Heavy ion charge exchange reactions and the link with β decay processes

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Abstract
Within the DWBA framework, we develop a theoretical description of single and double heavy ion charge exchange (CE) reactions. We show that absorption effects are particularly important for heavy ion reactions, leading to a noticeable reduction of the CE cross sections. At low momentum transfer, the single CE cross section can be factorised, thus allowing to evaluate corresponding distortion factors and access β decay strengths. Applications are shown for a system of experimental interest. Preliminary results are discussed also for double CE reactions, modeled as a two-step mechanims, i.e. a sequence of two charge-changing processes.

1 Introduction

Nuclear charge exchange reactions offer the possibility to explore the nuclear interaction in the spin-isospin channel, being related to excitations inducing isospin flip (with possibly also spin flip), such as the Gamow-Teller resonance (GTR). Over the years, a wealth of data has been accumulated as reviewed e.g. in [1–5]. Beyond using nucleonic probes, light ion reactions as e.g. $(^3He, ^3H)$ have become another workhorse of the field, now reaching accuracies allowing to investigate subtle details of spectral distributions in both the τ_+ and the τ_- branches. Soon after the first light ion studies, also heavy ions were used in charge exchange studies, as in [6, 7].

While the past experiments have been focused on single charge exchange (SCE) reactions, new territory was entered by the pilot experiment of Cappuzzello et al. [8], studying for the first time a nuclear double charge exchange (DCE) reaction. The reaction $^{18}O + ^{40}Ca \rightarrow ^{18}Ne + ^{40}Ar$ gave strong evidence for a direct reaction mechanism even for double charge exchange processes. Quite recently, the NUMEN project at LNS Catania was initiated, dedicated to investigations of SCE and DCE heavy ion reactions, elucidating and optimizing their potential for spectroscopic studies [9], also in the perspective of probing the single/double β-like nuclear response. DCE experiments have been recently performed also with different goals: search for exotic systems (such as the tetra-neutron (4n) system in $^4He(^8He,^8Be)4n$ reactions); search for the Double GT resonance for quantitative information about the corresponding sum-rule (for example in $(^{12}C,^{12}Be)$ reactions) [10, 11].

A thorough theoretical investigation of the dynamics of heavy-ion CE reactions, concerning in particular the possibility of extracting the nuclear structure information out of the total reaction cross section, is still missing, though some progress has been made over the past year [12]. This is especially important for DCE reactions, that necessarily involve heavier projectiles than the light ones traditionally employed in CE reactions. The possibility to single out the relevant structure information is an essential point if one wishes to exploit the measured cross sections as stringent benchmarks to constrain the theoretical structure models and the predicted transition matrix elements. This would allow to shed light on yet unknown aspects of charge-exchange transitions and of the underlying nuclear effective interaction. Moreover, owing to the analogies between strong and weak charge exchange processes, DCE studies can provide useful information to improve the accuracy of the calculations of the nuclear matrix elements responsible for neutrinoless double-beta decay, a quite hot subject of investigation nowadays.

In this contribution, we briefly review some aspects of the theory for heavy ion charge exchange reactions. In particular, we discuss the possibility to single out the structure information from the reaction cross section. Illustrative results are shown for systems recently investigated by the NUMEN collaboration.

2 Theory of Heavy Ion Charge Exchange Reactions

Charge changing reactions require two reaction partners, which are acting mutually as the source or sink, respectively, of the charge-changing virtual meson fields. Let us start considering ion-ion SCE reactions according to

$$ {}^{a}_{z}a + {}^{A}_{Z}A \rightarrow {}^{a}_{z\pm1}b + {}^{A}_{Z\mp1}B \tag{1}$$

which change the charge partition by a balanced redistribution of protons and neutrons.

The differential SCE cross section for a reaction connecting the channels α and β is defined as

$$ d^2\sigma_{\alpha\beta} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{1}{(2J_a+1)(2J_A+1)} \times \sum_{M_a,M_A\in\alpha;M_b,M_B\in\beta} |M_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta)|^2 d\Omega, \tag{2}$$

where $\mathbf{k}_\alpha$ ($\mathbf{k}_\beta$) and m_α (m_β) denote the relative 3-momentum and reduced mass in the entrance (exit) channel $\alpha = \{a, A\}$ ($\beta = \{b, B\}$). $\{J_aM_a, J_AM_A\cdots\}$ and $\{J_bM_b, J_BM_B\cdots\}$ account for the full set of (intrinsic) quantum numbers specifying the initial and final channel states, respectively.

In distorted wave approximation, the direct charge exchange reaction amplitude is given by the expression

$$ M_{\alpha\beta}(\mathbf{k}_\beta,\mathbf{k}_\alpha) = \langle \chi^{(-)}_\beta, bB|T_{NN}|aA,\chi^{(+)}_\alpha\rangle. \tag{3}$$

The distorted waves, denoted by $\chi^{(\pm)}_{\alpha,\beta}$ for asymptotically outgoing and incoming spherical waves, respectively, depend on the respective channel momenta $\mathbf{k}_{\alpha,\beta}$ and the optical potentials, thus accounting for initial state and final state interactions.

The charge-changing process is described by the nucleon-nucleon (NN) T-matrix T_{NN}. The anti-symmetrized T-matrix is given in non-relativistic momentum representation by

$$ T_{NN}(\mathbf{p}) = \sum_{S,T}\left\{V^{(C)}_{ST}(p^2)O_{ST}(1)\cdot O_{ST}(2) + \delta_{S1}V^{(Tn)}_{T}(p^2)\sqrt{\frac{24\pi}{5}}Y^*_2(\hat{\mathbf{p}})\cdot[O_{ST}(1)\otimes O_{ST}(2)]_2\right\}, \tag{4}$$

including isoscalar and isovector central spin-independent ($S = 0$) and spin-dependent ($S = 1$) interactions with form factors $V^{(C)}_{ST}(p^2)$, respectively, and rank-2 tensor interactions with form factors $V^{(Tn)}_{T}(p^2)$. O_{ST} denotes the spin-isospin operators $O_{ST}(i) = (\boldsymbol{\sigma}_i)^S(\boldsymbol{\tau}_i)^T$.

In Eq.(4) scalar products are indicated as a dot-product and the rank-2 tensorial coupling affects only the spin degrees of freedom. The subset of isovector operators, corresponding to Fermi-type $S = 0, T = 1$ and Gamow-Teller-type $S = 1$, $T = 1$ operators contributes to the charge-changing reaction amplitudes.

The matrix element of a single charge exchange reaction, Eq.(3), can be written in slightly different form as:

$$ M_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \langle \chi^{(-)}_\beta|\mathcal{U}_{\alpha\beta}|\chi^{(+)}_\alpha\rangle. \tag{5}$$

Then the nuclear structure information on multipolarities, transition strength and interactions are contained in the (anti-symmetrized) transition potential $\mathcal{U}_{\alpha\beta}$, depending on the channel coordinates $\mathbf{r}_{\alpha,\beta}$.

In the momentum representation, the full reaction amplitude can be rewritten as:

$$ M_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta) = \int d^3p\mathcal{U}_{\alpha\beta}(\mathbf{p})N_{\alpha\beta}(\mathbf{k}_\alpha,\mathbf{k}_\beta,\mathbf{p}), \tag{6}$$

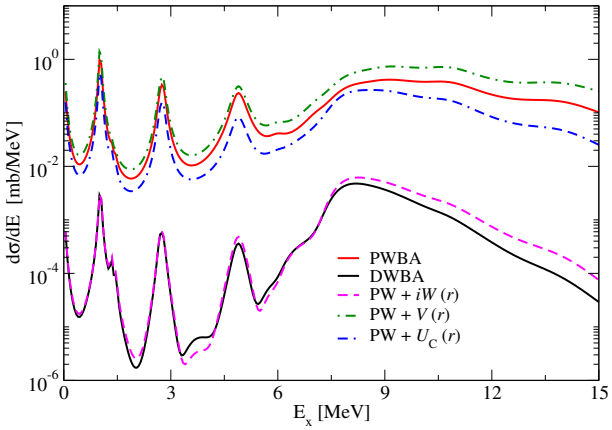

Fig. 1: (Color online) Cross sections as a function of the target excitation energy, E_x, for the $J^P = 1^+$ transition, for the SCE reaction $^{40}Ca\left(^{18}O,^{18}F\right)^{40}K$ reaction at $T_{lab} = 270\,MeV$, integrated over the full angular range. The different curves show the effect of Coulomb potential ($U_C(r)$), of real ($V(r)$) and imaginary ($W(r)$) components of the optical potential and of the full potential (DWBA), with respect to PWBA calculations.

where the distortion coefficient

$$N_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta, \mathbf{p}) = \frac{1}{(2\pi)^3}\langle\chi_\beta^{(-)}|e^{-i\mathbf{p}\cdot\mathbf{r}}|\chi_\alpha^{(+)}\rangle, \tag{7}$$

has been introduced, showing the dressing of the nuclear transition potential by initial and final state ion-ion interactions.

The reaction kernel is given by a product of form factors:

$$\begin{aligned} U_{\alpha\beta}^{(ST)}(\mathbf{p}) &= (4\pi)^2 (V_{ST}^{(C)}(p^2) F_{ST}^{(ab)\dagger}(\mathbf{p}) \cdot F_{ST}^{(AB)}(\mathbf{p}) \\ &+ \delta_{S1}\sqrt{\frac{24\pi}{5}} V_{ST}^{(Tn)}(p^2) Y_2^*(\hat{\mathbf{p}}) \cdot \left[F_{ST}^{(ab)\dagger}(\mathbf{p}) \otimes F_{ST}^{(AB)}(\mathbf{p})\right]_2), \end{aligned} \tag{8}$$

where $F_{ST}(\mathbf{p})$ represents the Fourier transform of the (projectile or target) transition density.

A possible extension of the formalism to DCE reactions is based on second order perturbation theory, thus double charge exchange is depicted as a two-step process, i.e. a sequence of two single charge exchange transitions. This implies a summation over (virtual) intermediate states, and the treatment of the relative distortion effects in the intermediate channel.

3 Results for single CE

As stressed above, our guiding principle to describe charge exchange reactions is direct nuclear reaction theory, based on the Distorted Wave Born Approximation (DWBA). Initial and final state ion-ion interactions are described by optical potentials. Microscopic optical potentials are used, obtained in the impulse approximation, by folding projectile and target Hartree-Fock-Bogoliubov (HFB) ground state densities with free space nucleon-nucleon T-matrices. QRPA calculations are performed for nuclear SCE transition densities and response functions, employing a G-Matrix interaction (see Ref. [12] and Refs. therein). For the sake of consistency, the same iteraction is considered for the evaluation of the reaction kernel, Eq.(8). More details can be found in Ref. [12].

As a case of physical interest, we consider throughout the SCE reaction $^{18}O + ^{40}Ca \rightarrow ^{18}F + ^{40}K$ at $T_{lab} = 15$ AMeV. In particular, we consider transitions leading to the ^{18}F g.s., that is a 1^+ state, and to $J^P = 1^+$ states for the target. Fig.1 represents the SCE cross section, integrated over the full angular range, as a function of the target excitation energy, as obtained with the HIDEX code [13]. One can

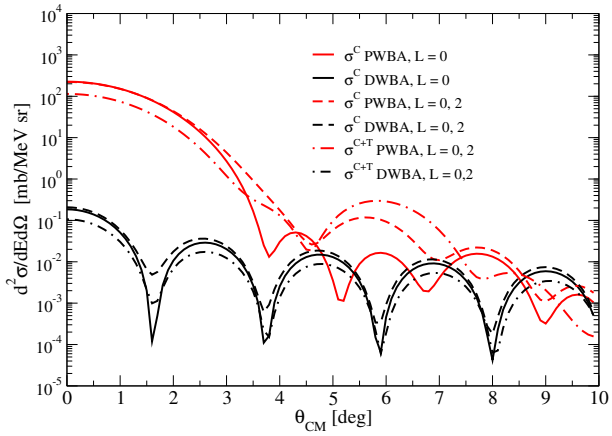

Fig. 2: (Color online) Angular distribution for the target state at E_x = 0. The plot shows the effects related to central and tensor components of the nuclear interaction, for the two multipolarities allowed by $J^\pi = 1^+$ transitions: $L = 0, 2$. The system is the same as in the previous figure.

identify several peaks, associated with corresponding excited states. For comparison, calculations are performed also in the Born approximation (PWBA), and isolating the effect of the different parts of the optical potential. One can notice that the DWBA cross section is quite suppressed with respect to PWBA results, pointing to strong absorption effects. Indeed results very close to the full DWBA calculations are obtained by simply considering the imaginary part of the optical potential. We notice that strong absorption effects are peculiar of heavy ion reactions. Fig.2 displays the angular distribution obtained for the 1^+ target state at the lowest excitation energy. The contribution of the two allowed multipolarities, as well as the effect of central and tensor components of the nuclear interaction, are shown on the figure. Also in this case PWBA and DWBA results are compared, allowing one to appreciate how the diffraction pattern is affected by the distortion effects. One can also observe that at small angles L = 0 transitions dominate and that the effects of the tensor component of the nuclear interaction are rather small.

4 Cross section factorization

Explicit simplified expressions can be derived for forward angle cross sections, owing to the small momentum transfer $\mathbf{q}_{\alpha\beta}$. Indeed, in this case the reaction kernel in Eq.(8) can be factorized into the product of its on-shell value, $\mathcal{U}(\mathbf{q}_{\alpha\beta})$, and a coefficient, $h(\mathbf{q})$, depending on the off-shell momentum $\mathbf{q} = \mathbf{p} - \mathbf{q}_{\alpha\beta}$. Then, after the integration is performed, Eq.(6) can be written as $M_{\alpha\beta} = \mathcal{U}(\mathbf{q}_{\alpha\beta})(1 - n_{\alpha\beta})$, leading to the distortion factor $f_{\alpha\beta} = |1 - n_{\alpha\beta}|^2$ in the cross section.

Analytical calculations can be performed in the black disk (BD) approximation and considering a Gaussian fit of the transition form factors. Results for the distortion factor are displayed in Fig.3 (left panel), as a function of the BD radius R_{abs}. For the reaction considered in our study, $R_{abs} \approx 8$ fm, as it can be extracted from the absorption cross section. Correspondingly, the suppression factor is found to be 8.14 10^{-4}, in good agreement with the numerical DWBA/PWBA result, $f_{BD}(numerical)|_{Ex=0} =$ 8.35 10^{-4}, as it can be extracted from the ratio between DWBA and PWBA calculations at zero angle (see Fig.2). Fig.3 (right panel) shows the distortion factor obtained for the reaction considered so far and for the system $^{18}O + ^{116}Sn$, as a function of the beam energy. Larger distortion effects are observed for the heavier system.

Thanks to the factorization of the distortion effects, it is possible to isolate the form factor, containing projectile and target transition densities, from the reaction cross section. It should be noticed that for L = 0 transitions, the latter are directly connected to β decay strengths. Thus heavy ion SCE reactions are indeed providing access to nuclear matrix elements relevant also for β-decay. The results derived in this section are of special importance since they are showing explicitly the potential of heavy ion SCE

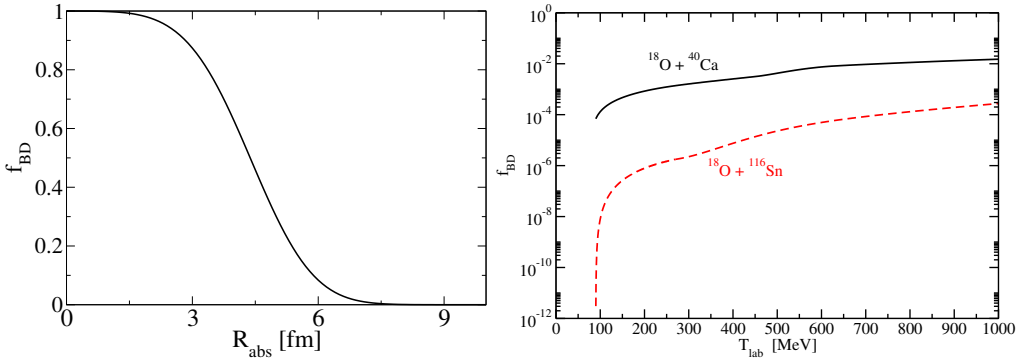

Fig. 3: (Color online) Left panel: Distortion factor as a function of R_{abs}, for the same system as in the previous figures. Right panel: The distortion factor f_{BD} is displayed as a function of the beam energy for two reaction systems: $^{18}O + ^{40}Ca$ and $^{18}O + ^{116}Sn$. The results were obtained numerically by the ratio of the quantum mechanical DWBA and PWBA forward angle cross sections.

reactions for spectral investigations, including the deduction of nuclear matrix elements for β-decay.

5 Results for DCE cross sections

Within the hypothesis of a two-step process, calculations have been performed for the double charge exchange reaction $^{18}O + ^{40}Ca \rightarrow ^{18}\ Ne + ^{40}Ar$ considering just one intermediate state (a 1+ state for both projectile and target). Results have been obtained using the FRESCO code [14] or by combining the results given by HIDEX for the two single SCE steps. The same form factors are employed in the two cases. Calculations have been performed in PWBA and DWBA approximations, as shown in Fig.4. The two methods (HIDEX double step and FRESCO) are in excellent agreement at the PWBA level. As far as DWBA calculations are concerned, two approximations are employed in the HIDEX case to deal with the distortion effects in the intermediate channel: i) plane waves are considered; ii) distorted waves associated with the full (absorptive) optical potential are employed. By comparing with the FRESCO results, one observes that the ii) option gives the right diffraction pattern, though the cross section is underestimated by a factor $f_{p.w.} \approx 10$. This indicates that distortion effects mainly act only in the entrance and exit channels and should not affect much the virtual intermediate states. On the other hand, the option i) reproduces the correct cross section order of magnitude (as given by FRESCO), though the angular pattern is not well described. A quite good agreement with FRESCO is obtained by scaling the results of ii) by the factor $f_{p.w.}$. The conditions allowing to factorize the DCE cross section described above, that would enable to access nuclear matrix elements of similar structure as (two neutrino) double β decay, are presently under investigations [15, 16].

6 Conclusions

In this contribution we discuss new results for heavy ion SCE and DCE reactions. As a general feature, heavy ion reactions are characterized by quite large absorption effects, justifying the use of the black disk approximation to account for initial and final state interactions. Then predictions are derived for the behavior of the distortion factor as a function of beam energy and projectile/target combinations.

We show that at forward angles the SCE cross section can be factorised, allowing one to isolate the CE transition matrix element. Preliminary results are shown also for DCE reactions, depicted as a sequence of two single CE steps. The cross section is further reduced for DCE reactions, with respect to SCE processes, as one can realize by comparing DWBA to PWBA calculations in each case.

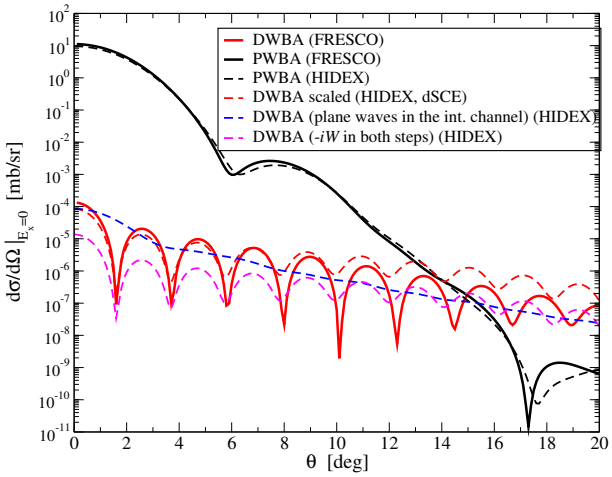

Fig. 4: (Color online) Angular distribution for the DCE reaction $^{18}O + ^{40}Ca \rightarrow ^{18}\ Ne + ^{40}Ar$ at 15 AMeV, as obtained with FRESCO and with double-step Hidex simulations (with two approximations for the intermediate channel, see text), in PWBA and in DWBA.

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References

[1] M. Ichimura, H. Sakai and T. Wakasa, *Prog. Part. Nucl. Phys.* **56** (2006) 446.

[2] J. H. Thies *et al.*, *Phys. Rev. C* **86** (2012) 044309.

[3] D. Frekers, P. Puppe, J. H. Thies and H. Ejiri, *Nucl. Phys. A* **916** (2013) 219.

[4] D. Frekers *et al.*, *Phys. Rev. C* **91** (2015) 034608.

[5] Y. Fujita, B. Rubio, and W. Gelletly, *Prog. Part. Nucl. Phys.* **66**, (2011) 549.

[6] C. Brendel, P. von Neumann-Cosel, A. Richter, G. Schrieder, H. Lenske, H. H. Wolter, J. Carter and D. Schüll, *Nucl. Phys. A* **477** (1988) 162.

[7] C. Bérat *et al.*, *Phys. Lett. B* **218** (1989) 299.

[8] F. Cappuzzello, M. Cavallaro, C. Agodi, M. Bondi, D. Carbone, A. Cunsolo and A. Foti, *Eur. Phys. J. A* **51** (2015) 145.

[9] F. Cappuzzello *et al.*, *Eur. Phys. J. A* **54** (2018) 72.

[10] K. Kisamori *et al.*, *Phys. Rev. Lett.* **116** (2016) 052501.

[11] M. Takaki *et al.*, CNS Ann. Rep. 94 (2014) 9.

[12] H. Lenske, J.I. Bellone, M. Colonna, J-A. Lay, Theory of Single Charge Exchange Heavy Ion Reactions, arXiv:1804.04827, accepted for publication in *Phys. Rev. C*.

[13] F. Cappuzzello *et al.*, *Nucl. Phys. A* **739** (2004) 30.

[14] I. Thompson, *Comp. Phys. Rep.* **7** (1988) 167.

[15] H. Lenske, *J. Phys. Conf. Ser.* **1056** (2018) 012030; J.I. Bellone, M. Colonna, H. lenske, J-A. Lay, *J. Phys. Conf. Ser.* **1056** (2018) 012004.

[16] E. Santopinto, H. García-Tecocoatzi, R. I. Magana-Vsevolodovna, J. Ferretti, Heavy-ion double-charge-exchange and its relation to neutrinoless double-beta decay, arXiv:1806.03069.