$B \to K^*\gamma$ decay on APE *

As.Abada[a] , Ph.Boucaud[a], M.Crisafulli[b] , J.P.Leroy[a], V.Lubicz[b], G.Martinelli[b,c] ,
F. Rapuano[b,c], M. Serone[b], N.Stella[b],
and
A Bartoloni[b], C. Battista[b], S. Cabasino[b], N. Cabibbo[d] , E. Panizzi[b], P.S. Paolucci[b], R. Sarno[b],
G.M. Todesco[b], M. Torelli[b], P. Vicini[b].
The APE Collaboration

[a] LPTHE, Université Paris-XI, 91405 Orsay, France†

[b] Dip. di Fisica, Univ. di Roma 'La Sapienza'

[c] Theory Division, CERN, 1211 GENEVA 23, Switzerland.

[d] Dip. di Fisica, Univ. di Roma 'Tor Vergata' and INFN , Sezione di Roma II,
Via della Ricerca Scientifica 1, I-00133 Roma,Italy .

We present results for the radiative decay $B \to K^*\gamma$, obtained by using the Clover action at $\beta = 6.0$ on APE. The compatibility between the scaling laws predicted by the Heavy Quark Effective Theory (HQET) and pole dominance is discussed. The final result depends crucially on the assumed q^2-dependence of the form factors.

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1. Introduction

The hadronic matrix element which governs the radiative decay $B \to K^*\gamma$ is parametrised in terms of three form factors:

$$\langle K^*_r(\eta,k)|J_\mu|B(p)\rangle = C_1^\mu T_1(q^2) + iC_2^\mu T_2(q^2) + iC_3^\mu T_3(q^2), \quad (1)$$

where

$$\begin{aligned} C_1^\mu &= 2\epsilon^{\mu\alpha\rho\sigma}\eta_r(k)_\alpha p_\rho k_\sigma, \\ C_2^\mu &= \eta_r(k)(M_B^2 - M_{K^*}^2) - (\eta_r(k).q)(p+k)^\mu, \\ C_3^\mu &= \eta_r(k).q\big(q^\mu - \frac{q^2}{M_B^2 - M_{K^*}^2}(p+k)^\mu\big). \quad (2) \end{aligned}$$

and $J_\mu = \bar{s}\sigma_{\mu\nu}\frac{1+\gamma_5}{2}q^\nu b$; η is the polarization vector of the K^* and q the momentum transfer. When the emitted photon is real, T_3 does not contribute to the physical rate and $T_1(0) = T_2(0)$. At $q^2 = 0$, the physics of this decay is thus described by only one form factor, T_1. The feasibility of the lattice approach has been demonstrated first by the work of Bernard et al. [1].

2. Scaling laws and q^2 dependence of the form factors

In order to obtain the form factors at the physical point, we need to extrapolate both to large meson masses and small values of q^2. The final results critically depend on the assumptions made on the q^2- and heavy mass-dependence. At fixed $\vec{p}_{K^*}$, with $|\vec{p}_{K^*}| \ll M_B$ in the B-meson rest frame, the following scaling laws can be derived [2]:

$$\begin{aligned} \frac{T_1}{\sqrt{M_B}} &= \gamma_1 \times \left(1 + \frac{\delta_1}{M_B} + \dots\right) \\ T_2\sqrt{M_B} &= \gamma_2 \times \left(1 + \frac{\delta_2}{M_B} + \dots\right) \quad (3) \end{aligned}$$

which are valid up to logarithmic corrections. On the other hand, "scaling" laws for the form factors at $q^2 = 0$ can only be found by using extra assumptions for their q^2 dependence. This procedure is acceptable, provided the "scaling" laws derived in this way respect the exact condition $T_1(0) = T_2(0)$. This is a non-trivial constraint: the q^2 behaviour of T_1 and T_2 has to compensate for the different mass dependence of the two form factors near the zero recoil point given in eq.(3). For example, the popular assumption of pole dominance for both T_1 and T_2 would give

*Talk presented by Ph. Boucaud
† Laboratoire associé au CNRS

that $T_1(0) \sim M_B^{-1/2}$ and $T_2(0) \sim M_B^{-3/2}$, which is inconsistent.

The assumptions on the q^2-dependence of the form factors can be tested directly on the numerical results, although only in a small domain of momenta.

3. Lattice set-up

The numerical simulation was performed on the 6.4 Gigaflops version of the APE machine, at $\beta = 6.0$, on a $18^3 \times 64$ lattice, using the SW-Clover action [3] in the quenched approximation. The results have been obtained from a sample of 170 gauge configurations and the statistical errors estimated by a jackknife procedure with a decimation of 10 configurations from the total set. For each configuration we have computed the quark propagators for seven values of the Wilson hopping parameter K_W, corresponding to "heavy" quarks, $K_H = 0.1150$, 0.1200, 0.1250, 0.1330, and "light" quarks, $K_L = 0.1425$, 0.1432 and 0.1440. Due to memory limitations, the propagators are "thinned". The matrix elements have been computed for an initial meson at rest and a final vector meson with momentum $\vec{p}_{K^*}$. We have taken $\vec{p}_{K^*} = 2\pi/(La)\ (0,0,0)$, $(1,0,0)$, $(1,1,0)$, $(1,1,1)$, and $(2,0,0)$, where L is the spatial extension of the lattice. The initial (final) meson was created (annihilated) by using a pseudoscalar (local vector) density inserted at a time $t_B/a = 28$ ($t_{K^*} = 0$), and we have varied the time position of the current in the interval $t_J/a = 10-14$. Two procedures, denoted by "ratio" and "analytic" in the tables, have been used to extract the plateaux: the three point functions are divided either by the numerical two point functions ("ratio") or by an analytical expression ("analytic"). Details can be found in ref [4].

4. Results

From our data, if we assume a pole dominance behaviour for T_2, the mass extracted from the fits with the pole mass as a free parameter is larger than the mass obtained from the axial two-point correlation functions. As a consequence $T_2(q^2)$ is flatter than predicted by pole dominance (see fig. 1). The values found in this way at $q^2 = 0$ are reported in table 1 as $T_2^{free}(0)$.

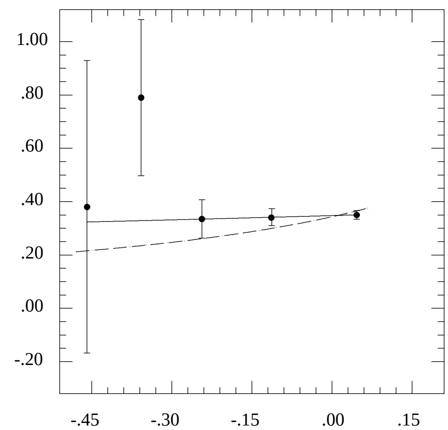

Figure 1. $T_2(q^2)$ as a function of q^2 for $K_H = .1330$. The curves show the pole dominance with either the lattice axial pole mass (dashed line) or with a free pole mass (full line).

In the case of T_1, the absence of data at q^2_{max} and the large errors in the data at high momenta make it difficult to test directly the validity of the pole dominance hypothesis. We can use the value for $T_1(q^2 = 0)$ obtained from the the condition $T_1(0) = T_2^{free}(0)$, together with the point at $\vec{p}_{K^*} = 2\pi/(La)\ (1,0,0)$ in a fit of T_1 to a pole dominance behaviour; the pole mass determined along this way for T_1 is compatible, though with large errors, with the mass of the corresponding lattice vector meson. More data (i.e. with a moving B meson) are needed to test this point more accurately. As a matter of comparison, we give in table 1 the values for $T_1^{pole}(q^2 = 0)$ ($T_2^{pole}(q^2 = 0)$) obtained under the assumption of a pole dominance with the lattice vector (axial) meson mass. Although the quality of our data is not accurate enough to draw a definite conclusion, they suggest that assuming T_2 flatter than pole dominance and T_1 following pole dominance gives a good description of our data. We call this option $m^{-1/2}$-scaling. In fig. 1, the curve corresponding to the pole dominance for T_2 is also given ($m^{-3/2}$-scaling); this scaling law would follow from a dipolar q^2-dependence for T_1. We take the two possibilities, $m^{-1/2}$- and $m^{-3/2}$-

scaling, as representatives of a whole class of possible "scaling" laws.

Table 1
Form factors at $q^2 = 0$ extrapolated to the strange quark, assuming independence on the spectator quark. "ratio" and "analytic" are explained in the text. T_1^{pole} and T_2^{pole} are computed with the appropriate lattice meson mass for the pole dominance, T_2^{free} with the pole mass as a free parameter.

	ratio	analytic
$T_1^{pole}(0)\kappa_h = .1150$	.286(35)	.297(34)
$T_2^{free}(0)\kappa_h = .1150$	.280(52)	.301(56)
$T_2^{pole}(0)\kappa_h = .1150$	.238(17)	.242(17)
$T_1^{pole}(0)\kappa_h = .1200$	.298(33)	.309(37)
$T_2^{free}(0)\kappa_h = .1200$	.293(40)	.309(40)
$T_2^{pole}(0)\kappa_h = .1200$	.262(16)	.265(17)
$T_1^{pole}(0)\kappa_h = .1250$	.311(32)	.322(31)
$T_2^{free}(0)\kappa_h = .1250$	.310(30)	.320(28)
$T_2^{pole}(0)\kappa_h = .1250$	.288(17)	.292(17)
$T_1^{pole}(0)\kappa_h = .1330$	.331(31)	.339(30)
$T_2^{free}(0)\kappa_h = .1330$	.345(19)	.348(18)
$T_2^{pole}(0)\kappa_h = .1330$	.340(19)	.343(19)

The extrapolation to the physical region (i.e. the B mass) is performed following these two hypothesis, $m^{-1/2}$- and $m^{-3/2}$-scaling with linear and quadratic fits. The results are presented in table 2. We give also δ_1, the coefficient of the $1/M$ corrections in the linear fit. It should be noted that in the $m^{-1/2}$-scaling case, the $1/M$ corrections are smaller and the extrapolated value is less affected by the adjunction of a quadratic term than in the $m^{-3/2}$-scaling hypothesis.

As a consistency check, we can first extrapolate T_1 to the B, at (small) fixed momentum, following the scaling law of eq. 3. We have used $\vec{p}_{K^*} = 2\pi/(La)\ (1,0,0)$. Then from a pole dominance with the physical vector meson mass ($M_V \sim 5.4$ GeV) we get for the value of $T_1(0)$ the results 0.192(44) and .200(44) for the "ratio" and "analytic" methods respectively, in agreement with the results in table 2 ($m^{-1/2}$-scaling). The same game can be played with $T_2(q^2_{max})$; the

Table 2
The form factors at $q^2 = 0$ extrapolated to the physical B mass. δ_1 is the coefficient of the $1/M$ correction.

	ratio	analytic
$T_1^{pole}(0)$ fit $m^{-\frac{1}{2}}$ lin.	.203(28)	.213(27)
$T_1^{pole}(0)$ fit $m^{-\frac{1}{2}}$ quad.	.191(40)	.200(40)
δ_1 (MeV)	310(109)	339(97)
$T_1^{pole}(0)$ fit $m^{-\frac{3}{2}}$ lin.	.102(11)	.106(12)
$T_1^{pole}(0)$ fit $m^{-\frac{3}{2}}$ quad.	.135(20)	.140(21)
δ_1(MeV)	871(34)	879(34)
$T_2^{pole}(0)$ fit $m^{-\frac{3}{2}}$ lin.	.082(7)	.083(7)
$T_2^{pole}(0)$ fit $m^{-\frac{3}{2}}$ quad.	.091(12)	.092(13)
δ_1(MeV)	735(51)	737(54)

results in this case are $T_2(q^2_{max}, B) = .217(15)$ and $T_2(q^2 = 0, B) = .090(6)$ for a value of $M_A \sim 5.7$ GeV for the axial pole mass; this is in agreement with the results in table 2 for the $m^{-3/2}$-scaling.

From table 2, we quote:

$$T_1^{pole}(0) = .196(45) \quad (m^{-1/2} \;\; scaling)$$
$$T_2^{pole}(0) = .090(15) \quad (m^{-3/2} \;\; scaling)$$

Clearly, the final result depends crucially on the assumption made for the q^2-dependence. Given the statistical errors, the systematic uncertainty in the extraction of the form factors, the effects of $O(a)$ terms and the limited range in q^2 and masses, the study of the q^2- and mass-dependence of the form factors, remains a crucial challenge for lattice calculations.

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