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Observation of several sources of *CP* violation in $B^+ \to \pi^+\pi^+\pi^-$ decays

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Abstract

Observations are reported of different sources of *CP* violation from an amplitude analysis of $B^+ \to \pi^+\pi^+\pi^-$ decays, based on a data sample corresponding to an integrated luminosity of $3\,\text{fb}^{-1}$ of pp collisions recorded with the LHCb detector. A large *CP* asymmetry is observed in the decay amplitude involving the tensor $f_2(1270)$ resonance, and in addition significant *CP* violation is found in the $\pi^+\pi^-$ S-wave at low invariant mass. The presence of *CP* violation related to interference between the $\pi^+\pi^-$ S-wave and the P-wave $B^+ \to \rho(770)^0\pi^+$ amplitude is also established; this causes large local asymmetries but cancels when integrated over the phase space of the decay. The results provide both qualitative and quantitative new insights into *CP*-violation effects in hadronic B decays.

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Violation of symmetry under the combined charge-conjugation and parity-transformation operations, *CP* violation, gives rise to differences between matter and antimatter. Violation of *CP* symmetry can occur in the amplitudes that describe hadron decay, in neutral hadron mixing, or in the interference between mixing and decay (for a review, see, *e.g.*, Ref. [1]). For charged mesons, only *CP* violation in decay is possible, where an asymmetry in particle and antiparticle decay rates can arise when two or more different amplitudes contribute to a transition. In particular, the phase of each complex amplitude can be decomposed into a weak phase, which changes sign under *CP*, and a strong phase, which is *CP* invariant. Differences in both the weak and strong phases of the contributing amplitudes are required for an asymmetry to occur.

In the Standard Model (SM), weak phases arise from the elements of the Cabibbo–Kobayashi–Maskawa matrix [2,3] that are associated with quark-level transition amplitudes. Decays of B hadrons that do not contain any charm quarks in the final state, such as $B^+ \to \pi^+\pi^+\pi^-$, are of particular interest as both tree-level and loop-level amplitudes are expected to contribute with comparable magnitudes, so that large *CP*-violation effects are possible. Indeed, significant asymmetries have been observed in the two-body $B^0 \to K^+\pi^-$ [4–6] and $B^0 \to \pi^+\pi^-$ [4,6,7] decays. In two-body decays, nontrivial strong phases can arise from rescattering or other hadronic effects. In three-body or multibody decays, variation of the strong phase is also expected due to the intermediate resonance structure, and hence amplitude analyses can provide additional sensitivity to *CP*-violation effects.

Analysis of the distribution of $B^+ \to \pi^+\pi^+\pi^-$ decays[1] across the Dalitz plot [8,9], which provides a representation of the two-dimensional phase space for the decays, has been previously performed by the BaBar collaboration [10,11]. A model-independent analysis by the LHCb collaboration, with over an order of magnitude more signal decays and much better signal purity compared to the BaBar data sample, subsequently observed an intriguing pattern of *CP* violation in its phase space, notably in regions not associated to any known resonant structure [12,13]. The observed variation of the *CP* asymmetry across the Dalitz plot is expected to be related to the changes in strong phase associated with hadronic resonances, but, to date, has not yet been explicitly described with an amplitude model. Many phenomenological studies [14–20] have provided possible interpretations of the asymmetries. Particular attention has been devoted to whether large *CP*-violation effects could arise from the interference between the broad low-mass spin-0 contributions and the spin-1 $\rho(770)^0$ resonance [21–24], from mixing between the $\rho(770)^0$ and $\omega(782)$ resonances [25–27], or from $\pi\pi \leftrightarrow K\overline{K}$ rescattering [21,23,24,28]. Further experimental studies are needed to clarify which of these sources are connected to the observed *CP* asymmetries.

In this Letter, results are reported on the amplitude structure of $B^+ \to \pi^+\pi^+\pi^-$ decays, obtained by employing decay models that account for *CP* violation. The results are based on a data sample corresponding to $3\,\text{fb}^{-1}$ of pp collisions at centre-of-mass energies of 7 and 8 TeV, collected with the LHCb detector. A more detailed description of the analysis is given in a companion paper [29]. The LHCb detector is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, described in detail in Refs. [30,31]

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[1] The inclusion of charge-conjugated processes is implied throughout this Letter, except where asymmetries are discussed.

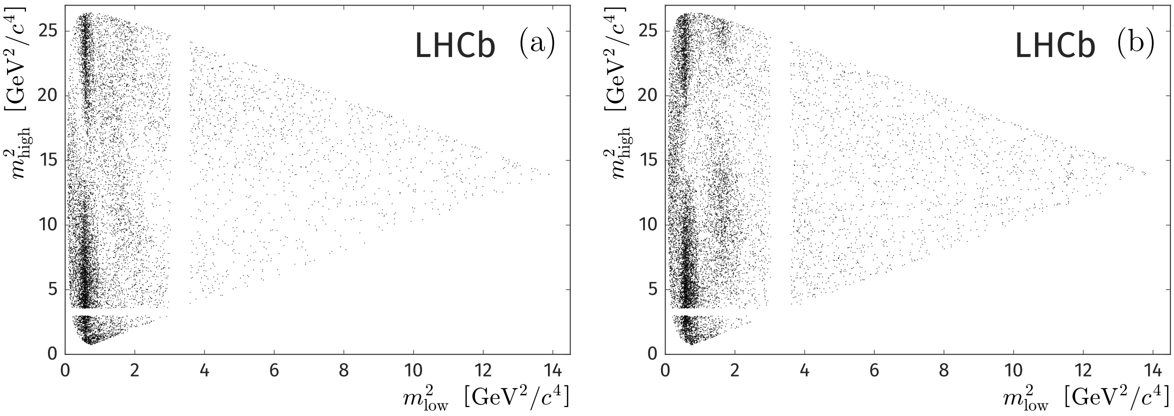

Figure 1: Dalitz-plot distributions for (a) B^+ and (b) B^- candidate decays to $\pi^\pm\pi^+\pi^-$. Depleted regions are due to the $\overline{D}^0$ veto.

The selection of signal candidates closely follows the procedure used in the model-independent analysis of the same data sample [12], with minor enhancements. Events containing candidates are selected online by a trigger [32] that includes a hardware and software stage. The hardware stage requires either energy deposits in the calorimeters associated to signal particles or a trigger caused by other particles in the event. The software triggers require that the signal tracks come from a secondary vertex consistent with the decay of a b hadron. In the offline selection, two multivariate algorithms are used to separate the $B^+ \to \pi^+\pi^+\pi^-$ signal from background formed from random combinations of tracks, and from other B decays with misidentified final state particles, such as $B^+ \to K^+\pi^+\pi^-$. Candidates that originate from $B^+ \to \overline{D}^0\pi^+$ with subsequent $\overline{D}^0 \to \pi^+\pi^-$ or misidentified $K^+\pi^-$ decays are removed with a veto on both $\pi^+\pi^-$ invariant mass combinations.

After application of all selection requirements, the B^+-candidate mass distribution is fitted to obtain signal and background yields. The fit function includes components for signal decays, combinatorial background and misidentified $B^+ \to K^+\pi^+\pi^-$ decays. The signal region in the B^+ candidate mass, $5.249 < m(\pi^+\pi^+\pi^-) < 5.317\,\text{GeV}/c^2$, which is used for the Dalitz-plot analysis, is estimated to contain $20\,600 \pm 1\,600$ signal, $4\,400 \pm 1\,600$ combinatorial background, and 143 ± 11 $B^+ \to K^+\pi^+\pi^-$ decays, where the uncertainties reflect the combination of statistical and systematic effects. The Dalitz-plot distributions of selected B^+ and B^- candidates are displayed in Fig. 1, where the phase space is folded by ordering the $\pi^+\pi^-$ pairs by their invariant mass, $m_{\rm low} < m_{\rm high}$.

Given the large number of broad overlapping resonances and decay-channel thresholds, it is particularly challenging to model the $B^+ \to \pi^+\pi^+\pi^-$ decay phenomenologically. Therefore, on top of the conventional "isobar" model using a coherent sum of all non-zero spin resonances, three complementary approaches are used to describe the S-wave amplitude. The first continues in the isobar approach, comprising the coherent sum of a σ pole [33] together with a $\pi\pi \leftrightarrow K\overline{K}$ rescattering term [34]; the second uses the K-matrix formalism with parameters obtained from scattering data [35–37]; and the third implements a "quasi-model-independent" (QMI) approach, inspired by previous QMI analyses [38], where the dipion mass spectrum is divided into bins with independent magnitudes and phases that are free to vary in the amplitude fit.

The amplitude for B^+ and B^- signal decays is constructed as the sum over N resonant contributions and the S-wave component,

$$A^{\pm}(m_{13}^2, m_{23}^2) = \sum_{j=1}^{N} c_j^{\pm} F_j(m_{13}^2, m_{23}^2) + A_{\rm S}^{\pm}(m_{13}^2, m_{23}^2)\,, \tag{1}$$

where m_{13} and m_{23} denote the $\pi^+\pi^-$ invariant mass combinations. Bose symmetry is accounted for by enforcing the amplitude to be identical under interchange of the two like-sign pions, making the labelling of the two combinations arbitrary. The F_j term is the normalised dynamical amplitude of resonance j, represented by a mass lineshape multiplied by the spin-dependent angular distribution using the Zemach tensor formalism [39, 40] and Blatt–Weisskopf barrier factors [41]. The complex coefficients, $c_j^{\pm}$, give the relative contribution of each resonance, and $A_{\rm S}^{\pm}$ is the S-wave amplitude (isobar, K-matrix or QMI). The amplitude models account for CP-violating differences between the distributions of B^+ and B^- decays by allowing the $c_j^{\pm}$ coefficients, and relevant parameters in $A_{\rm S}^{\pm}$, to take different values in the two cases. A likelihood function is constructed from the squared magnitude of the signal amplitude, accounting for efficiency effects and normalisation, and including background contributions modelled from data sidebands and simulation. The signal parameters are evaluated in the fit by minimising the negative logarithm of the total likelihood, calculated for all candidates in the signal region. The `Laura`$^{++}$ package [42] is used for the isobar and K-matrix approaches, while a GPU-accelerated version of the `Mint2` fitter [43] is used for the QMI approach.

With the exception of the S-wave, the included components are identical in each approach and consist of the $\rho(770)^0$ and $\omega(782)$ resonances described by a coherent ρ–ω mixing model [44], and the $f_2(1270)$, $\rho(1450)^0$, and $\rho_3(1690)^0$ resonances. These latter three resonances are all described by relativistic Breit–Wigner lineshapes. The choice of which resonances to include is made starting from the model obtained in the BaBar analysis [11], with additional contributions included if they cause a significant improvement in the fit to data.

In each approach, model coefficients for B^+ and B^- decays are obtained simultaneously. The amplitude coefficients extracted from the fit, $c_j^{\pm} = (x \pm \delta x) + i(y \pm \delta y)$, where positive (negative) signs are used for B^+ (B^-) decays, are defined such that CP violation is permitted. For the dominant ρ–ω mixing component, the magnitude of the coefficient in the B^+ amplitude is fixed to unity to set the scale, while both B^+ and B^- coefficients are aligned to the real axis as the absolute phase carries no physical meaning.

Good overall agreement between the data and the model is obtained for all three S-wave approaches, with some localised discrepancies that are discussed below. Moreover, the values for the CP-averaged fit fractions and quasi-two-body CP asymmetries (rate asymmetries between a quasi-two-body decay and its CP conjugate), derived from the fit coefficients and given in Table 1, show good agreement between the three approaches.

Projections of the data and the fit models are shown in regions of the data with $m(\pi^+\pi^-) < 1\,{\rm GeV}/c^2$ in Fig. 2. The $\rho(770)^0$ resonance is found to be the dominant component in all models, with a fit fraction of around 55% and a quasi-two-body CP asymmetry that is consistent with zero. The effect of ρ–ω mixing is very clear in the data (Fig. 2(b)) and is well described by the models. Contrary to some theoretical predictions [25–27], there is no evident CP-violation effect associated with ρ–ω mixing. However, a clear CP asymmetry is seen at values of $m(\pi^+\pi^-)$ below the $\rho(770)^0$ resonance,

Table 1: Results for *CP*-conserving fit fractions, quasi-two-body *CP* asymmetries, and phases for each component relative to the combined $\rho(770)^0$–$\omega(782)$ model, given for each S-wave approach. The $\rho(770)^0$ and $\omega(782)$ values are extracted from the combined $\rho(770)^0$–$\omega(782)$ mixing model. The first uncertainty is statistical while the second is systematic.

Contribution	Fit fraction (10^{-2})	A_{CP} (10^{-2})	B^+ phase (°)	B^- phase (°)
Isobar model				
$\rho(770)^0$	$55.5 \pm 0.6 \pm 2.5$	$+0.7 \pm 1.1 \pm 1.6$	—	—
$\omega(782)$	$0.50 \pm 0.03 \pm 0.05$	$-4.8 \pm 6.5 \pm 3.8$	$-19 \pm 6 \pm 1$	$+8 \pm 6 \pm 1$
$f_2(1270)$	$9.0 \pm 0.3 \pm 1.5$	$+46.8 \pm 6.1 \pm 4.7$	$+5 \pm 3 \pm 12$	$+53 \pm 2 \pm 12$
$\rho(1450)^0$	$5.2 \pm 0.3 \pm 1.9$	$-12.9 \pm 3.3 \pm 35.9$	$+127 \pm 4 \pm 21$	$+154 \pm 4 \pm 6$
$\rho_3(1690)^0$	$0.5 \pm 0.1 \pm 0.3$	$-80.1 \pm 11.4 \pm 25.3$	$-26 \pm 7 \pm 14$	$-47 \pm 18 \pm 25$
S-wave	$25.4 \pm 0.5 \pm 3.6$	$+14.4 \pm 1.8 \pm 2.1$	—	—
Rescattering	$1.4 \pm 0.1 \pm 0.5$	$+44.7 \pm 8.6 \pm 17.3$	$-35 \pm 6 \pm 10$	$-4 \pm 4 \pm 25$
σ	$25.2 \pm 0.5 \pm 5.0$	$+16.0 \pm 1.7 \pm 2.2$	$+115 \pm 2 \pm 14$	$+179 \pm 1 \pm 95$
K-matrix				
$\rho(770)^0$	$56.5 \pm 0.7 \pm 3.4$	$+4.2 \pm 1.5 \pm 6.4$	—	—
$\omega(782)$	$0.47 \pm 0.04 \pm 0.03$	$-6.2 \pm 8.4 \pm 9.8$	$-15 \pm 6 \pm 4$	$+8 \pm 7 \pm 4$
$f_2(1270)$	$9.3 \pm 0.4 \pm 2.5$	$+42.8 \pm 4.1 \pm 9.1$	$+19 \pm 4 \pm 18$	$+80 \pm 3 \pm 17$
$\rho(1450)^0$	$10.5 \pm 0.7 \pm 4.6$	$+9.0 \pm 6.0 \pm 47.0$	$+155 \pm 5 \pm 29$	$-166 \pm 4 \pm 51$
$\rho_3(1690)^0$	$1.5 \pm 0.1 \pm 0.4$	$-35.7 \pm 10.8 \pm 36.9$	$+19 \pm 8 \pm 34$	$+5 \pm 8 \pm 46$
S-wave	$25.7 \pm 0.6 \pm 3.0$	$+15.8 \pm 2.6 \pm 7.2$	—	—
QMI				
$\rho(770)^0$	$54.8 \pm 1.0 \pm 2.2$	$+4.4 \pm 1.7 \pm 2.8$	—	—
$\omega(782)$	$0.57 \pm 0.10 \pm 0.17$	$-7.9 \pm 16.5 \pm 15.8$	$-25 \pm 6 \pm 27$	$-2 \pm 7 \pm 11$
$f_2(1270)$	$9.6 \pm 0.4 \pm 4.0$	$+37.6 \pm 4.4 \pm 8.0$	$+13 \pm 5 \pm 21$	$+68 \pm 3 \pm 66$
$\rho(1450)^0$	$7.4 \pm 0.5 \pm 4.0$	$-15.5 \pm 7.3 \pm 35.2$	$+147 \pm 7 \pm 152$	$-175 \pm 5 \pm 171$
$\rho_3(1690)^0$	$1.0 \pm 0.1 \pm 0.5$	$-93.2 \pm 6.8 \pm 38.9$	$+8 \pm 10 \pm 24$	$+36 \pm 26 \pm 46$
S-wave	$26.8 \pm 0.7 \pm 2.2$	$+15.0 \pm 2.7 \pm 8.1$	—	—

where only the S-wave amplitude contributes significantly (Fig. 2(a)). A detailed inspection of the behaviour of the S-wave, given in Ref. [29], shows that this *CP* asymmetry remains approximately constant up to the inelastic threshold $2m_K$, where it appears to change sign; this is seen in all three approaches to the S-wave description. Estimates of the significance of this *CP*-violation effect give values in excess of ten Gaussian standard deviations (σ) in all the S-wave models. These estimates are obtained from the change in negative log-likelihood between, for each S-wave approach, the baseline fit and alternative fits where no such *CP* violation is allowed.

An additional source of *CP* violation, associated principally with the interference between S- and P-waves, is clearly visible when inspecting the $\cos\theta_{\rm hel}$ distributions separately in regions above and below the $\rho(770)^0$ peak (Fig. 3(a) and (b)). Here, $\theta_{\rm hel}$ is the angle, evaluated in the $\pi^+\pi^-$ rest frame, between the pion with opposite charge to the B and the third pion from the B decay. These asymmetries are modelled well in all three approaches to the S-wave description. Evaluation of the significance of *CP* violation in the interference between S- and P-waves gives values in excess of 25σ in all the S-wave models.

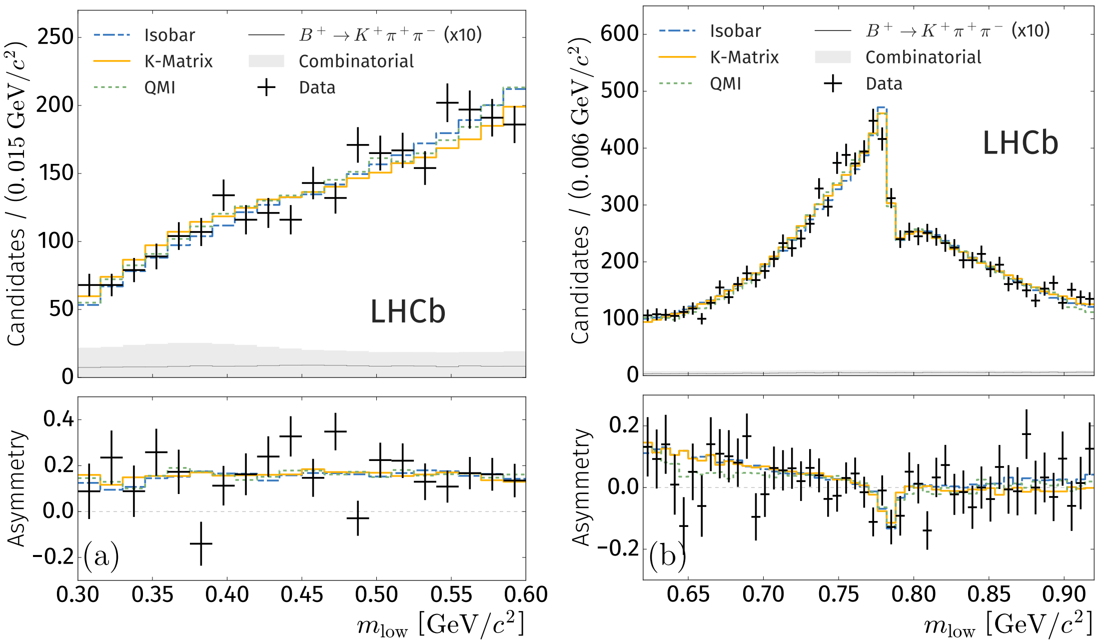

Figure 2: Projections of data and fits (top) on $m_{\rm low}$ in (a) the low $m(\pi^+\pi^-)$ region and (b) the ρ–ω region, with (bottom) the corresponding *CP* asymmetries in these ranges.

At higher $m(\pi^+\pi^-)$ values, the $f_2(1270)$ component is found to have a *CP*-averaged fit fraction of around 9% and a very large quasi-two-body *CP* asymmetry of around 40%, as can be seen in Fig. 4 and Table 1. This is the first observation of *CP* violation in any process involving a tensor resonance. The central value of the *CP* asymmetry is consistent with some theoretical predictions [19, 45, 46] that, however, have large uncertainties. The significance of *CP* violation in the complex amplitude coefficients of the $f_2(1270)$ component is in excess of 10σ. This conclusion holds in all the S-wave models and is robust against variations of the models performed to evaluate systematic uncertainties.

The parameters associated to the $\rho(1450)^0$ and $\rho_3(1690)^0$ resonances agree less well, but are nevertheless broadly consistent, between the different models. The small $\rho_3(1690)^0$ contribution exhibits a large quasi-two-body *CP* asymmetry; however this result is subject to significant systematic uncertainties, particularly due to ambiguities in the amplitude model, and therefore is not statistically significant.

The main sources of experimental systematic uncertainty are related to the signal, combinatorial and peaking background parameterisation in the B^+ invariant-mass fit, and the description of the efficiency variation across the Dalitz plot. Also considered, and found to be numerically larger for most results, are systematic uncertainties related to the physical amplitude models. These comprise the variation of masses and widths, according to the world averages [47], of established resonances, in addition to the inclusion of more speculative resonant structures. A small contribution from the $\rho(1700)^0$ resonance is expected by some theory predictions [48] and is considered a source of systematic uncertainty since the inclusion of this term did not significantly improve the models' agreement with data.

A clear discrepancy between all three modelling approaches and the data can be observed in the $f_2(1270)$ region (Fig. 4). This discrepancy can be resolved by freeing

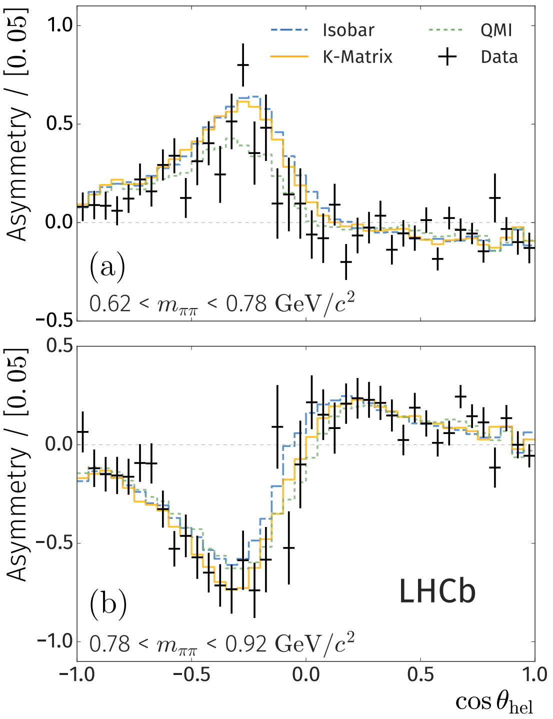

Figure 3: Projections of the CP asymmetry for data and fits as a function of $\cos\theta_{\mathrm{hel}}$ in the regions (a) below and (b) above the $\rho(770)^0$ resonance pole.

the $f_2(1270)$ mass parameter in the fit, however, the values obtained are significantly different from the world-average value. The discrepancy could arise from interference with an additional spin-2 resonance in this region, but all well established states are either too high in mass or too narrow in width to be likely to cause a significant effect. The inclusion of a second spin-2 component in this region, with free mass and width parameters, results in values of the $f_2(1270)$ mass consistent with the world average, where parameters of the additional state are broadly consistent with those of the speculative $f_2(1430)$ resonance; however the values obtained for the mass and width of the additional state are inconsistent between fits with different approaches to the S-wave description. Subsequent analysis of larger data samples will be required to obtain a more detailed understanding of the $\pi\pi$ D-wave in $B^+ \to \pi^+\pi^+\pi^-$ decays. Variation of the $f_2(1270)$ mass with respect to the world-average value, along with the addition of a second spin-2 resonance in this region, are taken into account in the systematic uncertainties.

In summary, an amplitude analysis of the $B^+ \to \pi^+\pi^+\pi^-$ decay is performed with data corresponding to $3\,\mathrm{fb}^{-1}$ of LHCb Run 1 data, using three complementary approaches to describe the large S-wave contribution to this decay. Good agreement is found between all three models and the data. In all cases, significant CP violation is observed in the decay amplitudes associated with the $f_2(1270)$ resonance and with the $\pi^+\pi^-$ S-wave at low invariant mass, in addition to CP violation characteristic of interference between the spin-1 $\rho(770)^0$ resonance and the spin-0 S-wave contribution. Violation of CP symmetry is previously unobserved in these processes and, in particular, this is the first observation of CP violation in the interference between two quasi-two-body decays. As such, these results provide significant new insight into how CP violation manifests in multi-body

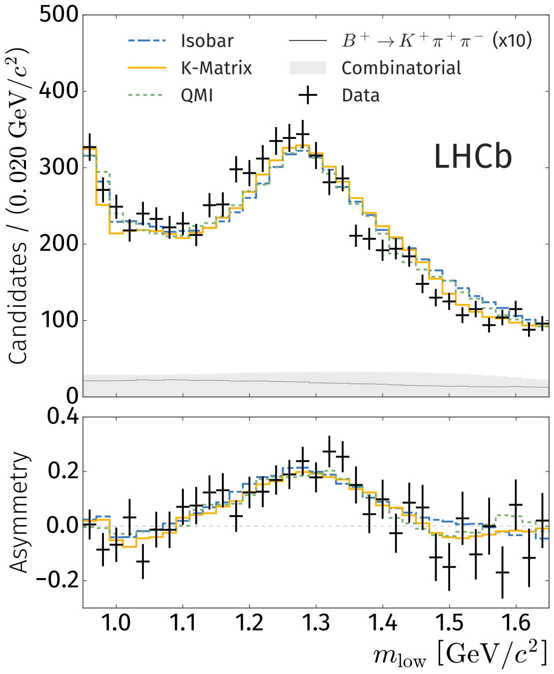

Figure 4: Projections of data and fits (top) on m_{low} in the $f_2(1270)$ mass region, with (bottom) the corresponding CP asymmetry.

B-hadron decays, and motivate further study into the processes that govern CP violation at low $\pi\pi$ invariant mass.

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