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LONGITUDINAL COUPLED-BUNCH INSTABILITY EVALUATION FOR FCC-hh

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Abstract

High-order modes (HOM) of the accelerating rf structures and other machine elements, if not sufficiently damped, can drive longitudinal coupled-bunch instabilities (CBI). Their thresholds can be accurately obtained from macroparticle simulations using the detailed impedance model containing many different contributions. This method, however, is very difficult to apply for synchrotrons with a large number of bunches, as it is the case for the Future Circular hadron-hadron Collider (FCC-hh) with up to 10400 circulating bunches per beam. In this paper the semi-analytical approach is used for calculations of the instability thresholds during the acceleration cycle of the FCC-hh. As the result, we define requirements for the HOM damping that would be sufficient to prevent development of longitudinal CBI in the presence of weak synchrotron radiation damping.

INTRODUCTION

The Future Circular hadron-hadron Collider (FCC-hh) is proposed to be built using the CERN infrastructure [1]; its main parameters relevant for the present work are summarised in Table 1. In this high-energy machine, synchrotron radiation becomes non-negligible during the acceleration cycle and especially at the collision energy. As high-intensity beams will be used in the FCC-hh, the evaluation of the high-order modes (HOM) damping is necessary to avoid longitudinal coupled-bunch instabilities (CBI).

Unlike electron-positron colliders where a strong synchrotron radiation damping can help in suppression of longitudinal CBI, in this machine we still need to rely on Landau damping due to a synchrotron frequency spread, which depends on beam and machine parameters. They should be carefully chosen to maintain single-bunch stability during injection, acceleration, and physics. In this work, we first present criteria which were used for the design of voltage and emittance programs for the FCC-hh cycle. Then the thresholds of longitudinal CBI are evaluated and requirements for HOM damping are defined.

RF AND BEAM PARAMETERS DURING THE CYCLE

In high-energy accelerators, Landau damping (LD) provides single-bunch stability in the presence of the constant inductive longitudinal impedance $\mathrm{Im}Z/n$ [2, 3] if

$$\mathrm{Im}Z/n < \frac{F|\eta|E}{eI_\mathrm{b}}\left(\frac{\Delta E_\mathrm{max}}{E}\right)^2\frac{\Delta f_{s,\mathrm{max}}}{f_{s0}}f_0\tau_\mathrm{b}\,. \quad (1)$$

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Here, F is the form-factor defined by particle distribution in the bunch, $\eta = 1/\gamma_t^2 - 1/\gamma^2$ the slip factor, γ the Lorentz factor, $I_\mathrm{b} = eN_\mathrm{p}f_0$ the single bunch current, f_0 the revolution frequency, and f_{s0} the frequency of small-amplitude synchrotron oscillations,

$$f_{s0}^2 = -\frac{hf_0^2\eta eV_\mathrm{rf}\cos\phi_s}{2\pi E}\,, \quad (2)$$

with the rf voltage V_rf, and the synchronous phase ϕ_s. In Eq. (1), ΔE_max is the maximum energy deviation in the bunch, $\Delta f_{s,\mathrm{max}}$ is the full synchrotron frequency spread due to non-linearity of the rf field, $\tau_\mathrm{b} = (\phi_2 - \phi_1)/(2\pi f_\mathrm{rf})$ is the full bunch length, ϕ_1 and ϕ_2 are synchrotron oscillation excursions in rf radians of the particle with ΔE_max.

In the present work we consider a stationary binomial particle distribution in action-angle variables $(\mathcal{J}, \psi)$

$$\mathcal{F}_\mu(\mathcal{J}) = \frac{1}{2\pi S_\mu}\left(1 - \frac{\mathcal{J}}{\mathcal{J}_\mathrm{max}}\right)^\mu, \quad (3)$$

where the normalisation factor $S_\mu = \int \mathcal{F}_\mu(\mathcal{J})d\mathcal{J}$, $\mathcal{J}_\mathrm{max}$ is the action of the particle with ΔE_max and for short symmetric bunches $\mathcal{J}_\mathrm{max} = \pi f_{s0}(\pi f_\mathrm{rf}\tau_\mathrm{b})^{-2}$. The distribution function with $\mu = 2$ fits well to the bunch profiles measured at flat top and flat bottom in the Large Hadron Collider (LHC) [4, 5]. According to the measured loss of LD threshold on the LHC flat top, the form-factor $F = 0.21$ [4, 5] and below it is assumed to be the same for the FCC-hh.

The choice of the rf voltage program and beam parameters during the cycle is based on an assumption for the inductive impedance budget $\mathrm{Im}Z/n = 0.2\,\Omega$. In the LHC, calculated and measured $\mathrm{Im}Z/n = 0.09\,\Omega$ and it will be 20% more in High-Luminosity LHC (HL-LHC) [5]. The threshold of longitudinal CBI depends on τ_b, which for short symmetric

Table 1: Main Ring and Beam Parameters [1]

Parameter	Unit	Value
Circumference	km	97.75
Energy, E (injection/physics)	TeV	3.3/50
Transition gamma, γ_t		99.33
Energy loss at 50 TeV, U_SR	MeV/turn	4.67
Longitudinal emittance damping time at 50 TeV, τ_ϵ	h	0.54
Bunch spacing, t_bb	ns	25
4σ Gaussian equivalent bunch length at 50 TeV, $\tau_{4\sigma}$	ns	1.07
Bunch intensity, N_p	ppb	1.0×10^{11}
rf frequency, f_rf	MHz	400.79
Harmonic number, h		130680

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bunches can be calculated from the full-width half-maximum (FWHM) bunch length τ_{FWHM} (used in LHC operation) as

$$\tau_b = \tau_{\mathrm{FWHM}} \left(1 - 2^{\frac{2}{2\mu+1}}\right)^{-\frac{1}{2}}. \tag{4}$$

Note that, in Ref. [1] the 4σ Gaussian equivalent bunch length $\tau_{4\sigma} = \tau_{\mathrm{FWHM}}\sqrt{2/\ln 2}$ and corresponding emittance of 2σ contour $\epsilon_{2\sigma}$ are used. In the present work the beam parameters are re-scaled to the full bunch length and the full longitudinal emittance ϵ assuming distribution with $\mu = 2$.

The minimum rf voltage of 42 MV and $\epsilon_{\mathrm{FT}} = 13.9$ eVs at the flat-top energy E_{FT} are defined by stability of bunches with the bunch length $\tau_{4\sigma} = 1.07$ ns required for physics and a margin for ±10% bunch length variation. As the loss of LD threshold $\propto \epsilon^{2.5}$ [2, 3], at 50 TeV synchrotron radiation damping with a characteristic time τ_ϵ of a half-hour can result in fast threshold reduction [6]. Thus a controlled emittance blow-up is needed to maintain longitudinal beam stability not only during the ramp but also at flat top. However, after the end of the ramp the preparation of the collider for physics might take up to 13 min [7] when the usage of the emittance blow-up is questionable because of a potential impact of the beam excitation. The solution could be an arrival to flat top with longer bunches having higher emittances for the same rf voltage, which ensures the beam stability during this process.

At the flat-bottom energy E_{FB} there is a flexibility for the value of emittance provided by the LHC as an injector. For the moment, $V_{\mathrm{rf}} = 12$ MV and corresponding $\epsilon_{\mathrm{FB}} = 3$ eVs were chosen providing longitudinal single-bunch stability with the same margin as at flat top.

The proposed 20 min momentum ramp [6] consists of parabolic, linear, and again parabolic parts with 10%, 80%, and 10% of the total energy increase $E_{\mathrm{FT}} - E_{\mathrm{FB}}$, respectively. The maximum voltage during the ramp depends on both the longitudinal emittance ϵ and the bucket filling factor in energy $q = \Delta E_{\max}/\Delta E_{\mathrm{B}}$, where ΔE_{B} is the bucket half-height. The scaling of the threshold impedance with the beam energy suggests the use of the controlled emittance blow-up with $\epsilon \propto \sqrt{E}$ [8], so the following emittance evolution was used in the calculations

$$\epsilon(E) = \epsilon_{\mathrm{FB}} + (\epsilon_{\mathrm{FT}} - \epsilon_{\mathrm{FB}})\sqrt{\frac{E - E_{\mathrm{FB}}}{E_{\mathrm{FT}} - E_{\mathrm{FB}}}}, \tag{5}$$

and as a function of time it is shown in Fig. 1 (solid blue line). The bucket filling factors at flat bottom ($q = 0.88$) and flat top ($q = 0.78$) are defined by the bunch lengths $\tau_{b,\mathrm{FB}} = 1.69$ ns and $\tau_{b,\mathrm{FT}} = 1.27$ ns, respectively. According to the LHC experience, we assume $q = 0.96$ in the linear part of the ramp as shown in Fiq. 1.

The calculated rf voltage program and the corresponding loss of LD threshold for baseline and alternative scenarios of the emittance blow-up during the cycle (see Fig. 1) are shown in Fig. 2. The voltage programs slightly differ in the final parabolic part (iii). The loss of LD threshold is always above the assumed impedance budget and there is a

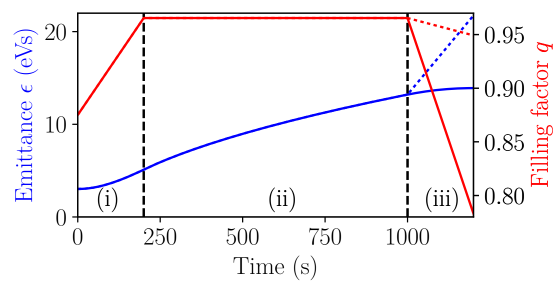

Figure 1: The longitudinal emittance (blue) and the bucket filling factor (red) during cycle for baseline (solid lines) and alternative (dotted lines) scenarios. The dashed vertical lines separate different parts of the ramp: (i) the parabolic, (ii) linear, and (iii) parabolic parts.

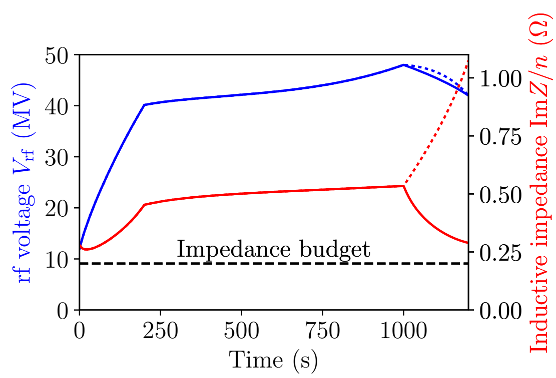

Figure 2: The rf voltage program (blue) and the threshold for the loss of LD (red) during the ramp for two scenarios from Fig. 1.

margin for ±10% bunch length variation at flat top and flat bottom energies. For the alternative scenario, the loss of LD threshold towards the end of the ramp is significantly higher. After 13 min of preparation for collisions, the threshold will be reduced to the minimum value needed for beam stability in physics, and the controlled emittance blow-up should be turned on again. The present parameters also provide single-bunch stability in the transverse plane as was checked in previous studies [9].

LONGITUDINAL COUPLED-BUNCH INSTABILITY

For evaluation of the longitudinal CBI threshold, we follow an approach discussed in [8, 10, 11]. We assume a ring uniformly filled with short bunches, so that the bunch asymmetry can be neglected when $\phi_s \neq \pi$. Introducing a new variable $\varphi = \phi - \phi_s$, the phase deviation of the particle with respect to the synchronous phase, we get $\phi_2 - \phi_s \approx \phi_s - \phi_1 = \varphi_{\max} = \pi f_{\mathrm{rf}} \tau_b$, where $\varphi_{\max}$ is the amplitude of synchrotron oscillations of the particle with $\Delta E_{\max}$. For short bunches with the binomial particle distribution, Eq. (3), in a single rf system the threshold shunt impedance

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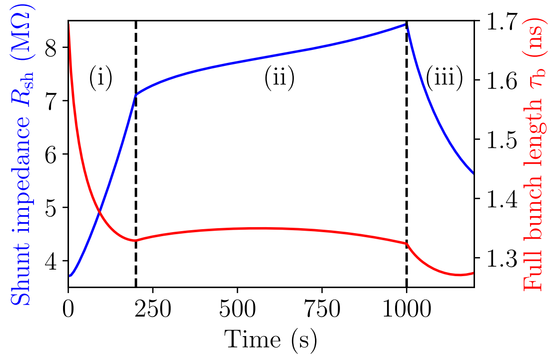

Figure 3: The minimum threshold shunt impedance during the cycle (parameters from solid curves in Figs. 1 - 2).

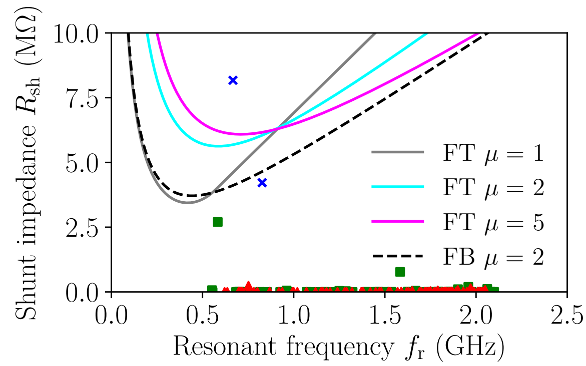

Figure 4: The threshold shunt impedance at 50 TeV (FT) for different μ and the same FWHM bunch length (obtained from Fig. 3 using Eq. (4)) and at 3.3 TeV (FB) for $\mu = 2$. The HOMs of WOW (blue crosses), DQW (green squares), and RFD (red triangles) crab cavities without frequency spread are also shown.

at the resonant frequency f_r is the lowest for the dipole mode and it is

$$R_{sh} < \frac{f_0 t_{bb} \varphi_{max} V_{rf} \cos\phi_s}{2 I_b} \frac{\Delta f_{s,max}}{f_{s0}} G_\mu(f_r \tau_b)\,, \text{ where} \quad (6)$$

$$\Delta f_{s,max} = \frac{f_{s0}}{16}\left(1 + \frac{5}{3}\tan^2\phi_s\right)\varphi_{max}^2, \text{ and} \quad (7)$$

$$G_\mu(x) = \frac{x}{\mu(\mu+1)} \min_{y\in[0,1]}\left[\left(1-y^2\right)^{\mu-1} J_1^2(\pi x y)\right]^{-1}, \quad (8)$$

with $J_1(x)$ the Bessel function of the first kind of the first order. The function $G_\mu(f_r\tau_b)$ has a minimum G_μ^{min}, which depends on μ and the bunch length. For example, $G_2^{min} = 0.66$ at $f_r = 0.75/\tau_b$, which differs from the minimum 0.5 at $f_r = 0.43/\tau_b$ obtained for different distribution function in [8,11]. Equation (6) is valid if the following conditions are satisfied for the bandwidth Δf_r of the resonator impedance

$$\Delta f_r = \frac{f_r}{2Q} \ll \frac{1}{t_{bb}}, \text{ and } \Delta f_r \ll \left|f_r - \frac{k}{2t_{bb}}\right|, \quad (9)$$

where Q is the quality factor, and k is an integer.

The minimum threshold shunt impedance calculated from Eq. (6) during ramp for $\mu = 2$ and the baseline scenario is the lowest at flat bottom (see Fig. 3). As controlled emittance blow-up is required during physics, the particle distribution can be different from the one observed in the LHC ($\mu = 2$) [12]. In LHC operation the FWHM bunch length is measured rather than the full bunch length because of noise in the bunch tails. In Fig. 4 we present the threshold as a function of the resonant frequency at flat top for distributions with different μ but the same τ_{FWHM} and also at flat bottom for $\mu = 2$. For $\mu = 1$, the minimum R_{sh} is the lowest and similar to that at flat bottom but it significantly increases at higher frequencies. For $\mu \geq 2$ the threshold curves are very similar and the minimum is about 50% higher than for distribution with $\mu = 1$.

In the present FCC-hh impedance model [1], HOMs of 24 (per beam) 400 MHz wide opened waveguide (WOW) crab cavities [13-15] have the largest contribution. The first mode at 667 MHz ($R/Q = 6.95\,\Omega$ in circuit definition, $Q = 4.8\times10^4$ [14]) is above the minimum R_{sh} at bottom and top energies for the worst case scenario when f_r is the same in all cavities (see Fig. 4). To have some stability margin (at least a factor of 2 for the lowest threshold) a relative spread of resonant frequencies $\delta f_r/f_r > 1\times10^{-4}$ needs to be introduced. The second HOM at 827.2 MHz ($R/Q = 12.55\,\Omega$, $Q = 4.8\times10^3$ [14]) is below the longitudinal CBI threshold at both energies and the frequency spread would further improve beam stability. In HL-LHC, the Double Quarter Wave (DQW) and RF-Dipole (RFD) crab cavities will be used. To provide the same total kick as WOW crab cavities, 12 cavities of each type would be needed for the FCC-hh. For comparison, their HOMs without frequency spread [16, 17] are also shown in Fig. 4. The LHC-like cavities can be used in the FCC-hh and 24 cavities per beam are required to provide up to 48 MV of the total rf voltage [1]. Their HOMs are well below the longitudinal CBI threshold [18].

SUMMARY

The longitudinal coupled-bunch instability thresholds were evaluated for the FCC-hh cycle, which is optimised for longitudinal and transverse single-bunch stability. For the considered family of the binomial particle distributions, bunches with different μ (except $\mu = 1$) but the same FWHM bunch length have similar threshold shunt impedances. Finally, to prevent longitudinal CBI in FCC-hh due to HOMs of WOW crab cavities, the relative spread of resonant frequencies $> 1\times10^{-4}$ is required.

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