# AMPLITUDE DEPENDENT CLOSEST TUNE APPROACH GENERATED BY NORMAL AND SKEW OCTUPOLES

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*Abstract*

Amplitude dependent closest tune approach, an action dependent analogue of the $\Delta Q_{min}$ generated by linear coupling, was observed in the LHC in 2012. It restricts the accessible resonance free area of the tune diagram and by altering tune spread can impact upon Landau damping. A theoretical description of such behaviour, generated by normal octupoles and linear coupling, was recently validated in the LHC. Simulation has also established that amplitude-dependent closest approach can be generated by a combination of normal and skew octupoles. This paper summarizes these simulation based observations.

## INTRODUCTION

Transverse linear coupling, generated by skew quadrupole and solenoid fields, creates a closest approach ($\Delta Q_{min}$) of the horizontal and vertical tunes [1]. To wit, the working point cannot move closer to the difference coupling resonance ($Q_{x,frac} - Q_{y,frac} = 0$) than the width of the linear coupling stop-band, $|C^-| = \Delta Q_{min}$. During nonlinear dynamics studies of the LHC at injection in 2012, it was observed that as octupolar detuning forced $Q_x$ and $Q_y$ together as a function of increasing amplitude, tune separation did not saturate to the well defined value of the linear coupling as expected, but to a significantly larger separation [2]. The observation was qualitatively reproduced in simulation and clearly associated with coupling between horizontal and vertical planes. The observation was interpreted as an amplitude dependent $\Delta Q_{min}$ [2, 3]. The initial observations in measurement and simulation are shown in Fig. 1, reproduced from [2, 4].

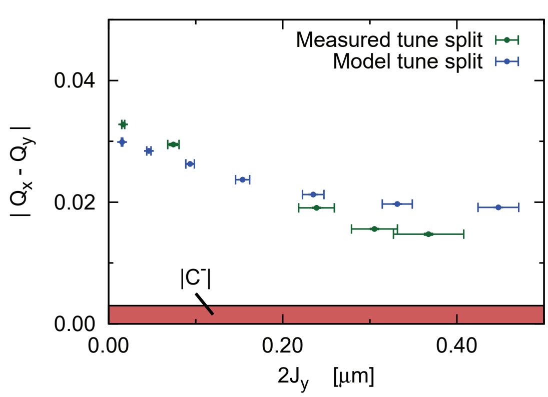


Figure 1: First observation of amplitude dependent closest tune approach [2].

Having observed this previously unconsidered behaviour in the LHC, extensive simulations were carried out to determine potential sources [4]. It was shown that linear coupling in combination with normal octupoles could not only substantially increase $\Delta Q_{min}$ at large amplitudes, but also allow penetration inside the linear coupling stop-band [4]. A theory for the mechanism behind this behaviour was proposed in terms of the interaction of linear coupling with the $h_{1111}$ Hamiltonian coefficient generated by normal octupoles [5]. Specific predictions of this theory have now been validated in the LHC [6].

Amplitude-dependent $\Delta Q_{min}$ can lead to highly nonlinear distortions of the tune footprint in the vicinity of the $Q_x - Q_y$ resonance, and is of concern in regard to Landau damping. Fortunately, control of linear coupling is becoming a priority for LHC operation. In 2016 correction of $|C^-|$ below the per-mil level was demonstrated [7]. New operational tools should allow tighter control of $|C^-|$ during regular operation [6, 8–11]. Generation of amplitude-dependent $\Delta Q_{min}$ by linear coupling and normal octupoles is likely to be better suppressed in future LHC and HL-LHC operation. This motivates the search for additional sources of amplitude-dependent $\Delta Q_{min}$ in simulation, which may become meaningful contributions as $|C^-|$ is reduced.

## AMPLITUDE DEPENDENT $\Delta Q_{min}$ FROM NORMAL AND SKEW OCTUPOLES

Inclusion of skew octupole ($a_4$) errors in an LHC model, with Landau octupoles (MO) powered, prevented detuning of particles to the $Q_x - Q_y$ resonance, even though no sources of linear coupling were included in the simulation. This is shown in Fig. 2. In the absence of $a_4$ errors the coupling resonance could be reached as a function of increasing particle amplitude. That normal octupoles on their own do not create an amplitude dependence of the closest tune approach was also well established in earlier studies of amplitude-dependent $\Delta Q_{min}$ generated by linear coupling in conjunction with $b_4$ [4]. Inclusion of $a_4$ errors into simulation appears therefore to have generated a nonlinear $\Delta Q_{min}$.

Before pursuing further study of the $\Delta Q_{min}$ generated by inclusion of $a_4$ errors, the relevance in relation to the already established source of amplitude-dependent $\Delta Q_{min}$ was assessed. Figure 3 shows the approach of tracking simulations to the $Q_x - Q_y$ resonance at injection, for particles of increasing amplitude. Landau octupoles are powered in all simulations at their values for 2012 LHC operation. Blue data shows the amplitude-dependent $\Delta Q_{min}$ created by linear coupling and normal octupoles. In this case $|C^-| = 0.001$, and for zero amplitude one could approach to the pale blue coupling stop-band. This represents an optimistic value of linear coupling for LHC operation at injection. Red data shows the approach to $Q_x - Q_y$ for a model containing only Landau octupoles and $a_4$ errors, with no linear coupling sources. In this case the nonlinear $\Delta Q_{min}$ corresponds to the




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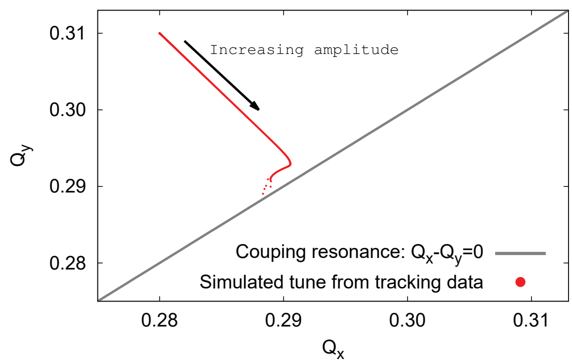


Figure 2: Detuning of particles towards the linear coupling resonance, with $a_4$ errors and Landau octupoles included in the simulation. $|C^-| = 0.000$ in this simulation.

closest approach of red tracking data to the gray resonance line. The nonlinear $\Delta Q_{min}$ generated by inclusion of $a_4$ errors is comparable to that generated by linear coupling and octupoles for a realistic operational scenario. Green data demonstrates that in a model combining linear coupling, $a_4$ errors, and Landau octupoles, the nonlinear closest approach is enhanced relative to either individual source. There is a clear motivation therefore for further understanding of the $a_4$ source.

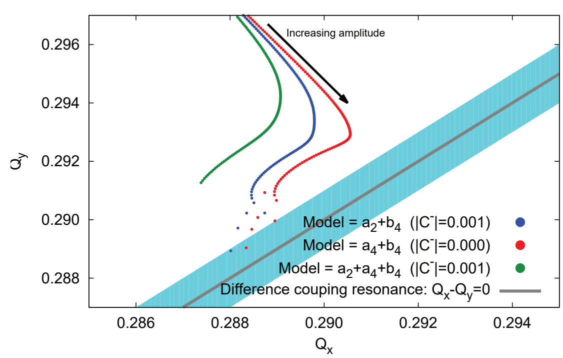


Figure 3: Approach to the $Q_x - Q_y$ resonance for LHC simulations at injection, including various multipole sources.

The nonlinear $\Delta Q_{min}$ generated by $a_4$ errors has only been studied in simulation, and confirmation in a real accelerator will be desired. A nonlinear $\Delta Q_{min}$ at the level of $1-3\times10^{-3}$, as predicted from the $a_4$ error simulations, is unlikely to be measurable. The LHC is equipped with skew octupole correctors, located on the left and right sides its four experimental insertions. These are intended for compensation of resonances driven by $a_4$ errors in the Insertion Regions (IR) at low-$\beta^*$, but have never been used operationally. Figure 4 shows the approach to the $Q_x - Q_y$ resonance for LHC simulations at injection, with MO powered as per 2012 operation, and with various configurations of the IR-$a_4$ correctors. No other multipole sources are included in the simulation. The top plot shows an increasing nonlinear $\Delta Q_{min}$ as $a_4$ corrector strengths are uniformly increased to their maximum current in all circuits. The lower plot shows various configurations of the $a_4$ correctors. Fig. 5 shows an example of the simulated spectrum obtained as a function of particle amplitude for a uniform 50 % powering of the $a_4$ correctors. On the level demonstrated by these simulations a beam-based validation should be possible in the LHC.

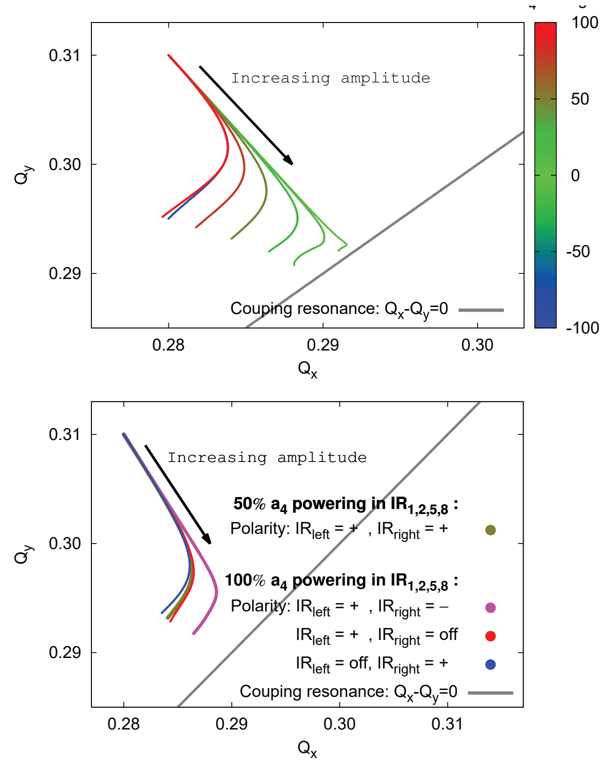


Figure 4: Detuning of particles towards the $Q_x - Q_y$ resonance with increasing strength of $a_4$ correctors in LHC insertions at injection (top). Detuning for various configurations of $a_4$ circuits (bottom). $|C^-|$=0.000 in all cases.

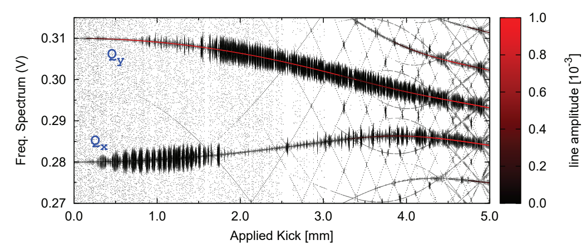


Figure 5: Frequency spectrum vs kick amplitude with 50 % powering in $a_4$ correctors.

While measurements with beam will be a necessary part of future studies of nonlinear $\Delta Q_{min}$ generated by $a_4$, the mechanism can also be explored further in simulation. Conventionally amplitude-dependent $\Delta Q_{min}$ has been studied, both in simulation and with beam, via saturation of tune separation while amplitude detuning from normal octupoles force $Q_{x,y}$ towards the coupling resonance. In such studies the effect of $a_4$ cannot be separated from the normal octupoles, which provide the mechanism to move towards the linear $\Delta Q_{min}$. An alternative study can be considered, where a tune trim is used to move the working point iteratively closer to the $Q_x - Q_y$ resonance, prior to tracking simulations with a particle of well defined amplitude. This represents an application of the classical coupling measurement, wherein tunes are forced together to determine $\Delta Q_{min}$, to tracking simulations with non-zero particle amplitude. Figure 6 shows the result of these tracking simulations.

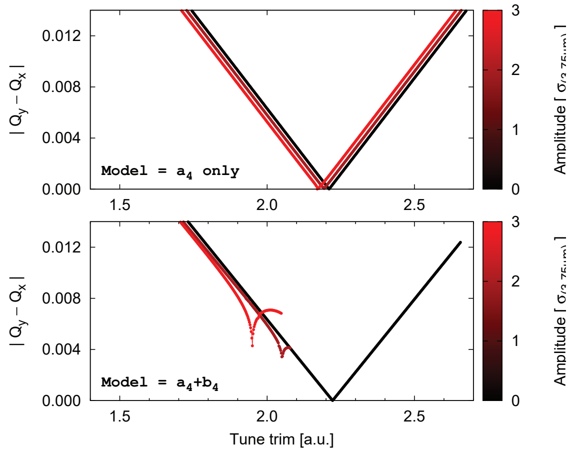


Figure 6: Tune separation in LHC models with only $a_4$, and with $a_4$ plus $b_4$ sources included.

Figure 6 (top) shows tune separation as a function of the working point trim, in a model containing only $a_4$ sources. Regardless of particle amplitude the tunes approach $Q_x - Q_y$ unhindered. In contrast when both $a_4$ and $b_4$ sources are included (Fig. 6, bottom), particles at zero amplitude behave according to the linear expectation while particles at large amplitudes are unable to approach within some threshold of the $Q_x - Q_y$ resonance. The amplitude dependent closest tune approach observed in Fig. 2-4 is therefore not generated by $a_4$ sources in isolation, but only in combination between normal and skew octupole fields.

In Fig. 4 (bottom) a slight asymmetry was seen between the amplitude-dependent $\Delta Q_{min}$ generated with $a_4$ correctors on the left side of the IRs powered (red), and simulations where only the right side of the IPs were powered (blue). This may provide first hints as to a mechanism generating the nonlinear $\Delta Q_{min}$, since $a_4$ correctors either side of the IP lie at differing ratios of $\beta_x/\beta_y$, and will therefore favourably drive RDTs corresponding to either the $x^3y$ or $y^3x$ skew octupole Hamiltonian terms. This feature should be enhanced at low-$\beta^*$ optics. Figure 7 shows simulations at $\beta^* = 0.4$ m, with octupole powering from 2016 operation and strong $a_4$ sources left or right of IR1 and IR5 (the LHC's low-$\beta$ IRs). With negative octupole polarity (red and blue data in Fig. 7), and large vertical kicks, there is a clear enhancement to footprint distortion for $a_4$ correctors on the right side of the IR1 and IR5 (blue) where $\beta_y \gg \beta_x$. When octupole polarity is reversed (purple and green data in Fig. 7), and kicks are performed horizontally, the opposite situation arises and correctors on the left side of IR1 and 5 (purple), with $\beta_x \gg \beta_y$, generate the greater distortion.

Results of Fig. 7 should not be over-interpreted. The simulated behaviour for opposite octupole polarities are quite different and simply powering correctors with $\beta_{x,y} \gg \beta_{y,x}$

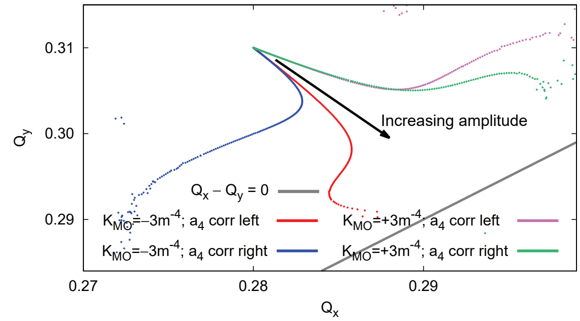


Figure 7: Approach to $Q_x - Q_y = 0$, for negative MO polarity with vertical kicks, and positive MO polarity with horizontal kicks. Only $a_4$ correctors left of IR1 and 5, or only right of IR1 and 5, are powered. $|C^-|$=0.0 in all cases.

does not fully suppress any particular RDTs. However, this may still hint that Hamiltonian terms from both the $x^3y$ and $y^3x$ skew octupole monomials contribute. Further studies will be required however, to isolate the specific mechanism at work.

## CONCLUSIONS

In isolation normal octupole fields do not affect observed $\Delta Q_{min}$. Similarly, skew octupoles alone do not hinder particles approach to the $Q_x - Q_y$ resonance. In combination however, normal and skew octupoles have been observed in simulation to generate an amplitude dependence of the closest tune approach. In the LHC at injection, the measured skew octupole errors and operation configurations of the normal octupoles create a nonlinear $\Delta Q_{min}$ in simulation which is comparable with already established and understood sources. By using $a_4$ correctors in the experimental insertions to artificially enhance the skew octupole content of the machine, validation of the amplitude dependent closest tune approach generated by normal and skew octupoles should be possible in the LHC at injection.

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