Comparative study of charged multiplicities and moments in *pp* collisions at $\sqrt{s} = 7$ TeV in the forward region at the LHC

R. Aggarwal[1] and M. Kaur[2,*]

[1]*Department of Technology, Savitribai Phule Pune University, Pune-411 007, India*
[2]*Department of Physics, Panjab University, Chandigarh-160 014, India*

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Charged particle multiplicities in proton-proton collisions measured in the LHCb detector at a center-of-mass energy of $\sqrt{s} = 7$ TeV in different windows of pseudorapidity, η, in the forward region of the vertex detector are studied by using different statistical distributions. Three distributions are compared with the data, and the moments of the distributions are calculated. The data constituting two sets, one of minimum bias events and another of hard QCD events, are analyzed. The distributions considered derive from different functional forms based on underlying interaction dynamics. The analysis complements the multiplicity analysis done by LHCb in terms of Monte Carlo event generators. The present analysis is from a different perspective, using statistical distributions.

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I. INTRODUCTION

The focus of the high energy experimentation has undergone a paradigm shift from fixed target experiments to collider experiments in pursuit of increasing energy in the center-of-mass system (c.m.s). The latest experiments at the large hadron collider (LHC) have lead to several new results. The energy available for particle production in pp collisions at the LHC results in a multitude of charged particles, the number of which is often the first observable measured in all experimental setups. The increase in numbers follows a logarithmic rise in the average values with the increasing energy of collision in the center of mass system. The number of charged particles is predicted to be connected with the underlying dynamics of interactions. Numerous theoretical, phenomenological, and statistical models have been proposed to develop an understanding of the interaction dynamics. In high energy physics, the negative binomial distribution (NBD), which exhibits approximate Koba-Nielsen-Olesen (KNO) scaling has been used since very early times [1–9]. The first failure of KNO was reported in the analysis of $\bar{p}p$ data obtained by the UA5 Collaboration [10], followed by similar observations made by other experiments such as UA1 [11,12]. As a result, the probability versus the number of charged particles could not follow the negative binomial behavior, due to the appearance of a shoulder structure. This triggered the interest in modifying the negative binomial distribution. The first suggestion was put forth by C. Fuglesang [13], who proposed to consider the NB distribution as composed of weighted superposition of two components, soft events (events without minijets) and semihard events (events with minijets). The fraction of soft events α is taken as a weight and multiplicity distribution of each component being NB type. So that the P(total distribution, NB type) = α P(soft event distribution-NB) + $(1-\alpha)$P(semihard event distribution-NB), where P stands for the probability. Such a distribution was referred to as a modified NBD. Since then, several of the statistical distributions have been modified in the similar way and used for describing the multiplicity distributions at different energies. Some of these are gamma distribution [14,15], Tsallis distribution [16,17], the shifted Gompertz distribution [18,19], and the Weibull distribution [20,21] for the description of particle production which successfully explain the multiplicity distributions in different kinds of collisions. The charged particle multiplicity at the LHC has been measured by CMS, ATLAS, and ALICE experiments [22–24] mainly in the central region. While the LHCb experiment is the only experiment to measure it in the forward region at $\sqrt{s} = 7$ TeV [25], it studied the multiplicity distributions in different phase space slices, in comparison to predictions from several Monte Carlo event generators.

In the present analysis, the first study of multiplicity distributions in pp data collected by the LHCb Collaboration in the forward region of the detector in terms of three distributions namely, the negative binomial (NB), shifted Gompertz (SG), and Weibull (WB) distributions, is reported. The forward region spanning the

*manjit@pu.ac.in

pseudorapidity η range between $-2.0 < \eta < -2.5$ and $2.0 < \eta < 4.5$ with a further division into smaller pseudorapidity windows is studied. The study of the forward region is particularly interesting as the region is sensitive to low Bjorken-x QCD, which plays an important role in multipartonic interactions (MPI) and in the understanding of interaction dynamics. In addition to the multiplicity distributions, the other standard physical observables are the normalized moments (C_q), the normalized factorial moments (F_q), and normalized factorial cumulants (K_q). The ratio of the cumulants to factorial moments has also been widely studied. We give an outline of the models used in the following section.

The paper is organized as follows: we describe the essential details required for the calculations of probability distributions NBD, SGD, and WB and the data used in analysis in Sec. II. Section III gives results obtained from the comparison of our analysis of the three distributions, analysis of moments, and cumulants of moments followed by the conclusions in Sec. IV.

II. DISTRIBUTIONS AND THE PARAMETRIZATIONS

Charged particle multiplicity can be characterized by a function $P(n|\tilde{\omega})$, which determines the probability of producing n charged particles in an interaction, given a set of parameters $\tilde{\omega}$. The function $P(n|\tilde{\omega})$ may represent a probability distribution function (PDF) predicting the distribution of number of particles according to the distribution. The mean of this distribution gives the average number of particles produced. We discuss PDFs of the three distributions below.

A. NBD

The following probability distribution function defines the distribution known as the negative binomial distribution in the variable n:

PDF,

$$P(n|\langle n\rangle, k) = \binom{n+k-1}{k-1}\left(\frac{\langle n\rangle/k}{1+\langle n\rangle/k}\right)^n \times (1+\langle n\rangle/k)^{-k}. \quad (1)$$

In the general case, the binomial coefficient is written as $k(k+1)\ldots..(k+n-1)/n!$ when the positive parameter k is not an integer. The first parameter, n determines the position, being equal to the expected average, $\langle n\rangle$, and k influences the shape of the distribution.

B. SGD

The shifted Gompertz distribution has two independent random variables, one of which has an exponential distribution with a parameter b and the other has a Gumbel distribution, also known as log-Weibull distribution, with parameters β and b. We proposed to use this distribution for studying the collision data obtained from the high energy colliders, the Super Proton Synchrotron (SPS), the Large Electron Positron Collider (LEP), and the Large Hadron Collider (LHC). These data are for $\bar{p}p$, e^+e^-, and pp collisions at various c.m.s. energies. In the detailed studies, we have shown that SGD describes the data trends very well [18,19,26]. To fit the data, the probability density function (PDF) is described by two nonnegative free parameters, b, the scale parameter and η, determining the shape of the distribution. The following equations define the distribution:

PDF,

$$P(n|b,\beta) = be^{-bn}e^{-\beta e^{-bn}}[1+\beta(1-e^{-bn})] \quad \text{for } n > 0. \quad (2)$$

The mean of the distribution,

$$\left(-\frac{1}{b}\right)(E[ln(Y)] - ln(\beta)) \quad \text{where } Y = \beta e^{-bn} \quad (3)$$

$$E[ln(Y)] = \left[1+\frac{1}{\beta}\right]\int_0^\infty e^{-Y}[ln(Y)]dY - \frac{1}{\beta}\int_0^\infty Ye^{-Y}[ln(Y)]dY, \quad (4)$$

where $b \geq 0$ and $\beta \geq 0$.

Though the validity of SGD has been tested by us recently for the charged particle multiplicity distribution in the pp collision data at $\sqrt{s} \sim 7$ TeV, collected by the CMS Collaboration [22] in the central region, this is the first analysis of the data collected by the LHCb experiment [25] in the forward region.

C. WB

The charged multiplicity data from variety of collision types and energies, as mentioned before, have also been analyzed using Weibull distribution [20,21,27]. The Weibull distribution is also a two parameter distribution. In its standard form, these two parameters represent the scale and shape of the distribution. The two parameter Weibull has been used during the last few years to describe the collision data from high energy experiments. The probability distribution function can be defined as below.

PDF,

$$P_N(N|\lambda, K) = \begin{cases} \frac{K}{\lambda}\left(\frac{N}{\lambda}\right)^{(K-1)}\exp^{-\left(\frac{N}{\lambda}\right)^K} & N \geq 0 \\ 0 & N < 0 \end{cases}. \quad (5)$$

$\lambda > 0$ is a scale parameter $\lambda > 0$, and $K > 0$ is the shape parameter.

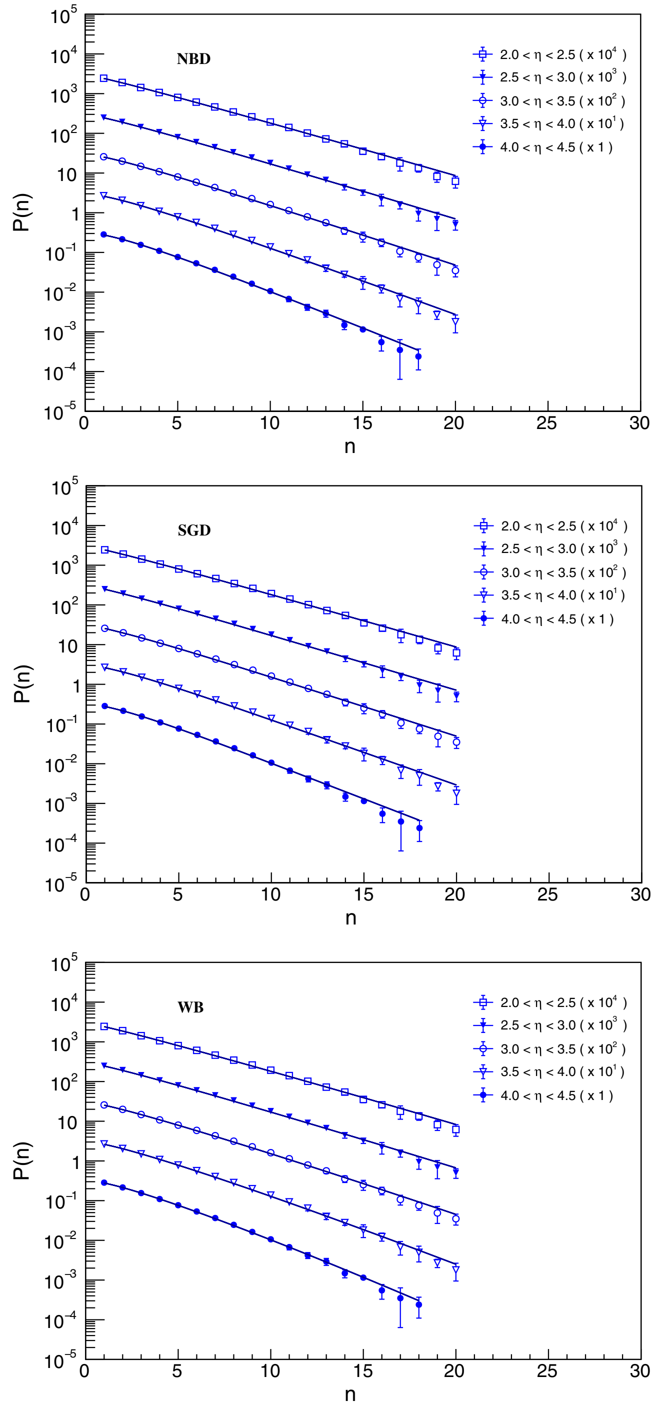

FIG. 1. Data on charged particle multiplicity distributions in pp minimum bias events at $\sqrt{s} = 7$ TeV. Points show the data and solid lines are the fits for various distributions (top to bottom) in different pseudorapidity intervals.

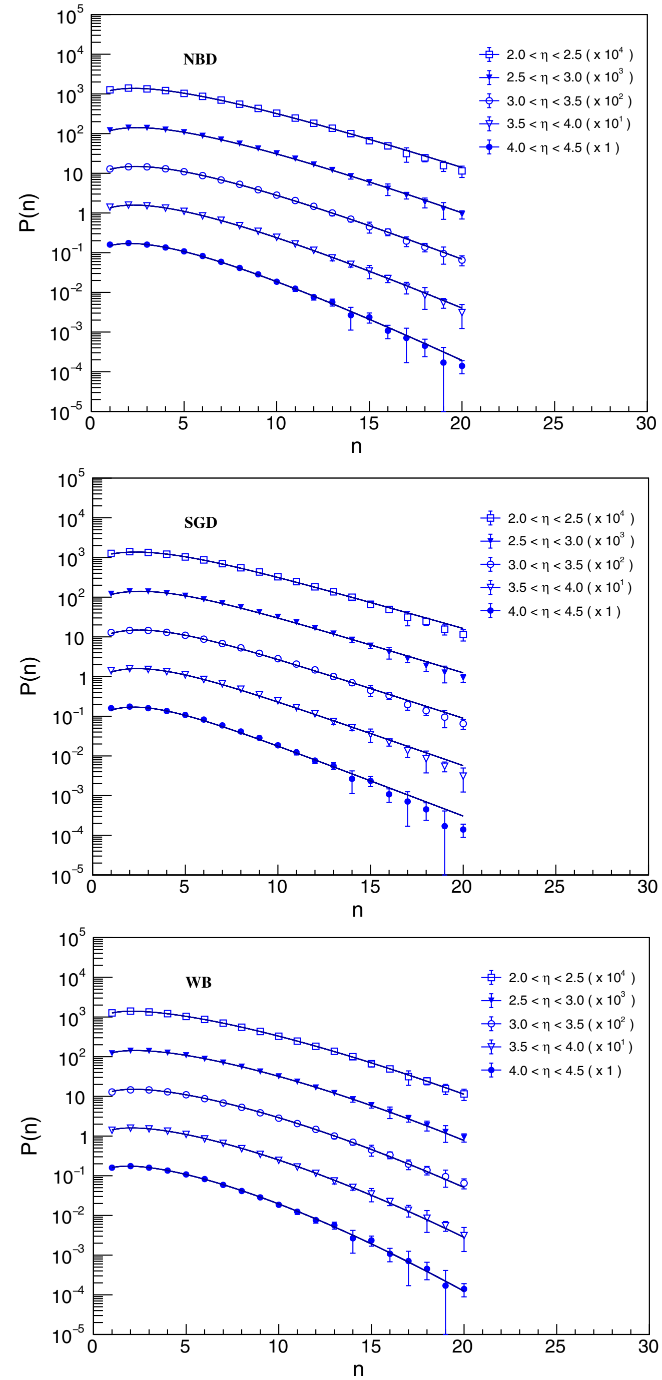

FIG. 2. Data on charged particle multiplicity distributions in pp hard QCD events at $\sqrt{s} = 7$ TeV. Points show the data and solid lines are the fits for various distributions (top to bottom) in different pseudorapidity intervals.

Mean of the distribution function is given by

$$\bar{N} = \lambda\Gamma(1 + 1/K). \tag{6}$$

For a multiplicity distribution, the normalized moments C_q, normalized factorial moments (F_q), normalized factorial cumulants (K_q), and ratio of the two (H_q) moments are defined as

$$C_q = \frac{\sum_{n=1}^{\infty} n^q P(n)}{(\sum_{n=1}^{\infty} nP(n))^q} \tag{7}$$

TABLE I. Fit parameters of three distributions for minimum bias events.

NBD				
η	k	$\langle n\rangle$	χ^2/ndf	p value
$-2.5 < \eta < -2.0$	1.059 ± 0.060	3.085 ± 0.055	2.79/15	1.00
$2.0 < \eta < 2.5$	1.116 ± 0.040	3.045 ± 0.045	14.16/17	0.66
$2.5 < \eta < 3.0$	1.149 ± 0.041	2.955 ± 0.045	14.52/17	0.63
$3.0 < \eta < 3.5$	1.226 ± 0.045	2.829 ± 0.043	12.63/17	0.76
$3.5 < \eta < 4.0$	1.350 ± 0.049	2.689 ± 0.041	15.83/17	0.54
$4.0 < \eta < 4.5$	1.451 ± 0.067	2.508 ± 0.043	8.18/14	0.88
$2.0 < \eta < 4.5$	1.294 ± 0.020	12.956 ± 0.102	213.82/36	<0.01
SGD				
η	β	b	χ^2/ndf	p value
$-2.5 < \eta < -2.0$	0.061 ± 0.058	0.292 ± 0.009	2.70/15	1.00
$2.0 < \eta < 2.5$	0.110 ± 0.037	0.305 ± 0.005	14.18/17	0.65
$2.5 < \eta < 3.0$	0.137 ± 0.038	0.318 ± 0.005	14.80/17	0.61
$3.0 < \eta < 3.5$	0.200 ± 0.039	0.343 ± 0.005	13.72/17	0.69
$3.5 < \eta < 4.0$	0.300 ± 0.042	0.378 ± 0.005	19.07/17	0.32
$4.0 < \eta < 4.5$	0.370 ± 0.052	0.414 ± 0.007	10.48/14	0.73
$2.0 < \eta < 4.5$	0.409 ± 0.029	0.092 ± 0.002	329.70/36	<0.01
WB				
η	K	λ	χ^2/ndf	p value
$-2.5 < \eta < -2.0$	1.021 ± 0.019	3.569 ± 0.057	2.58/15	1.00
$2.0 < \eta < 2.5$	1.040 ± 0.012	3.536 ± 0.047	12.33/17	0.78
$2.5 < \eta < 3.0$	1.049 ± 0.012	3.446 ± 0.047	12.23/17	0.79
$3.0 < \eta < 3.5$	1.068 ± 0.012	3.319 ± 0.044	9.60/17	0.92
$3.5 < \eta < 4.0$	1.097 ± 0.012	3.176 ± 0.042	10.62/17	0.88
$4.0 < \eta < 4.5$	1.120 ± 0.015	2.991 ± 0.044	4.78/14	0.99
$2.0 < \eta < 4.5$	1.125 ± 0.008	13.662 ± 0.103	254.03/36	<0.01

$$F_q = \frac{\sum_{n=q}^{\infty} n(n-1)\ldots\ldots(n-q+1)P(n)}{(\sum_{n=1}^{\infty} nP(n))^q} \tag{8}$$

$$K_q = F_q - \sum_{m=1}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} K_{q-m} F_m \tag{9}$$

$$H_q = K_q/F_q. \tag{10}$$

D. The data used

Charged particle multiplicity distributions at $\sqrt{s} = 7$ TeV, collected by the LHCb Collaboration [25] using the vertex detector (VELO), have been analyzed. The vertex detector has been designed to provide a uniform acceptance in the forward region with additional coverage of the backward region. Particle multiplicity is measured using only tracks reconstructed with the VELO. Further the tracks are considered only if their pseudorapdity lies either in the range $-2.5 < \eta < -2.0$ or $2.0 < \eta < 4.5$. The measurements are done in the forward range divided into five pseudorapidity windows with a size $\Delta\eta = 0.5$. We have analyzed the distributions in each of these windows separately. Two samples of data are available: (i) the minimum bias events which have one or more reconstructed tracks in the vertex detector and (ii) the hard QCD events with each event having at least one track with transverse momentum >1 GeV/c.

III. RESULTS

LHCb studied the experimentally measured charged particle multiplicity distributions in different pseudorapidity windows and also in the full forward region by comparing with several event generators [25]. None are able to describe fully the multiplicity distributions as a function of η. In general, the models were found to underestimate the data. In the present paper, we study the experimental distributions from a different perspective.

The experimental charged multiplicity distributions are studied with the PDFs from the negative binomial, shifted Gompertz, and Weibull distributions. All these distributions are two parameter distributions, namely scale and shape

TABLE II. Fit parameters of three distributions for hard QCD events.

NBD				
η	k	$\langle n\rangle$	χ^2/ndf	p value
$-2.5 < \eta < -2.0$	1.748 ± 0.076	4.645 ± 0.061	6.34/16	0.98
$2.0 < \eta < 2.5$	2.396 ± 0.090	4.846 ± 0.054	6.91/17	0.98
$2.5 < \eta < 3.0$	2.693 ± 0.101	4.754 ± 0.053	2.42/17	1.00
$3.0 < \eta < 3.5$	2.821 ± 0.111	4.520 ± 0.052	1.58/17	1.00
$3.5 < \eta < 4.0$	2.964 ± 0.123	4.229 ± 0.050	2.47/17	1.00
$4.0 < \eta < 4.5$	3.078 ± 0.140	3.859 ± 0.049	6.63/17	0.99
$2.0 < \eta < 4.5$	2.997 ± 0.051	21.926 ± 0.215	12.77/36	1.00
SGD				
η	β	b	χ^2/ndf	p value
$-2.5 < \eta < -2.0$	0.732 ± 0.063	0.273 ± 0.006	4.69/16	1.00
$2.0 < \eta < 2.5$	1.266 ± 0.071	0.304 ± 0.004	13.46/17	0.71
$2.5 < \eta < 3.0$	1.493 ± 0.076	0.326 ± 0.004	10.31/17	0.89
$3.0 < \eta < 3.5$	1.570 ± 0.081	0.347 ± 0.005	9.56/17	0.92
$3.5 < \eta < 4.0$	1.636 ± 0.086	0.374 ± 0.005	15.78/17	0.54
$4.0 < \eta < 4.5$	1.649 ± 0.092	0.409 ± 0.005	34.36/17	0.01
$2.0 < \eta < 4.5$	3.202 ± 0.059	0.095 ± 0.001	79.50/36	<0.01
WB				
η	K	λ	χ^2/ndf	p value
$-2.5 < \eta < -2.0$	1.239 ± 0.019	5.196 ± 0.063	3.23/16	1.00
$2.0 < \eta < 2.5$	1.368 ± 0.017	5.452 ± 0.058	1.32/17	1.00
$2.5 < \eta < 3.0$	1.413 ± 0.017	5.367 ± 0.057	2.63/17	1.00
$3.0 < \eta < 3.5$	1.421 ± 0.018	5.123 ± 0.056	3.86/17	1.00
$3.5 < \eta < 4.0$	1.426 ± 0.018	4.816 ± 0.054	2.42/17	1.00
$4.0 < \eta < 4.5$	1.420 ± 0.019	4.414 ± 0.052	1.95/17	1.00
$2.0 < \eta < 4.5$	1.826 ± 0.014	23.565 ± 0.231	64.72/36	<0.01

parameters. The PDFs are calculated by using Eqs. (1)–(6) and matching with the data by carrying out minimum χ^2 fits using ROOT6.18.

A. Comparison of PDFs of different distributions of multiplicities

Fits to the data for minimum bias events are shown in Fig. 1 for five pseudorapidity windows. Figure 2 shows the similar figures for the hard QCD events. Tables I and II give the parameters of the fits and the corresponding χ^2/ndf and p values for both minimum bias events and hard QCD events, for all the distributions.

One finds that in comparison to the minimum bias data, the multiplicity distributions for hard QCD events have larger high-multiplicity tails. For almost all finely binned pseudorapidity intervals, the NBD, SGD, and WB distributions reproduce the data very well. The hard QCD fits are far better than the corresponding minimum bias distributions. However, in the wider pseudorapidity interval, $2.0 < \eta < 4.5$, SGD and WB show a very large χ^2/ndf and are statistically excluded with p values corresponding to $CL < 0.10\%$ for both inelastic and NSD events. The exception is the case of NBD which fits the multiplicity distribution of hard QCD events in $2.0 < \eta < 4.5$ very well but fails in the case of minimum bias events. In addition, the pseudorapidity range $4.0 < \eta < 4.5$ remains poorly described in SGD for this category. Most of these observations agree with the observations made by LHCb [25] but in a different study using event generators.

It has been well established, since the observations made by UA5 Collaboration, [10] that the multiplicity distributions at higher collision energies show a shoulder structure. This feature however is not present in the data for finely sliced η bins as shown in Figs. 1 and 2. However, in the wider η range of 2.0-to-4.5, the shoulder structure in the multiplicity distribution is present, as shown in Fig. 3. We also observe that all of the three fits fail in this η range, of the forward region for the minimum bias events. It was proposed by A. Giovannini *et al.* [28] that the observed shoulder structure can be described by using a weighted superposition of two component distributions. One describing the soft event distribution and another describing the semihard event distribution, with each distribution

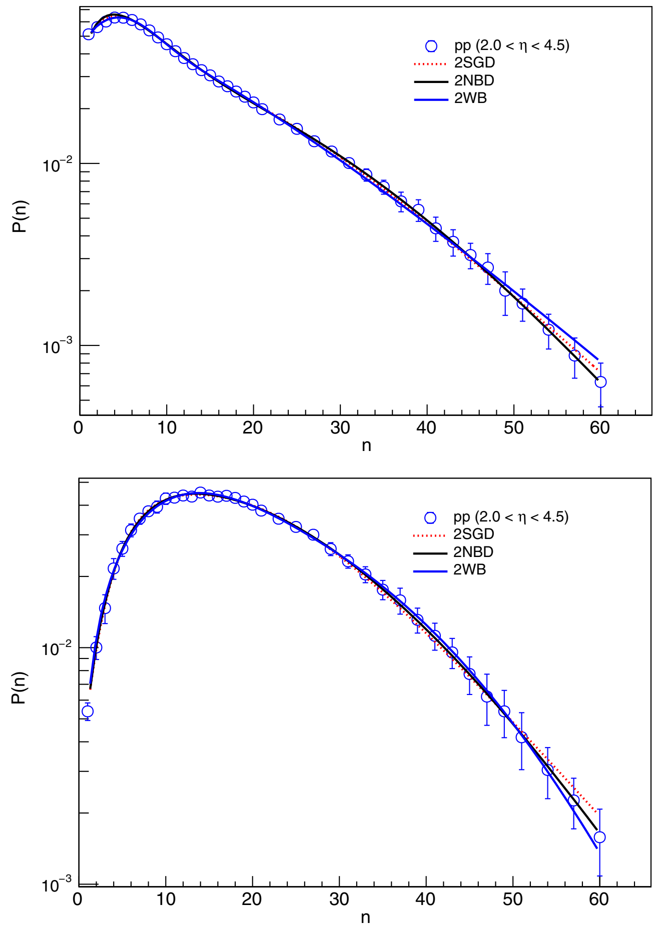

FIG. 3. Data on charged particle multiplicity distributions in pp minimum bias (top) and hard QCD events (bottom) at $\sqrt{s} = 7$ TeV with fits from 2NBD, 2SGD, and 2WB distributions in the forward pseudorapidity region ($\eta = 2.0$ to 4.5.)

following the NBD. Adopting this approach, we redefine each of the probability distributions, NBD, SGD, and WB in terms of weighted superposition of two component distributions and fit the data accordingly as follows:

$$P(n)^X = \alpha P(n)^X_{\text{soft}} + (1-\alpha)P(n)^X_{\text{semihard}}, \qquad (11)$$

where X stands for NBD, SGD, or WB distribution.

Figure 3 shows the fits of the convolution of two component distributions, and we label them as 2NBD, 2SGD, and 2WB. The corresponding fit parameters are given in Table III. It is observed that the data fit perfectly well with each of the modified distributions with minimum χ^2/ndf values, due to which the p values in each case turn out to be very nearly 1.0.

An interesting observation is the oscillations of H_q as a function of the rank q obtained from data. These are reproduced by the ratio of cumulants, K_q/F_q calculated from the 2NBD, 2SGD, or 2WB. This leads to the fact that the second multiplicity component is connected with cumulants, each of which involves an infinite cumulative sum over all multiplicity probabilities, as shown in Eq. (8). The next section describes the moment analysis.

B. Moments of multiplicity distributions

The possibility of discovering correlations amongst the charged particles produced in collisions, higher-order moments and the cumulants are the precise tools [29]. The deviation with respect to independent and uncorrelated production of particles can be measured well using the factorial moments, F_q [30]. Figures 4 and 5 show for NBD, SGD, and WB distributions, the normalized moments C_q [Eq. (7)] for the two categories of events: minimum bias and hard-QCD events. Similarly Figs. 6 and 7 show the normalized factorial moments F_q [Eqs. (8)]. The values for all the moments are given in Tables IV and V. Table VI

TABLE III. Fit parameter values, χ^2/ndf, and p values obtained for the minimum bias and hard QCD events from 2NBD, 2SGD, and 2WB models.

	2 NBD						
Events	$k1$	$\langle n1\rangle$	α	$k2$	$\langle n2\rangle$	χ^2/ndf	p value
Minimum bias	2.157 ± 0.161	7.595 ± 0.845	0.623 ± 0.086	4.398 ± 1.139	24.068 ± 2.238	14.90/33	1.00
Hard QCD	3.754 ± 0.608	13.321 ± 7.313	0.402 ± 0.663	5.502 ± 3.940	27.277 ± 8.259	4.41/33	1.00
	2 SGD						
Events	$\beta 1$	$b1$	α	$\beta 2$	$b2$	χ^2/ndf	p value
Minimum bias	1.362 ± 0.072	0.198 ± 0.015	0.640 ± 0.057	5.434 ± 1.552	0.098 ± 0.004	7.75/33	1.00
Hard QCD	3.572 ± 0.194	0.184 ± 0.040	0.285 ± 0.204	5.490 ± 2.720	0.094 ± 0.004	6.91/33	1.00
	2 WB						
Events	$K1$	$\lambda 1$	α	$K2$	$\lambda 2$	χ^2/ndf	p value
Minimum bias	2.248 ± 0.236	6.797 ± 0.222	0.086 ± 0.018	1.171 ± 0.018	15.728 ± 0.314	6.84/33	1.00
Hard QCD	2.023 ± 0.143	14.736 ± 1.362	0.258 ± 0.150	2.001 ± 0.181	27.644 ± 2.311	6.42/33	1.00

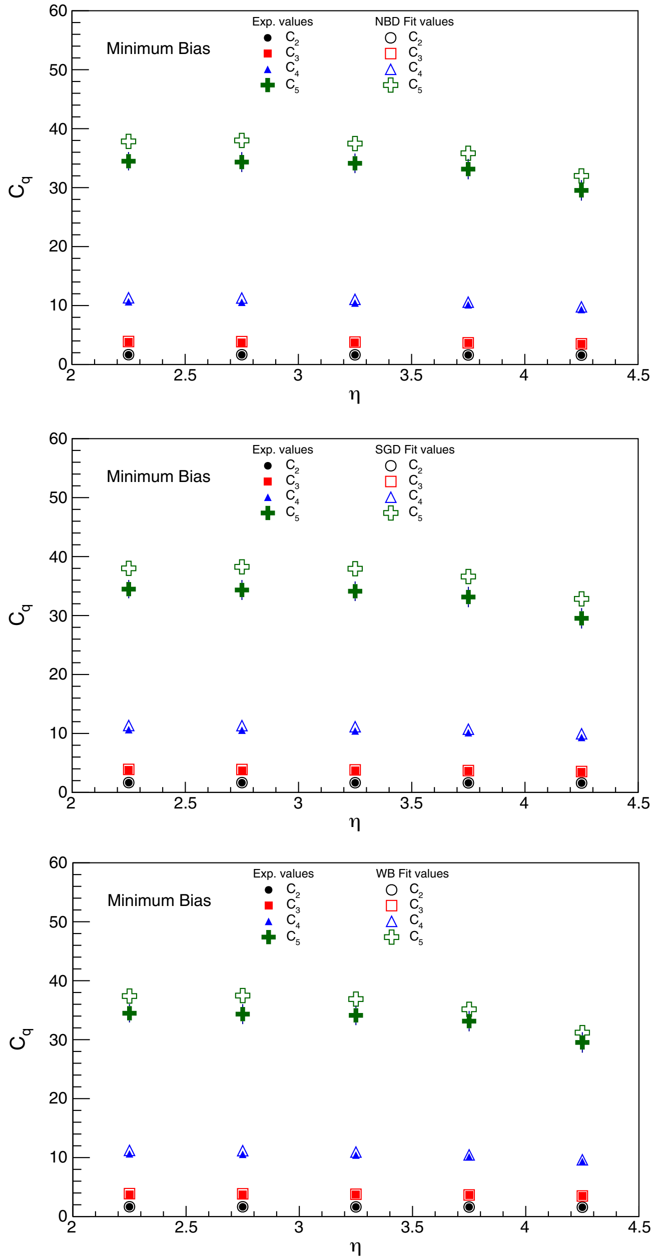

FIG. 4. Normalized moments of multiplicity distributions for minimum bias events in different pseudorapidity bins with bin size $\Delta\eta = 0.5$. Experimental values are shown in comparison to the fit values.

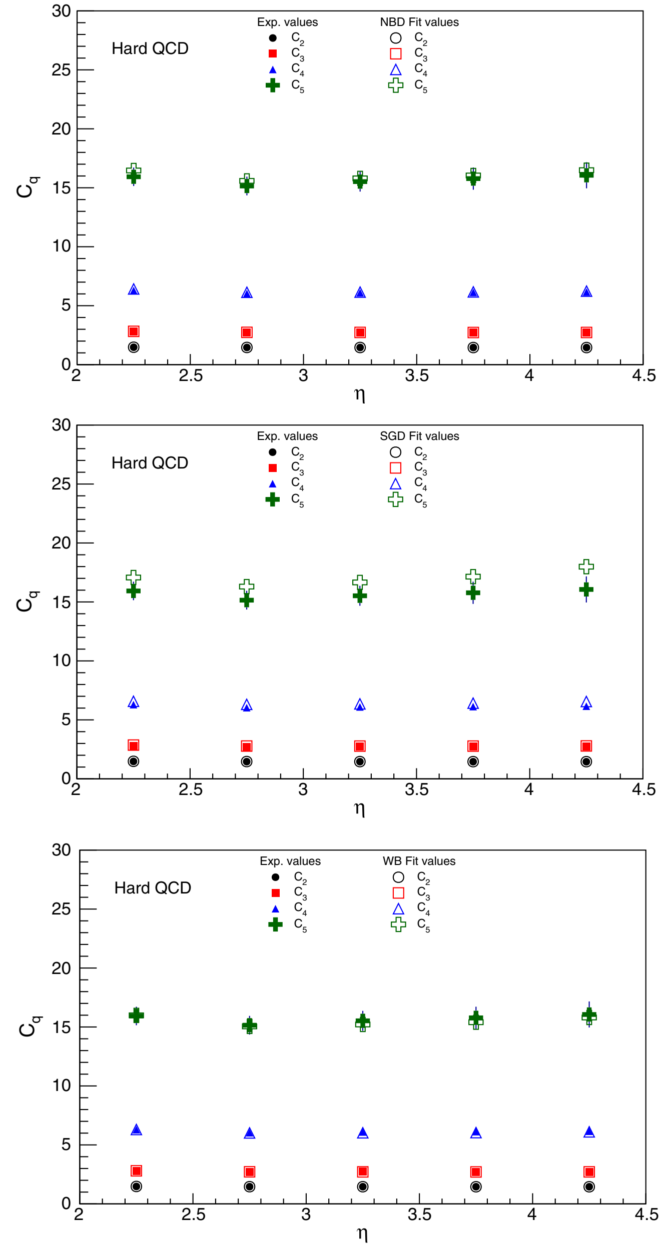

FIG. 5. Normalized moments of multiplicity distributions for hard-QCD events in different pseudorapidity bins with bin size $\Delta\eta = 0.5$. Experimental values are shown in comparison to the fit values.

gives the values of the normalized moments C_q and normalized factorial moments F_q for the distributions 2NBD, 2SGD, and 2WB. Table IV summarize the values of normalized moments C_q and normalized factorial moments F_q with q 2,3,4,5. The following observations can be made: (i) the values of moments, both C_q and F_q remain constant in different η bins, with the exception of the bin $4.0 < \eta < 4.5$, in which the value is consistently lower for all moments. Although the value is low, it agrees with the experimental value. (ii) All values of moments C_q and F_q calculated from different distributions agree very well within the limits of error for $q = 2$, 3, 4 with the experimental values. However, the fit distributions overestimate the values of moments for $q = 5$. On comparison between the two categories of events, it is observed that the discrepancies between the fit values and data for $q = 5$ moments are more pronounced for minimum bias events. The shape of the charged-particle multiplicity distribution analyzed in terms of the H_q shows quasioscillations. As early as 1975, following the solution of QCD equations for the generating function, a special oscillation pattern for the

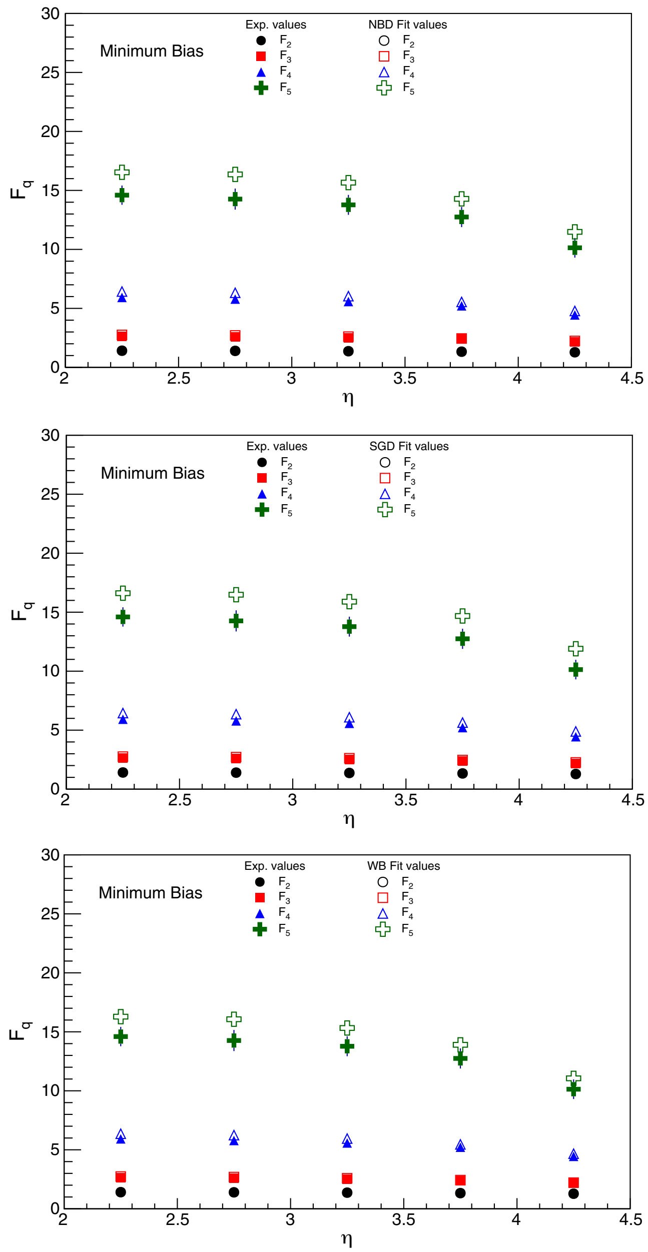

FIG. 6. Normalized factorial moments of multiplicity distributions for minimum bias events in different pseudorapidity bins with bin size $\Delta\eta = 0.5$. Experimental values are shown in comparison to the fit values.

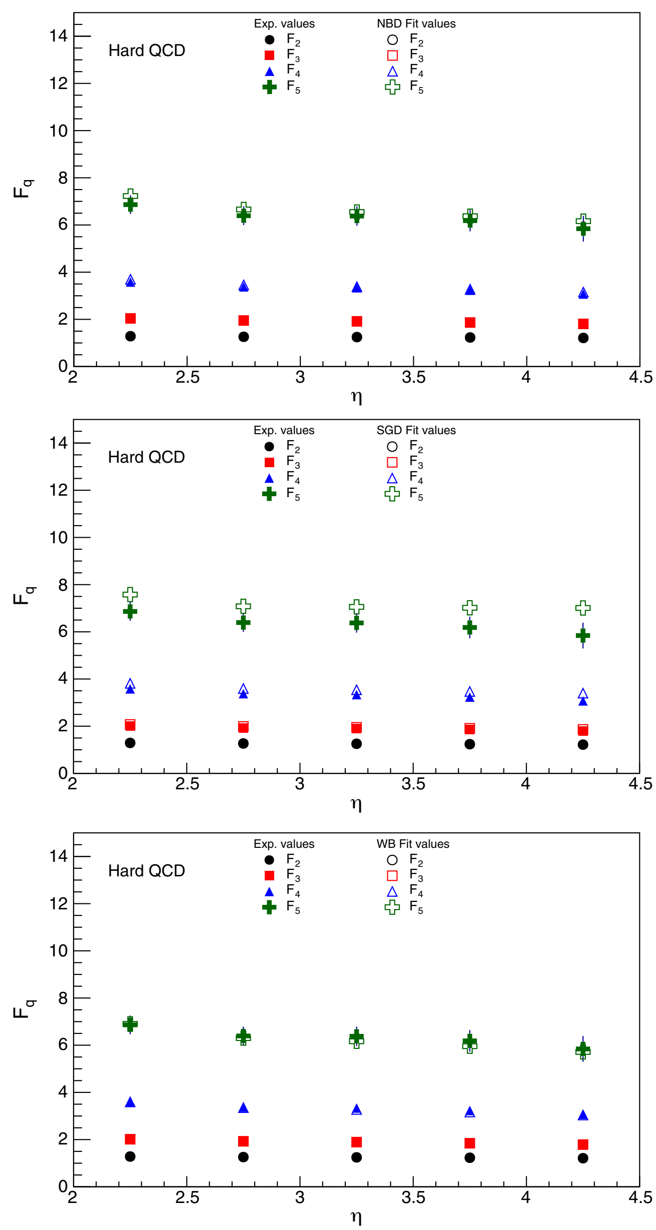

FIG. 7. Normalized factorial moments of multiplicity distributions for hard-QCD events in different pseudorapidity bins with bin size $\Delta\eta = 0.5$. Experimental values are shown in comparison to the fit values.

ratio of cumulants to factorial moments $H_q = K_q/F_q$ was predicted with the first minimum occurring around $q_{\min} \sim 5$ and determined by the inverse value of the QCD anomalous dimension,

$$\gamma_0 = (2N_c\alpha_s/\pi)^{1/2}, \tag{12}$$

where α_s is the QCD running coupling constant, $N_c = 3$ the number of colors. Details can be found in Refs. [31–35]. However, this prediction was supposedly valid for the moments of parton multiplicity distributions, especially for gluons. When the same analysis was done for the final hadronic multiplicities in e^+e^- and $pp/\bar{p}p$ experiments [36], similar oscillations in the H_q ratio were observed. In the present work, we study the behavior of moments to check whether the multiplicity distributions in the forward region also follow the same trends. We analyze both minimum bias events and hard QCD events using a single as well as two component distributions.

Figures 8 and 9 show the ratio H_q [Eq. (10)] as a function of the rank q for the data, NBD, SGD, and WB distributions, for the two categories of events: minimum bias and

TABLE IV. Comparison of experimental normalized moments and normalized factorial moments of multiplicity distributions of minimum bias events with fit values from three distributions.

η	C_2	C_3	C_4	C_5	F_2	F_3	F_4	F_5
Experimental values (minimum bias)								
$-2.5 < \eta < -2.0$	1.648 ± 0.029	3.72 ± 0.14	10.19 ± 0.63	31.60 ± 2.75	1.393 ± 0.027	2.59 ± 0.11	5.57 ± 0.41	12.79 ± 1.38
$2.0 < \eta < 2.5$	1.652 ± 0.013	3.78 ± 0.07	10.65 ± 0.33	34.48 ± 1.56	1.395 ± 0.012	2.64 ± 0.05	5.93 ± 0.22	14.60 ± 0.82
$2.5 < \eta < 3.0$	1.644 ± 0.013	3.75 ± 0.07	10.56 ± 0.35	34.33 ± 1.69	1.381 ± 0.012	2.59 ± 0.06	5.79 ± 0.24	14.26 ± 0.89
$3.0 < \eta < 3.5$	1.632 ± 0.014	3.70 ± 0.07	10.43 ± 0.34	34.12 ± 1.66	1.359 ± 0.013	2.51 ± 0.06	5.58 ± 0.22	13.78 ± 0.84
$3.5 < \eta < 4.0$	1.614 ± 0.014	3.62 ± 0.07	10.13 ± 0.35	33.14 ± 1.73	1.327 ± 0.013	2.40 ± 0.06	5.22 ± 0.23	12.75 ± 0.85
$4.0 < \eta < 4.5$	1.580 ± 0.015	3.44 ± 0.07	9.33 ± 0.35	29.53 ± 1.74	1.272 ± 0.013	2.17 ± 0.06	4.44 ± 0.22	10.14 ± 0.83
$2.0 < \eta < 4.5$	1.665 ± 0.009	3.86 ± 0.05	11.15 ± 0.25	37.06 ± 1.20	1.579 ± 0.009	3.45 ± 0.05	9.29 ± 0.22	28.45 ± 0.99
NBD fit values								
$-2.5 < \eta < -2.0$	1.656 ± 0.009	3.78 ± 0.04	10.53 ± 0.17	33.24 ± 0.71	1.402 ± 0.008	2.65 ± 0.03	5.84 ± 0.10	13.79 ± 0.32
$2.0 < \eta < 2.5$	1.667 ± 0.006	3.90 ± 0.03	11.31 ± 0.12	37.84 ± 0.53	1.412 ± 0.005	2.75 ± 0.02	6.43 ± 0.07	16.53 ± 0.26
$2.5 < \eta < 3.0$	1.660 ± 0.006	3.88 ± 0.03	11.28 ± 0.12	38.00 ± 0.56	1.398 ± 0.006	2.71 ± 0.02	6.32 ± 0.07	16.36 ± 0.27
$3.0 < \eta < 3.5$	1.643 ± 0.007	3.80 ± 0.03	11.05 ± 0.13	37.47 ± 0.58	1.370 ± 0.006	2.60 ± 0.02	6.04 ± 0.08	15.66 ± 0.28
$3.5 < \eta < 4.0$	1.617 ± 0.007	3.68 ± 0.03	10.57 ± 0.13	35.83 ± 0.59	1.329 ± 0.006	2.45 ± 0.02	5.55 ± 0.07	14.28 ± 0.27
$4.0 < \eta < 4.5$	1.587 ± 0.008	3.51 ± 0.03	9.78 ± 0.14	31.99 ± 0.63	1.280 ± 0.007	2.24 ± 0.02	4.78 ± 0.08	11.49 ± 0.27
$2.0 < \eta < 4.5$	1.668 ± 0.002	3.90 ± 0.01	11.51 ± 0.06	39.74 ± 0.31	1.579 ± 0.002	3.47 ± 0.01	9.56 ± 0.05	30.46 ± 0.25
SGD fit values								
$-2.5 < \eta < -2.0$	1.656 ± 0.009	3.78 ± 0.04	10.52 ± 0.17	33.22 ± 0.70	1.402 ± 0.008	2.65 ± 0.03	5.84 ± 0.10	13.77 ± 0.32
$2.0 < \eta < 2.5$	1.669 ± 0.006	3.91 ± 0.03	11.35 ± 0.11	38.01 ± 0.53	1.413 ± 0.005	2.76 ± 0.02	6.45 ± 0.07	16.61 ± 0.26
$2.5 < \eta < 3.0$	1.662 ± 0.006	3.89 ± 0.03	11.33 ± 0.12	38.26 ± 0.55	1.399 ± 0.006	2.71 ± 0.02	6.35 ± 0.07	16.48 ± 0.27
$3.0 < \eta < 3.5$	1.646 ± 0.007	3.82 ± 0.03	11.14 ± 0.13	37.94 ± 0.59	1.372 ± 0.006	2.62 ± 0.02	6.09 ± 0.08	15.89 ± 0.28
$3.5 < \eta < 4.0$	1.621 ± 0.007	3.70 ± 0.03	10.72 ± 0.13	36.62 ± 0.60	1.332 ± 0.006	2.47 ± 0.02	5.64 ± 0.07	14.68 ± 0.27
$4.0 < \eta < 4.5$	1.591 ± 0.008	3.54 ± 0.03	9.94 ± 0.14	32.83 ± 0.64	1.284 ± 0.007	2.27 ± 0.02	4.89 ± 0.08	11.90 ± 0.27
$2.0 < \eta < 4.5$	1.676 ± 0.002	3.93 ± 0.01	11.62 ± 0.07	40.31 ± 0.34	1.585 ± 0.002	3.49 ± 0.01	9.63 ± 0.06	30.84 ± 0.28
WB fit values								
$-2.5 < \eta < -2.0$	1.655 ± 0.009	3.78 ± 0.04	10.49 ± 0.17	33.09 ± 0.71	1.401 ± 0.008	2.64 ± 0.03	5.81 ± 0.10	13.70 ± 0.33
$2.0 < \eta < 2.5$	1.665 ± 0.006	3.88 ± 0.03	11.22 ± 0.11	37.40 ± 0.53	1.409 ± 0.005	2.73 ± 0.02	6.36 ± 0.07	16.29 ± 0.26
$2.5 < \eta < 3.0$	1.657 ± 0.006	3.86 ± 0.03	11.17 ± 0.12	37.48 ± 0.55	1.395 ± 0.006	2.69 ± 0.02	6.24 ± 0.07	16.07 ± 0.27
$3.0 < \eta < 3.5$	1.641 ± 0.006	3.78 ± 0.03	10.94 ± 0.12	36.88 ± 0.58	1.367 ± 0.006	2.59 ± 0.02	5.95 ± 0.08	15.32 ± 0.28
$3.5 < \eta < 4.0$	1.615 ± 0.007	3.66 ± 0.03	10.45 ± 0.12	35.15 ± 0.57	1.327 ± 0.006	2.43 ± 0.02	5.46 ± 0.07	13.90 ± 0.26
$4.0 < \eta < 4.5$	1.584 ± 0.008	3.49 ± 0.03	9.62 ± 0.14	31.20 ± 0.62	1.277 ± 0.007	2.22 ± 0.02	4.67 ± 0.08	11.06 ± 0.26
$2.0 < \eta < 4.5$	1.670 ± 0.002	3.89 ± 0.01	11.44 ± 0.06	39.35 ± 0.33	1.579 ± 0.002	3.46 ± 0.01	9.48 ± 0.06	30.08 ± 0.27

hard-QCD events for different pseudorapdity windows with $\Delta\eta = 0.5$. We find that the dependence of H_q on q is very similar in all the η bins with a minimum value around $q = 6$–7. For minimum bias events, there is a disagreement between the data and the fit values at the highest q values for all distributions. But for the hard QCD events, the agreement between the data and the fit values is very good for WB and NBD, with SGD also following the data closely for all distributions. There are slight discrepancies again towards the highest q values in SGD.

In one of the studies by I. M. Dremin [33], it was pointed that in gluodynamics, the gluon ratio H_q has the minimum around $q \approx 4$ or 5. The quark factorial moments are larger than those of gluon jets. First minimum of quark cumulants and of their ratio to factorial moments is positioned at $q \approx 8$. To translate the theoretical predictions to experimentally measured values implies a transition from the parton to the particle level and hence, perturbative QCD to nonperturbative QCD. This involves implementation of some hadronization models. The perturbative QCD (pQCD) controls the relevant observables to be measured at colliders, but its applicability fails to describe the evolution of partons into final hadrons that hit the detectors. That is why one advocates the local parton hadron duality hypothesis, which mainly consists in comparing the shape and normalization of the obtained distribution with the corresponding data sets. Therefore, the normalized moments of gluons and quarks should be just related to the moments of the observed processes. In a detailed study [37] of the e^+e^-, pp, and $\bar{p}p$ in a wide range of energies, it has been concluded that the qualitative features of the behavior of H_q are very similar in all processes. It is observed from the analysis of $\bar{p}p$ from the UA5 Collaboration, an abrupt descent and the subsequent

TABLE V. Comparison of experimental normalized moments and normalized factorial moments of multiplicity distributions of hard QCD events with fit values from three distributions.

η	C_2	C_3	C_4	C_5	F_2	F_3	F_4	F_5
Experimental values (hard QCD)								
$-2.5 < \eta < -2.0$	1.525 ± 0.016	3.01 ± 0.08	7.00 ± 0.34	18.23 ± 1.36	1.322 ± 0.016	2.17 ± 0.07	3.99 ± 0.26	7.78 ± 0.82
$2.0 < \eta < 2.5$	1.477 ± 0.014	2.80 ± 0.06	6.29 ± 0.21	15.93 ± 0.78	1.281 ± 0.012	2.01 ± 0.04	3.58 ± 0.14	6.86 ± 0.40
$2.5 < \eta < 3.0$	1.457 ± 0.015	2.72 ± 0.06	6.03 ± 0.22	15.15 ± 0.79	1.258 ± 0.013	1.93 ± 0.05	3.38 ± 0.14	6.39 ± 0.40
$3.0 < \eta < 3.5$	1.456 ± 0.015	2.73 ± 0.06	6.10 ± 0.23	15.52 ± 0.85	1.248 ± 0.013	1.91 ± 0.05	3.34 ± 0.14	6.38 ± 0.41
$3.5 < \eta < 4.0$	1.453 ± 0.016	2.72 ± 0.07	6.12 ± 0.25	15.78 ± 0.94	1.233 ± 0.014	1.86 ± 0.05	3.23 ± 0.15	6.18 ± 0.46
$4.0 < \eta < 4.5$	1.451 ± 0.016	2.72 ± 0.07	6.16 ± 0.28	16.06 ± 1.11	1.211 ± 0.015	1.79 ± 0.05	3.07 ± 0.18	5.84 ± 0.55
$2.0 < \eta < 4.5$	1.363 ± 0.012	2.33 ± 0.04	4.65 ± 0.14	10.39 ± 0.44	1.308 ± 0.011	2.10 ± 0.04	3.92 ± 0.12	8.04 ± 0.35
NBD fit values								
$-2.5 < \eta < -2.0$	1.535 ± 0.006	3.07 ± 0.02	7.28 ± 0.09	19.29 ± 0.33	1.337 ± 0.005	2.24 ± 0.02	4.24 ± 0.06	8.50 ± 0.17
$2.0 < \eta < 2.5$	1.479 ± 0.006	2.83 ± 0.02	6.42 ± 0.07	16.47 ± 0.26	1.285 ± 0.005	2.04 ± 0.02	3.70 ± 0.05	7.23 ± 0.13
$2.5 < \eta < 3.0$	1.459 ± 0.006	2.74 ± 0.02	6.14 ± 0.07	15.59 ± 0.25	1.261 ± 0.005	1.95 ± 0.02	3.46 ± 0.04	6.66 ± 0.12
$3.0 < \eta < 3.5$	1.456 ± 0.006	2.73 ± 0.02	6.15 ± 0.08	15.80 ± 0.27	1.248 ± 0.006	1.91 ± 0.02	3.38 ± 0.05	6.55 ± 0.13
$3.5 < \eta < 4.0$	1.451 ± 0.007	2.73 ± 0.02	6.17 ± 0.08	16.04 ± 0.30	1.231 ± 0.006	1.87 ± 0.02	3.28 ± 0.05	6.37 ± 0.14
$4.0 < \eta < 4.5$	1.450 ± 0.007	2.73 ± 0.03	6.23 ± 0.09	16.49 ± 0.33	1.211 ± 0.006	1.81 ± 0.02	3.16 ± 0.05	6.16 ± 0.14
$2.0 < \eta < 4.5$	1.365 ± 0.003	2.34 ± 0.01	4.72 ± 0.03	10.69 ± 0.11	1.309 ± 0.003	2.12 ± 0.01	3.99 ± 0.03	8.30 ± 0.09
SGD fit values								
$-2.5 < \eta < -2.0$	1.539 ± 0.006	3.09 ± 0.02	7.35 ± 0.09	19.53 ± 0.33	1.340 ± 0.005	2.25 ± 0.02	4.28 ± 0.06	8.62 ± 0.16
$2.0 < \eta < 2.5$	1.486 ± 0.006	2.86 ± 0.02	6.58 ± 0.08	17.07 ± 0.27	1.292 ± 0.005	2.07 ± 0.02	3.81 ± 0.05	7.58 ± 0.13
$2.5 < \eta < 3.0$	1.465 ± 0.006	2.78 ± 0.02	6.31 ± 0.07	16.30 ± 0.26	1.267 ± 0.005	1.99 ± 0.02	3.60 ± 0.04	7.08 ± 0.12
$3.0 < \eta < 3.5$	1.461 ± 0.006	2.78 ± 0.02	6.35 ± 0.08	16.66 ± 0.28	1.254 ± 0.006	1.95 ± 0.02	3.54 ± 0.05	7.06 ± 0.13
$3.5 < \eta < 4.0$	1.457 ± 0.007	2.77 ± 0.03	6.42 ± 0.09	17.15 ± 0.32	1.238 ± 0.006	1.91 ± 0.02	3.47 ± 0.05	7.02 ± 0.14
$4.0 < \eta < 4.5$	1.456 ± 0.007	2.79 ± 0.03	6.56 ± 0.09	17.99 ± 0.35	1.218 ± 0.007	1.86 ± 0.02	3.40 ± 0.05	7.01 ± 0.15
$2.0 < \eta < 4.5$	1.347 ± 0.003	2.27 ± 0.01	4.51 ± 0.03	10.09 ± 0.11	1.291 ± 0.003	2.05 ± 0.01	3.79 ± 0.03	7.80 ± 0.09
WB fit values								
$-2.5 < \eta < -2.0$	1.530 ± 0.006	3.04 ± 0.02	7.16 ± 0.09	18.86 ± 0.34	1.330 ± 0.005	2.21 ± 0.02	4.13 ± 0.06	8.22 ± 0.17
$2.0 < \eta < 2.5$	1.477 ± 0.006	2.80 ± 0.02	6.31 ± 0.07	15.99 ± 0.26	1.282 ± 0.005	2.02 ± 0.02	3.60 ± 0.05	6.90 ± 0.13
$2.5 < \eta < 3.0$	1.458 ± 0.005	2.72 ± 0.02	6.02 ± 0.07	15.05 ± 0.24	1.259 ± 0.005	1.93 ± 0.02	3.35 ± 0.04	6.30 ± 0.12
$3.0 < \eta < 3.5$	1.455 ± 0.006	2.71 ± 0.02	6.03 ± 0.07	15.21 ± 0.26	1.247 ± 0.005	1.89 ± 0.02	3.28 ± 0.04	6.17 ± 0.12
$3.5 < \eta < 4.0$	1.451 ± 0.007	2.71 ± 0.02	6.04 ± 0.08	15.40 ± 0.29	1.231 ± 0.006	1.84 ± 0.02	3.17 ± 0.05	5.96 ± 0.13
$4.0 < \eta < 4.5$	1.450 ± 0.007	2.71 ± 0.02	6.11 ± 0.09	15.81 ± 0.32	1.211 ± 0.006	1.79 ± 0.02	3.04 ± 0.05	5.71 ± 0.14
$2.0 < \eta < 4.5$	1.348 ± 0.003	2.24 ± 0.01	4.36 ± 0.03	9.45 ± 0.11	1.291 ± 0.003	2.02 ± 0.01	3.64 ± 0.03	7.22 ± 0.09

TABLE VI. Comparison of experimental normalized moments and normalized factorial moments of multiplicity distributions of minimum bias and hard QCD events with fit values from 2NBD, 2SGD, and 2 WB distributions.

	C_2	C_3	C_4	C_5	F_2	F_3	F_4	F_5
Minimum bias								
Experiment	1.665 ± 0.009	3.86 ± 0.05	11.15 ± 0.25	37.06 ± 1.20	1.579 ± 0.009	3.45 ± 0.05	9.29 ± 0.22	28.45 ± 0.99
2NBD fit	1.668 ± 0.003	3.88 ± 0.02	11.22 ± 0.09	37.40 ± 0.46	1.582 ± 0.003	3.46 ± 0.02	9.35 ± 0.08	28.70 ± 0.39
2SGD fit	1.666 ± 0.003	3.88 ± 0.02	11.24 ± 0.09	37.67 ± 0.42	1.580 ± 0.003	3.46 ± 0.02	9.37 ± 0.08	28.94 ± 0.35
2 WB fit	1.668 ± 0.003	3.90 ± 0.02	11.43 ± 0.08	38.72 ± 0.37	1.583 ± 0.003	3.49 ± 0.02	9.55 ± 0.07	29.86 ± 0.30
Hard QCD								
Experiment	1.363 ± 0.012	2.33 ± 0.04	4.65 ± 0.14	10.39 ± 0.44	1.308 ± 0.011	2.10 ± 0.04	3.92 ± 0.12	8.04 ± 0.35
2NBD fit	1.364 ± 0.004	2.33 ± 0.01	4.67 ± 0.05	10.47 ± 0.16	1.308 ± 0.004	2.11 ± 0.01	3.94 ± 0.04	8.10 ± 0.13
2SGD fit	1.366 ± 0.004	2.35 ± 0.01	4.74 ± 0.05	10.73 ± 0.15	1.310 ± 0.004	2.12 ± 0.01	4.00 ± 0.04	8.33 ± 0.12
2 WB fit	1.363 ± 0.004	2.32 ± 0.01	4.63 ± 0.05	10.27 ± 0.16	1.308 ± 0.004	2.10 ± 0.01	3.90 ± 0.04	7.94 ± 0.13

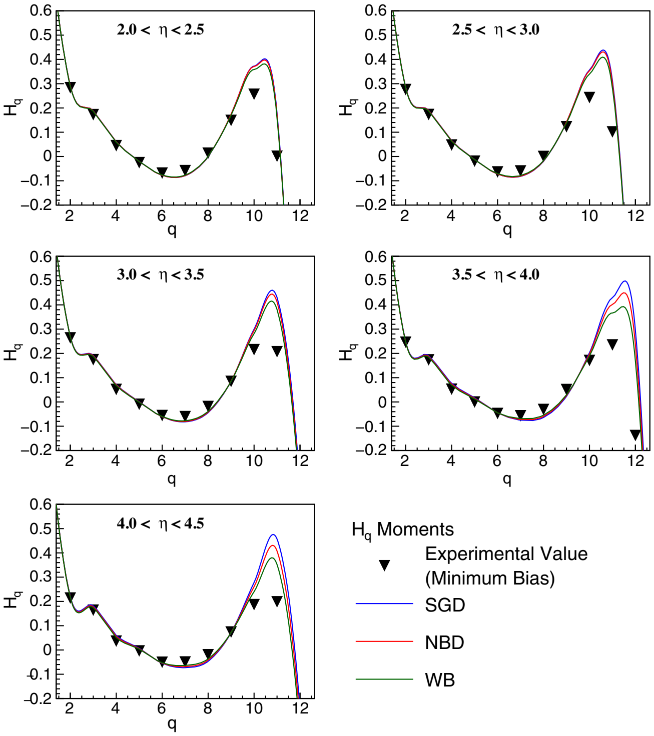

FIG. 8. Dependence of experimental values of H_q moments on the rank q in comparison to the values predicted by various distributions, in different η windows for minimum bias events.

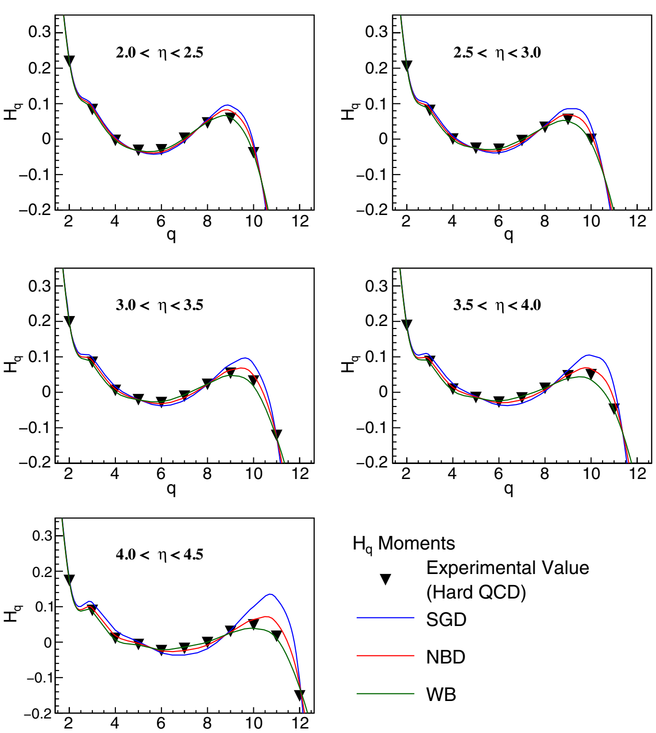

FIG. 9. Dependence of experimental values of H_q moments on the rank q in comparison to the values predicted by various distributions, in different η windows for hard QCD events.

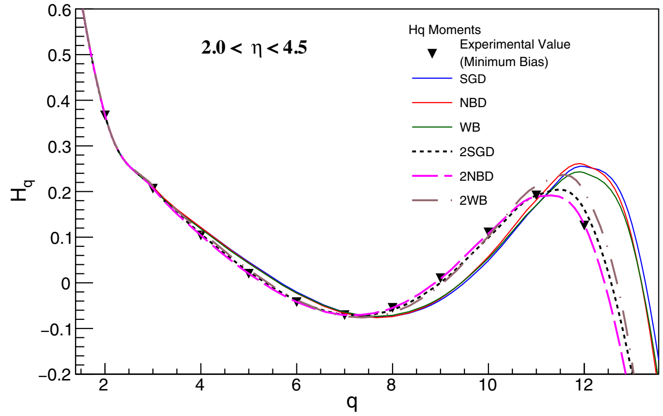

FIG. 10. Dependence of experimental values of H_q moments on the rank q in comparison to the values predicted by various distributions, in the forward region ($2.0 < \eta < 4.5$) for the minimum bias events.

oscillations are observed with minima at $q \approx 4$ and 12, while the maxima at $q \approx 9$ and 15.

Figures 10 shows the H_q moments versus q value calculated from the data and compared with NBD, SGD, WB, 2NBD, 2SGD, 2WB distributions, for the minimum bias events for the full forward region in the $2.0 < \eta < 4.5$ interval. 2NBD followed by 2SGD best describe the data with a minimum around $q \sim 7$. Figure 11 shows the shape of the charged-particle multiplicity distribution of hard QCD events in the full forward region, analyzed in terms of the variation of H_q moments as a function of q. A comparison with NBD, SGD, WB, 2NBD, 2SGD, 2WB distributions shows that the 2NBD best describes the data, closely followed by 2WB and 2SGD. The two minima appear at $q \approx 6$ and 12 and the maxima appear at $q \sim 9$ and 15, with quasioscillations about zero for larger values of q. These observations confirm the predictions from quantum chromodynamics and also the next-to-next-to-leading

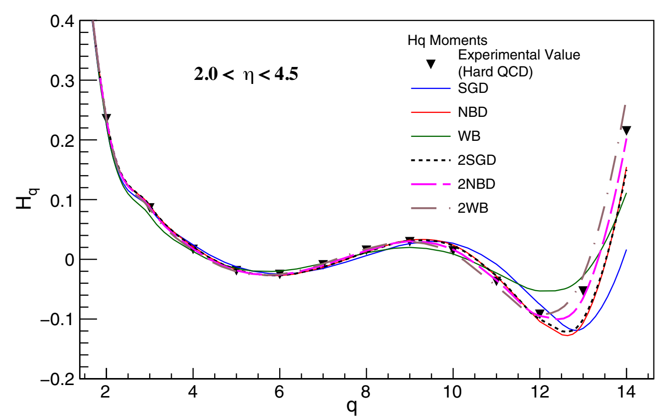

FIG. 11. Dependence of experimental values of H_q moments on the rank q in comparison to the values predicted by various distributions, in the forward region ($2.0 < \eta < 4.5$) for the hard QCD events.

TABLE VII. Comparison of experimental average charged particle multiplicity $\langle n_{ch} \rangle$ with the corresponding values from three fitted distributions.

$\langle n_{ch} \rangle$ (minimum bias)				
η	Data	SGD	NBD	WB
$-2.5 < \eta < -2.0$	3.913 ± 0.070	3.936 ± 0.019	3.938 ± 0.019	3.936 ± 0.019
$2.0 < \eta < 2.5$	3.902 ± 0.042	3.904 ± 0.012	3.908 ± 0.012	3.907 ± 0.012
$2.5 < \eta < 3.0$	3.804 ± 0.042	3.804 ± 0.012	3.809 ± 0.012	3.808 ± 0.012
$3.5 < \eta < 3.5$	3.661 ± 0.041	3.649 ± 0.012	3.655 ± 0.012	3.654 ± 0.012
$3.5 < \eta < 4.0$	3.482 ± 0.040	3.462 ± 0.012	3.470 ± 0.012	3.470 ± 0.012
$4.0 < \eta < 4.5$	3.248 ± 0.038	3.257 ± 0.014	3.259 ± 0.014	3.254 ± 0.014
$2.0 < \eta < 4.5$	11.672 ± 0.099	11.018 ± 0.021	11.160 ± 0.019	11.093 ± 0.020
$\langle n_{ch} \rangle$ (hard QCD)				
η	Data	NBD	SGD	WB
$-2.5 < \eta < -2.0$	4.941 ± 0.095	5.026 ± 0.023	5.035 ± 0.024	4.999 ± 0.025
$2.0 < \eta < 2.5$	5.114 ± 0.069	5.155 ± 0.020	5.152 ± 0.020	5.123 ± 0.021
$2.5 < \eta < 3.0$	5.037 ± 0.069	5.050 ± 0.019	5.043 ± 0.020	5.016 ± 0.020
$3.5 < \eta < 3.5$	4.820 ± 0.067	4.828 ± 0.019	4.821 ± 0.020	4.800 ± 0.020
$3.5 < \eta < 4.0$	4.534 ± 0.065	4.553 ± 0.019	4.545 ± 0.020	4.528 ± 0.020
$4.0 < \eta < 4.5$	4.164 ± 0.064	4.201 ± 0.018	4.198 ± 0.019	4.178 ± 0.020
$2.0 < \eta < 4.5$	17.955 ± 0.235	17.856 ± 0.051	17.958 ± 0.052	17.692 ± 0.061

TABLE VIII. Average charged particle multiplicity $\langle n_{ch} \rangle$ for double distributions.

$\langle n_{ch} \rangle$ (minimum bias)				
η	Data	2SGD	2NBD	2WB
$2.0 < \eta < 4.5$	11.672 ± 0.099	11.619 ± 0.029	11.617 ± 0.030	11.638 ± 0.025
$\langle n_{ch} \rangle$ (hard QCD)				
η	Data	2SGD	2NBD	2WB
$2.0 < \eta < 4.5$	17.955 ± 0.235	17.909 ± 0.069	17.937 ± 0.076	17.982 ± 0.077

logarithm approximation (NNLLA) of perturbative QCD [31,33,38].

Tables VII and VIII show the average charged particle multiplicity ($\langle n_{ch} \rangle$) for the data in all pseudorapidity bins and the full-forward region, in the two categories of events. The average charged multiplicity is calculated from the probability distributions as $\langle n_{ch} \rangle = \sum nP(n)/\sum P(n)$. Interesting observations reveal that the $\langle n_{ch} \rangle$ in the η intervals $-2.5 < \eta < -2.0$ and $2.0 < \eta < 2.5$ are very nearly the same within the error limits. This indicates that the forward and backward regions are identical. This result confirms the observation made in the paper by LHCb [25]. Average multiplicity changes minimally over the η intervals for the minimum bias events. For the hard QCD events, it decreases from the bin $2.0 < \eta < 2.5$ to $4.0 < \eta < 4.5$ as expected due to the criterion of having at least one track with $P_T > 1$ GeV in each event. Overall, the values obtained from the fit values of the distributions NBD, SGD, WB, 2NBD, 2SGD, 2WB agree with the data values. Finally, the $\langle n_{ch} \rangle$ for the hard QCD events in the full forward region is larger than the minimum bias events. But in each case, the values agree with the fit values from all the distributions, with WB giving the closest agreement.

IV. CONCLUSION

Comparison of multiplicity distributions at $\sqrt{s} = 7$ TeV in restricted pseudorapidity (η) windows in the forward region obtained by the LHCb experiment is performed with three statistics inspired distributions, namely, negative binomial, shifted Gompertz, and Weibull distributions. Although the distributions fit the data very well in smaller η windows (typically $\Delta\eta = 0.5$), they all fail in the full forward region ($2.0 < \eta < 4.5$). This kind of violation was observed with NBD at energies as low as 200–900 GeV [10,12]. A possible explanation of the effect was suggested by C. Fuglesang [13] in terms of purely phenomenological considerations indicating the presence of a substructure. To overcome the violation, the multiplicity distribution was proposed to be a superposition of two component distributions. Using this approach, we find that in the forward region, there is manifold reduction in the χ^2/ndf values, and the distributions become statistically significant with a p value corresponding to $CL > 0.1\%$.

Shape of the charged-particle multiplicity distribution is related to the particle production. To study the shape, normalized factorial moments are used. If particles produced are correlated, the distribution is broader, and the F_q are greater than unity; if the particles are anticorrelated, the distribution becomes narrower reducing the F_q values to less than unity. In the analysis of these moments, we find that values of C_q and F_q moments remain constant in different η bins, with the exception of bin $4.0 < \eta < 4.5$, in which the value is consistently lower for all moments. This observation is consistent with the results from LHCb Collaboration [25]. Study of H_q moments shows that dependence of H_q on q is very similar in all the η bins with a minimum value around $q = 6$–7. For the hard QCD events, the agreement between the data and the fit values is very good for all distributions shown in Fig. 8. For minimum bias events, there is a disagreement between the data and the fit values at the highest q values for all fitted distributions. The values of all F_q are greater than unity, indicating the presence of correlations.

For the full forward region, $2.0 < \eta < 4.5$, the H_q moments versus q analysis shows that the two component multiplicity distributions best describe the data for results obtained for hard QCD events. In agreement with the QCD predictions, the two minima appear at $q \approx 6$ and 12 and the maxima appear at $q \approx 9$ and 15. In particular, this observation also confirms the expectations of the next-to-next-to-leading logarithm approximation (NNLLA) from perturbative QCD. The 2NBD best describes the data, closely followed by 2WB and 2SGD. The distributions in forward and backward regions in the pseudorapidity $2.0 < |\eta| < 2.5$ are found to be nearly identical. The $\langle n_{ch} \rangle$ for the hard QCD events in the full forward region is larger than the minimum bias events. However, in each case, the values agree with the fit values from all the distributions, with 2WB giving the closest agreement. In a detailed study of the charged particle multiplicities at 900 GeV, 2.36 TeV, and 7 TeV [22], the results from the CMS experiment confirm the change of slope in the probability distribution in the widest central pseudorapidity intervals observed at 7 TeV, a strong linear increase of the C_q moments, and a clear indication of violation of KNO scaling with respect to lower energies. The results presented in this paper also agree with the results measured by the CMS in central pseudorapidity range of $|\eta| < 0.5$ to $|\eta| < 2.4$.

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