpha)\bigg), \qquad \text{(B1)}
\end{aligned}
$$

$$
\begin{aligned}
G^S_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c'^+\pi^-} f_{\Xi_c'^0[\frac{1}{2}^-]} f_{\Xi_c'^+}}{(\bar\Lambda_{\Xi_c'^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c'^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{3 f_\pi m_\pi^2}{4\pi^2(m_u+m_d)t^4}\phi^p_{3;\pi}(u) + \frac{i f_\pi m_\pi^2 v\cdot q}{8\pi^2(m_u+m_d)t^3}\phi^\sigma_{3;\pi}(u) \\
&\quad - \frac{3 i f_\pi m_s}{16\pi^2 t^3 v\cdot q}\psi_{4;\pi}(u) - \frac{i f_\pi}{16 t v\cdot q}\langle\bar s s\rangle\psi_{4;\pi}(u) + \frac{f_\pi m_s m_\pi^2}{32(m_u+m_d)}\phi^p_{3;\pi}(u) - \frac{i f_\pi t}{256 v\cdot q}\langle g_s\bar s\sigma G s\rangle\psi_{4;\pi}(u) \\
&\quad + \frac{i f_\pi m_s m_\pi^2 t v\cdot q}{192(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u)\bigg). \qquad \text{(B2)}
\end{aligned}
$$

The sum rule equations for the $\Xi_b'^-[\frac{3}{2}^-]$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ are

$$
\begin{aligned}
G^D_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c'^+\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c'^+\pi^-} f_{\Xi_c'^0[\frac{3}{2}^-]} f_{\Xi_c'^+}}{(\bar\Lambda_{\Xi_c'^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c'^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{f_\pi m_s u}{8\pi^2 t^2}\phi_{2;\pi}(u) + \frac{f_\pi m_\pi^2 u}{24(m_u+m_d)\pi^2 t^2}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{f_\pi m_s u}{128\pi^2}\phi_{4;\pi}(u) + \frac{f_\pi u}{24}\langle\bar s s\rangle\phi_{2;\pi}(u) + \frac{f_\pi m_s m_\pi^2 u t^2}{576(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\phi}(u) + \frac{f_\pi u t^2}{384}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad + \frac{f_\pi u t^2}{384}\langle g_s\bar s\sigma G s\rangle\phi_{2;\pi}(u) + \frac{f_\pi u t^4}{6144}\langle g_s\bar s\sigma G s\rangle\phi_{4;\pi}(u)\bigg) \\
&\quad - \int_0^\infty dt\int_0^1 du\int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \bigg(-\frac{f_{3\pi} u}{4\pi^2 t^2}\Phi_{3;\pi}(\underline\alpha) + \frac{i f_{3\pi}\alpha_3 u^2 v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha) \\
&\quad + \frac{i f_{3\pi}\alpha_2 u v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha) + fracif_{3\pi}\alpha_3 uv\cdot q 4\pi^2 t\Phi_{3;\pi}(\underline\alpha) - \frac{i f_{3\pi} u v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha) + \frac{i f_{3\pi}\alpha_3 v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha) \\
&\quad - \frac{i f_{3\pi} v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline\alpha)\bigg), \qquad \text{(B3)}
\end{aligned}
$$

$$
\begin{aligned}
G^{D}_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c'^0[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{f_\pi m_s u}{24\pi^2 t^2}\phi_{2;\pi}(u) + \frac{f_\pi m_\pi^2 u}{72(m_u+m_d)\pi^2 t^2}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{f_\pi m_s u}{384\pi^2}\phi_{4;\pi}(u) + \frac{f_\pi u}{72}\langle\bar s s\rangle\phi_{2;\pi}(u) + \frac{f_\pi m_s m_\pi^2 u t^2}{1728(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u) + \frac{f_\pi u t^2}{1152}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad + \frac{f_\pi u t^2}{1152}\langle g_s\bar s\sigma G s\rangle\phi_{2;\pi}(u) + \frac{f_\pi u t^4}{18432}\langle g_s\bar s\sigma G s\rangle\phi_{4;\pi}(u)\bigg) \\
&\quad - \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \bigg(\frac{i f_{3\pi}\alpha_3 u^2 v\cdot q}{12\pi^2 t}\Phi_{3;\pi}(\underline{\alpha}) + \frac{f_{3\pi} u v\cdot q}{12\pi^2 t}\Phi_{3;\pi}(\underline{\alpha}) \\
&\quad + \frac{i f_{3\pi}\alpha_2 u v\cdot q}{12\pi^2 t}\Phi_{3;\pi}(\underline{\alpha}) + \frac{i f_{3\pi}\alpha_3 u v\cdot q}{12\pi^2}\Phi_{3;\pi}(\underline{\alpha}) - \frac{i f_{3\pi} u v\cdot q}{12\pi^2 t}\Phi_{3;\pi}(\underline{\alpha}) + \frac{i f_{3\pi}\alpha_2 v\cdot q}{12\pi^2 t}\Phi_{3;\pi}(\underline{\alpha}) \\
&\quad - \frac{i f_{3\pi} v\cdot q}{12\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})\bigg),
\end{aligned} \tag{B4}
$$

$$
\begin{aligned}
G^{S}_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c'^0[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(-\frac{f_\pi m_\pi^2}{6\pi^2(m_u+m_d)t^4}\phi^p_{3;\pi}(u) - \frac{i f_\pi m_\pi^2 v\cdot q}{36\pi^2(m_u+m_d)t^3}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{f_\pi m_s}{24\pi^2 t^3 v\cdot q}\psi_{4;\pi}(u) + \frac{i f_\pi}{72 t v\cdot q}\langle\bar s s\rangle\psi_{4;\pi}(u) - \frac{f_\pi m_s m_\pi^2}{144(m_u+m_d)}\phi^p_{3;\pi}(u) + \frac{i f_\pi t}{1152 v\cdot q}\langle g_s\bar s\sigma G s\rangle\psi_{4;\pi}(u) \\
&\quad - \frac{i f_\pi m_s m_\pi^2 t v\cdot q}{864(m_u+m_d)\pi^2 t^2}\phi^\sigma_{3;\pi}(u)\bigg).
\end{aligned} \tag{B5}
$$

The sum rule equations for the $\Xi_b'^-[\frac{1}{2}^-]$ belonging to $[\mathbf{6}_F, 0, 1, \lambda]$ are

$$
\begin{aligned}
G^{S}_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c^{+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c^{+}\pi^-} f_{\Xi_c'^0[\frac{1}{2}^-]} f_{\Xi_c^{+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(-\frac{3 f_\pi m_\pi^2}{4\pi^2(m_u+m_d)t^4}\phi^p_{3;\pi}(u) - \frac{i f_\pi m_\pi^2 v\cdot q}{8\pi^2(m_u+m_d)t^3}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{i f_\pi}{16 t v\cdot q}\langle\bar s s\rangle\psi_{4;\pi}(u) + \frac{i f_\pi t}{256 v\cdot q}\langle g_s\bar s\sigma G s\rangle\psi_{4;\pi}(u) + \frac{3 i f_\pi m_s}{16\pi^2 t^3 v\cdot q}\psi_{4;\pi}(u) \\
&\quad - \frac{f_\pi m_s m_\pi^2}{32(m_u+m_d)}\langle\bar s s\rangle\phi^p_{3;\pi}(u) - \frac{i f_\pi m_s m_\pi^2 t v\cdot q}{192(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u)\bigg),
\end{aligned} \tag{B6}
$$

$$
\begin{aligned}
G^{S}_{\Xi_c'^0[\frac{1}{2}^-]\to\Lambda_c^{+}K^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{1}{2}^-]\to\Lambda_c^{+}K^-} f_{\Xi_c'^0[\frac{1}{2}^-]} f_{\Lambda_c^{+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^{+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(-\frac{3 f_K m_K^2}{4\pi^2(m_u+m_s)t^4}\phi^p_{3;K}(u) - \frac{i f_K m_K^2 v\cdot q}{8\pi^2(m_u+m_s)t^3}\phi^\sigma_{3;K}(u) \\
&\quad + \frac{i f_K}{16 t v\cdot q}\langle\bar q q\rangle\psi_{4;K}(u) + \frac{i f_K t}{256 v\cdot q}\langle g_s\bar q\sigma G q\rangle\psi_{4;K}(u)\bigg).
\end{aligned} \tag{B7}
$$

The sum rule equations for the $\Xi_b'^{-}[\frac{1}{2}^-]$ belonging to $[\mathbf{6}_F, 1, 1, \lambda]$ are

$$
\begin{aligned}
G^D_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c'^0[\frac{1}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(-\frac{f_\pi u}{4\pi^2 t^3}\phi_{2;\pi}(u) + \frac{f_\pi m_s m_\pi^2 u}{48(m_u+m_d)\pi^2 t}\phi^\sigma_{3;\pi}(u) - \frac{f_\pi u}{64\pi^2 t}\phi_{4;\pi}(u) \\
&\quad - \frac{f_\pi m_s u t}{96}\langle\bar s s\rangle\phi_{2;\pi}(u) + \frac{f_\pi m_\pi^2 u t}{144(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u) - \frac{f_\pi m_s t^3}{1536}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad + \frac{f_\pi m_\pi^2 t^3}{2304(m_u+m_d)}\langle g_s\bar s\sigma G s\rangle\phi^\sigma_{3;\pi}(u)\bigg) - \int_0^\infty dt\int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \\
&\quad \times \bigg(-\frac{f_\pi \alpha_3 u^2}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi \alpha_2 u}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi\alpha_3 u}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi\alpha_3 u}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{f_\pi\alpha_2}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi\alpha_2}{16\pi^2 t}\tilde\Phi 4;\pi(\underline{\alpha}) + \frac{f_\pi}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha}) + \frac{i f_\pi u}{8\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad + \frac{3 i f_\pi}{8\pi^2 t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha}) - \frac{i f_\pi}{8\pi^2 t^2 v\cdot q}\Phi_{4;\pi}(\underline{\alpha}) - \frac{3 i f_\pi}{8\pi^2 t^2 v\cdot q}\tilde\Phi_{4;\pi}(\underline{\alpha}) \\
&\quad + \frac{i f_\pi}{8\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) - \frac{i f_\pi}{8\pi^2 t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha})\bigg), \qquad \text{(B8)}
\end{aligned}
$$

$$
\begin{aligned}
G^S_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c'^{+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{1}{2}^-]\to\Xi_c'^{+}\pi^-} f_{\Xi_c'^0[\frac{1}{2}^-]} f_{\Xi_c'^{+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c'^{+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{3 i f_\pi v\cdot q}{2\pi^2 t^4}\phi_{2;\pi}(u) + \frac{3 i f_\pi v\cdot q}{32\pi^2 t^2}\phi_{4;\pi}(u) - \frac{i f_\pi m_\pi^2 v\cdot q}{24(m_u+m_d)}\phi^\sigma_{3;\pi}(u) \\
&\quad - \frac{i f_\pi m_\pi^2 v\cdot q t^2}{384(m_u+m_d)}\langle g_s\bar s\sigma G s\rangle\phi^\sigma_{3;\pi}(u) + \frac{i f_\pi m_s v\cdot q}{16}\langle\bar s s\rangle\phi_{2;\pi}(u) - \frac{i f_\pi m_s m_\pi^2 v\cdot q}{8\pi^2(m_u+m_d)t^2}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{i f_\pi m_s t^2 v\cdot q}{256}\langle\bar s s\rangle\phi_{4;\pi}(u)\bigg) - \int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \\
&\quad \times \bigg(-\frac{3 i f_\pi u v\cdot q}{4\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) + \frac{i f_\pi u v\cdot q}{2\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) - \frac{i f_\pi v\cdot q}{8\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) - \frac{3 i f_\pi v\cdot q}{8\pi^2 t^2}\tilde\Phi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{i f_\pi v\cdot q}{4\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) + \frac{i f_\pi v\cdot q}{4\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha})\bigg). \qquad \text{(B9)}
\end{aligned}
$$

The sum rule equations for the $\Xi_b'^{-}[\frac{3}{2}^-]$ belonging to $[\mathbf{6}_F, 1, 1, \lambda]$ are

$$
\begin{aligned}
G^D_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c'^{+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c'^{+}\pi^-} f_{\Xi_c'^0[\frac{3}{2}^-]} f_{\Xi_c'^{+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c'^{+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{i f_\pi u}{4\pi^2 t^3}\phi_{2;\pi}(u) - \frac{i f_\pi m_s m_\pi^2 u}{48(m_u+m_d)\pi^2 t}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{i f_\pi u}{64\pi^2 t}\phi_{4;\pi}(u) + \frac{i f_\pi m_s u t}{96}\langle\bar s s\rangle\phi_{2;\pi}(u) - \frac{i f_\pi m_\pi^2 u t}{144(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u) + \frac{i f_\pi m_s u t^3}{1536}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad - \frac{i f_\pi m_\pi^2 u t^3}{2304(m_u+m_d)}\langle g_s\bar s\sigma G s\rangle\phi^\sigma_{3;\pi}(u)\bigg) - \int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2}
\end{aligned}
$$

$$\times \left(\frac{if_\pi \alpha_3 u^2}{8\pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) + \frac{if_\pi \alpha_2 u}{8\pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) + \frac{if_\pi \alpha_3 u}{16\pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) + \frac{if_\pi \alpha_3 u}{16\pi^2 t} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) - \frac{if_\pi u}{8\pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) \right.$$
$$+ \frac{if_\pi \alpha_2}{16\pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) + \frac{if_\pi \alpha_2}{16\pi^2 t} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) - \frac{if_\pi}{16\pi^2 t} \Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi}{16\pi^2 t} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u}{8\pi^2 t^2 v\cdot q} \Psi_{4;\pi}(\underline{\alpha})$$
$$+ \frac{3f_\pi u}{8\pi^2 t^2 v\cdot q} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) - \frac{f_\pi}{8\pi^2 t^2 v\cdot q} \Phi_{4;\pi}(\underline{\alpha}) - \frac{3f_\pi}{8\pi^2 t^2 v\cdot q} \tilde{\Phi}_{4;\pi}(\underline{\alpha})$$
$$\left. + \frac{f_\pi}{8\pi^2 t^2 v\cdot q} \Psi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi}{8\pi^2 t^2 v\cdot q} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) \right), \tag{B10}$$

$$G^D_{\Xi_c'^0[\frac{3}{2}^-]\to\Sigma_c^+K^-}(\omega,\omega') = \frac{g_{\Xi_c'^0[\frac{3}{2}^-]\to\Sigma_c^+K^-} f_{\Xi_c'^0[\frac{3}{2}^-]} f_{\Sigma_c^+}}{(\bar{\Lambda}_{\Xi_c'^0[\frac{3}{2}^-]} - \omega')(\bar{\Lambda}_{\Sigma_c^+} - \omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(\frac{if_K u}{4\pi^2 t^3} \phi_{2;K}(u) + \frac{if_K u}{64\pi^2 t} \phi_{4;K}(u) - \frac{if_K m_K^2 u t}{144(m_u+m_s)} \langle \bar{q}q\rangle \phi^\sigma_{3;K}(u) \right.$$
$$\left. - \frac{if_K m_K^2 u t^3}{2304(m_u+m_s)} \langle g_s \bar{q}\sigma G q\rangle \phi^\sigma_{3;K}(u) \right)$$
$$- \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \left(\frac{if_K \alpha_3 u^2}{8\pi^2 t} \Phi_{4;K}(\underline{\alpha}) + \frac{if_K \alpha_2 u}{8\pi^2 t} \Phi_{4;K}(\underline{\alpha}) \right.$$
$$+ \frac{if_K \alpha_3 u}{16\pi^2 t} \Phi_{4;K}(\underline{\alpha}) + \frac{if_K \alpha_3 u}{16\pi^2 t} \tilde{\Phi}_{4;K}(\underline{\alpha}) - \frac{if_K u}{8\pi^2 t} \Phi_{4;K}(\underline{\alpha}) + \frac{if_K \alpha_2}{16\pi^2 t} \Phi_{4;K}(\underline{\alpha}) + \frac{if_K \alpha_2}{16\pi^2 t} \tilde{\Phi}_{4;K}(\underline{\alpha})$$
$$- \frac{if_K}{16\pi^2 t} \Phi_{4;K}(\underline{\alpha}) - \frac{if_K}{16\pi^2 t} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u}{8\pi^2 t^2 v\cdot q} \Psi_{4;K}(\underline{\alpha}) + \frac{3f_K u}{8\pi^2 t^2 v\cdot q} \tilde{\Psi}_{4;K}(\underline{\alpha}) - \frac{f_K}{8\pi^2 t^2 v\cdot q} \Phi_{4;K}(\underline{\alpha})$$
$$\left. - \frac{3f_K}{8\pi^2 t^2 v\cdot q} \tilde{\Phi}_{4;K}(\underline{\alpha}) + \frac{f_K}{8\pi^2 t^2 v\cdot q} \Psi_{4;K}(\underline{\alpha}) - \frac{f_K}{8\pi^2 t^2 v\cdot q} \tilde{\Psi}_{4;K}(\underline{\alpha}) \right), \tag{B11}$$

$$G^S_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') = \frac{g_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c'^0[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar{\Lambda}_{\Xi_c'^0[\frac{3}{2}^-]} - \omega')(\bar{\Lambda}_{\Xi_c^{*+}} - \omega)}$$
$$= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(\frac{f_\pi v\cdot q}{3\pi^2 t^4} \phi_{2;\pi}(u) - \frac{f_\pi v\cdot q}{48\pi^2 t^2} \phi_{4;\pi}(u) \right.$$
$$+ \frac{f_\pi m_\pi^2 v\cdot q}{108(m_u+m_d)} \langle \bar{s}s\rangle \phi^\sigma_{3;\pi}(u) + \frac{f_\pi m_\pi^2 t^2 v\cdot q}{1728(m_u+m_d)} \langle g_s \bar{s}\sigma G s\rangle \phi^\sigma_{3;\pi}(u) - \frac{f_\pi m_s v\cdot q}{72} \langle \bar{s}s\rangle \phi_{2;\pi}(u)$$
$$\left. - \frac{f_\pi m_s t^2 v\cdot q}{1152} \langle \bar{s}s\rangle \phi_{4;\pi}(u) + \frac{f_\pi m_s m_\pi^2 v\cdot q}{36\pi^2(m_u+m_d)t^2} \phi^\sigma_{3;\pi}(u) \right)$$
$$- \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \left(\frac{f_\pi u v\cdot q}{6\pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) - \frac{7f_\pi u v\cdot q}{72\pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) \right.$$
$$\left. + \frac{f_\pi u v\cdot q}{24\pi^2 t^2} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) + \frac{f_\pi v\cdot q}{72\pi^2 t^2} \Phi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi v\cdot q}{24\pi^2 t^2} \tilde{\Phi}_{4;\pi}(\underline{\alpha}) + \frac{5f_\pi v\cdot q}{72\pi^2 t^2} \Psi_{4;\pi}(\underline{\alpha}) - \frac{5f_\pi v\cdot q}{72\pi^2 t^2} \tilde{\Psi}_{4;\pi}(\underline{\alpha}) \right). \tag{B12}$$

The sum rule equations for the $\Xi_b'^-[\frac{3}{2}^-]$ belonging to $[\mathbf{6}_F, 2, 1, \lambda]$ are

$$\begin{aligned}
G^D_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^+\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c^+\pi^-} f_{\Xi_c'^0[\frac{3}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Xi_c'^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{f_\pi m_s u}{4\pi^2 t^2}\phi_{2;\pi}(u) + \frac{f_\pi m_\pi^2 u}{12(m_u+m_d)\pi^2 t^2}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{f_\pi m_s u}{64\pi^2}\phi_{4;\pi}(u) + \frac{f_\pi u}{12}\langle\bar s s\rangle\phi_{2;\pi}(u) + \frac{f_\pi m_s u t^2}{288(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u) + \frac{f_\pi u t^2}{192}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad + \frac{f_\pi u t^2}{192}\langle g_s\bar s\sigma G s\rangle\phi_{2;\pi}(u) + \frac{f_\pi u t^4}{3072}\langle g_s\bar s\sigma G s\rangle\phi_{4;\pi}(u)\bigg) \\
&\quad - \int_0^\infty dt\int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2}\times\bigg(\frac{f_{3\pi}u}{2\pi t^2}\Phi_{3;\pi}(\underline{\alpha}) - \frac{f_{3\pi}}{2\pi^2 t^2}\Phi_{3;\pi}(\underline{\alpha}) \\
&\quad + \frac{i f_{3\pi} u^2\alpha_3 v\cdot q}{2\pi^2 t}\Phi_{3;\pi}(\underline{\alpha}) + \frac{i f_{3\pi} u\alpha_2 v\cdot q}{2\pi^2 t}\Phi_{3;\pi}(\underline{\alpha}) - \frac{i f_{3\pi} u v\cdot q}{2\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})\bigg),
\end{aligned} \tag{B13}$$

$$\begin{aligned}
G^D_{\Xi_c'^0[\frac{3}{2}^-]\to\Lambda_c^+K^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{3}{2}^-]\to\Lambda_c^+K^-} f_{\Xi_c'^0[\frac{3}{2}^-]} f_{\Lambda_c^+}}{(\bar\Lambda_{\Xi_c'^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{f_K m_K^2 u}{12(m_u+m_s)\pi^2 t^2}\phi^\sigma_{3;K}(u) + \frac{f_K u}{12}\langle\bar q q\rangle\phi_{2;K}(u) \\
&\quad + \frac{f_K u t^2}{192}\langle\bar q q\rangle\phi_{4;K}(u) + \frac{f_K u t^2}{192}\langle g_s\bar q\sigma G q\rangle\phi_{2;K}(u) + \frac{f_K u t^4}{3072}\langle g_s\bar q\sigma G q\rangle\phi_{4;K}(u)\bigg) \\
&\quad - \int_0^\infty dt\int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2}\times\bigg(\frac{f_{3K}u}{2\pi t^2}\Phi_{3;K}(\underline{\alpha}) - \frac{f_{3K}}{2\pi^2 t^2}\Phi_{3;K}(\underline{\alpha}) \\
&\quad + \frac{i f_{3K} u^2\alpha_3 v\cdot q}{2\pi^2 t}\Phi_{3;K}(\underline{\alpha}) + \frac{i f_{3K} u\alpha_2 v\cdot q}{2\pi^2 t}\Phi_{3;K}(\underline{\alpha}) - \frac{i f_{3K} u v\cdot q}{2\pi^2 t}\Phi_{3;K}(\underline{\alpha})\bigg),
\end{aligned} \tag{B14}$$

$$\begin{aligned}
G^D_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c'^+\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{3}{2}^-]\to\Xi_c'^+\pi^-} f_{\Xi_c'^0[\frac{3}{2}^-]} f_{\Xi_c'^+}}{(\bar\Lambda_{\Xi_c'^0[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c'^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{3 i f_\pi u}{4\pi^2 t^3}\phi_{2;\pi}(u) - \frac{i f_\pi m_\pi^2 m_s u}{16(m_u+m_d)\pi^2 t}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{3 i f_\pi u}{64\pi^2 t}\phi_{4;\pi}(u) + \frac{i f_\pi m_s u t}{32}\langle\bar s s\rangle\phi_{2;\pi}(u) - \frac{i f_\pi m_\pi^2 u t}{48(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u) + \frac{i f_\pi m_s u t^3}{512}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad - \frac{i f_\pi m_\pi^2 u t^3}{768(m_u+m_d)}\langle g_s\bar s\sigma G s\rangle\phi^{\mathrm{sigma}}_{3;\pi}(u)\bigg) \\
&\quad - \int_0^\infty dt\int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2}\times\bigg(\frac{3 f_\pi u}{8\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) - \frac{3 f_\pi u}{8\pi^2 t62 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{3 f_\pi}{8\pi^2 t^2 v\cdot q}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 f_\pi}{8\pi^2 t^2 v\cdot q}\tilde\Phi_{4;\pi}(\underline{\alpha}) - \frac{3 f_\pi}{8\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) + \frac{3 f_\pi}{8\pi^2 t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad + \frac{3 i f_\pi \alpha_3 u^2}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi \alpha_2 u}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi \alpha_3 u}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi \alpha_3 u}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{3 i f_\pi u}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi \alpha_2}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi \alpha_2}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha}) - \frac{3 i f_\pi}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{3 i f_\pi}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha})\bigg),
\end{aligned} \tag{B15}$$

$$
\begin{aligned}
G^D_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^+K^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Sigma_c^+K^-} f_{\Xi_c^{\prime 0}[\frac{3}{2}^-]} f_{\Sigma_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(\frac{3if_K u}{4\pi^2 t^3}\phi_{2;K}(u) + \frac{3if_K u}{64\pi^2 t}\phi_{4;K}(u) - \frac{if_K m_K^2 ut}{48(m_u+m_s)}\langle\bar q q\rangle \phi_{3;K}^\sigma(u)\right. \\
&\quad \left. - \frac{if_K m_K^2 ut^3}{768(m_u+m_s)}\langle g_s\bar q\sigma G q\rangle \phi_{3;K}^{\mathrm{sigma}}(u)\right) \\
&\quad - \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \left(\frac{3f_K u}{8\pi^2 t^2 v\cdot q}\Psi_{4;K}(\underline{\alpha}) - \frac{3f_K u}{8\pi^2 t62 v\cdot q}\tilde\Psi_{4;K}(\underline{\alpha})\right. \\
&\quad - \frac{3f_K}{8\pi^2 t^2 v\cdot q}\Phi_{4;K}(\underline{\alpha}) + \frac{3f_K}{8\pi^2 t^2 v\cdot q}\tilde\Phi_{4;K}(\underline{\alpha}) - \frac{3f_K}{8\pi^2 t^2 v\cdot q}\Psi_{4;K}(\underline{\alpha}) + \frac{3f_K}{8\pi^2 t^2 v\cdot q}\tilde\Psi_{4;K}(\underline{\alpha}) \\
&\quad + \frac{3if_K\alpha_3 u^2}{8\pi^2 t}\Phi_{4;K}(\underline{\alpha}) + frac3if_K\alpha_2 u 8\pi^2 t\Phi_{4;K}(\underline{\alpha}) + \frac{3if_K\alpha_3 u}{16\pi^2 t}\Phi_{4;K}(\underline{\alpha}) + \frac{3if_K\alpha_3 u}{16\pi^2 t}\tilde\Phi_{4;K}(\underline{\alpha}) \\
&\quad \left. - \frac{3if_K u}{8\pi^2 t}\Phi_{4;K}(\underline{\alpha}) + \frac{3if_K\alpha_2}{16\pi^2 t}\Phi_{4;K}(\underline{\alpha}) + \frac{3if_K\alpha_2}{16\pi^2 t}\tilde\Phi_{4;K}(\underline{\alpha}) - \frac{3if_K}{16\pi^2 t}\Phi_{4;K}(\underline{\alpha}) - \frac{3if_K}{16\pi^2 t}\tilde\Phi_{4;K}(\underline{\alpha})\right), \qquad \text{(B16)}
\end{aligned}
$$

$$
\begin{aligned}
G^D_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c^{\prime 0}[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(-\frac{if_\pi u}{4\pi^2 t^3}\phi_{2;\pi}(u) + \frac{if_\pi m_s m_\pi^2 u}{48(m_u+m_d)\pi^2 t}\phi_{3;\pi}^\sigma(u)\right. \\
&\quad - \frac{if_\pi u}{64\pi^2 t}\phi_{4;\pi}(u) - \frac{if_\pi m_s ut}{96}\langle\bar s s\rangle\phi_{2;\pi}(u) + \frac{if_\pi m_\pi^2 ut}{144(m_u+m_d)}\langle\bar s s\rangle\phi_{3;\pi}^\sigma(u) - \frac{if_\pi m_s ut^3}{1536}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad \left. + \frac{if_\pi m_\pi^2 ut^3}{2304(m_u+m_d)}\langle g_s\bar s\sigma G s\rangle\phi_{3;\pi}^\sigma(u)\right) \\
&\quad - \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \left(-\frac{f_\pi u}{8\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi u}{8\pi^2 t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha})\right. \\
&\quad + \frac{f_\pi}{8\pi^2 t^2 v\cdot q}\Phi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi}{8\pi^2 t^2 v\cdot q}\tilde\Phi_{4;\pi}(\underline{\alpha}) + \frac{f_\pi}{8\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) - \frac{f_\pi}{8\pi^2 t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{if_\pi\alpha_3 u^2}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi\alpha_2 u}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi\alpha_3 u}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi\alpha_3 u}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha}) \\
&\quad \left. + \frac{if_\pi u}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi\alpha_2}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi\alpha_2}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha}) + \frac{if_\pi}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{if_\pi}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha})\right), \qquad \text{(B17)}
\end{aligned}
$$

$$
\begin{aligned}
G^S_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{3}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c^{\prime 0}[\frac{3}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= -\int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \left(-\frac{uv\cdot q}{24\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) + \frac{uv\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha})\right. \\
&\quad \left. + \frac{v\cdot q}{24\pi^2 t^2}\Phi_{4;\pi}(\underline{\alpha}) - \frac{v\cdot q}{24\pi^2 t^2}\tilde\Phi_{4;\pi}(\underline{\alpha}) + \frac{v\cdot q}{24\pi^2 t^2}\Psi_{4;\pi}(\underline{\alpha}) - \frac{v\cdot q}{24\pi^2 t^2}\tilde\Psi_{4;\pi}(\underline{\alpha})\right). \qquad \text{(B18)}
\end{aligned}
$$

The sum rule equations for the $\Xi_b^{\prime -}[\frac{5}{2}^-]$ belonging to $[\mathbf{6}_F, 2, 1, \lambda]$ are

$$
\begin{aligned}
G^D_{\Xi_c^{\prime 0}[\frac{5}{2}^-]\to\Xi_c^+\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{5}{2}^-]\to\Xi_c^+\pi^-} f_{\Xi_c^{\prime 0}[\frac{5}{2}^-]} f_{\Xi_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{5}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{3 f_\pi m_s u}{20\pi^2 t^2}\phi_{2;\pi}(u) + \frac{f_\pi m_\pi^2 u}{20(m_u+m_d)\pi^2}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{3 f_\pi m_s u}{320\pi^2}\phi_{4;\pi}(u) + \frac{f_\pi u}{20}\langle\bar s s\rangle\phi_{2;\pi}(u) + \frac{f_\pi m_s m_\pi^2 u t^2}{480(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u) + \frac{f_\pi u t^2}{320}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad + \frac{f_\pi u t^2}{320}\langle g_s\bar s\sigma G s\rangle\phi_{2;\pi}(u) + \frac{f_\pi u t^4}{5120}\langle g_s\bar s\sigma G s\rangle\phi_{4;\pi}(u)\bigg) \\
&\quad - \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \bigg(\frac{3 f_{3\pi} u}{10\pi^2 t^2}\Phi_{3;\pi}(\underline{\alpha}) - \frac{3 f_{3\pi}}{10\pi^2 t^2}\Phi_{3;\pi}(\underline{\alpha}) \\
&= \frac{3 i f_{3\pi}\alpha_3 u^2 v\cdot q}{10\pi^2 t}\Phi_{3;\pi}(\underline{\alpha}) + \frac{3 i f_{3\pi}\alpha_2 u v\cdot q}{10\pi^2 t}\Phi_{3;\pi}(\underline{\alpha}) - \frac{3 i f_{3\pi} u v\cdot q}{10\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})\bigg),
\end{aligned} \tag{B19}
$$

$$
\begin{aligned}
G^D_{\Xi_c^{\prime 0}[\frac{5}{2}^-]\to\Lambda_c^+ K^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{5}{2}^-]\to\Lambda_c^+ K^-} f_{\Xi_c^{\prime 0}[\frac{5}{2}^-]} f_{\Lambda_c^+}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{5}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_c^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{f_K m_K^2 u}{20(m_u+m_s)\pi^2}\phi^\sigma_{3;K}(u) \\
&\quad + \frac{f_K u}{20}\langle\bar q q\rangle\phi_{2;K}(u) + \frac{f_K u t^2}{320}\langle\bar q q\rangle\phi_{4;K}(u) \\
&\quad + \frac{f_K u t^2}{320}\langle g_s\bar q\sigma G q\rangle\phi_{2;K}(u) + \frac{f_K u t^4}{5120}\langle g_s\bar q\sigma G q\rangle\phi_{4;K}(u)\bigg) \\
&\quad - \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \bigg(\frac{3 f_{3K} u}{10\pi^2 t^2}\Phi_{3;K}(\underline{\alpha}) - \frac{3 f_{3K}}{10\pi^2 t^2}\Phi_{3;K}(\underline{\alpha}) \\
&= \frac{3 i f_{3K}\alpha_3 u^2 v\cdot q}{10\pi^2 t}\Phi_{3;K}(\underline{\alpha}) + \frac{3 i f_{3K}\alpha_2 u v\cdot q}{10\pi^2 t}\Phi_{3;K}(\underline{\alpha}) - \frac{3 i f_{3K} u v\cdot q}{10\pi^2 t}\Phi_{3;K}(\underline{\alpha})\bigg),
\end{aligned} \tag{B20}
$$

$$
\begin{aligned}
G^D_{\Xi_c^{\prime 0}[\frac{5}{2}^-]\to\Xi_c^{\prime +}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c^{\prime 0}[\frac{5}{2}^-]\to\Xi_c^{\prime +}\pi^-} f_{\Xi_c^{\prime 0}[\frac{5}{2}^-]} f_{\Xi_c^{\prime +}}}{(\bar\Lambda_{\Xi_c^{\prime 0}[\frac{5}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{\prime +}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \bigg(\frac{3 i f_\pi u}{10\pi^2 t^3}\phi_{2;\pi}(u) - \frac{i f_\pi m_s m_\pi^2 u}{40(m_u+m_d)\pi^2 t}\phi^\sigma_{3;\pi}(u) \\
&\quad + \frac{3 i f_\pi u}{160\pi^2 t}\phi_{4;\pi}(u) + \frac{i f_\pi m_s u t}{80}\langle\bar s s\rangle\phi_{2;\pi}(u) - \frac{i f_\pi m_\pi^2 u t}{120(m_u+m_d)}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u) + \frac{i f_\pi m_s u t^3}{1280}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad - \frac{i f_\pi m_\pi^2 u t^3}{1920}\langle g_s\bar s\sigma G s\rangle\phi^\sigma_{3;\pi}(u)\bigg) \\
&\quad - \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2} \times \bigg(\frac{3 f_\pi u}{20\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) - \frac{3 f_\pi u}{20\pi t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{3 f_\pi}{20\pi^2 t^2 v\cdot q}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 f_\pi}{20\pi^2 t^2 v\cdot q}\tilde\Phi_{4;\pi}(\underline{\alpha}) - \frac{3 f_\pi}{20\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) + \frac{3 f_\pi}{20\pi^2 t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha}) \\
&\quad + \frac{3 i f_\pi\alpha_3 u^2}{20\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi\alpha_2 u}{20\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi\alpha_3 u}{40\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi\alpha_3 u}{40\pi^2}\tilde\Phi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{3 i f_\pi u}{20\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi\alpha_2}{40\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{3 i f_\pi\alpha_2}{40\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha}) - \frac{3 i f_\pi}{40\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{3 i f_\pi}{40\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha})\bigg),
\end{aligned} \tag{B21}
$$

$$
\begin{aligned}
G^D_{\Xi_c'^0[\frac{5}{2}^-]\to\Sigma_c'^+K^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{5}{2}^-]\to\Sigma_c'^+K^-} f_{\Xi_c'^0[\frac{5}{2}^-]} f_{\Sigma_c'^+}}{(\bar\Lambda_{\Xi_c'^0[\frac{5}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c'^+}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(\frac{3if_K u}{10\pi^2 t^3}\phi_{2;K}(u) + \frac{3if_K u}{160\pi^2 t}\phi_{4;K}(u) - \frac{if_K m_K^2 ut}{120(m_u+m_s)}\langle\bar q q\rangle \phi^\sigma_{3;K}(u) \right. \\
&\quad \left. - \frac{if_K m_K^2 ut^3}{1920}\langle g_s\bar q\sigma G q\rangle\phi^\sigma_{3;K}(u)\right) \\
&\quad - \int_0^\infty dt\int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \frac{1}{2}\times \left(\frac{3f_K u}{20\pi^2 t^2 v\cdot q}\Psi_{4;K}(\underline{\alpha}) - \frac{3f_K u}{20\pi t^2 v\cdot q}\tilde\Psi_{4;K}(\underline{\alpha}) \right. \\
&\quad - \frac{3f_K}{20\pi^2t^2 v\cdot q}\Phi_{4;K}(\underline{\alpha}) + \frac{3f_K}{20\pi^2t^2 v\cdot q}\tilde\Phi_{4;K}(\underline{\alpha}) - \frac{3f_K}{20\pi^2t^2 v\cdot q}\Psi_{4;K}(\underline{\alpha}) + \frac{3f_K}{20\pi^2t^2 v\cdot q}\tilde\Psi_{4;K}(\underline{\alpha}) \\
&\quad + \frac{3if_K\alpha_3u^2}{20\pi^2 t}\Phi_{4;K}(\underline{\alpha}) + \frac{3if_K\alpha_2 u}{20\pi^2 t}\Phi_{4;K}(\underline{\alpha}) + \frac{3if_K\alpha_3 u}{40\pi^2 t}\Phi_{4;K}(\underline{\alpha}) + \frac{3if_K\alpha_3 u}{40\pi^2}\tilde\Phi_{4;K}(\underline{\alpha}) \\
&\quad \left. - \frac{3if_K u}{20\pi^2 t}\Phi_{4;K}(\underline{\alpha}) + \frac{3if_K\alpha_2}{40\pi^2 t}\Phi_{4;K}(\underline{\alpha}) + \frac{3if_K\alpha_2}{40\pi^2 t}\tilde\Phi_{4;K}(\underline{\alpha}) - \frac{3if_K}{40\pi^2 t}\Phi_{4;K}(\underline{\alpha}) - \frac{3if_K}{40\pi^2 t}\tilde\Phi_{4;K}(\underline{\alpha})\right), \qquad \text{(B22)}
\end{aligned}
$$

$$
\begin{aligned}
G^D_{\Xi_c'^0[\frac{5}{2}^-]\to\Xi_c^{*+}\pi^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{5}{2}^-]\to\Xi_c^{*+}\pi^-} f_{\Xi_c'^0[\frac{5}{2}^-]} f_{\Xi_c^{*+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{5}{2}^-]}-\omega')(\bar\Lambda_{\Xi_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(\frac{4if_\pi u}{15\pi^2 t^3}\phi_{2;\pi}(u) + \frac{if_\pi m_s m_\pi^2 u}{45(m_u+m_d)\pi^2 t}\phi^\sigma_{3;\pi}(u) - \frac{if_\pi u}{60\pi^2 t}\phi_{4;\pi}(u) \right. \\
&\quad - \frac{if_\pi m_s ut}{90}\langle\bar s s\rangle\phi_{2;\pi}(u) + \frac{if_\pi m_\pi^2 ut}{135}\langle\bar s s\rangle\phi^\sigma_{3;\pi}(u) - \frac{if_\pi m_s ut^3}{1440}\langle\bar s s\rangle\phi_{4;\pi}(u) \\
&\quad \left. + \frac{if_\pi m_\pi^2 ut^3}{2160}\langle g_s\bar s\sigma G s\rangle\phi^\sigma_{3;\pi}(u)\right) - \int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times\frac{1}{2} \\
&\quad \times\left(-\frac{2f_\pi u}{15\pi^2t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) + \frac{2f_\pi u}{15\pi^2t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha}) + \frac{2f_\pi}{15\pi^2t^2 v\cdot q}\Phi_{4;\pi}(\underline{\alpha}) - \frac{2f_\pi}{15\pi^2t^2 v\cdot q}\tilde\Phi_{4;\pi}(\underline{\alpha}) \right. \\
&\quad + \frac{2f_\pi}{15\pi^2t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha}) - \frac{2f_\pi}{15\pi^2t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi\alpha_3u^2}{15\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{2f_\pi\alpha_2 u}{15\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) \\
&\quad - \frac{if_\pi\alpha_3 u}{15\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi\alpha_3 u}{15\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha}) + \frac{2if_\pi u}{15\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi\alpha_2}{15\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) - \frac{if_\pi\alpha_2}{15\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha}) \\
&\quad \left. + \frac{if_\pi}{15\pi^2 t}\Phi_{4;\pi}(\underline{\alpha}) + \frac{if_\pi}{15\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha})\right), \qquad \text{(B23)}
\end{aligned}
$$

$$
\begin{aligned}
G^D_{\Xi_c'^0[\frac{5}{2}^-]\to\Sigma_c^{*+}K^-}(\omega,\omega') &= \frac{g_{\Xi_c'^0[\frac{5}{2}^-]\to\Sigma_c^{*+}K^-} f_{\Xi_c'^0[\frac{5}{2}^-]} f_{\Sigma_c^{*+}}}{(\bar\Lambda_{\Xi_c'^0[\frac{5}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_c^{*+}}-\omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(\frac{4if_K u}{15\pi^2 t^3}\phi_{2;K}(u) - \frac{if_K u}{60\pi^2 t}\phi_{4;K}(u) + \frac{if_K m_\pi^2 ut}{135}\langle\bar q q\rangle\phi^\sigma_{3;K}(u) \right. \\
&\quad \left. + \frac{if_K m_K^2 ut^3}{2160}\langle g_s\bar q\sigma G q\rangle\phi^\sigma_{3;K}(u)\right) - \int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times\frac{1}{2} \\
&\quad \times\left(-\frac{2f_K u}{15\pi^2t^2 v\cdot q}\Psi_{4;K}(\underline{\alpha}) + \frac{2f_K u}{15\pi^2t^2 v\cdot q}\tilde\Psi_{4;K}(\underline{\alpha}) + \frac{2f_K}{15\pi^2t^2 v\cdot q}\Phi_{4;K}(\underline{\alpha}) - \frac{2f_K}{15\pi^2t^2 v\cdot q}\tilde\Phi_{4;K}(\underline{\alpha})\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{2f_K}{15\pi^2t^2v\cdot q}\Psi_{4;K}(\underline{\alpha})-\frac{2f_K}{15\pi^2t^2v\cdot q}\tilde{\Psi}_{4;K}(\underline{\alpha})-\frac{if_K\alpha_3u^2}{15\pi^2t}\Phi_{4;K}(\underline{\alpha})-\frac{2f_K\alpha_2u}{15\pi^2t}\Phi_{4;K}(\underline{\alpha})\\
&-\frac{if_K\alpha_3u}{15\pi^2t}\Phi_{4;K}(\underline{\alpha})-\frac{if_K\alpha_3u}{15\pi^2t}\tilde{\Phi}_{4;K}(\underline{\alpha})+\frac{2if_Ku}{15\pi^2t}\Phi_{4;K}(\underline{\alpha})-\frac{if_K\alpha_2}{15\pi^2t}\Phi_{4;K}(\underline{\alpha})-\frac{if_K\alpha_2}{15\pi^2t}\tilde{\Phi}_{4;K}(\underline{\alpha})\\
&+\frac{if_K}{15\pi^2t}\Phi_{4;K}(\underline{\alpha})+\frac{if_K}{15\pi^2t}\tilde{\Phi}_{4;K}(\underline{\alpha})\bigg). \qquad \text{(B24)}
\end{aligned}
$$

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