PROCEEDINGS OF SCIENCE

Parton distribution functions of Δ^+ on the lattice

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We present results for renormalized matrix elements related to the unpolarized quasi-distribution function of the Δ^+ baryon making use of the large momentum effective theory. Two ensembles of $N_f = 2+1+1$ twisted mass fermions with a clover term and pion masses of 250 MeV and 330 MeV are analyzed. We employ momentum smearing to improve the overlap with the boosted Δ state significantly reducing in this way the statistical error of both two- and three-point functions.

The 37th International Symposium on Lattice Field Theory - LATTICE2019
16-22 July, 2019
Central China Normal University, Hilton Hotel Wuhan Riverside, Wuhan, China.

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1. Introduction

Quantum Chromodynamics (QCD), the fundamental theory which describes the strong interaction among quarks and gluons, can be solved non-perturbatively by numerical simulation of the theory defined on a 4-dimensional Euclidean lattice. Parton distribution functions (PDFs) encode important information on the internal structure of hadrons such as how the spin and momentum are distributed among the quarks and gluons. However, these key quantities are presently only determined from global fits to experimental data mostly for the proton. Lattice QCD is thus the natural candidate for a reliable computation of PDFs. However, since they are defined on the light-cone it was thought until recently to be impossible to compute PDFs directly on the lattice. A method to extract PDFs from lattice QCD was proposed by Ji few years ago and is based on the large-momentum effective theory (LaMET) [1, 2]. In this approach, instead of calculating the light-cone quark correlations in a hadron, one computes matrix elements with quarks separated by a space-like distance in a boosted hadron, which is feasible on the lattice. Fourier transforming these spatial matrix elements yields –after a suitable non-perturbative renormalization– the so-called quasi-PDFs that have the same infrared physics as the light-cone PDFs. Because of this property, one can relate quasi-PDFs and PDFs using perturbation theory, through a procedure called matching. If the hadron is boosted with high enough momentum, the matching equations should correctly bring the quasi-PDFs to the PDFs, up to $O(M^2/P_z^2, \Lambda^2_{QCD}/P_z^2)$ corrections, where M and P_z are the mass and momentum of the boosted hadron. The hadron mass corrections are known to all orders in M^2/P_z^2, while the higher twist corrections are still not calculated. Besides quasi-PDFs, there are other approaches to accessthe x-dependence PDFs from lattice QCD[3, 4, 5, 6, 7]. We refer the interested Reader to the recent review of Ref. [8].

Since Ji's proposal, considerable progress has been made. In particular progress in the non-perturbative renormalization of quasi-PDFs [9, 10, 11] and in the calculation of the one-loop matching coefficient [12, 13] has enabled the extraction of PDFs from quasi-PDFs for the proton and pion. Results on the proton unpolarized PDFs have been obtained using also simulations with physical pion mass [14, 15, 13]. In this work we present first results on the $\Delta(1232)$ baryon, the lightest baryon resonance of a nucleon-pion system. Being a strongly decaying resonance its structure is not directly accessible experimentally. Thus lattice QCD can provide important information on its structure by evaluating the Δ electromagnetic and axial form factors and the Δ-nucleon transition form factors. In this work we present first results on the PDFs of the $\Delta(1232)$ baryon. Such a study is important in order to test the conjecture made in Ref. [16] that spontaneous breaking of chiral symmetry should lead to a significant difference between the proton and the Δ^+ flavor asymmetry for the isovector distribution $\overline{d}(x) - \overline{u}(x)$. Due to the short lifetime of the Δ, lattice QCD offers the exciting possibility to study the Δ unpolarized PDF in order to understand the mechanism behind the breaking of the flavor symmetry between light quarks in the nucleon sea.

2. Theoretical setup

The unpolarized PDF, denoted by $q(x)$, is defined on the light cone as [17]

$$q(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \left\langle h \middle| \overline{\psi}(0)\, \gamma^+ W\left(0, \xi^-\right) \psi(\xi^-) \middle| h \right\rangle \tag{2.1}$$

where the light-cone vectors are taken as $\xi^{\pm} = \left(\xi^0 \pm \xi^3\right)/\sqrt{2}$. $W\left(0,\xi^-\right) = e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)}$ is the Wilson line required for gauge invariance and x is the momentum fraction carried by the quarks in the hadron. The plus component of the momentum P^+ is $\left(P^0+P^3\right)/\sqrt{2}$ and $|h\rangle$ the hadron state of interest. Deep inelastic scattering is light-cone dominated, meaning that $\xi^2 = t^2 - \vec{r}^2 \approxeq 0$. However, in Euclidean space time, this reduces to a single point, which makes it impossible to compute PDFs in lattice QCD directly. In the large momentum effective theory, the PDF can be extracted from the quasi-PDF defined by

$$\tilde{q}\left(x,P_z,\mu\right) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-ixP_z z} \langle h(P_z)|\overline{\psi}(0)\Gamma W(0,z)\psi(z)|h(P_z)\rangle, \tag{2.2}$$

where $|h(P_z)\rangle$ is the boosted hadron state with finite momentum $P = (E,0,0,P_z)$, and $W(0,z)$ is the Wilson line along the boosted direction. μ is the renormalization scale.The Dirac structure Γ defines the type of PDF. For the unpolarized PDF, Γ is chosen to be γ^0 to avoid operator mixing [18].

Since the infrared physics is the same for quasi-PDFs and light-cone PDFs and the difference is only in the perturbative region [2], the quasi-PDFs are related to light-cone PDFs through the matching equation

$$\tilde{q}\left(x,P_z,\mu\right) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) q\left(y,\mu\right) + O\left(\frac{M^2}{P_z^2},\frac{\Lambda_{QCD}^2}{P_z^2}\right), \tag{2.3}$$

where $q(y,\mu)$ is the light-cone PDF at the scale μ, C is the matching kernel, which can be calculated perturbatively and has been evaluated to one-loop level. How large the momentum P_z needs to be for the validity of LaMET has to be tested. Exploratory studies have been performed showing a great potential for this approach, as e.g. in Ref. [19, 15, 20, 14].

3. Lattice details

In this study, we evaluate the isovector unpolarized quasi-PDFs, $\overline{u}(x) - \overline{d}(x)$ of Δ^+. The matrix elements of interest are given by

$$h(P_z,z) = \left\langle h\left|\overline{\psi}(0)\gamma^0 W(0,z)\psi(z)\right|h\right\rangle \tag{3.1}$$

for a straight Wilson line W, with varying length from $z=0$ up to half of the spatial extension $L/2$. In order to extract the matrix element of Eq. (3.1) we construct ratios of suitable three-point functions and two-point functions, averaged over the gauge field ensemble. The two-point and three-point functions are given by

$$C^{2\text{pt}}_{\sigma\rho}(\mathscr{P},\mathbf{P};t_s,0) = \mathscr{P}_{\alpha\beta}\sum_{\mathbf{x}} e^{-i\mathbf{P}\cdot\mathbf{x}}\left\langle 0\left|\mathscr{J}_{\sigma\alpha}(\mathbf{x},t_s)\overline{\mathscr{J}}_{\rho\beta}(\mathbf{0},0)\right|0\right\rangle \tag{3.2}$$

$$C^{3\text{pt}}_{\sigma0\rho}\left(\tilde{\mathscr{P}},\mathbf{P};t_s,\tau,0\right) = \tilde{\mathscr{P}}_{\alpha\beta}\sum_{\mathbf{x},\mathbf{y}} e^{-i\mathbf{P}\cdot\mathbf{x}}\left\langle 0\left|\mathscr{J}_{\sigma\alpha}\left(\mathbf{x},t_s\right)\mathscr{O}(\mathbf{y},\tau;z)\overline{\mathscr{J}}_{\rho\beta}(\mathbf{0},0)\right|0\right\rangle \tag{3.3}$$

where t_s is the time separation of the sink relative to the source, τ is the insertion time of the $\mathscr{O}$ operator, and $\mathscr{J}_{\sigma\alpha}(x)$ is the Δ^+ interpolating operator,

$$\mathscr{J}_{\sigma\alpha}(x) = \frac{1}{\sqrt{3}}\varepsilon^{abc}\left[2\left(u^{a^T}(x)C\gamma_\sigma d^b(x)\right)u^c_\alpha(x) + \left(u^{a^T}(x)C\gamma_\sigma u^b(x)\right)d^c_\alpha(x)\right], \tag{3.4}$$

with $C = \gamma^0\gamma^2$ being the charge conjugation matrix.

We use $\mathscr{P} = \tilde{\mathscr{P}} = \frac{1+\gamma^0}{4}$, and sum over the space components of σ,ρ when computing the ratio of the three to the two point functions:

$$h(\mathbf{P},z) \overset{\tau,t_s \gg 1}{=} \frac{\sum_{\sigma=1}^{3} C^{3pt}_{\sigma 0 \sigma}\left(\tilde{\mathscr{P}},\mathbf{P};t_s,\tau,0\right)}{\sum_{\sigma=1}^{3} C^{2pt}_{\sigma\sigma}(\mathscr{P},\mathbf{P};t_s,0)} . \tag{3.5}$$

In principle, the interpolating field also overlaps with the heavier spin-1/2 excitations. However, it is known form previous studies that the overlap with spin-1/2 state is small, and can be neglected.

4. Results at $m_\pi = 330$ MeV

As a first benchmark computation, we use a $24^3 \times 48$ twisted mass ensemble with $a = 0.096$ fm generated by the Extended Twisted Mass (ETM) collaboration [21]. The twisted mass parameter is $\mu = 0.0053$, which corresponds to a pion mass of $m_\pi = 330$ MeV. Since this is a first computation of the Δ PDF we are interested in exploring the feasibility of the calculation and, in particular, the momentum smearing technique to reduce the statistical errors due to a boosted Δ.

It is well known that the overlap with the ground state is improved by smearing the quark fields of the interpolating function. We use Gaussian smearing on the quark fields accompanied with APE smearing on the gauge fields that enter the Gaussian smearing function. For stationary hadron states this provides an efficient approach to improve ground state dominance. However, in the computation of matrix elements of quasi-PDFs, we need the overlap of a boosted particle state with the interpolating field. Thus, we use momentum smearing [22] which has proven to improve significantly the overlap of a boosted nucleon state with the interpolating field. The momentum smearing modifies the standard Gaussian smearing by a complex phase, given by

$$S_{mom} = \frac{1}{1+6\alpha}\Big(\psi(x) + \alpha\sum_j U_j(x)e^{-i\xi P\cdot j}\psi(x+\hat{j})\Big), \tag{4.1}$$

where U_j is the gauge link in the spatial j-direction. The momentum smearing parameter ξ needs to be tuned in order to optimize the overlap with the boosted particle state.

The necessity to boost the particle with relatively large momentum makes the excited state contamination more severe, since the spectrum gets denser. We use the source-sink time separations of 9a, 10a, 11a, 12a corresponding to 0.86, 0.96, 1.05, 1.15 fm, in order to study the excited state contamination. In this preliminary study we employ momentum $P_z = 2\pi/L \approx 0.54$ GeV. In Fig. 1 we show the comparison of results among four different separations. The results for both the real part of the matrix element, for the four source-sink time separations are all consistent within our current precision. For the imaginary part there is a decreasing trend becoming consistent for source-sink time separation $t_s = 10a$. Therefore, we fix the t_s to $10a \approx 0.96$ fm throughout this work.

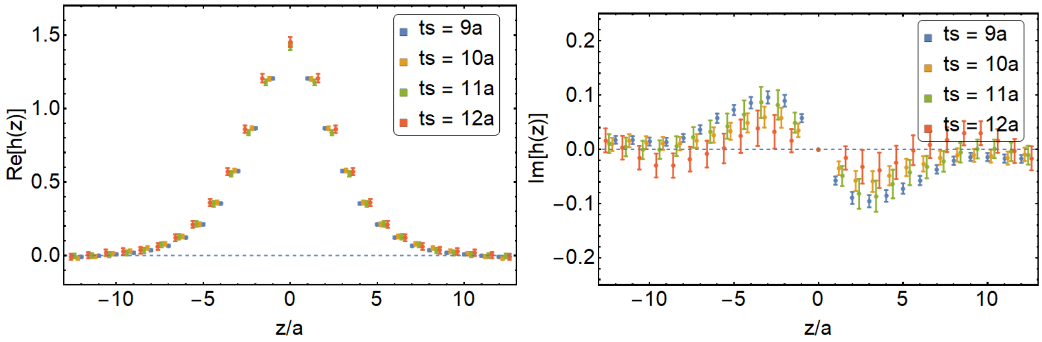

Fig. 1: The real (left) and imaginary (right) part of the ratio in eq. (3.5) yielding the isovector unpolarized quasi-PDFs of the Δ with boosted momentum $P_z = 2\pi/L$ for different values of the source-sink time separation as a function of z/a.

5. Results at $m_\pi = 250$ MeV

In this section we present results using a $32^3 \times 64$ lattice with lattice spacing $a = 0.096$ fm and pion mass $m_\pi = 250$ MeV. At this value of the pion mass the $(\bar{d} - \bar{u})$ asymmetry is expected to be larger, as pointed out in Ref. [16]. All the results presented here are computed with a source-sink time separation of $10a$. We extract matrix elements for the first two momenta $\{2\pi/L, 4\pi/L\}$, which correspond to $\{0.41, 0.82\}$ GeV with $\{906, 3600\}$ measurements.

In the computation of the matrix elements, we apply three-dimensional stout smearing to the gauge links that enter in the Wilson line of the operator. This reduces the power divergence in the matrix elements and brings the necessary renormalization factors closer to tree-level value. In Fig. 2, we show the bare matrix elements for momentum $4\pi/L$ with different stout smearing steps. As can be seen, the stout smearing increases the value of matrix elements. The results for different stout smearing converge as the smearing steps increase.

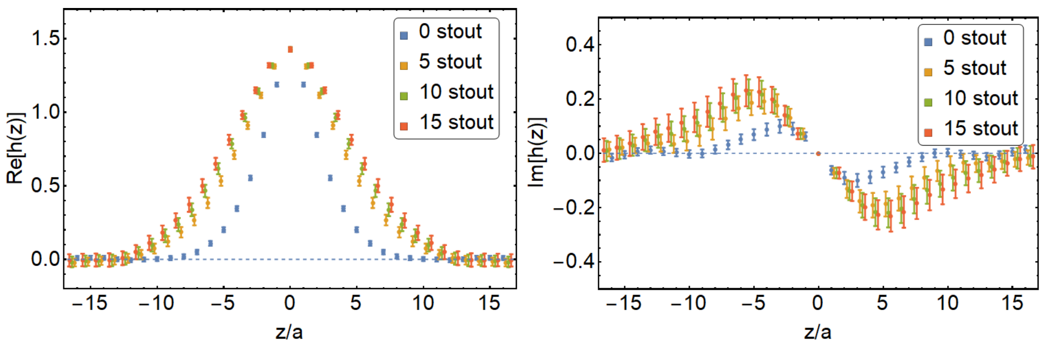

Fig. 2: The real (left) and imaginary (right) part of the ratio yielding the isovector unpolarized quasi-PDF of the Δ for different smearing steps, for momentum $P_z = 4\pi/L$.

For obtaining physical results, the operator must be renormalized to eliminate divergences. We adopt a non-perturbative RI-MOM renormalization scheme [23, 9]. The renormalization factors Z are converted to the $M\overline{MS}$ scheme [15] and evolved to $\mu = 2$ GeV. The final values of the Z-factors are extracted by chirally extrapolating to zero pion mass. The Z-factors are shown in

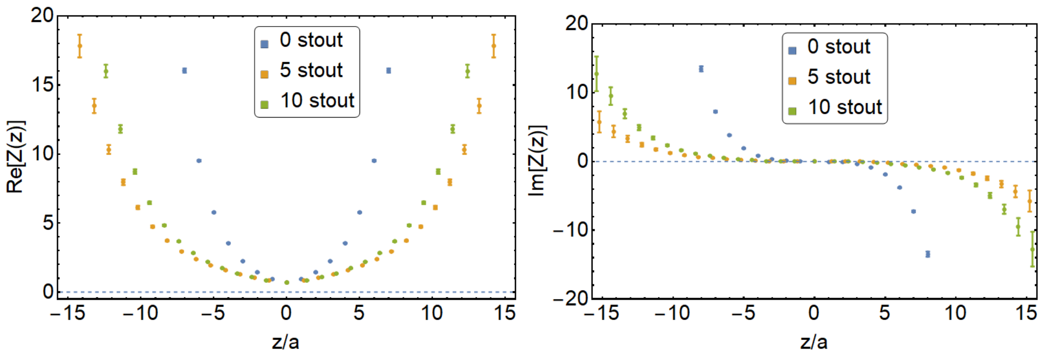

Fig. 3: Real part (left) and imaginary (right) parts of the renormalization factor for different stout smearing steps.

Fig.3 for different stout smearing steps. The renormalized matrix elements are shown in Fig.4 for momentum $P_z = 4\pi/L \approx 0.82$ GeV. As expected, the renormalized matrix elements for different stout smearing steps are consistent, with stout smearing clearly reducing the errors in the renormalization matrix elements. The agreement between different smearing steps verifies the effectiveness of our renormalization procedure.

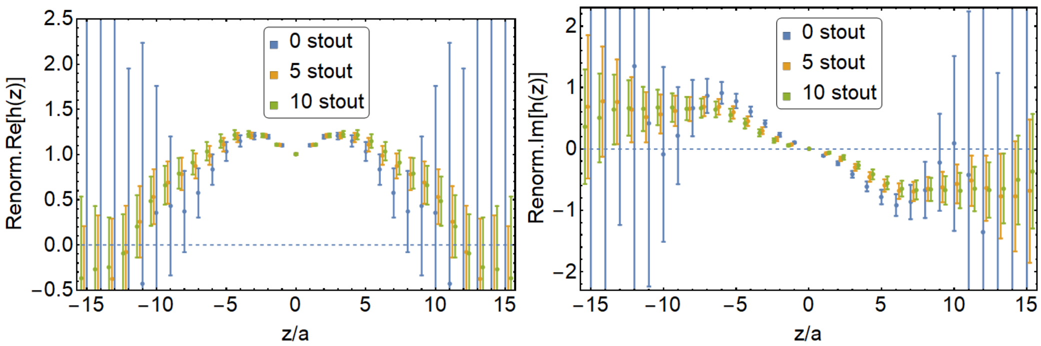

Fig. 4: Renormalized matrix of quasi-PDF with boosted momentum $4\pi/L$

6. Summary and outlook

In this study, we perform a first and exploratory study of the renormalized matrix elements for the unpolarized isovector PDF of the Δ^+. The momentum boosts used are 0.41 GeV and 0.82 GeV for the $m_\pi = 250$ MeV ensemble, and 0.54 GeV for the $m_\pi = 330$ MeV ensemble. These values are rather small and the next step is therefore to boost the Δ^+ to higher momentum. In particular, we aim at reaching values of the momentum similar to those used in our studies of the nucleons quasi-PDFs. This will enable us to compute the ratio $(\bar{d}(x) - \bar{u}(x))^{\Delta^+}/(\bar{d}(x) - \bar{u}(x))^{p^+}$ in order to check the asymmetry enhancement prediction. If the light quark asymmetry in the nucleon sea has its origin in the spontaneous breaking of chiral symmetry, this ratio should be as large as 2 for $x \approxeq 0.1$. This would provide a striking explanation of the physical mechanism responsible for $\bar{d}(x) > \bar{u}(x)$ in the nucleon.

Acknowledgments: We would like to thank all members of Extended Twisted Mass Collaboration for their constant and pleasant collaboration. Y.C., X.F., C.L., Y.L. and S.X. were supported in part by NSFC of China under Grant No. 11775002 and No. 2015CB856702. It is also supported in part by DFG grant no. TRR 110 and NSFC grant No. 11621131001. This work has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 642069 (HPC-LEAP). K.C. was supported by National Science Centre (Poland) grant SONATA BIS no. 2016/22/E/ST2/00013. F.S. was funded by DFG project number 392578569. M.C. acknowledges financial support by the U.S. Department of Energy, Office of Nuclear Physics, within the framework of the TMD Topical Collaboration, as well as, by the National Science Foundation under Grant No. PHY-1714407. This research used resources of TianHe-3 (prototype) at Chinese National Supercomputer Center in Tianjin, and JUWELS at the Juelich Supercomputing Centre, under project id ECY00.

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