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# SIGNATURES OF SUPERSYMMETRY IN B PHYSICS [1]

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ABSTRACT

We discuss $B^0$-$\bar{B}^0$ mixing and radiative $B$-meson decay within the framework of the supersymmetric standard model (SSM). In sizable ranges of SSM parameters, $B^0$-$\bar{B}^0$ mixing receives large new contributions, leading to a value, for the $CP$-violating phase of the Cabibbo-Kobayashi-Maskawa matrix, different from its value in the standard model (SM). This difference may be detected by measuring $CP$-asymmetries in $B^0$-meson decays or amount of $B_s^0$-$\bar{B}_s^0$ mixing. The radiative $B$-meson decay also receives new contributions from the SSM, which could make its branching ratio deviate from the SM prediction.

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## 1. Introduction

The supersymmetric standard model (SSM) is a plausible extension of the standard model (SM). This model can be directly tested by searching for supersymmetric particles in their production and decay processes. Another way of testing the SSM is to examine its possible effects at loop level. In particular, a big opportunity could be provided by processes of flavor chaging neutral current (FCNC) in the $B$-meson system, for which precise experimental measurements have been achieved and their further improvements are expected in the near future. Observation of discrepancies within SM predictions may turn out to indicate the validity of the SSM.

In this article we discuss SSM contributions to $B^0$-$\bar{B}^0$ mixing [1] and radiative $B$-meson decay [2], searching for signatures of supersummetry. It is shown that SSM predictions are deviated from SM ones in sizable ranges of SSM parameters. For definiteness, we assume the framework of grand unification theories coupled to $N = 1$ supergravity [3].

In the SSM there are several new sources which can induce FCNC processes, in addition to the standard $W$-boson interactions. These are the interactions in which a quark $q$ couples to a squark $\tilde{q}$ of a different generation and a chargino $\tilde{\omega}$, a neutralino $\tilde{\chi}$, or a gluino $\tilde{g}$ [4]. Since the SSM contains two doublets of Higgs bosons, charged Higgs bosons $H^{\pm}$ also mediate FCNC processes [5].

Among these new sources, however, sizable new contributions to FCNC processes in the $B$-meson system can only be expected from the interactions mediated by the charginos and the charged Higgs bosons [1,2,6]. The reasons are as follows: Since the $t$-quark has a large mass, the coupling strengths of the related Yukawa interactions for the chargino, the $t$-squark, and the down-type quark and for the charged Higgs boson, the $t$-quark, and the down-type quark are comparable to that of the SU(2) gauge interaction. Consequently, the chargino interaction strengths for the $t$-squarks are made different from those for the $u$- or $c$-squarks which are determined by the gauge interaction alone. Besides, if the squark masses of the first two generations are not much different from the $t$-quark mass, one of the $t$-squarks can be lighter than the other squarks. These effects soften the cancellation among different squark contributions for the chargino loop diagrams, which otherwise is rather severe. In the loop diagrams exchanging charged Higgs bosons, the charged Higgs boson interactions for the $t$-quark are no longer weak, compared to the standard $W$-boson interactions, while those for the $u$- or $c$-quark are much weaker. Therefore, the chargino and the charged Higgs boson contributions could naturally be the same order of magnitude as the $W$-boson contributions. On the other hand, the gluino and the neutralino contributions are mediated by the down-type squarks, whose masses are quite degenerated. Although the interaction strength for the gluinos is stronger than for the charginos, in the grand unification scheme the gluino mass is proportionally larger than the chargino masses. Therefore, the gluino and the neutralino contributions become smaller than the chargino ones and thus can be ignored.

In sect. 2 we briefly review the model. In sect. 3 we discuss the SSM contribution to $B^0$-$\bar{B}^0$ mixing and its effect on the evaluation of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In sect. 4 we discuss the inclusive branching ratio of radiative $B$-meson decay. Summary is given in sect. 5. In Appendix A we list some equations.

## 2. Supersymmetric Standard Model

In the SSM down-type quarks interact with charginos and up-type squarks. The charginos are the mass eigenstates of the SU(2) charged gauginos and the charged higgsinos. Their mass matrix is given by

$$M^- = \begin{pmatrix} \tilde{m}_2 & -\frac{1}{\sqrt{2}}gv_1 \\ -\frac{1}{\sqrt{2}}gv_2 & m_H \end{pmatrix}, \tag{1}$$

where $v_1$ and $v_2$ are the vacuum expectation values of the Higgs bosons, and $\tilde{m}_2$ and $m_H$ respectively denote the SU(2) gaugino mass and the higgsino mass parameter. In the ordinary scheme for generating the gaugino masses, $\tilde{m}_2$ is smaller than or around the gravitino mass $m_{3/2}$. If the SU(2)$\times$U(1) symmetry is broken through radiative corrections, $\tan\beta$ ($\equiv v_2/v_1$) $\gtrsim 1$ holds and the magnitude of $m_H$ is at most of order of $m_{3/2}$. The chargino mass eigenstates are obtained by diagonalizing the matrix $M^-$ as

$$C_R^\dagger M^- C_L = \mathrm{diag}(\tilde{m}_{\omega 1}, \tilde{m}_{\omega 2}) \quad (\tilde{m}_{\omega 1} < \tilde{m}_{\omega 2}), \tag{2}$$

$C_R$ and $C_L$ being unitary matrices.

The quark fields and the squark fields are respectively mixed in generation space. Since the left-handed squarks and the right-handed ones are also mixed, the mass-squared matrices for the up-type squarks $M_U^2$ is expressed by a $6 \times 6$ matrix:

$$M_U^2 = \begin{pmatrix} M_{U11}^2 & M_{U12}^2 \\ M_{U21}^2 & M_{U22}^2 \end{pmatrix};$$

$$
\begin{aligned}
M^2_{U11} &= \cos 2\beta(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)M_Z^2 + \tilde{m}_Q^2 + (1+c)m_U m_U^\dagger, \\
M^2_{U22} &= \frac{2}{3}\cos 2\beta \sin^2\theta_W M_Z^2 + \tilde{m}_U^2 + (1+2c)m_U^\dagger m_U, \\
M^2_{U12} &= M^{2\dagger}_{U21} = \cot\beta m_H m_U + a^* m_{3/2} m_U,
\end{aligned} \qquad (3)
$$

where $m_U$ denotes the mass matrix of the up-type quarks. The mass parameters $m_{3/2}$, $\tilde{m}_Q$, $\tilde{m}_U$ and the dimensionless constants $a$, $c$ come from the terms in the SSM lagrangian which break supersymmetry softly: $\tilde{m}_Q$ and $\tilde{m}_U$ are determined by the gravitino and gaugino masses and $\tilde{m}_Q \simeq \tilde{m}_U \sim m_{3/2}$; $a$ is related to the breaking of local supersymmetry; and $c$ is related to radiative corrections to the squark masses. At the electroweak energy scale, $a$ is of order unity and $c = -1 - (-0.1)$.

The squark mixings among different generations in Eq. (3) are removed by the same matrices that diagonalize the up-type quark mass matrix. As a result, the generation mixings in the lagrangian between the down-type quarks and the up-type squarks in mass eigenstates are described by the CKM matrix of the quarks. The mixings between the left-handed and right-handed squarks can be neglected for the first two generations because of the smallness of the corresponding quark masses. The masses of the left-handed squarks $\tilde{u}_L, \tilde{c}_L$ and the right-handed squarks $\tilde{u}_R, \tilde{c}_R$ are given by

$$
\begin{aligned}
\tilde{M}^2_{uL} = \tilde{M}^2_{cL} &= \tilde{m}_Q^2 + \cos 2\beta(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)M_Z^2, \\
\tilde{M}^2_{uR} = \tilde{M}^2_{cR} &= \tilde{m}_U^2 + \frac{2}{3}\cos 2\beta \sin^2\theta_W M_Z^2.
\end{aligned} \qquad (4)
$$

For the third generation, the large $t$-quark mass leads to an appreciable mixing between $\tilde{t}_L$ and $\tilde{t}_R$. The mass-squared matrix for the $t$-squarks becomes

$$
M_t^2 = \begin{pmatrix} \tilde{M}^2_{uL} + (1-|c|)m_t^2 & (\cot\beta m_H + a^* m_{3/2})m_t \\ (\cot\beta m_H^* + a m_{3/2})m_t & \tilde{M}^2_{uR} + (1-2|c|)m_t^2 \end{pmatrix}. \qquad (5)
$$

The mass eigenstates of the $t$-squarks are obtained by diagonalizing the matrix $M_t^2$ as

$$
S_t M_t^2 S_t^\dagger = \mathrm{diag}(\tilde{M}^2_{t1}, \tilde{M}^2_{t2}) \quad (\tilde{M}^2_{t1} < \tilde{M}^2_{t2}), \qquad (6)
$$

where $S_t$ is a unitary matrix.

Down-type quarks interact also with charged Higgs bosons and up-type quarks. The generation mixings in these interactions are described by the CKM matrix. The parameters which determine FCNC processes, other than the SM parameters, are only the charged Higgs boson mass $M_{H^\pm}$ and $\tan\beta$.

## 3. $B^0$-$\bar{B}^0$ Mixing

The SSM gives new contributions to $B^0$-$\bar{B}^0$ mixing through box diagrams mediated by the charginos or the charged Higgs bosons. For the chargino contribution exchanged bosons are up-type squarks. For the charged Higgs boson contribution exchanged fermions are up-type quarks and exchanged bosons are either only charged Higgs bosons or charged Higgs bosons and $W$-bosons.

The effective lagrangian for $B^0_d$-$\bar{B}^0_d$ mixing induced by the chargino interactions becomes [1,6]

$$
\begin{aligned}
L^C_{B^0_d} &= \frac{1}{8M_W^2}\left(\frac{g^2}{4\pi}\right)^2 (V^*_{31}V_{33})^2 \\
&\quad [A^C_V \bar{d}\gamma^\mu \frac{1-\gamma_5}{2} b \bar{d}\gamma_\mu \frac{1-\gamma_5}{2} b + A^C_S \bar{d}\frac{1+\gamma_5}{2} b \bar{d}\frac{1+\gamma_5}{2} b], \\
A^C_n &= \sum_{i,j}\sum_{k,l}[F^C_n(3,k;3,l;i,j) + F^C_n(1,k;1,l;i,j) \\
&\quad - F^C_n(1,k;3,l;i,j) - F^C_n(3,k;1,l;i,j)] \quad (n=V,S), \\
F^C_V(a,k;b,l;i,j) &= \frac{1}{4} G^{(a,k)i} G^{(a,k)j*} G^{(b,l)i*} G^{(b,l)j} Y_1(r_{(a,k)}, r_{(b,l)}, s_i, s_j), \\
F^C_S(a,k;b,l;i,j) &= H^{(a,k)i} G^{(a,k)j*} G^{(b,l)i*} H^{(b,l)j} Y_2(r_{(a,k)}, r_{(b,l)}, s_i, s_j),
\end{aligned} \tag{7}
$$

where $a,b$ are generation indices, and $i,j$ and $k,l$ respectively stand for the two charginos and the two squarks in each flavor. Coupling constants $G^{(a,k)i}$, $H^{(a,k)i}$ are given by

$$
\begin{aligned}
G^{(1,1)i} &= G^{(2,1)i} = \sqrt{2}C^*_{R1i}, \quad G^{(1,2)i} = G^{(2,2)i} = 0, \\
G^{(3,k)i} &= \sqrt{2}C^*_{R1i}S_{tk1} - \frac{C^*_{R2i}S_{tk2}}{\sin\beta}\frac{m_t}{M_W}, \\
H^{(1,1)i} &= H^{(2,1)i} = \frac{C^*_{L2i}}{\cos\beta}\frac{m_b}{M_W}, \quad H^{(1,2)i} = H^{(2,2)i} = 0, \\
H^{(3,k)i} &= \frac{C^*_{L2i}S_{tk1}}{\cos\beta}\frac{m_b}{M_W}.
\end{aligned} \tag{8}
$$

Functions $Y_1$ and $Y_2$ are defined in Appendix A, their arguments being given by

$$
\begin{aligned}
r_{(1,1)} &= r_{(2,1)} = \frac{\tilde{M}^2_{uL}}{M_W^2}, \quad r_{(1,2)} = r_{(2,2)} = \frac{\tilde{M}^2_{uR}}{M_W^2}, \quad r_{(3,k)} = \frac{\tilde{M}^2_{tk}}{M_W^2}, \\
s_i &= \frac{\tilde{m}^2_{\omega i}}{M_W^2}.
\end{aligned} \tag{9}
$$

The CKM matrix is denoted by $V$. The box diagram with the squarks of the $a$-th and $b$-th generations gives a term proportional to $V^*_{a1}V_{a3}V^*_{b1}V_{b3}$ for $L^C_{B^0_d}$. Since the squarks belonging to the first two generations have the same mass, their contributions are summed up to be proportional to $(V^*_{31}V_{33})^2$ by the unitarity of the CKM matrix. The effective lagrangian $L^C_{B^0_d}$ contains a new quark operator proportional to $A^C_S$ which is not yielded by the $W$-boson interactions. However, since $A^C_S$ is suppressed by $(m_b/M_W)^2$ compared to $A^C_V$, this quark operator can be neglected unless $\tan\beta$ is extremely larger than unity.

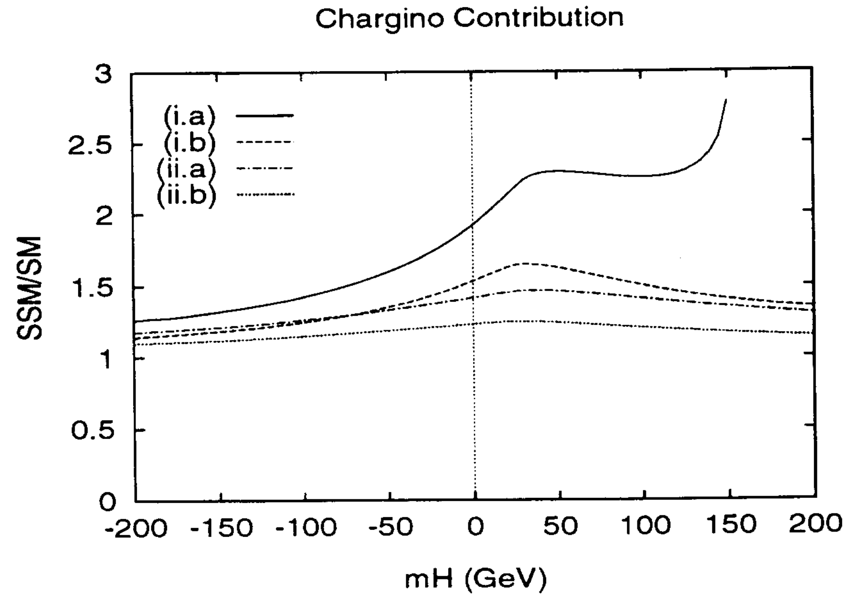


Fig. 1. The ratio $R$ due to the chargino contribution for the parameter values in Table 1. Other parameters are fixed as $\tilde{m}_2 = 200$ GeV, $\tilde{m}_Q = \tilde{m}_U = am_{3/2}$, and $|c| = 0.3$.

For the charged Higgs boson interactions, the effective lagrangian for $B_d^0$-$\bar{B}_d^0$ mixing is given by [1,5]

$$
\begin{aligned}
L^H_{B^0_d} &= \frac{1}{8M_W^2}\left(\frac{g^2}{4\pi}\right)^2 (V_{31}^* V_{33})^2 \\
&\quad [F_V^H(3;3)\bar{d}\gamma^\mu\frac{1-\gamma_5}{2}b\bar{d}\gamma_\mu\frac{1-\gamma_5}{2}b + F_S^H(3;3)\bar{d}\frac{1+\gamma_5}{2}b\bar{d}\frac{1+\gamma_5}{2}b], \\
F_V^H(a;b) &= \frac{1}{4\tan^4\beta}s_a s_b Y_1(r_H, r_H, s_a, s_b) \\
&\quad + \frac{1}{2\tan^2\beta}s_a s_b Y_1(1, r_H, s_a, s_b) - \frac{2}{\tan^2\beta}\sqrt{s_a s_b}Y_2(1, r_H, s_a, s_b), \\
F_S^H(a;b) &= \frac{m_b^2}{M_W^2}\sqrt{s_a s_b}Y_2(r_H, r_H, s_a, s_b), \\
r_H &= \frac{M_{H^\pm}^2}{M_W^2}, \quad s_a = \frac{m_{ua}^2}{M_W^2},
\end{aligned}
\tag{10}
$$

where $m_{ua}$ denotes the up-type quark mass of the $a$-th generation. Since the box diagrams with $t$-quarks give a dominant contribution, $L^H_{B^0_d}$ is approximately proportional to $(V_{31}^* V_{33})^2$. Except for a large value of $\tan\beta$, $F_S^H$ is negligible.

We now consider the physical effects of the SSM contribution. One observable for $B_d^0$-$\bar{B}_d^0$ mixing is the mixing parameter $x_d = \Delta M_{B_d}/\Gamma_{B_d}$ [7], where $\Delta M_{B_d}$ and $\Gamma_{B_d}$ denote the mass difference and the average width for the $B_d^0$-meson mass eigenstates.

Table 1. The values of $\tilde{m}_Q$ and $\tan\beta$ for curves (i.a)–(ii.b) in Fig. 1.

| | (i.a) | (i.b) | (ii.a) | (ii.b) |
|---|---|---|---|---|
| $\tilde{m}_Q$ (GeV) | 200 | 200 | 300 | 300 |
| $\tan\beta$ | 1.2 | 2.0 | 1.2 | 2.0 |

The mass difference is induced dominantly by the short distance contributions of box diagrams. We can express the mixing parameter as

$$x_d = \frac{G_F^2}{6\pi^2} M_W^2 \frac{M_{B_d}}{\Gamma_{B_d}} f_{B_d}^2 B_{B_d} |V_{31}^* V_{33}|^2 \eta_{B_d} |F_V^W(3;3) + A_V^C + F_V^H(3;3)|, \qquad (11)$$

where $G_F$, $f_{B_d}$, $B_{B_d}$, and $\eta_{B_d}$ represent the Fermi constant, the $B_d^0$-meson decay constant, the bag factor for $B_d^0$-$\bar{B}_d^0$ mixing, and the QCD correction factor. The contribution of the standard box diagram with $W$-bosons and $t$-quarks is expressed by $F_V^W(3;3)$. We have neglected $A_S^C$ and $F_S^H$. Assuming that QCD corrections for $F_V^W(3;3)$, $A_V^C$, and $F_V^H(3;3)$ do not much differ from each other, the same QCD factor has been multiplied. The new contributions to $x_d$ from the SSM are $A_V^C$ and $F_V^H(3;3)$. The amount of the SSM contribution can be measured by the ratio

$$R = \frac{F_V^W(3;3) + A_V^C + F_V^H(3;3)}{F_V^W(3;3)}. \qquad (12)$$

If there exists no sizable new contribution, $R$ becomes unity.

In Fig. 1 we show $R$ as a function of the higgsino mass parameter $m_H$, taking $F_V^H(3;3) = 0$ to see the amount of the chargino contribution alone. The values for $\tilde{m}_Q$ and $\tan\beta$ are listed in Table 1. The other parameters are set, as typical values, for $\tilde{m}_2 = 200$ GeV, $\tilde{m}_Q = \tilde{m}_U = a m_{3/2}$, and $|c| = 0.3$. For the $t$-quark mass we use $m_t = 170$ GeV [8] throughout this paper. The mass of the lighter chargino is smaller than 100 GeV for $-80$ GeV $\lesssim m_H \lesssim 160$ GeV. The mass of the lighter $t$-squark is smaller than 100 GeV for $m_H \gtrsim 80$ GeV in case (i.a) and for $m_H \gtrsim 120$ GeV in case (i.b). The sign of the chargino contribution is the same as that of the $W$-boson contribution, and these contributions interfere constructively. In case (i.a) $R$ is larger than 1.5 for $m_H \gtrsim -80$ GeV and $R \simeq 2.3$ in the region with the lighter $t$-squark mass 50 GeV $\lesssim \tilde{M}_{t1} \lesssim 130$ GeV. The ratio $R$ can also have a value larger than 1.5 in case (i.b). The manifest dependence of $R$ on $\tan\beta$ arises from the chargino Yukawa interactions, as seen in Eq. (8): $R$ increases as $\tan\beta$ decreases, since a smaller value for $v_2$ enhances the Yukawa couplings of the charginos to the $t$-squarks. In Fig. 2 we show $R$ as a function of the charged Higgs boson mass for (a) $\tan\beta = 1.2$ and (b) $\tan\beta = 2$, taking $A_V^C = 0$ to see the amount of the charged Higgs boson contribution alone. The charged Higgs bosons also contribute constructively. The ratio $R$ is larger than 1.5 for $M_{H^\pm} \lesssim 180$ GeV in case (a). Similarly to the chargino contribution, the value of $R$ increases as $\tan\beta$ decreases. The net amount of the SSM contribution is given by the sum of all the contributions. The ratio $R$ in Eq. (12) is larger than that

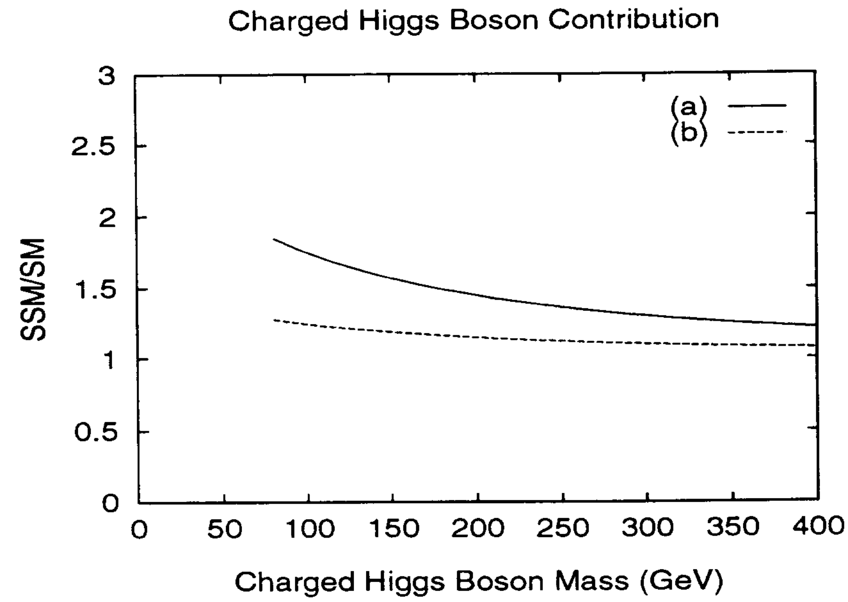


Fig. 2. The ratio $R$ due to the charged Higgs boson contribution for (a) $\tan\beta = 1.2$ and (b) $\tan\beta = 2$.

shown in Fig. 1 or Fig. 2. For example, in case (i.a) with $M_{H^\pm} = 200$ GeV, the ratio $R$ becomes $R \simeq 1.9$ for $m_H = -100$ GeV ($\tilde{M}_{t1} \simeq 188$ GeV, $\tilde{m}_{\omega 1} \simeq 119$ GeV) and $R \simeq 2.7$ for $m_H = 100$ GeV ($\tilde{M}_{t1} \simeq 85$ GeV, $\tilde{m}_{\omega 1} \simeq 56$ GeV).

The value of $x_d$ has been experimentally measured as $x_d = 0.71 \pm 0.06$ [9]. An enhanced value of $R$ is considered to give a prediction, for the CKM matrix, different from the SM prediction through Eq. (11). Let us express the CKM matrix by the standard parametrization [9]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \qquad (13)$$

where $c_{ab} = \cos\theta_{ab}$ and $s_{ab} = \sin\theta_{ab}$. Without loss of generality, the angles $\theta_{12}$, $\theta_{23}$, and $\theta_{13}$ can be taken to lie in the first quadrant, leading to $\sin\theta_{ab} > 0$ and $\cos\theta_{ab} > 0$. At present, the experiments give $|V_{12}| = 0.22$, $|V_{23}| = 0.04 \pm 0.004$, and $|V_{13}/V_{23}| = 0.08 \pm 0.02$ [9,10] within the framework of the SM. Since these values have been measured through the processes for which new contributions by the SSM, if any, are negligible, they are also valid in the SSM. Among the four independent parameters of the CKM matrix, the value of $\sin\theta_{13}$ is determined by $\sin\theta_{13} = |V_{13}|$. Owing to this smallness of $\sin\theta_{13}$, the values of $\sin\theta_{12}$ and $\sin\theta_{23}$ are given by $\sin\theta_{12} = |V_{12}|$ and $\sin\theta_{23} = |V_{23}|$. The remaining undetermined parameter is the $CP$-violating phase $\delta$, which can be measured by $x_d$. Therefore, the value of $\delta$ depends on $R$ and the SSM with $R > 1$ and the SM predict different values for it.

The $CP$-violating phase $\delta$ can also be measured by the $CP$ violation parameter $\epsilon$

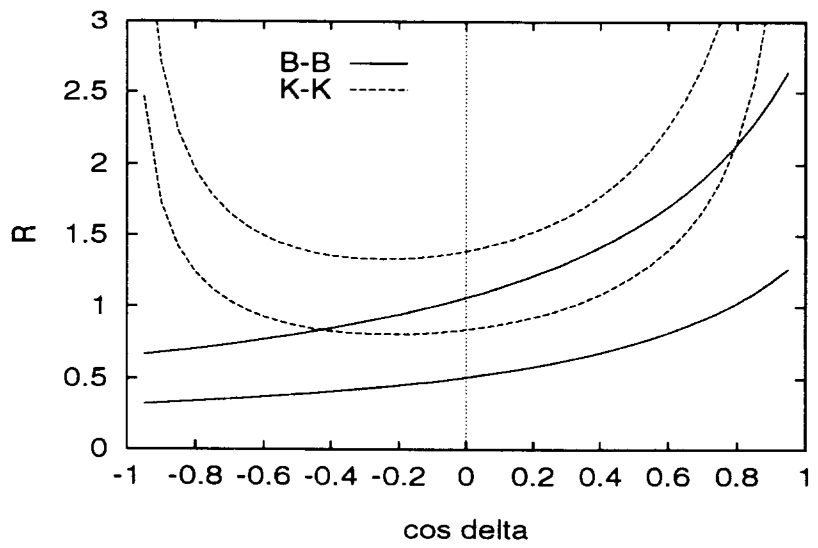


Fig. 3. The ratio $R$ allowed by the experimental values of $x_d$ and $\epsilon$ for $|V_{13}/V_{23}| = 0.08$ and $|V_{23}| = 0.04$.

of $K^0$-$\bar{K}^0$ mixing, which receives new contributions from the SSM similarly to $B^0$-$\bar{B}^0$ mixing. The parameter $\epsilon$ can be written as

$$\begin{aligned}\epsilon &= -e^{i\pi/4}\frac{G_F^2}{12\sqrt{2}\pi^2}M_W^2\frac{M_K}{\Delta M_K}f_K^2 B_K \\ &\quad \mathrm{Im}[(V_{31}^*V_{32})^2\eta_{K33}(F_V^W(3;3)+A_V^C+F_V^H(3;3)) \\ &\quad +(V_{21}^*V_{22})^2\eta_{K22}(F_V^W(2;2)+F_V^H(2;2)) \\ &\quad +2V_{31}^*V_{32}V_{21}^*V_{22}\eta_{K32}(F_V^W(3;2)+F_V^H(3;2))], \qquad (14)\end{aligned}$$

where $f_K$ and $B_K$ represent the decay constant and the bag factor. The QCD correction factors are denoted by $\eta_{Kab}$. The standard $W$-boson box diagram with $c$-quarks and that with $t$- and $c$-quarks give $F_V^W(2;2)$ and $F_V^W(3;2)$. The new contributions are $A_V^C$ and $F_V^H$. However, as long as $\tan\beta \gtrsim 1$, which is favored by the SSM. $F_V^H(2;2)$ and $F_V^H(3;2)$ are respectively much smaller than $F_V^W(2;2)$ and $F_V^W(3;2)$. Thus, the difference between the SSM and the SM contributions to $\epsilon$ is only in the term proportional to $(V_{31}^*V_{32})^2$, which is described by $R$ of Eq. (12).

The value of $\delta$ is determined as a function of $R$ or vice versa independently by $x_d$ and $\epsilon$. Using consistency in these two evaluations, we can specify the values of $\delta$ and $R$. In Fig. 3 we show the allowed range for the ratio $R$ derived from the experimental values of $x_d$, $\epsilon$, and CKM matrix elements, as a function of $\cos\delta$. We have assumed the experimental central values $|V_{12}| = 0.22$, $|V_{23}| = 0.04$, $|V_{13}/V_{23}| = 0.08$, $x_d = 0.71$ and $|\epsilon| = 2.26\times10^{-3}$ [9]. The theoretical uncertainties of $B_K$, $B_{B_d}$, and $f_{B_d}$ are incorporated as $0.6 < B_K < 0.9$ from a combined result of lattice and 1/N calculations [11] and 180

Table 2. The values of $\sin 2\phi_\alpha$, $\sin 2\phi_\beta$, $\sin 2\phi_\gamma$, and $x_s/x_d$ predicted by the SSM and the SM for $|V_{13}/V_{23}| = 0.08$, $|V_{23}| = 0.04$.

| | SSM | SM |
|---|---|---|
| $\sin 2\phi_\alpha$ | $-0.96 - 0.74$ | $0.15 - 0.74$ |
| $\sin 2\phi_\beta$ | $0.55 - 0.66$ | $0.61 - 0.66$ |
| $\sin 2\phi_\gamma$ | $-0.18 - 1.00$ | $-0.18 - 0.54$ |
| $x_s/x_d$ | $17 - 36$ | $17 - 21$ |

MeV $< f_{B_d}\sqrt{B_{B_d}} <$ 260 MeV from a lattice calculation [12]. For the QCD correction factors we have used $\eta_{B_d} = 0.55$ in Eq. (11) and $\eta_{K33} = 0.57$, $\eta_{K22} = 1.1$, and $\eta_{K32} = 0.36$ in Eq. (14) [13]. The region between the solid curves and that between the dashed curves are respectively allowed by $x_d$ and $\epsilon$. In the region consistent with both $x_d$ and $\epsilon$, the allowed ranges are $0.8 \lesssim R \lesssim 2.1$ and $-0.5 \lesssim \cos\delta \lesssim 0.8$, while $-0.1 \lesssim \cos\delta \lesssim 0.3$ for the SM of $R = 1$. The ratio $R$ cannot be much larger than 2, which rules out some regions in the SSM parameter space. Within the possible ranges for $|V_{13}/V_{23}|$ and $|V_{23}|$ taking into account the experimental uncertainties, the ratio $R$ is shown to be at most $R \sim 3$.

We can see from Fig. 3 that the value of $\cos\delta$ in the SSM could be larger than that allowed in the SM. In near future experiments, $CP$ asymmetries in $B^0$-meson decays and amount of $B_s^0$-$\bar{B}_s^0$ mixing will be measured, which also depend on $\delta$. It is possible that these physical quantities have values outside the ranges predicted by the SM. The $CP$ asymmetries enable to measure the angles of the unitarity triangle given by

$$\phi_\alpha = \arg\left(-\frac{V_{31}V_{33}^*}{V_{11}V_{13}^*}\right), \quad \phi_\beta = \arg\left(-\frac{V_{21}V_{23}^*}{V_{31}V_{33}^*}\right), \quad \phi_\gamma = \arg\left(-\frac{V_{11}V_{13}^*}{V_{21}V_{23}^*}\right). \tag{15}$$

For instance, the decays $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \psi K_S$, and $B_s^0 \to \rho K_S$ can be used to determine $\sin 2\phi_\alpha$, $\sin 2\phi_\beta$, and $\sin 2\phi_\gamma$, respectively [14]. The mixing parameter $x_s$ for $B_s^0$-$\bar{B}_s^0$ mixing is given by an equation analogous to Eq. (11). The ratio of $x_s$ to $x_d$ becomes

$$\frac{x_s}{x_d} = \frac{|V_{32}|^2}{|V_{31}|^2}, \tag{16}$$

where we have neglected the small differences between $B_d^0$ and $B_s^0$ caused by the $SU(3)_{flavor}$ breaking.

In Table 2 we give the predicted values of $\sin 2\phi_i$ ($i = \alpha, \beta, \gamma$) and $x_s/x_d$ in the SSM ($R \geq 1$) and the SM ($R = 1$) for $|V_{13}/V_{23}| = 0.08$ and $|V_{23}| = 0.04$. There are wide ranges of $\sin 2\phi_\alpha$, $\sin 2\phi_\gamma$, and $x_s/x_d$ which are allowed only in the SSM. The experimental results $\sin 2\phi_\alpha < 0$, $\sin 2\phi_\gamma \sim 1$, and $x_s/x_d \sim 30$ could implicate $R > 1$ and indirectly suggest the validity of the SSM. Note, however, that the predicted ranges vary with the values of $|V_{13}/V_{23}|$ and $|V_{23}|$. More precise measurements for these values are necessary to make predictions definitely.

## 4. Radiative $B$-Meson Decay

The $b$-quark can decay into the $s$-quark and the photon at one-loop level. The new contributions by the SSM are through the diagrams in which charginos and up-type squarks, or charged Higgs bosons and up-type quarks are exchanged. The decay width is expressed in terms of dipole form factors $F_2$ and $G_2$ as

$$\Gamma(b \to s\gamma) = \frac{\alpha_{EM}}{2} m_b(|F_2|^2 + |G_2|^2)\left(1 - \frac{m_s}{m_b}\right)^2 \left(1 - \frac{m_s^2}{m_b^2}\right) \tag{17}$$

at the rest frame of the $b$-quark. The dipole form factors induced by the chargino and the charged Higgs boson loop diagrams become the followings.
*The chargino contribution:*

$$\begin{aligned} F_2^{\tilde{\omega}} &= G_2^{\tilde{\omega}} \\ &= \frac{g^2}{32\pi^2} V_{32}^* V_{33} \sum_{i=1}^{2} \frac{m_b^2}{\tilde{m}_{\omega i}^2} \\ & [-|C_{R1i}|^2 r_{ui} K_1(r_{ui}) - \frac{C_{R1i} C_{L2i}^*}{\sqrt{2}\cos\beta} \frac{\tilde{m}_{\omega i}}{M_W} r_{ui} K_2(r_{ui}) \\ & + \sum_{k=1}^{2} \{ |C_{R1i} S_{tk1}^* - \frac{C_{R2i} S_{tk2}^*}{\sqrt{2}\sin\beta} \frac{m_t}{M_W}|^2 r_{ki} K_1(r_{ki}) \\ & + \frac{C_{L2i}^* S_{tk1}}{\sqrt{2}\cos\beta} \left( C_{R1i} S_{tk1}^* - \frac{C_{R2i} S_{tk2}^*}{\sqrt{2}\sin\beta} \frac{m_t}{M_W} \right) \frac{\tilde{m}_{\omega i}}{M_W} r_{ki} K_2(r_{ki}) \}]; \\ K_i(r) &= I_i(r) + \frac{2}{3} J_i(r) \quad (i = 1, 2), \\ & r_{ui} = \frac{\tilde{m}_{\omega i}^2}{\tilde{M}_{uL}^2}, \quad r_{ki} = \frac{\tilde{m}_{\omega i}^2}{\tilde{M}_{tk}^2}, \end{aligned} \tag{18}$$

where $I_i(r)$ and $J_i(r)$ are defined in Appendix A. The unitarity of the CKM matrix has been used and $F_2^{\tilde{\omega}}$ is proportional to $V_{32}^* V_{33}$.
*The charged Higgs boson contribution:*

$$\begin{aligned} F_2^{H^\pm} &= G_2^{H^\pm} \\ &= -\frac{g^2}{64\pi^2} V_{32}^* V_{33} \frac{m_b^2}{M_W^2} r_H \\ & [\frac{1}{\tan^2\beta} \{ \frac{2}{3} I_1(r_H) + J_1(r_H) \} + \frac{2}{3} I_2(r_H) + J_2(r_H)]; \\ & r_H = \frac{m_t^2}{M_{H^\pm}^2}. \end{aligned} \tag{19}$$

Since the loop diagram with $t$-quarks makes a dominant contribution. $F_2^{H^\pm}$ is proportional to $V_{32}^* V_{33}$.

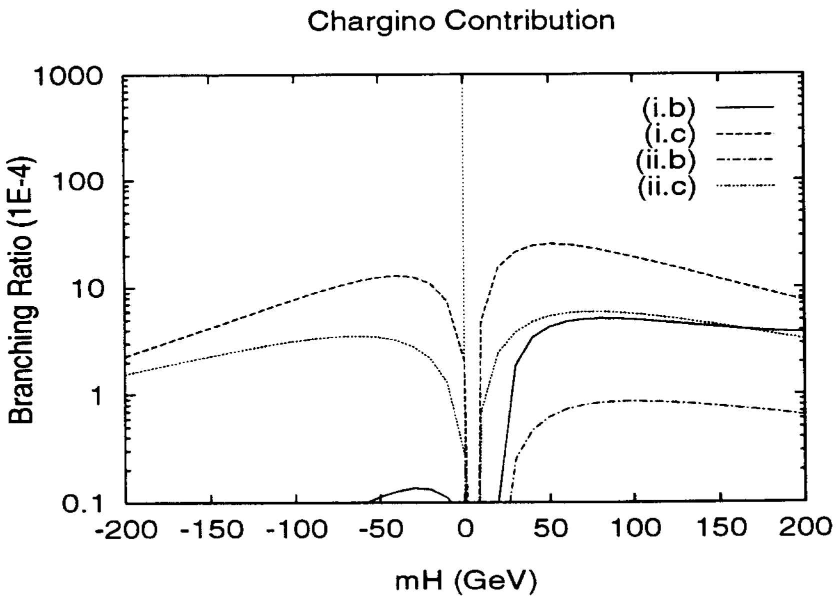


Fig. 4. The inclusive branching ratio of $B \to X_s\gamma$ due to the chargino contribution for the parameter sets in Table 3 with $\tilde{m}_2$ =200 GeV.

Table 3. The values of $\tilde{m}_Q$ and $\tan\beta$ for curves (i.b)–(ii.c) in Fig. 4.

| | (i.b) | (i.c) | (ii.b) | (ii.c) |
|---|---|---|---|---|
| $\tilde{m}_Q$ (GeV) | 200 | 200 | 300 | 300 |
| $\tan\beta$ | 2.0 | 10.0 | 2.0 | 10.0 |

The radiative decay $b \to s\gamma$ is observed as a $B$-meson decay into a hadronic state involving a strange particle and a hard photon. One of the dominant decay modes for the $b$-quark is a three-body decay into the $c$-quark, the electron, and the neutrino. Assuming that the $B$-meson decays are determined by the $b$-quark decays, we can calculate the branching ratio for the inclusive radiative decay $B \to X_s\gamma$ by

$$\mathrm{BR}(B \to X_s\gamma) = \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\bar{\nu}_e)}\mathrm{BR}(B \to X_c e\bar{\nu}_e), \tag{20}$$

where $\mathrm{BR}(B \to X_c e\bar{\nu}_e)$ is known experimentally to be 0.11 [9]. The CKM matrix elements appearing in our calculations are $V_{23}, V_{32}$, and $V_{33}$. We take $|V_{23}| = |V_{32}|$ and $|V_{33}| = 1$, which are in good approximation. No further evaluation is necessary for the CKM matrix. We neglect QCD corrections, which does not change much the results.

We give the numerical results for the branching ratio (BR) of $B \to X_s\gamma$. In order to see each effect of the various SSM contributions, the evaluation is made assuming only one contribution instead of summing all. The BR due to the chargino contribution is shown in Fig. 4 for $\tilde{m}_2 = 200$ GeV. For $\tilde{m}_Q$ and $\tan\beta$ we take four sets

interferes constructively. Compared with the SM prediction, the branching ratio of

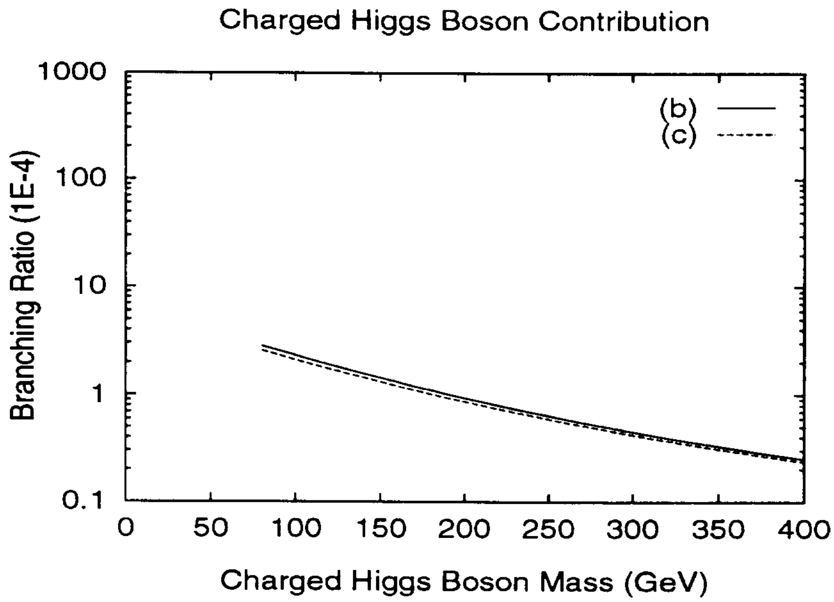


Fig. 5. The inclusive branching ratio of $B \to X_s\gamma$ due to the charged Higgs boson contribution for (b) $\tan\beta = 2$ and (c) $\tan\beta = 10$.

of values listed in Table 3. The other parameters are set for $\tilde{m}_Q = \tilde{m}_U = am_{3/2}$ and $|c| = 0.3$. The $t$-quark mass is taken for $m_t = 170$ GeV. The BR due to the standard $W$-boson contribution with QCD corrections is $(2-5) \times 10^{-4}$. Compared to this value, the chargino contribution could give an equal or a larger amount of branching ratio, if $\tan\beta$ is large or $\tilde{M}_{t1}$ is small. The dependence of the BR on $\tan\beta$ comes from the chargino Yukawa interactions. The form factor $F_2^{\tilde{\omega}}$ in Eq. (18) is roughly proportional to the Yukawa coupling constant which gives a mass to the $b$-quark and is enhanced for a smaller value of $v_1$. The sign of the dipole form factor $F_2^{\tilde{\omega}}$ depends on the parameters. It is the same as the sign of the $W$-boson contribution $F_2^W$ for $m_H \lesssim 0$ GeV and opposite for $m_H \gtrsim 50$ GeV in our examples. In Fig. 5 we show the BR due to the charged Higgs boson contribution for (b) $\tan\beta = 2$ and (c) $\tan\beta = 10$. As long as $\tan\beta > 1$, the BR does not much depend on the value of $\tan\beta$. The sign of $F_2^{H^\pm}$ is the same as $F_2^W$. Taking into account all the contributions, the BR in the SSM is either larger or smaller than the SM prediction in sizable regions of its parameter space.

## 5. Summary

In the SSM there exist several new interactions which can induce FCNC processes. We have discussed their effects on $B^0$-$\bar{B}^0$ mixing. This mixing receives contributions from box diagrams in which charginos and up-type squarks, or charged Higgs bosons and up-type quarks are exchanged. We have calculated the ratio $R$ of the SSM contribution to the contribution in the SM. The new SSM contributions interfere constructively with the standard $W$-boson contribution. The ratio $R$ is sizably larger than unity, if $\tan\beta$ has a value around unity and a chargino, a $t$-squark, and/or a charged Higgs boson are not much heavier than 100 GeV.

The enhanced SSM contribution to $B^0$-$\bar{B}^0$ mixing makes the $CP$-violating phase $\delta$ of the CKM matrix have a value different from the SM prediction. We have discussed the ranges of $\cos\delta$ and $R$ derived from $x_d$ and $\epsilon$. The present uncertainties in $|V_{13}/V_{23}|$, $|V_{23}|$, $f_{B_d}\sqrt{B_{B_d}}$, and $B_K$ are still large, and the allowed ranges for $\cos\delta$ and $R$ vary with the values of those quantities. However, if the present central values for $|V_{13}/V_{23}|$ and $|V_{23}|$ are in the vicinities of actual values, $\cos\delta$ and $R$ should lie in the ranges $-0.1 - 0.8$ and $1.0 - 2.1$, respectively, while $-0.1 \lesssim \cos\delta \lesssim 0.3$ should hold for the SM of $R = 1$. This $CP$-violating phase $\delta$ can be probed by $CP$ asymmetries in $B^0$-meson decays and amount of $B_s^0$-$\bar{B}_s^0$ mixing. We have shown the possibility that the measurements of $\sin 2\phi_\alpha$, $\sin 2\phi_\beta$, $\sin 2\phi_\gamma$, and $x_s/x_d$ disclose values of $\cos\delta$ and $R$ which are not allowed in the SM, thereby implicating the validity of the SSM.

The SSM interactions mediated by the charginos and the charged Higgs bosons could also sizably contribute to radiative $B$-meson decay. The chargino contribution interferes with the standard $W$-boson contribution either constructively or destructively depending on the parameter values, while the charged Higgs boson contribution interferes constructively. Compared with the SM prediction, the branching ratio of $B \to X_s\gamma$ becomes either enhanced or reduced, if $\tan\beta$ is much larger than unity or the relevant particles are not heavy. Radiative $B$-meson decay therefore provides information on the SSM complimentarily to $B^0$-$\bar{B}^0$ mixing.

## Appendix A

The functions $Y_1$ and $Y_2$ in Eqs. (7) and (10) coming from loop integrals for $B^0$-$\bar{B}^0$ mixing are given by

$$\begin{aligned}
&Y_1(r_\alpha, r_\beta, s_i, s_j) = \\
&\quad \frac{r_\alpha^2}{(r_\beta - r_\alpha)(s_i - r_\alpha)(s_j - r_\alpha)} \ln r_\alpha + \frac{r_\beta^2}{(r_\alpha - r_\beta)(s_i - r_\beta)(s_j - r_\beta)} \ln r_\beta \\
&\quad + \frac{s_i^2}{(r_\alpha - s_i)(r_\beta - s_i)(s_j - s_i)} \ln s_i + \frac{s_j^2}{(r_\alpha - s_j)(r_\beta - s_j)(s_i - s_j)} \ln s_j, \\
&Y_2(r_\alpha, r_\beta, s_i, s_j) = \\
&\quad \sqrt{s_i s_j}\Big[\frac{r_\alpha}{(r_\beta - r_\alpha)(s_i - r_\alpha)(s_j - r_\alpha)} \ln r_\alpha + \frac{r_\beta}{(r_\alpha - r_\beta)(s_i - r_\beta)(s_j - r_\beta)} \ln r_\beta \\
&\quad + \frac{s_i}{(r_\alpha - s_i)(r_\beta - s_i)(s_j - s_i)} \ln s_i + \frac{s_j}{(r_\alpha - s_j)(r_\beta - s_j)(s_i - s_j)} \ln s_j\Big]. \qquad \text{(A.1)}
\end{aligned}$$

The functions $I_i$ and $J_i$ in Eqs. (18) and (19) for $b \to s\gamma$ are given by

$$I_1(r) = \frac{1}{12(1-r)^4}(2 + 3r - 6r^2 + r^3 + 6r\ln r) \quad \left(I_1(1) = \frac{1}{24}\right).$$

$$
\begin{aligned}
I_2(r) &= \frac{1}{2(1-r)^3}(-3+4r-r^2-2\ln r) \quad (I_2(1)=\frac{1}{3}), \\
J_1(r) &= \frac{1}{12(1-r)^4}(1-6r+3r^2+2r^3-6r^2\ln r) \quad (J_1(1)=\frac{1}{24}), \\
J_2(r) &= \frac{1}{2(1-r)^3}(1-r^2+2r\ln r) \quad (J_2(1)=\frac{1}{6}).
\end{aligned} \tag{A.2}
$$


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