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Renormalons on the Lattice

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We present the first lattice calculation of the B-meson binding energy $\overline{\Lambda}$ and of the kinetic energy $\lambda_1/2m_Q$ of the heavy-quark inside the pseudoscalar B-meson. In order to cancel the ambiguities due to the ultraviolet renormalons present in the operator matrix elements, this calculation has required the non-perturbative subtraction of the power divergences present in the Lagrangian operator $\bar{Q}(x)\,D_4\,Q(x)$ and in the kinetic energy operator $\bar{Q}(x)\,\vec{D}^2\,Q(x)$. The non-perturbative renormalization of the relevant operators has been implemented by imposing suitable renormalization conditions on quark matrix elements in the Landau gauge.

1. Introduction

Among the quantities which cannot be predicted on the basis of the Heavy Quark Effective Theory (HQET) [1] there are several parameters which characterize the dynamics of strong interactions, such as the heavy quark binding energy $\overline{\Lambda}$, relevant for higher order corrections to the semileptonic form factors, and the heavy quark kinetic energy $\lambda_1/2m_Q$, which enters in the predictions of many inclusive decay rates. Lattice HQET offers the possibility of a numerical, non -perturbative determination of these quantities from first principles and without free parameters.

The parameter $\overline{\Lambda}$ denotes the asymptotic value of the difference between the hadron and the heavy quark "pole" mass m_Q

$$\overline{\Lambda} = \lim_{m_Q\to\infty} (M_H - m_Q)\,. \qquad (1)$$

It has been recently shown that the pole mass is ambiguous due to the presence of infrared renormalon singularities [2]. At lowest order in $1/m_Q$, the infrared renormalon ambiguity appearing in the definition of the pole mass is closely related to the ultra-violet renormalon singularity present in the matrix elements of the operator $\bar{Q}(x)\,D_4\,Q(x)$. This singularity is due to the linear power divergence of $\bar{Q}(x)\,D_4\,Q(x)$, induced by its mixing with the lower dimensional operator $\bar{Q}(x)Q(x)$. On the lattice the linear divergence manifests itself as a factor proportional to the inverse lattice spacing $1/a$ in the mixing coefficient of the operator $\bar{Q}(x)Q(x)$. In ref. [3], it was stressed that these divergences must be subtracted non-perturbatively since factors such as

$$\frac{1}{a}\exp\Big(-\int^{g_0(a)}\frac{dg'}{\beta(g')}\Big) \sim \Lambda_{QCD}\,, \qquad (2)$$

which do not appear in perturbation theory, give non-vanishing contributions as $a\to 0$. Renormalons represent an explicit example of non-perturbative effects of this kind.

The matrix elements of the kinetic energy operator also contain power divergent contributions. In this case, the origin of the divergences is the mixing of $\bar{Q}(x)\,\vec{D}^2\,Q(x)$ with the operator $\bar{Q}(x)\,D_4\,Q(x)$, with a coefficient that diverges linearly, and with the scalar density $\bar{Q}(x)\,Q(x)$, with a quadratically divergent coefficient [3].

Following the non-perturbative method for eliminating the power divergences proposed in ref. [4], we have computed the "physical" values of $\overline{\Lambda}$ and λ_1. This method will be explained in sec. 2.

By fixing the non-perturbative renormalization conditions of $\bar{Q}(x)\,D_4\,Q(x)$ by using the heavy

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quark propagator in the Landau gauge, we found

$$\bar{\Lambda} = (232 \pm 22 \pm 30 \pm 25)\,\mathrm{MeV} \qquad (3)$$

where the first error is statistical and the others systematic and will be explained in sec. 3.

In order to remove the power divergences from the kinetic energy operator, we have imposed to the relevant operator a renormalization condition which corresponds to the "physical" requirement $\langle Q(\vec{p}\,=\,0)|\bar{Q}(x)\,\vec{D}^2\,Q(x)|Q(\vec{p}\,=\,0)\rangle\;=\;0$. This renormalization condition has been used to extract the values of the renormalization constants, that have been obtained with a small statistical error. Unfortunately, after the subtraction of the power divergences, we were only able to obtain a loose upper bound $\lambda_1 < 1.0\,\mathrm{GeV}^2$.

2. Non-perturbative definition of $\overline{\Lambda}$ and λ_1

The cancellation of the power divergences of the operators $\bar{Q}(x)\,D_4\,Q(x)$ and $\bar{Q}(x)\,\vec{D}^2\,Q(x)$ is achieved by imposing appropriate renormalization conditions on the quark matrix elements [4]. In numerical simulations, quark and gluon propagators can be computed non-perturbatively by working in a fixed gauge, typically the Landau gauge [5]. On general grounds, we expect that the heavy quark propagator, at lowest order in $1/m_Q$ has the form

$$S(x) \;=\; \delta(\vec{x})\,\theta(t)\,A(t)\,\exp(-\lambda t), \qquad (4)$$

where $S(x) \;=\; \langle S(\vec{x},t|\vec{0},0)\rangle$,

$$S(\vec{x},t|\vec{0},0) = \delta(\vec{x})\,\theta(t)\,\exp\Big(i\int_0^t A_0(t')dt'\Big)$$

being the non-translational invariant propagator for a given gauge field configuration. $\langle\ldots\rangle$ represents the average over the gauge field configurations and $A(t)$ is an unknown smooth function of t, such that $\ln\Big(A(t+a)/A(t)\Big) \;\to 0$ as $t\;\to\infty$. The constant λ is linearly divergent in $1/a$ and is associated with the ultraviolet renormalon in of the heavy-quark propagator. We can remove it by using

$$\mathcal{L}_{\mathrm{eff}} \;=\; \frac{1}{1+\delta ma}\Big(\bar{Q}(x)\,D_4\,Q(x)\;+\;\delta m\,\bar{Q}(x)\,Q(x)\Big),$$

which corresponds to the propagator

$$S'(\vec{x},t) \;=\; \delta(\vec{x})\,\theta(t)\,A(t)\exp\left(-[\lambda-\delta\overline{m}]\,t\right). \qquad (5)$$

with

$$-\,\delta\overline{m} \;\equiv\; \frac{\ln(1+\delta ma)}{a} \;=\; \lim_{t\to\infty}\delta\overline{m}(t) =$$

$$\lim_{t\to\infty}\frac{1}{a}\,\ln\left(\frac{S(\vec{x},t+a)}{S(\vec{x},t)}\right) \to -\lambda\;+\;O(\frac{1}{t}). \qquad (6)$$

We are now in a position of defining the renormalized binding energy $\overline{\Lambda}$ using $\mathcal{E}$, the bare "binding" energy usually computed from the two point heavy-light meson correlation functions [6]

$$C(t) = \sum_{\vec{x}}\langle 0|\,\bar{Q}(\vec{x},t)\Gamma q(\vec{x},t)\;\bar{q}(\vec{0},0)\Gamma Q(\vec{0},0)\,|0\rangle$$

$$\to Z^2\exp(-\mathcal{E}t) \qquad (7)$$

Thus

$$\overline{\Lambda}\;\equiv\;\mathcal{E}\;-\;\delta\overline{m}\,, \qquad (8)$$

The renormalized kinetic operator, free of power divergences has the form

$$\bar{Q}(x)\,\vec{D}_R^2\,Q(x) = \bar{Q}(x)\,\vec{D}^2\,Q(x) - \frac{C_1}{a} \qquad (9)$$

$$\Big(\bar{Q}(x)\,D_4\,Q(x)\;+\delta m\bar{Q}(x)\,Q(x)\Big) - \frac{C_2}{a^2}\bar{Q}(x)\,Q(x),$$

where the constants C_1 and C_2 are a function of the bare lattice coupling constant $g_0(a)$. In order to eliminate the quadratic and linear power divergences, a possible non-perturbative renormalization condition for $\bar{Q}(x)\,\vec{D}_R^2\,Q(x)$ is that its subtracted matrix element, computed for a quark at rest in the Landau gauge, vanishes $\langle Q(\vec{p}\,=\,0)|\bar{Q}(x)\,\vec{D}_R^2\,Q(x)|Q(\vec{p}\,=\,0)\rangle\,=\,0$. This is equivalent to defining the subtraction constants through the relation

$$R_{\vec{D}^2}(t) \equiv C_1\;+\;C_2\,t =$$

$$\frac{\sum_{\vec{x},\vec{y},t'=0}^{t}\langle\,S'(\vec{x},t|\vec{y},t')\,\vec{D}_{\vec{y}}^2(t')\,S'(\vec{y},t'|\vec{0},0)\,\rangle}{\sum_{\vec{x}}\langle S'(\vec{x},t|\vec{0},0\rangle} \qquad (10)$$

By fitting the time dependence of $R_{\vec{D}^2}(t)$ to eq. (10), one obtains $C_{1,2}$. The relation between the

Figure 1. Effective mass of the heavy-quark propagator $S_H(t)$ as a function of the time. The curve represents a fit of the numerical results (in the improved case) to the expression given in eq. (12).

mass of the meson and the mass of the quark to order $1/m_Q$ is then given by

$$M_H \;=\; m_Q \;+\; \mathcal{E} \;-\; \delta\overline{m} \;+\; \frac{\lambda_1 - C_2}{2\, m_Q} + O(\frac{1}{m_Q^2}) \quad (11)$$

Notice that only the constant C_2 enters the eq. (11) because C_1 is eliminated by using the equations of motions.

3. Numerical implementation of the renormalization procedure

The non-perturbative, numerical renormalization of $\bar{Q}(x)\, D_4\, Q(x)$ and $\bar{Q}(x)\, \vec{D}^2\, Q(x)$ has been performed by using the heavy quark propagators and matrix elements computed on a statistical sample of 36 gluon configurations, generated by numerical simulation on a $16^3 \times 32$ lattice at $\beta = 6.0$. The heavy-light meson propagators have been computed using the improved SW-Clover action [7] for the light quarks, in the quenched approximation. For the binding energy $\mathcal{E}$ we made use of the high statistics results obtained by the APE collaboration at $\beta = 6.0$, using the Wilson [8] and the SW-Clover action [9].

In fig. 1, we present the values of $\delta\overline{m}(t)$ as a function of time. Inspired by one loop perturbation theory at small values of a/t, we made a fit

Figure 2. The ratio $R_{\vec{D}^2}(t)$ as a function of the time. The linear fit is also given.

to $\delta\overline{m}(t)$ using the expression

$$a\, \delta\overline{m}(t) = a\, \delta\overline{m} + \gamma \frac{a}{t}, \qquad (12)$$

where $\delta\overline{m}$ and γ are the parameters of the fit. We have also used different expressions to fit $\delta\overline{m}(t)$ and changed the interval of the fits in order to check the stability of the value of the results. Our best estimate of the mass counter-term is

$$a\, \delta\overline{m} = 0.50 \pm 0.01 \pm 0.02 \qquad (13)$$

where the first error is statistical and the second the systematic one from the different extrapolation procedures. In order to evaluate $\overline{\Lambda}$, we have used the results of the high statistics calculations of $\mathcal{E}$ given in refs. [8,9]. They obtained $a\,\mathcal{E}_W = 0.600(4)$. with the Wilson action and $a\,\mathcal{E}_{SW} = 0.616(4)$ in the Clover case. The difference between the results obtained with different actions $\mathcal{E}_{SW} - \mathcal{E}_W = (0.616 - 0.600)\, a^{-1} \sim 30$ MeV give us a conservative estimate of $O(a)$ effects in the determination of this quantity.

We are now ready to present our prediction for $\overline{\Lambda}$. Using $\delta\overline{m}$ from eq. (13) and the SW-Clover determination of $\mathcal{E}$, we quote

$$\overline{\Lambda} = (232 \pm 22 \pm 30 \pm 25)\, \mathrm{MeV}, \qquad (14)$$

where the first error is the statistical one, the second is our estimate of $O(a)$ effects and the third comes from the calibration of the value of the lattice spacing.

In fig. 2, we plot $R_{\vec{D}^2}(t)$, as defined in eq. (10), as a function of the time t. The numerical results are in remarkable agreement with the predicted linear behaviour. We notice that the constant C_2 is obtained, with a modest sample of configurations, with a precision of $\sim 5\%$. In order to compute λ_1, we have also computed the three-point correlation function

$$\sum_{\vec{x},\vec{y}} \langle 0|\, J(\vec{x},t)\, \left[\bar{Q}(\vec{y},t')\vec{D}^2_y Q(\vec{y},t')\right]\, J^\dagger(\vec{0},0)\,|0\rangle =$$

$$C_{\vec{D}^2}(t,t') \rightarrow Z^2\,\lambda_1\, \exp(-(\mathcal{E}-\delta\overline{m})\,t) \qquad (15)$$

for sufficiently large euclidean time distances t' and $|t-t'|$. Therefore, we can determine λ_1 by taking the ratio

$$R(t,t') = \frac{C_{\vec{D}^2}(t,t')}{C(t)} \rightarrow \lambda_1 \qquad (16)$$

as usually done in numerical simulations. We have obtained the unrenormalized value $a^2\,\lambda_1 = -0.75 \pm 0.15$, and hence

$$a^2\,\lambda_1 - C_2 = 0.06 \pm 0.15. \qquad (17)$$

From the above result, we can at most put a loose upper bound $|\lambda_1 - \frac{C_2}{a^2}| < 1.0\,\mathrm{GeV}^2$

We have shown that lattice numerical simulations give the opportunity of defining unambiguously the important phenomenological parameters $\overline{\Lambda}$ and λ_1. By matching the full to the effective theory, this will allow more accurate theoretical predictions of quantities relevant in heavy flavour physics.

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