3 Inclusion of mixed QCD-QED resummation effects at higher-orders

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In this document, we review some recent results concerning the inclusion of mixed QCD-QED corrections to the computation of physical observables. First, we comment on the extension of the DGLAP equations to deal with the presence of mixed QCD-QED interactions. We describe the calculation of the full set of higher-order corrections to the splitting kernels, through the Abelianization algorithm. This procedure allows to build the functional form of the QCD-QED corrections starting from pure QCD terms. As a practical application of this technique, we also explore the computation of fixed-order corrections to diphoton production, and the inclusion of higher-order mixed QCD-QED resummation effects to Z production. In both cases, we directly apply the Abelianization to the q_T-subtraction/resummation formalism, obtaining the universal ingredients that allow to compute the beforehand mentioned corrections to any process involving colorless and neutral particles in the final state.

3.1 Introduction and motivation

The large amount of data that high-energy experiments are collecting allows to increase notoriously the precision of several measurements. In consequence, theoretical predictions must be pushed forward by including previously neglected small effects. This is the case of electroweak (EW) or QED corrections, which are sub-dominating for collider physics. However, from a naive power-counting, it is easy to notice that $\mathcal{O}(\alpha) \approx \mathcal{O}(\alpha_S^2)$. On top of that, QED interactions (as well as the full set of EW ones) lead to novel effects that could interfere with the well-known QCD signals. Moreover, these effects might play a crucial role in the context of future lepton colliders, such as the FCC-ee. For these reasons, EW and QED higher-order corrections must be seriously studied in a fully consistent framework.

The aim of this brief document is presenting some results related to the impact of QED corrections in the calculation of physical observables for colliders. In Sec. 3.2, we recall the computation of the full set of QCD-QED splitting functions at $\mathcal{O}(\alpha\,\alpha_S)$ and $\mathcal{O}(\alpha^2)$, centering into the Abelianization algorithm and the relevance of the corrections to achieve a better determination of the photon PDF. Then, we apply the Abelianization to the well-established q_T-subtraction/resummation [1, 2] framework. In Sec. 3.3, we show the impact of the NLO QED corrections to diphoton production. After that, we characterize the mixed QCD-QED resummation of soft gluon/photons for Z boson production in Sec. 3.4. The conclusions and future research directions are discussed in Sec. 3.5.

3.2 Splittings and PDF evolution

Splitting functions are crucial to describe the singular collinear behavior of scattering amplitudes. On one side, they are used to build the counter-terms to subtract infrared (IR) singularities from cross-sections. On the other hand, they are the evolution kernels of the integro-differential DGLAP equations [3], which govern the perturbative evolution of PDFs. When taking into account QCD and EW/QED interactions, it is necessary to include photon and lepton PDFs, and this will lead to the presence of new splitting functions. In Refs. [4, 5] we computed the $\mathcal{O}(\alpha\,\alpha_S)$ and $\mathcal{O}(\alpha^2)$ corrections to the DGLAP equations, as well as the associated kernels. The strategy that we adopted was based on the implementation of a universal algorithm, called *Abelianization*, which aims to explode previously known pure QCD results to obtain the corresponding QCD-QED or QED expressions. Roughly speaking, the key idea behind this method is *transforming gluons into photons*: color factors are replaced by suitable electric charges, as well as symmetry or counting factors.

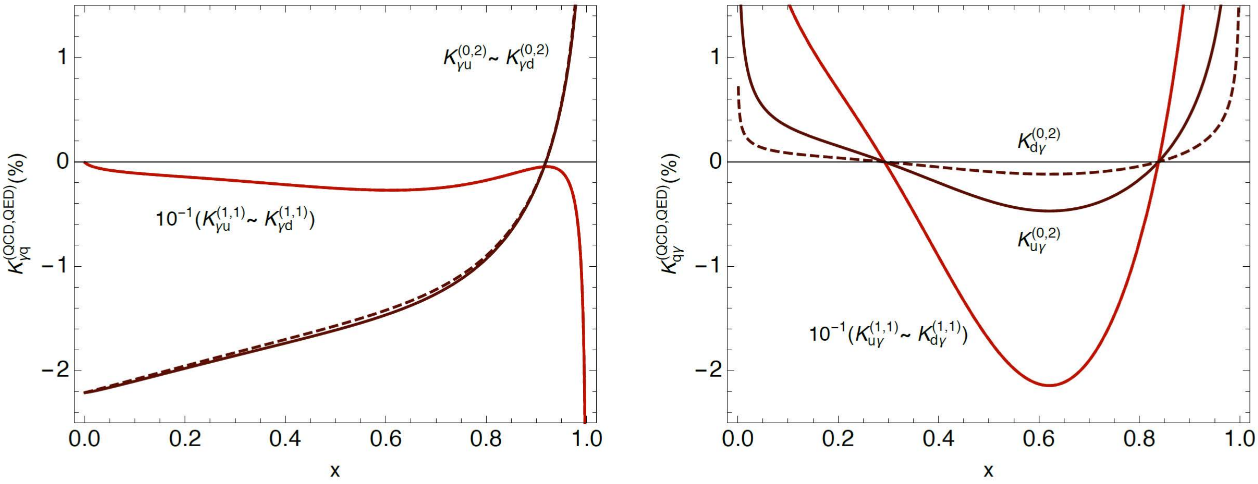

Fig. B.20: Corrections due to the inclusion of QED contributions in the $P_{q\gamma}$ (right) and $P_{\gamma q}$ (left) splitting kernels. We include both $\mathcal{O}(\alpha^2)$ (brown) and $\mathcal{O}(\alpha\,\alpha_{\mathrm{S}})$ (red) terms. K-ratio is defined using the leading order as normalization. To ease the visual presentation, we re-scaled the $\mathcal{O}(\alpha\,\alpha_{\mathrm{S}})$ terms by a factor 0.1.

With the purpose of exhibiting the quantitative effects that mixed QCD-QED or $\mathcal{O}(\alpha^2)$ corrections might have, we plot the K-ratio for quark-photon and photon-quark splitting functions in Fig. B.20. It is important to notice that these contributions are not present in pure QCD, which implies that the evolution of photon PDF is noticeably affected by the $\mathcal{O}(\alpha\,\alpha_{\mathrm{S}})$ splittings or even higher-orders in the mixed QCD-QED perturbative expansion. We would like to highlight that a precise determination of photon distributions is crucial to obtain more accurate predictions for several physical observables.

3.3 Fixed order effects: Application to diphoton production

The q_T-subtraction/resummation formalism [1, 2] is a powerful approach to compute higher-order corrections to physical observables. This formalism has been mainly applied to QCD calculations, and relies on the color-neutrality of the final state particles[‡]. So, we used the Abelianization algorithm to compute the universal coefficients required to implement NLO QED corrections to any process involving only neutral particles in the final state. In this way, we demonstrate that this extension can deal consistently with the cancellation of IR divergences in the limit $q_T \to 0$.

As a practical example, we used the public code `2gNNLO` [8,9], which provides up to NNLO QCD corrections to diphoton production, and we implemented the corresponding NLO QED corrections [10, 11]. We applied the default ATLAS cuts, with 14 TeV center-of-mass energy, and the `NNPDF3.1QED` [12, 13] PDF set. The transverse momentum and invariant mass spectra are shown in Fig. B.21. It is interesting to appreciate that, even if the corrections are small

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[‡] An extension to deal with massive or coloured particles in the final state was presented in Refs. [6, 7].

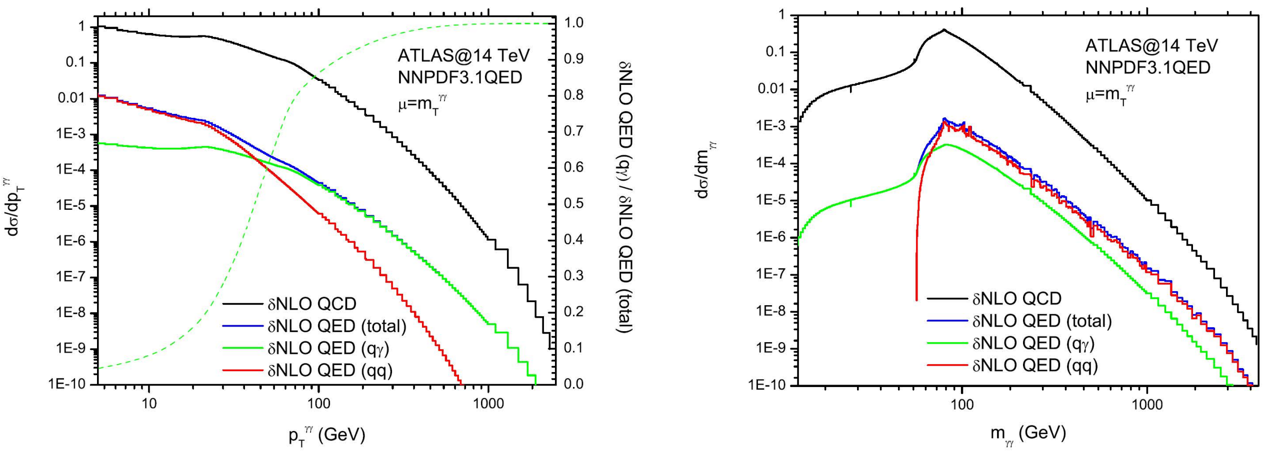

Fig. B.21: Impact of higher-order QED corrections on the transverse momentum (left) and invariant mass (right) distributions for diphoton production. The black (blue) curve shows the total NLO QCD (QED) prediction, without including the LO contribution. The dashed green line indicates the relative contribution of the $q\gamma$-channel to the total NLO QED correction.

compared to the QCD contributions, the QED interactions lead to novel features, such as a dynamical cut in the invariant mass spectrum. This is due to the fact that real radiation in the $q\bar{q}$-channel contains three final state photons, which must be ordered according to their transverse momenta before imposing the selection cuts. Besides that, introducing the QED corrections (or, even better, mixed NLO QCD-QED corrections) will allow to reduce the scale uncertainties and produce more reliable theoretical predictions.

3.4 Mixed resummation effects: Z-boson production

Finally, we studied the impact of including mixed QCD-QED terms within the q_T-resummation formalism. This is equivalent to consider the simultaneous emission of soft/collinear gluons and photons. A detailed description of the formalism is presented in Ref. [14], where we computed the modified Sudakov form factors as well as all the required universal coefficients to reach mixed NLL'+NLO accuracy in the double expansion in α and $\alpha_{\rm S}$. Explicitly, we obtained

$$\begin{aligned} \mathcal{G}'_N(\alpha_{\rm S},\alpha,L) &= \mathcal{G}_N(\alpha_{\rm S},L) + L\, g'^{(1)}(\alpha L) + g'^{(2)}_N(\alpha L) + \sum_{n=3}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}_N(\alpha L) \\ &+ g'^{(1,1)}(\alpha_{\rm S}L,\alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{\infty}\left(\frac{\alpha_{\rm S}}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g'^{(n,m)}_N(\alpha_{\rm S}L,\alpha L)\ , \end{aligned} \tag{3.35}$$

and

$$\begin{aligned} \mathcal{H}'^F_N(\alpha_{\rm S},\alpha) &= \mathcal{H}^F_N(\alpha_{\rm S}) + \frac{\alpha}{\pi}\,\mathcal{H}'^{F\,(1)}_N + \sum_{n=2}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n}\mathcal{H}'^{F\,(n)}_N \\ &+ \sum_{n,m=1}^{\infty}\left(\frac{\alpha_{\rm S}}{\pi}\right)^{n}\left(\frac{\alpha}{\pi}\right)^{m}\mathcal{H}'^{F\,(n,m)}_N\,, \end{aligned} \tag{3.36}$$

for the expansion of the Sudakov exponents and the hard-virtual coefficients, respectively. A similar expansion is available for the soft-collinear coefficients C_{ab}. Other important ingredients of the formalism are the mixed QCD-QED renormalization group equations (RGE), which include a double expansion of the corresponding β-functions [14].

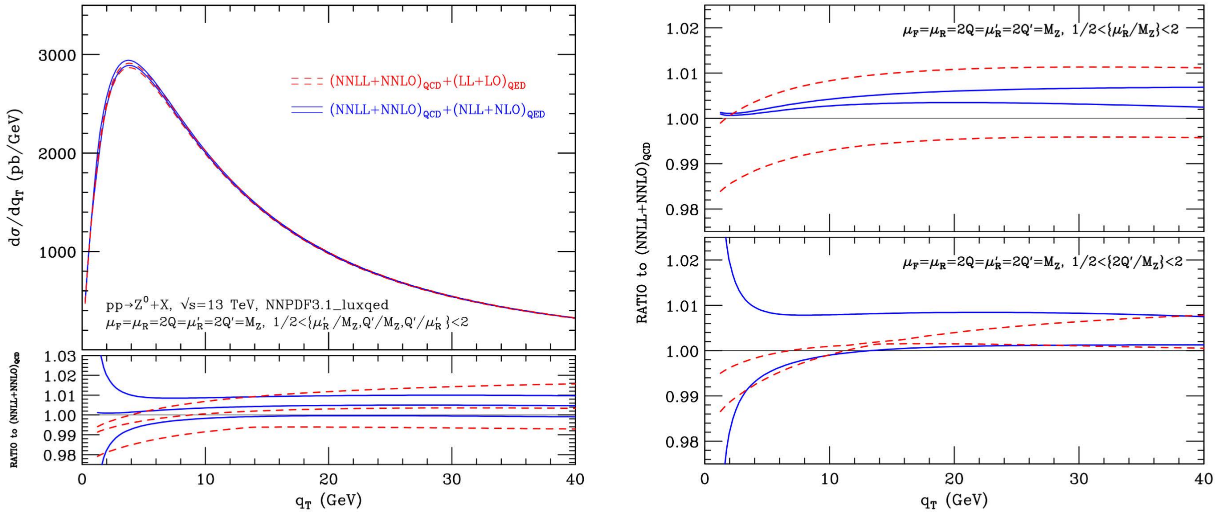

Fig. B.22: The q_T spectrum for Z boson production at the LHC with 13 TeV center-of-mass energy. In the left panel, we show the combination of NNLL+NNLO QCD contributions together with the LL (red dashed) and NLL'+NLO (blue solid) QED effects. We include the uncertainty bands that result from the full scale variation by a factor 2 (up and down). More details about scale uncertainties are shown in the right panel, where we modified independently the resummation (upper plot) and renormalization (lower plot) scales.

In order to test our formalism, we used Z boson production as a benchmark process. We started from the code `DYqT` [15] to compute the next-to-next-to-leading logarithmic QCD (NNLL) corrections properly matched to the fixed-order contribution (i.e. NNLO QCD in this case). In Fig. B.22, we show the combination of NNLL+NNLO QCD predictions for the q_T spectrum of the produced Z (in the narrow width approximation), together with the LL (red dashed) and mixed NLL'+NLO QED contributions (blue solid). The effects introduced by mixed QCD-QED terms reach the percent-level for $q_T \approx 20\,\text{GeV}$, when considering LHC kinematics at 13 TeV center-of-mass energy. However, the most noticeable consequence of introducing these corrections is the scale-dependence reduction. This means that our predictions are more stable when varying the electro-weak parameters or the factorization/renormalization/resummation scales.

3.5 Conclusions

In this brief document, we reviewed some of our recent efforts towards more precise phenomenological predictions for colliders. We centered the discussion into the inclusion of QED and mixed QCD-QED corrections to the evolution of PDFs (through the computation of novel splitting functions), QED fixed-order computations (using diphoton production as a benchmark) and mixed QCD-QED q_T-resummation (applied to Z boson production). In all these cases, the corrections constitute percent-level deviation from the dominant QCD one, but this could still be detected through an increased precision of the forthcoming experimental measurements (such as those provided by the FCC-ee). So, understanding how to extend the exposed frameworks to deal with even higher perturbative orders is crucial to match the quality of the experimental data, allowing to detect any possible deviation from the Standard Model and discover new physical phenomena.

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