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Q DRIFT DURING DEBUNCHING TIME

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Summary

It is shown that the Q drift observed during the debunching time can be explained by the slowly decaying eddy currents in the end laminations of the quadrupoles, in combination with the β variation in the quadrupole. A method for neutralizing the effect is suggested.

1. INTRODUCTION

Measurements of Q_H and Q_V during the debunching time (with RF off) have shown an upward drift of the order of $\dot{Q}$ = .02/s.

During this time, the field in dipoles and quadrupoles is kept constant by a feedback system that looks at the integrated field in reference dipoles and quadrupoles. The integrated field is obtained by using long measuring coils of uniform cross section. The fluctuations of this integrated field during the debunching time are random and too small to explain the Q drift.

However, even if the integrated gradient of the quadrupoles is constant, its longitudinal profile changes due to slowly decaying eddy currents in the plane of the laminations near the ends. These eddy currents exist because at the ends of the quadrupole the flux lines corresponding to the stray field are perpendicular to the laminations. The decay time of these eddy currents is of the order of seconds.

After the descent, the gradient at the ends will tend to decrease more slowly due to the eddy currents and the longitudinal gradient profile will show positive bumps at the ends.

If the betatron amplitude function β at the ends of quadrupole is equal to the average value inside, the Q value will not change due to the change in gradient profile, as long as the integrated gradient is kept constant. However, β varies and therefore the effect is not zero.

2. CALCULATION OF THE EFFECT

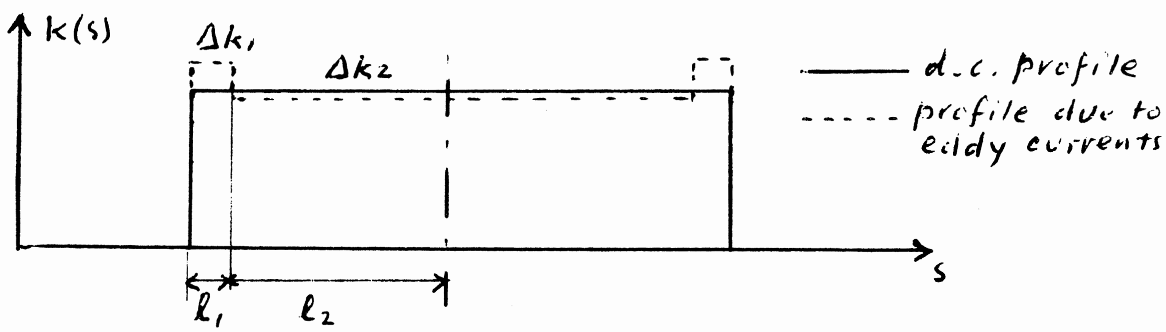
We shall use the following definitions :

$\beta_{0\,max}$ = average β value inside a quadrupole at β_{max} (e.g. QF for horizontal plane)

$\beta_{1\,max}$ = β at the end of this quadrupole

$\beta_{0\,min}$, $\beta_{1\,min}$ } the same for the quadrupole at β_{min}

We replace the longitudinal focusing profile by the following model :



We call Δk_1 and Δk_2 the changes in focusing strength due to the eddy currents at the ends and inside. Due to the feedback from the reference quads we have

$$\ell_1 \Delta k_1 + \ell_2 \Delta k_2 = 0 \quad \text{.......} (1)$$

The Q change due to 108 quadrupoles of each type, each with 2 ends, is :

$$\Delta Q = \frac{1}{4\pi} \int \beta(s) k(s) ds$$

$$= \frac{216}{4\pi} \left[\beta_{1\ max} \ell_1 \Delta k_1 + \beta_{0\ max} \ell_2 \Delta k_2 - \beta_{1\ min} \ell_1 \Delta k_1 - \beta_{0\ min} \ell_2 \Delta k_2 \right] \text{..} (2)$$

Note that the minus signs are due to the fact that we have used the same Δk for QF and QD although the sign of k is different for the two.

Using (1), we have

$$\Delta Q = \frac{54}{\pi} \ell_1 \Delta k_1 (\beta_{1_{max}} - \beta_{0_{max}} - \beta_{1_{min}} + \beta_{0_{min}}) \text{.......} (3)$$

For the SPS lattice, we have approximately

$\beta_{1_{max}}$ = 105.14 m
$\beta_{0_{max}}$ = 107.65 m
$\beta_{1_{min}}$ = 18.77 m
$\beta_{0_{min}}$ = 18.24 m.

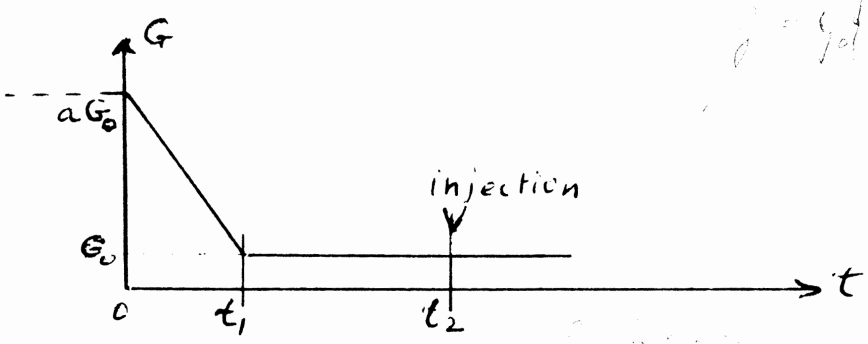
These values were obtained from the lattice table in LAB II/DI-PA/Note/71-2 (April 1972) by assuming a parabolic β profile inside the quadrupoles.

With these values we have

$$\Delta Q = -52 \ell_1 \Delta k_1 \quad \text{.......} (4)$$

with $\ell_1 \Delta k_1$ expressed in m^{-1}.

We now approximate the gradient at the end of the cycle by the following shape, assuming an infinitely long flat top.



We also assume that the transfer function

$$\frac{\text{gradient due to eddy currents}}{\text{main gradient}} = \frac{\Delta G}{G}$$

is equal to $-\frac{sT}{1 + sT}$, i.e. zero at zero frequency, full cancellation over a distance ℓ_1 at high frequencies, and decay with a single time constant T.

By Laplace transformation we then obtain for the response to a unit ramp starting at $t = 0$ ($G = t$)

$$\Delta G = - T(1 - e^{-t/T}) \quad \ldots\ldots (5)$$

Therefore with the cycle shape as above we get for $t > t_1$

$$\frac{\Delta G}{G_0} = (a-1) \frac{T}{t_1} \left[(1 - e^{-t/T}) - (1 - e^{-(t-t_1)/T})\right]$$

$$= (a-1) \frac{T}{t_1} e^{-t/T} (e^{t_1/T} - 1) \quad \ldots\ldots (6)$$

Equating $\frac{\Delta k_1 - \Delta k_2}{k}$ to $\frac{\Delta G}{G_0}$, using (1) and (4) and $\ell_1 << \ell_2$, we obtain at $t = t_2$

$$\Delta Q = - 52 \, \ell_1 k(a-1) \frac{T}{t_1} \ell^{-t_2/T} (e^{t_1/T} - 1) \quad \ldots\ldots (7)$$

Now $k = .0151 \text{ m}^{-2}$

and for the cycle used, $a = 20$
$t_1 = .6\text{s}$
$t_2 = 1.8\text{s}$

We can write (7) as

$$\Delta Q = - C\ell_1$$

with C depending on T as follows

T (s)	C (m^{-1})
.6	1.3
.8	2.3
1	3.4
1.5	5.5
2	7.1
3	9.1

To explain e.g. $\Delta Q = - .04$ at injection, with $T = 2$s, ℓ_1 (i.e. the equivalent length influenced by the eddy currents) only needs to be 5.6 mm With $T = .8$s (which seems to be about the minimum that could be reconciled with the measurements), we would need ℓ_1 = 17 mm. Even this seems perfectly possible. The conclusion is therefore that the order of magnitude of the effect is sufficient to explain the measurements.

3. COMPENSATION OF THE EFFECT

Evidently, the effect would be zero if we would ensure $\int \beta k ds = 0$, or

$(\beta_{1_{max}} - \beta_{1_{min}})\ \ell_1 \Delta k_1 + (\beta_{0_{max}} - \beta_{0_{min}})\ \ell_2 \Delta k_2 = 0$ instead of (1).

This can be done by making the ends of the measuring coils less sensitive to the gradient. The required factor is

$$\frac{\beta_{1_{max}} - \beta_{1_{min}}}{\beta_{0_{max}} - \beta_{0_{min}}} \approx .966 \qquad (8)$$

The easiest way to do this might be to shift the coil lengthwise so that one end does not measure the complete stray field. However, quite apart from practical problems in doing this, it would be difficult to determine in advance the exact shift required. Also, the decay time constant might vary longitudinally.

A better solution might therefore be to add small pickup coils near one end of the reference quadrupole, connected in series with the main coils in the opposite direction. Ideally, these coils could be shaped to compensate for the variations of (8) due to the longitudinal variation of β_1. This would cancel the effect of decay time variation with longitudinal position. The coils should extend inside the quadrupole up to the point where the gradient is no longer influenced by eddy currents.