What pions can tell us about the neutron skin of nuclei

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Abstract
We use the Isospin Quantum Molecular Dynamics approach to analyse the isospin ratios of pion production in collisions of heavy ions at incident energies below 2 AGeV. It is found that the comparison of the centrality dependence at different energies allows to gain information on the neutron skin of the nuclei, but also on other ingredients like the nuclear equation of state of asymmetric matter.

1 Introduction

One of the main interests of the study of relativistic heavy ion collisions is the investigation of the properties of nuclear matter at extreme densities and excitation energies. These investigations include the production of secondary particles, the properties of particles in a (dense) nuclear medium, the compression and repulsion of dense nuclear matter, its equilibration during the reaction and its decay into fragments and single particles. The most prominent secondary particle is the pion, a pseudoscalar meson which due to its very small mass can already be produced in heavy ion collisions of a few hundred AMeV of incident energies in the laboratory frame. This article will focus on the production of that particle in an energy range of several hundred AMeV to a few AGeV. For our calculations we use the IQMD approach [1], a microscopic transport model calculating heavy ion collisions on an event-by-event basis. We will first describe the production of pions in IQMD and the initialisation of protons and neutrons. In the next step we will discuss the rescattering of pions in nuclear matter and its effect on the isospin ratio to finally study the effect of the neutron skin and of the nuclear equation of state of asymmetric matter on these ratios.

1.1 Pion production in IQMD

The philosophy of IQMD follows the idea used by many other microscopic transport models [2–4] to decompose the interaction of nucleons into a long-range part described by local central force two-body potentials and a short-range part described by stochastic collisions. The long-range term leads to nuclear potentials, which become important for the stability of the nucleus and also touch topics like the nuclear equation of state [5]. We will shortly summarize the important part of the collision term and refer for a detailed description of both parts and their application in IQMD on [6]. Inside IQMD particles are described by Gaussian wave packages in coordinate and momentum space. Two particles collide if their minimum distance d, i.e. the minimum relative distance of the centroids of the Gaussians during their motion, in their CM frame fulfills the requirement:

$$d \le d_0 = \sqrt{\frac{\sigma_{\rm tot}}{\pi}}, \qquad \sigma_{\rm tot} = \sigma(\sqrt{s},\ \text{type}). \tag{1}$$

where the cross section is assumed to be the free cross section of the regarded collision type ($N - N$, $N - \Delta$, ...). The cross sections for elastic and inelastic collisions are obtained by a table lookup using experimentally measured cross sections (when available) or derived from available cross sections using symmetry assumptions and detailed balance.

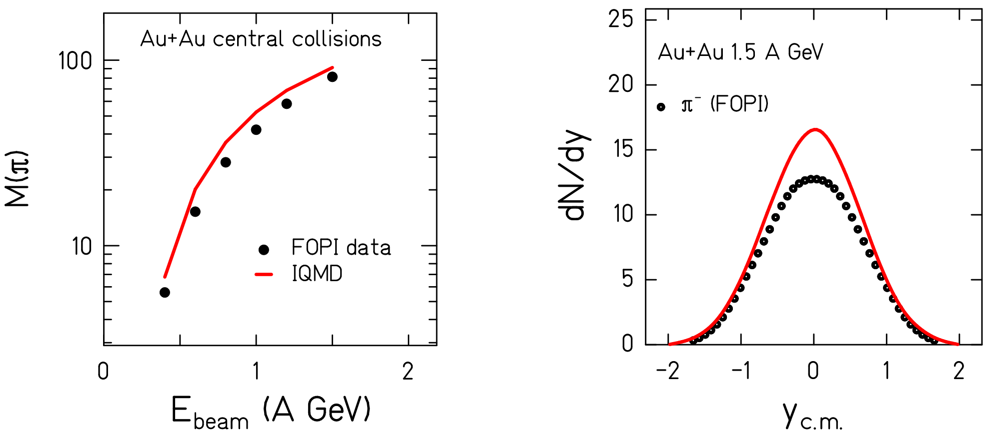

Fig. 1: Comparison between IQMD calculations and FOPI results on the absolute pion number as function of the incident energy (left) and of the rapidity distribution of π^- in collisions of Au+Au at 1.5 AGeV (right). (from [9])

The pion production in IQMD is done via the Δ-channel, where deltas can be produced in nucleon-nucleon (NN) collisions and be reabsorbed in $N\Delta$ collisions. The Δ decays and produces a free pion, which can be reabsorbed in collisions with a nucleon and form a Δ again:

$$NN \leftrightarrow N\Delta \qquad \Delta \leftrightarrow N\pi \tag{2}$$

These reactions have to comply with detailed balance and isospin effects have to be taken into account by the use of Clebsch-Gordon coefficients. For more details see [7].

The production of pions in IQMD has successfully been tested by various comparisons with experimental measurements performed by the FOPI collaboration at GSI [8]. Giving an example taken from [9], figure 1 compares the excitation function of the total pion yield in Au+Au collisions measured by FOPI to IQMD calculations (left hand side). For this purpose, the events calculated by IQMD have undergone the same analysis procedures as the experimental data. The multiplicities calculated by IQMD are slightly higher than those obtained by FOPI but the excitation functions show nicely the same behavior. Figure 1 compares on the right hand side the rapidity distributions of negative pions in central collisions. As already stated before, IQMD shows slightly larger absolute pion yields, therefore it is not astonishing that the absolute numbers of the rapidity distribution are also higher than the experimental points. However, the structure of the distribution is quite similar. The distribution is peaked at midrapidity underlining that most of the pions are produced by first collisions or in collisions of the stopped participant matter. Afterwards they will undergo rescattering and we will come to that point later on.

1.2 Density profiles of protons and neutrons

In standard IQMD calculations the centroids of the Gaussians are distributed inside a sphere in the rest frame of the nucleus according to

$$(\vec{r}_i - \vec{r}_{CM})^2 \leq R_A^2 \qquad R_A = R_0 \cdot A^{1/3} \tag{3}$$

where $\vec{r}_i$ and $\vec{r}_{CM}$ are the position vectors of particle i and of the center-of-mass of the nucleus, respectively, and $A = Z + N$ is the number of nucleons of the nucleus. The radius parameter is chosen as $R_0 = 1.12$ fm. This initialization, which we will call "R_P=R_N ", assures the same rms radius for protons and neutrons even for heavy isospin-asymmetric systems. Consequently, in the whole nucleus the neutrons have systematically a higher density than the protons. It should be noted that most other microscopic models with the exception of [10] use a similar procedure yielding the same rms radius for protons and neutrons. If we want to assume the same density of protons and neutrons at least in the centre of the nucleus we have to allow protons and neutrons to have different rms radii, which can be obtained by the distribution of the centroids of the Gaussians according to

$$R_P = R_0 \cdot (2Z)^{1/3} \qquad R_N = R_0 \cdot (2N)^{1/3} \tag{4}$$

with R_P and R_N denoting the radii for protons and neutrons. We will call this initialisation "R_P<R_N ". However, R_P<R_N yields a difference of the rms radii of protons and neutrons of around 0.5 fm in a system like ^{197}Au.

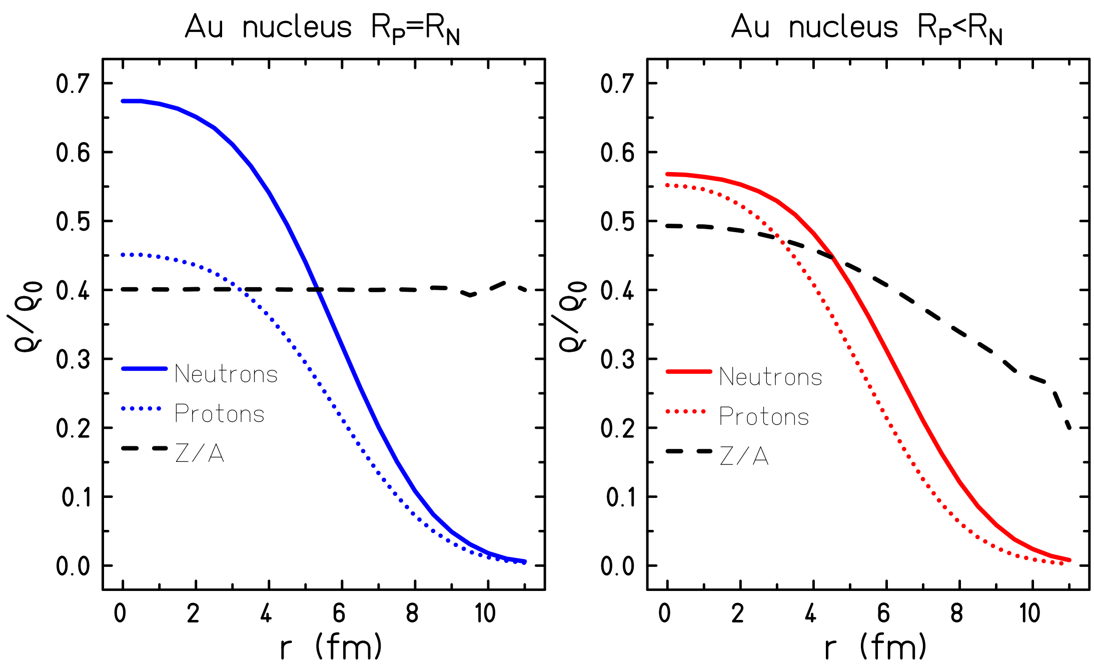

Fig. 2: Profiles of proton and neutron densities and for the charge ration $Z/A(r)$ for initialisations assuming R_P=R_N (left) and R_P<R_N (right).

Figure 2 shows the density profiles of protons (dotted lines) and neutrons (full lines) in a ^{197}Au nucleus using both initialisations R_P=R_N (left hand side) and R_P<R_N (right hand side). While for the first one the charge density $Z/A(R)$ (dashed line) remains constant over the whole nucleus, that ratio varies strongly for R_P<R_N . It should be noted that the rescattering cross sections of π^- with neutrons are higher than those with protons - and analogously those of π^+ with protons are higher than those with neutrons, therefore this difference will have an effect on the isospin ratios as we will see later on.

2 Dynamics of pion production and the importance of rescattering

Let us first focus on the collisions of Au(400 AMeV)+Au. This system has gained a lot of interest, since the experiment shows a strong enhancement of the ratio π^-/π^+ [8] which could not be completely explained by microscopic calculations and thus yielded a lot of speculations about its relation to the nuclear equation of state of asymmetric nuclear matter [11, 12].

Figure 3 shows the time evolution of a central collision of that system using both initialisations, R_P=R_N (blue lines) and R_P<R_N (red lines). The left hand side presents on the top the total nucleon density reached in the central point of the collisions. We see identical curves for both initialisations, reaching a maximum of $2.5\rho_0$ at a time of about 12 fm/c. On the bottom (still left hand side) we see that the isospin-integrated absolute numbers of free pions (dashed lines), deltas (dotted lines) and of their

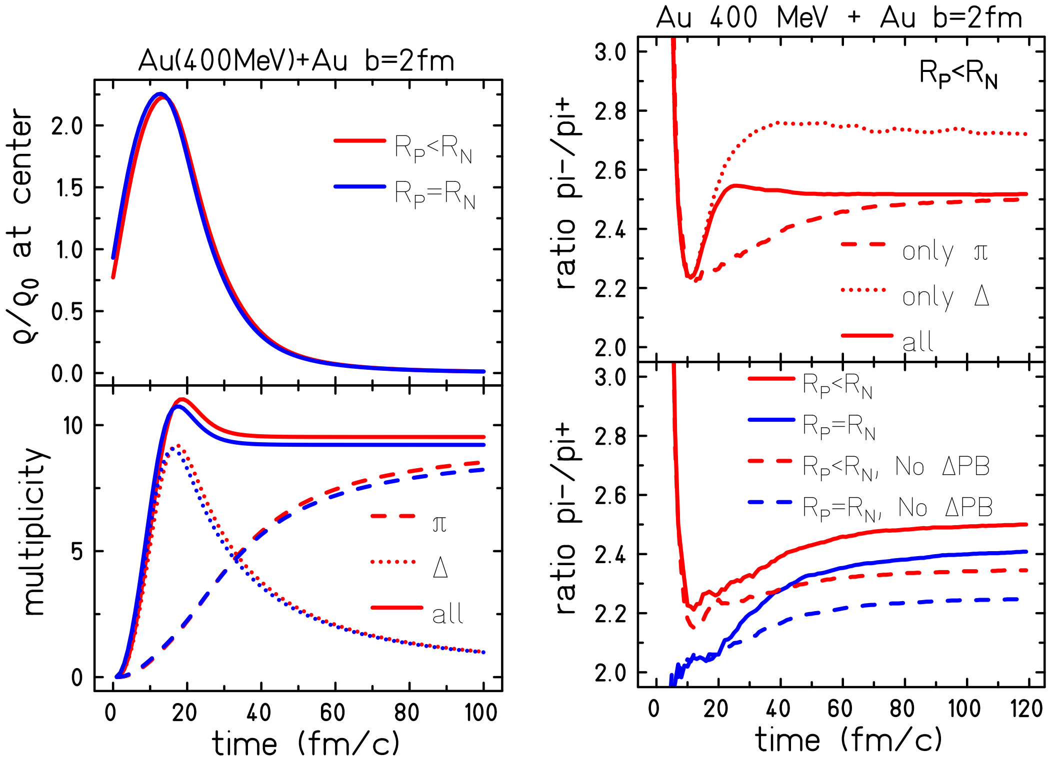

Fig. 3: Time evolution of a collision of Au(400 AMeV)+Au. Left: central density (top) and yields of deltas and pion (bottom), right: ratios π^-/π^+ assumed from deltas and free pions (top) and the for different assumptions on the initialisation and Pauli blocking in the delta decay.

sum (full lines) are also quite identical for both initialisations. The final total pion yields seems thus not to be affected by one or the other choice of the initialisation. The total pion numbers (full lines) rise up quite quickly during the compression stage to obtain its maximum at the moment of maximum compression but reaches its final value in the beginning of the expansion phase. We also see that the free pions (dashed lines) come out quite late, while in the high density stage most of the pions are "hidden" in the deltas (dotted lines). This is due to the huge reabsorption cross section $\pi N \to \Delta$.

On the right hand side we now study the time evolution of the isospin ratio π^-/π^+: on the top we distinguish that ratio by determining it by only counting the free pions (dashed line), taking only the deltas and applying the Clebsch-Gordon coefficients for the decay (dotted lines) or combining both calculations (full line). We see that the final value is determined quite early when taking the combined calculations, while the free pions smoothly converge to that value and the deltas always compile higher values. This may be interpreted such, as negative pions seem to "stay longer" in the delta, which may be a hint for higher rescattering and perhaps Pauli blocking in neutron rich matter. We will return to that point later.

The bottom part shows the influence of the initialisations on the ratio obtained from the free pions: the initialisation with $R_P{<}R_N$ (red lines) systematically yields higher values than that with $R_P{=}R_N$ (blue lines). This may be explained by the effect that neutron rich matter acts differently on π^- and π^+: while for π^- rescattering will lead to an intermediate Δ^- which decays only by $\Delta^- \to n\pi^-$ - and thus does not influence the π^- yield, a rescattering of a π^+ - yielding a Δ^+ - will reproduce a π^+ with a probability of only one third and thus penalises the π^+ yield. This effect is enhanced when the Pauli blocking of the delta decay is activated (full lines): the high density of neutrons will add a penalty to the the $\Delta^+ \to n\pi^+$ channel and reduce the π^+ yield even more with respect to calculations without Pauli blocking (dashed lines). The Pauli blocking also acts on the channel $\Delta^- \to n\pi^-$, but since there is

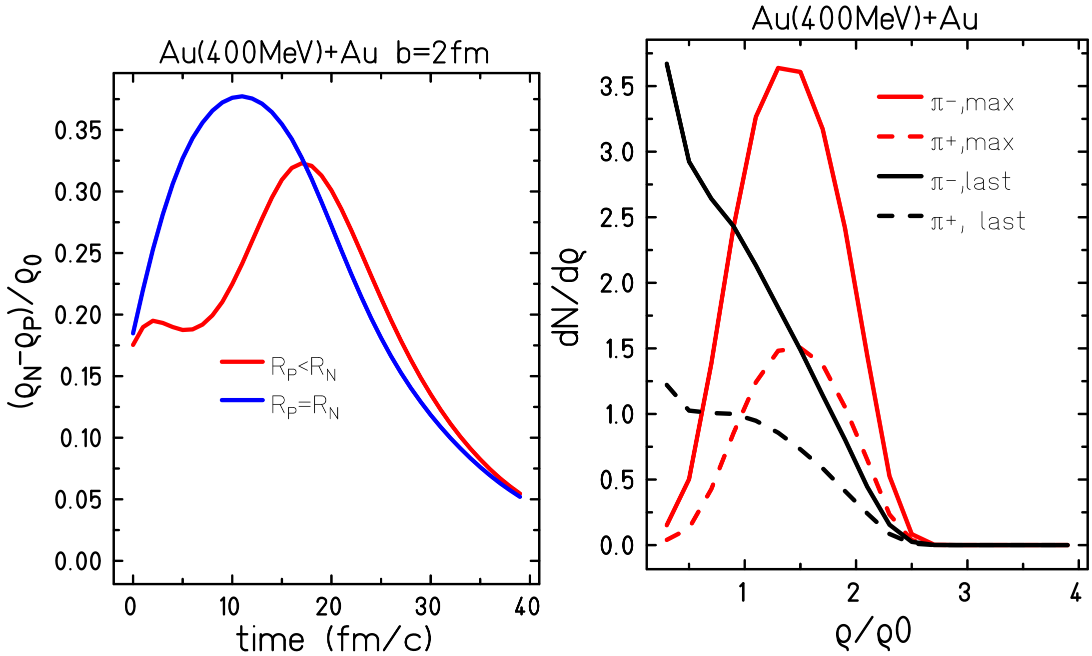

Fig. 4: Left: Time evolution of the density difference between neutrons and protons, right: distribution of maximum density and of freeze-out density of pions

no concurrent channel, this will only delay the decay of the delta, as we have seen this on the top graph. This could also explain the effect, why in the top part of Fig. 3 the ratio obtained from the deltas only than the ratio obtained from the free pions.

Another significant difference are the values of the ratios at very early times: while the calculation using R_P=R_N (blue line) starts with low values nearby 2 which smoothly rise, the calculation using R_P<R_N starts with very high values above 3 which rapidly fall down to rise smoothly again. This corresponds to the fact, the the very first deltas are created at the moment when the nuclei start to touch each other. At this time they feel the $Z/A(R = R_A)$ which is always around 0.4 for R_P=R_N but much lower for R_P<R_N as seen in figure 2. Here we see already an indication, that later on the study of very peripheral collisions might be quite interesting. The rise of the ratio after about 20 fm/c - common for both initialisations - is due to rescattering as explained before.

In order to test this assumption, we will inspect the rescattering of pions in more detail. Figure 4 analyses the time evolution of the central density in a different way: instead of summing up the densities for protons and neutrons (as done in fig.3) the difference $(\rho_n - \rho_p)/\rho_0$ is compared for the initialisation with R_P=R_N (blue line) and R_P<R_N (red line). We see that while in R_P=R_N the compression of matter with neutron excess ($Z/A(R) \approx 0.4$ everywhere) causes quite significant density differences, the nearly isospin-equilibrated center of the nuclei ($Z/A(R = 0) \approx 0.5$ yield much weaker values, which only enhance at late times, after the maximum compression had been reached. In the expansion stage both simulations reach similar values. From this finding we may assume that effects at the high density stage, e.g. the effects of asymmetry dependent potentials (like the equation of state of asymmetric matter) should be even weaker with R_P<R_N ! This would give a paradoxal result: the higher neutron densities obtained with R_P=R_N should lead to even a higher π^-/π^+ ratio, if the high density stage is decisive for the pion yields. The right hand side explains this paradoxal situation: here the maximum density (red lines) seen by a π^- (full line) and a π^+ (dashed line) is compared to the freeze-out density (black lines), i.e. the density of the last delta decay. While the pions are initiated in the compression phase and thus "felt" the high densities (with a maximum of the distribution nearby $1.5\rho_0$), the density signal from the last interaction (black lines) peak at lowest densities: the pions will only "memorize" low densities seen

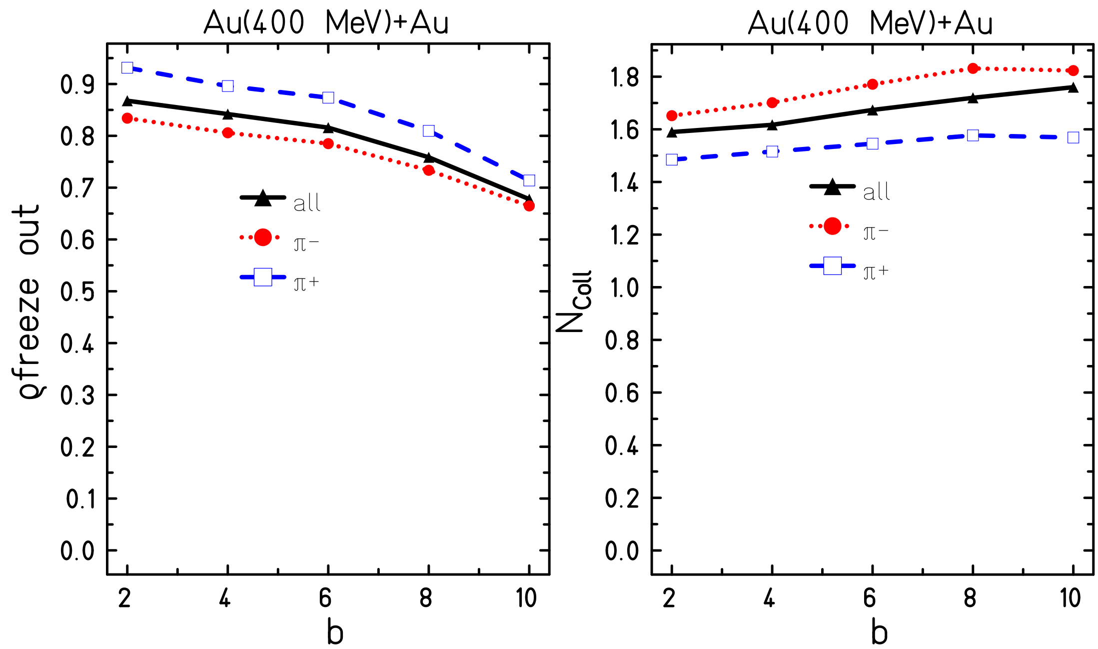

Fig. 5: Impact parameter dependence of π^- and π^+ freeze-out density (left) and number of collisions (right) in reactions of Au+Au at 400 AMeV

at the freeze-out.

This is nicely shown in figure 5 which describes on the left hand side the centrality dependence of the mean freeze-out density of π^- (red dotted line) and π^+ (blue dashed line): Both types freeze-out at densities below ρ_0. However, π^- systematically freeze-out at lower densities than π^+ which supports the idea of a late freeze-out in the expanding matter. Additional analysis show indeed that the freeze-out time of π^- is later than that of π^+. In this context it should be reminded, that the rescattering cross sections of π^- in neutron rich matter is higher than that of π^+. The right hand side supports this by presenting the mean number of collisions the pions have undergone: π^- show higher rescattering numbers than π^+ and values nearly up to 2 are obtained. Thus the idea of the importance of the neutron skin on the π^-/π^+ ratio seems to be coherent.

3 Centrality dependence of the isospin ratio

As shown in the precedent section the isospin ratio of pions in central collisions is influenced by the description of the neutron skin of the nucleus as well of the Pauli blocking in the delta decay. This seem to add an ingredient to other propositions [11–14] mainly addressing the nuclear equation of state of asymmetric matter and therefore that effect should be discussed as well. A very common description is to scale the density dependence of the asymmetry potential with an exponent γ. The (classical) linear dependence thus corresponds to $\gamma = 1$, a soft asy-eos to $\gamma < 1$ and a hard one to $\gamma > 1$.

Figure 6 presents on the left hand side the effects of the initialisations R_P<R_N (red full line) and R_P=R_N (blue dashed line) as well of the neglection of the Pauli blocking in the delta decay (black dotted line). For central collisions this effect has already discussed previously: the effect of rescattering in neutron rich matter penalises the π^+ - especially in case of R_P<R_N , where $Z/A(R)$ is quite low in the outer part, which is the region where the last rescattering happens. Since no penalty applies to the π^- this enhances the π^-/π^+ ratio. The Pauli blocking in the delta decay strengthens this penalty on the π^+ and thus enhances the ratio with respect to a calculation without Pauli blocking of the delta decay (black dotted line). However, a significant rise of the ratio can be seen for R_P<R_N when going to peripheral collisions. Here a strongly neutron rich environment is found even in the first initial collisions.

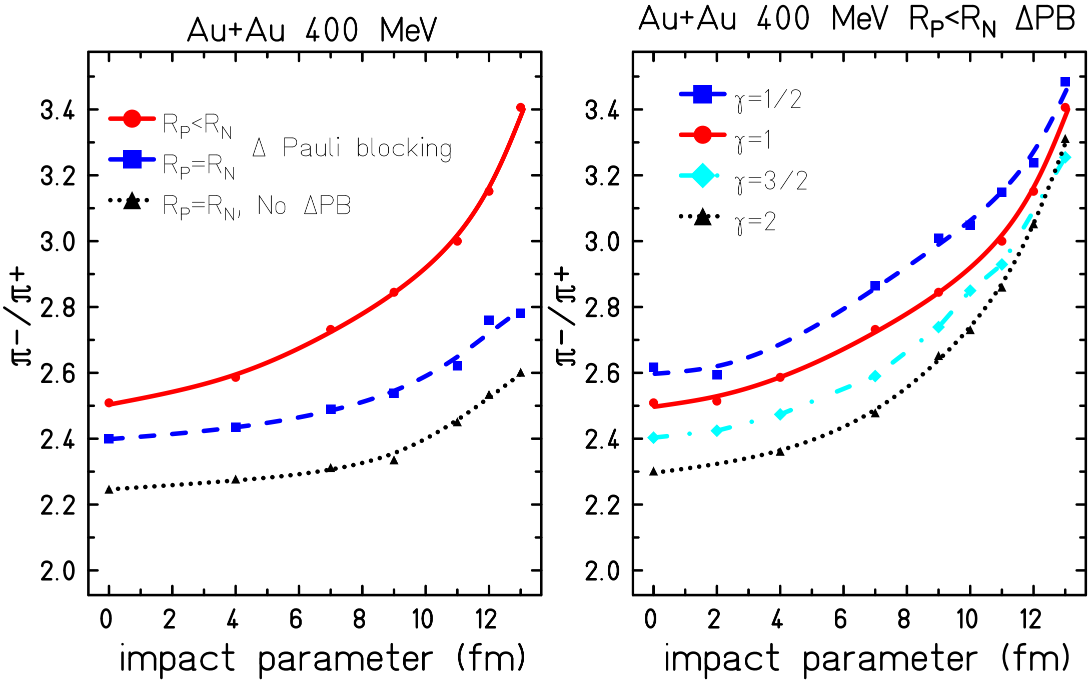

Fig. 6: Impact parameter dependence of the ratio π^-/π^+ in collisions of Au(400 AMeV)+Au. Left: influence of neutron skin and delta Pauli blocking right: influence of the equation of state of asymmetric matter.

We already indicated this when discussing the early time behavior of the ratio in figure 3.

The effect of the equation of state of asymmetric matter (right hand side, the initialisation $R_P{<}R_N$ is used) shows on the contrary a rather moderate influence at peripheral collisions but a significant effect for central collisions. Here a softer asy-eos (blue dashed line) enhances the π^-/π^+ ratio with respect to a hard one (black dotted line). This can be interpreted such a way, that a strong (hard) asymmetry potential tries to dilute regions with strong neutron enrichment and thus reduces the effects described above. For peripheral collisions less compression is reached and the effects become smaller. This gives us a first indication that the analysis of the centrality dependence may play a very important role for disentangling the different effects.

The effect of the neutron skin on the centrality dependence increases even at higher incident energies, as it can be depicted from figure 7 presenting the same analysis but for Au(1200 AMeV)+Au. While the effect of the neutron skin becomes very prominent, the influence of the Pauli blocking in the delta decay and the effects of the equation of state of asymmetric matter vanish completely. Here we get a very good handle to test the neutron skin from π^-/π^+ ratios at high energies: from comparison of central and very peripheral collisions we may estimate the thickness of the neutron skin. A more detailed investigation of that procedure is presented in [15], where more refined parametrisations of the neutron skin are studied and applied to different nuclei like ^{48}Ca or ^{208}Pb. Once the question of the neutron skin is fixed, one may attack the other effects at low energies.

4 Conclusion

In this articles the influence of the neutron skin on the isospin ratio of pions π^-/π^+ has been discussed. It has been shown that due to rescattering a neutron skin is able to enhance this ratio in collisions of Au (400 AMeV)+Au. This enhancement becomes very significant at very peripheral collisions and gets even more prominent when going up in the incident energy. The effects of the equation of state of asymmetric matter only show up at low incident energy and dominate in central collisions. Therefore, a procedure of disentangling the effects can be proposed by measuring the full impact parameter dependence of π^-/π^+

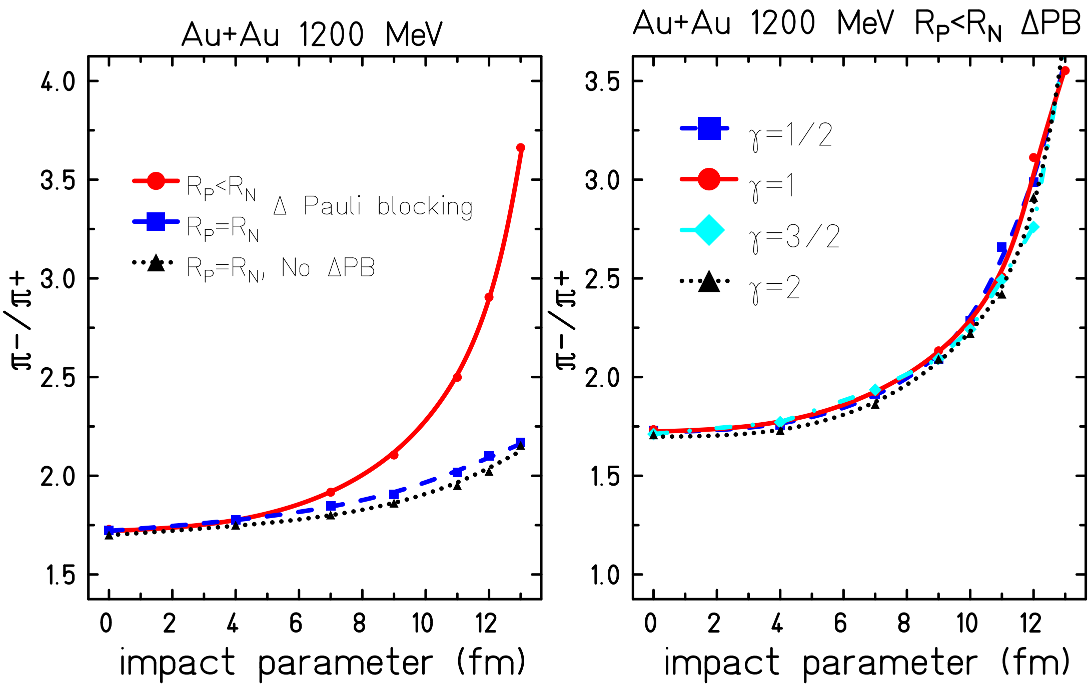

Fig. 7: Impact parameter dependence of the ratio π^-/π^+ in collisions of Au(400 AMeV)+Au. Left: influence of neutron skin and delta Pauli blocking right: influence of the equation of state of asymmetric matter.

at low and at high incident energies. From the comparison of high precision data to simulation one may thus reveal better information on the neutron skin and get another handle to attack the nuclear equation of state of asymmetric matter.

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