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BEAM LOADING AND LONGITUDINAL STABILITY EVALUATION FOR THE FCC-ee RINGS

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Abstract

In high-current accelerators, interaction of the beam with the fundamental impedance of the accelerating cavities can limit machine performance. It can result in a significant variation of bunch-by-bunch parameters (bunch length, synchronous phase, etc.) and lead to longitudinal coupled-bunch instability. In this work, these limitations are analysed together with possible cures for the high-current option (Z machine) of the future circular electron-positron collider (FCC-ee). The time-domain calculations of steady-state beam loading are presented and compared with frequency-domain analysis.

INTRODUCTION

The future circular electron-positron collider (FCC-ee) is considered to be built in four energy stages, defined by physics program [1]. To keep the same power loss budget for the synchrotron radiation in each machine, the beam current will be gradually reduced for each energy stage from 1.4 A to 5.4 mA. The Z machine, with parameters summarized in Table 1, can suffer from beam loading issues, which can result in modulation of the cavity voltage and beam parameters. The coupled-bunch instability due to fundamental cavity impedance can also be a limiting factor.

In general, there are two methods to calculate the beam induced transients: in frequency domain and time domain. The former, developed by Pedersen [2], is usually called a small-signal model. It allows to calculate the modulation of cavity voltage produced by modulation of the beam current. The latter method is the tracking of the beam and a simulation of the RF system evolution in time domain [3,4] which comes to the steady-state regime after many synchrotron periods. Considering a machine with large circumference, high beam current, and large number of bunches, as for the case of the Z machine, applicability of both existing approaches is questionable.

In this work, we present results of beam loading analysis in superconducting rf cavities modeled by a lumped circuit with a generator linked to the cavity via a circulator [5]. It allows us to get steady-state solution for beam and cavity parameters (beam phase, cavity voltage amplitude and phase) for arbitrary beam currents and filling schemes. The longitudinal coupled-bunch instability is estimated using the standard equations from Ref. [6]. Mitigations of both issues using the direct rf feedback around the cavity are also discussed.

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Table 1: The parameters of the Z machine of FCC-ee used for calculations in this work [7]. The bunch length is given for the case of non-colliding beams defined by equilibrium of quantum excitation and synchrotron radiation (SR).

Parameter	Unit	Value
Circumference, C	km	97.75
Harmonic number, h		130680
rf frequency, f_{rf}	MHz	400.79
(R/Q)	Ω	42.3
Beam energy, E	GeV	45.6
DC beam current, $I_{\mathrm{b,DC}}$	A	1.39
Number of bunches per beam, M		16640
Bunch population, N_{p}	10^{11}	1.7
rms bunch length, σ	ps	12
Momentum compaction factor, α_p	10^{-6}	14.79
Synchrotron tune, Q_s		0.025
Longitudinal damping time, τ_{SR}	ms	415.1
Total rf voltage, V_{tot}	MV	100
Number of cavities N_{cav}		52

BEAM LOADING BASICS

We consider short electron bunches for which the average of the rf component of the beam current $\langle I_{\mathrm{b,rf}} \rangle$ is twice the DC beam current $I_{\mathrm{b,DC}}$. For the steady-state beam loading, the generator current I_{g} can be derived from the lumped circuit model [5, 8] shown in Fig. 1:

$$I_{\mathrm{g}} e^{i\phi_{\mathrm{L}}} = \left[\frac{V_{\mathrm{cav}}}{2(R/Q)} \left(\frac{1}{Q_{\mathrm{L}}} + \frac{1}{Q_0} \right) + I_{\mathrm{b,DC}} \cos\phi_s \right] \tag{1}$$

$$- i \left[I_{\mathrm{b,DC}} \sin\phi_s + \frac{V_{\mathrm{cav}} \Delta\omega}{\omega_{\mathrm{rf}}(R/Q)} \right], \tag{2}$$

where ϕ_{L} is the loading angle, V_{cav} is the cavity voltage, (R/Q) is the ratio of the shunt impedance to the quality factor of the cavity fundamental mode, $Q_{\mathrm{L}} = Z_{\mathrm{c}}/(R/Q)$ is the loaded quality factor expressed using the coupler impedance Z_{c}, Q_0 is the cavity quality factor, $\omega_{\mathrm{rf}} = 2\pi f_{\mathrm{rf}}$ is the rf angular frequency, $\Delta\omega = \omega_0 - \omega_{\mathrm{rf}}$ is the cavity detuning, ω_0 is the cavity resonant frequency, ϕ_s is the bunch stable phase (electron machine convention). Considering superconducting cavity with $Q_0 \gg Q_{\mathrm{L}}$, the loading angle ϕ_{L} can be expressed from Eq. (2) as

$$\tan\phi_{\mathrm{L}} = -\frac{\tan\phi_{\mathrm{z}} + Y \sin\phi_s}{1 + Y \cos\phi_s}, \tag{3}$$

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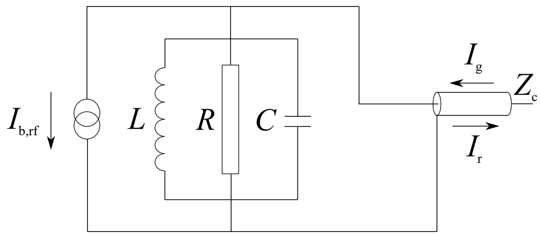

Figure 1: The lumped circuit model: a cavity is modeled by LCR-block, the coupler by a connected transmission line of impedance Z_c, and the beam by a current source. $I_{b,rf}$ is the beam current, I_g is the generator current, I_r is the reflected current, which is absorbed in the matched load (a generator is connected via a circulator).

where $Y = 2I_{b,DC}(R/Q)Q_L/V_{cav}$ is the relative beam loading, and the detuning angle is defined as

$$\tan\phi_z = \frac{2Q_L\Delta\omega}{\omega_{rf}}. \tag{4}$$

To minimize the generator power, one can define the optimum values for the cavity detuning

$$\Delta\omega_{opt} = -\omega_{rf}\frac{I_{b,DC}(R/Q)\sin\phi_s}{V_{cav}}, \tag{5}$$

and for the coupler quality factor

$$Q_{L,opt} \approx \frac{V_{cav}}{2(R/Q)I_{b,DC}\cos\phi_s}. \tag{6}$$

For the optimum parameters however, the beam is unstable because the second Robinson limit [9] is reached, which reads as

$$Y < -\frac{2\sin\phi_s}{\sin(2\phi_z)}, \tag{7}$$

when no loops around the cavity are present.

TRANSIENT BEAM LOADING

In operation of the Z machine different filling schemes can be used. To perform systematic analysis of the transient beam loading, we introduce the filling schemes following [10], which are schematically shown in Fig. 2. We consider a beam containing M bunches, which can be grouped in n_{tr} equal trains with a distance between first bunches of the consecutive trains t_{tt}. This distance should be a multiple of bunch spacings t_{bb}. Each train contains a number of filled buckets $M_b \leq t_{tt}/t_{bb}$. The beam has a regular filling, which can also contain an abort gap of length t_{gap}. Thus, the DC beam current depends on the filling scheme

$$I_{b,DC} = \frac{n_{tr}M_bN_pe}{T_{rev}} = \frac{\langle I_{b,rf}\rangle}{2}, \tag{8}$$

with $T_{rev} = 2\pi/\omega_{rev}$ - the revolution period, ω_{rev} - the angular revolution frequency, and N_p - the number of particles per bunch.

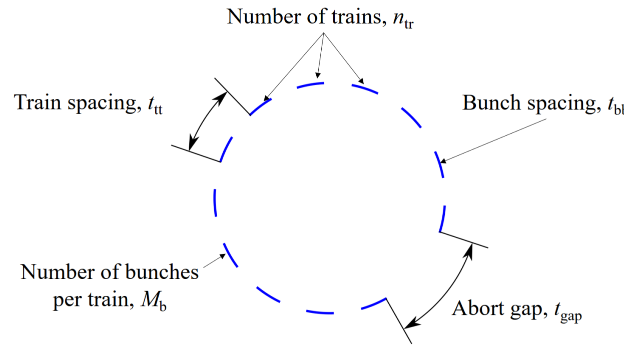

Figure 2: Sketch of the filling schemes used for the present study.

Steady-state Time-domain Approach

To calculate modulation of beam and cavity parameters due to modulation of the rf component of the beam current, the following equation can be used [8]

$$I_g e^{i\phi_L} = \frac{V(t)}{2(R/Q)}\left(\frac{1}{Q_L} - 2i\frac{\Delta\omega}{\omega_{rf}}\right) + \frac{dV(t)}{dt}\frac{1}{\omega_{rf}(R/Q)} + \frac{I_{b,rf}(t)e^{-i(\phi_s+\phi_b(t))}}{2}. \tag{9}$$

Here, the generator current is assumed be to constant, ϕ_b is the beam phase modulation, and the cavity voltage V is modulated in the form

$$V(t) = A(t)e^{i\phi(t)}, \tag{10}$$

where A is the amplitude of the cavity voltage, and ϕ is the phase of the cavity voltage with respect to rf phase $\omega_{rf}t$. Combining these equations and separating the real and imaginary parts, we get the following system of equations

$$\frac{dA(t)}{dt} = -\frac{A(t)}{\tau} + (R/Q)\omega_{rf} \times \left\{I_g\cos\left[\phi_L - \phi(t)\right] - \frac{I_{b,rf}\cos\left[\phi_s + \phi_b(t) + \phi(t)\right]}{2}\right\}, \tag{11}$$

$$\frac{d\phi(t)}{dt} = \Delta\omega + \frac{(R/Q)\omega_{rf}}{A} \times \left\{I_g\sin\left[\phi_L - \phi(t)\right] + \frac{I_{b,rf}(t)\sin\left[\phi_s + \phi_b(t) + \phi(t)\right]}{2}\right\}, \tag{12}$$

where $\tau = 2Q_L/\omega_{rf}$ is the cavity filling time. The stable phase in this case is not constant due to amplitude modulation of the cavity voltage, which should be taken into account by using the relation

$$N_{cav}A(t)\cos\left[\phi_s + \phi_b(t) + \phi(t)\right] = eU_0, \tag{13}$$

where eU_0 is the energy loss per turn due to synchrotron radiation, and N_{cav} is the number of cavities. Modulation of A, ϕ, and ϕ_b results in bunch-by-bunch variation of the bunch length, the synchrotron tune, and can lead to a collision point shift in the detectors. Unfortunately Eqs. (11-13) can not be solved analytically, but below the results of numerical calculations will be presented.

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Frequency Domain Calculations

The main equations from the small-signal model [11] are summarized below. The normalized modulations of the beam current a_b and the cavity voltage amplitude a_V are

$$I_{\mathrm{b,rf}} = 2I_{\mathrm{b,DC}}\left[1 + a_\mathrm{b}(t)\right], \quad A(t) = V_\mathrm{cav}\left[1 + a_V(t)\right], \quad (14)$$

correspondingly. Assuming small modulation ($|a_V| \ll 1$, $|\phi| \ll 1$, $|a_\mathrm{b}| \ll 1$, and $|\phi_\mathrm{b}| \ll 1$), the transfer functions from the beam amplitude to the cavity voltage amplitude, cavity voltage phase, and the beam phase are

$$\frac{\tilde{a}_V}{\tilde{a}_\mathrm{b}} = -\frac{\Delta\omega_\mathrm{opt}}{D(s)}\left[\frac{\Delta\omega_\mathrm{opt}}{\sin^2\phi_s} - \Delta\omega + \left(s + \frac{1}{\tau}\right)\cot\phi_s\right], \quad (15)$$

$$\frac{\tilde{\phi}}{\tilde{a}_\mathrm{b}} = -\frac{\Delta\omega_\mathrm{opt}}{D(s)}\left[\left(\frac{\Delta\omega_\mathrm{opt}}{\sin^2\phi_s} - \Delta\omega\right)\cot\phi_s + \left(s + \frac{1}{\tau}\right)\right], \quad (16)$$

$$\frac{\tilde{\phi}_\mathrm{b}}{\tilde{a}_\mathrm{b}} = \frac{\Delta\omega_\mathrm{opt}}{D(s)\sin^2\phi_s}\left(s + \frac{1}{\tau}\right), \quad (17)$$

$$\text{where } D(s) = \left(s + \frac{1}{\tau}\right)^2 - \Delta\omega\left[\frac{\Delta\omega_\mathrm{opt}}{\sin^2\phi_s} - \Delta\omega\right], \quad (18)$$

and $\tilde{a}$ is the Laplace image of the variable a. For the optimum detuning ($\Delta\omega = \Delta\omega_\mathrm{opt}$), there is no solution because these equations have a pole at $s = 0$, which corresponds to the second Robinson limit. For a slightly larger detuning, $\Delta\omega = \Delta\omega_\mathrm{opt}/\sin^2\phi_s$, simple first order responses can be obtained from Eqs. (15-17)

$$\frac{\tilde{a}_V}{\tilde{a}_\mathrm{b}} = \frac{-1}{1+\tau s}, \quad \frac{\tilde{\phi}}{\tilde{a}_\mathrm{b}} = -\frac{\Delta\omega_\mathrm{opt}\tau}{1+\tau s}, \quad \frac{\tilde{\phi}_\mathrm{b}}{\tilde{a}_\mathrm{b}} = \frac{\Delta\omega_\mathrm{opt}\tau}{(1+\tau s)\sin^2\phi_s}. \quad (19)$$

This detuning is used in calculations below with expense of slightly larger generator power.

Results and Comparisons

To solve Eqs. (11–13), the Euler method was used with total calculation time of 5 revolution periods, which was sufficient to get the steady-state solution since $\tau < T_\mathrm{rev}$. An example of a reasonable agreement of time-domain and frequency-domain (Eq. (19)) calculations is shown in Fig 3. There is a strong modulation due to the abort gap and a fine structure due to the gaps between trains.

To study systematically modulation of the beam and the cavity parameters, the scan for different train spacings was performed for a fixed bunch spacing $t_\mathrm{bb} = 15$ ns (Fig. 4). For each train spacing the peak-to-peak value of the beam phase modulation (the top plot) and the phase modulation of the cavity voltage (the bottom plot) are calculated. The results obtained in frequency domain are slightly larger than in time domain. We argue that the difference is due to a strong beam current modulation, while for the frequency-domain calculations it is assumed that $a_\mathrm{b} \ll 1$.

We propose to use the following equations to estimate the peak-to-peak beam phase modulation

$$\max\phi_\mathrm{b} - \min\phi_\mathrm{b} = \left|\frac{\Delta\omega_\mathrm{opt}t_\mathrm{gap}}{\sin^2\phi_s}\right|, \quad (20)$$

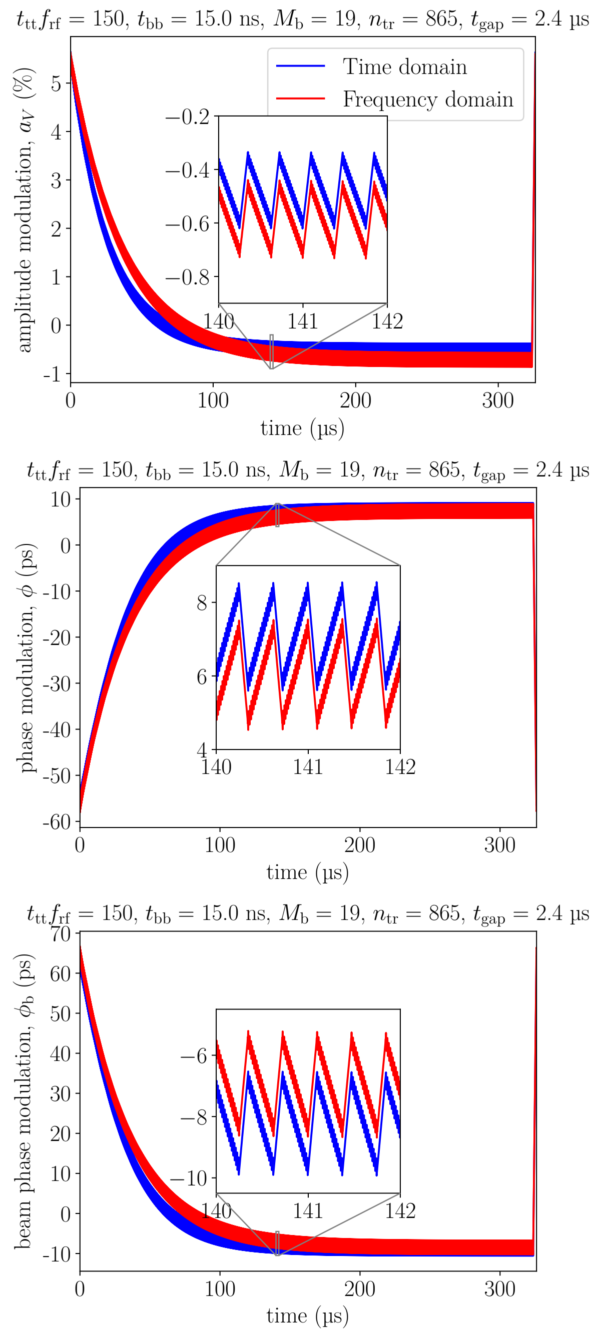

Figure 3: Comparison of the time-domain (Eqs. (11–13)) and the frequency-domain approaches (Eqs. (19)) for the Z machine for the detuning $\Delta\omega = \Delta\omega_\mathrm{opt}/\sin^2\phi_s = 13.1$ kHz. The amplitude modulation of the cavity voltage (the top plot), the phase modulation of the cavity voltage (the center plot), and the beam phase modulation (the bottom plot) within one turn are shown.

and peak-to-peak phase modulation of the cavity voltage

$$\max\phi - \min\phi = |\Delta\omega_\mathrm{opt}t_\mathrm{gap}|. \quad (21)$$

They agree very well with the results of the time-domain calculations (see the dashed black lines and the blue solid lines in Fig. 4, correspondingly). The dependence of the

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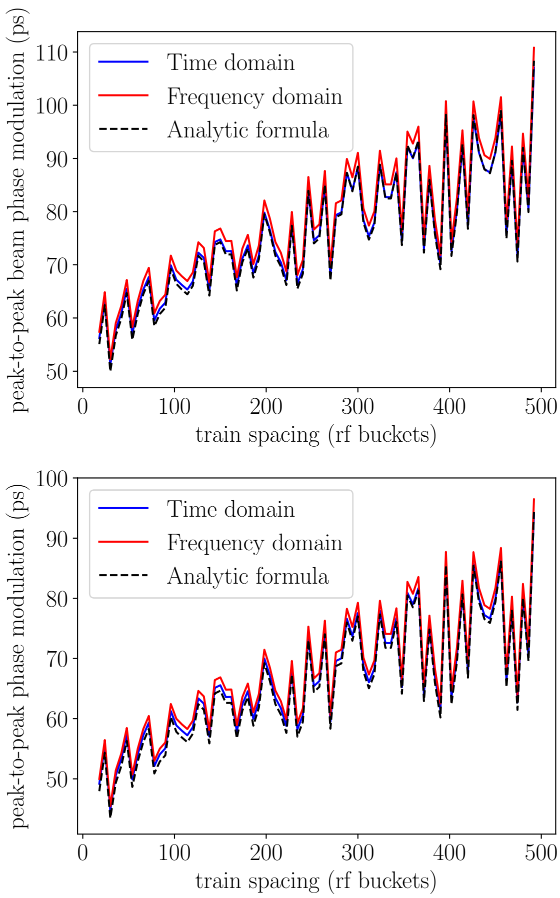

Figure 4: Dependence of the peak-to-peak values for the beam phase modulation (the top plot) and the phase modulation of the cavity voltage (the bottom plot) on the train spacing for the bunch spacing of 15 ns. The results from the analytic formulas are given by Eq. (20) (the top plot) and by Eq. (21) (the bottom plot).

product of the cavity detuning and the abort gap length on the train spacing for different bunch spacings is shown in Fig. 5. In general, larger bunch spacing leads to a smaller value of $|\Delta\omega_{\mathrm{opt}} t_{\mathrm{gap}}|$. For $t_{\mathrm{tt}} f_{\mathrm{rf}} > 100$, this product is larger than 0.15 for all bunch spacings, which corresponds to 60 ps peak-to-peak phase modulation of the cavity voltage and about 70 ps peak-to-peak beam phase modulation. However, in operation the shift of collision point can be eliminated by matching abort gap transients.

LONGITUDINAL COUPLED-BUNCH INSTABILITY

The equation used for calculation of the growth rate of longitudinal coupled-bunch instability can be found in textbooks (for example in [6]). For short Gaussian bunches and a mode m it is

$$\frac{1}{\tau_{\mathrm{inst},m}} = \frac{e\eta\omega_{\mathrm{rf}} I_{\mathrm{b,DC}} N_{\mathrm{cav}}}{4\pi E Q_s} \{\mathrm{Re}\left[Z(\omega_{\mathrm{rf}} + (m+Q_s)\omega_{\mathrm{rev}})\right] - \mathrm{Re}\left[Z(\omega_{\mathrm{rf}} - (m+Q_s)\omega_{\mathrm{rev}})\right]\}, \quad (22)$$

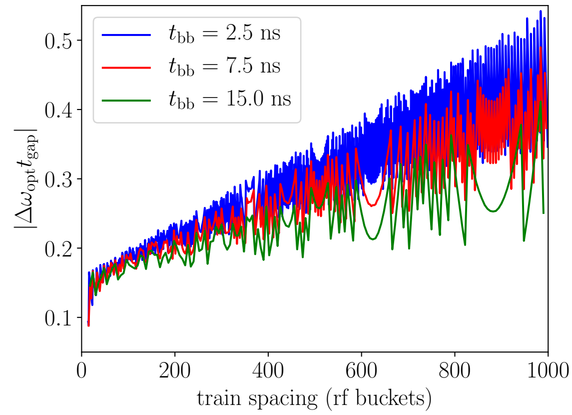

Figure 5: Dependence of the peak-to-peak phase modulation of the cavity voltage from Eq. (21) on the train spacings for different bunch spacings.

where $Z(\omega)$ is the longitudinal cavity impedance. Without detuning ($\Delta\omega = 0$), the growth rates of the modes are negligible in comparison to the synchrotron radiation damping rate $1/\tau_{\mathrm{SR}}$. For the detuning $\Delta\omega = \Delta\omega_{\mathrm{opt}}/\sin^2\phi_s$, the largest growth rate is for the mode $m = -4$ with a rise time of about 10 revolution periods (see Fig. 6).

To avoid longitudinal coupled-bunch instability one can use the direct rf feedback around the cavity [12]. It reduces the impedance seen by the beam in the region relevant for the beam stability. The impedance of the closed loop in this case is

$$Z_{\mathrm{cl}}(\omega) = \frac{Z(\omega)}{1 + GZ(\omega)e^{-i\tau_d\omega + i\phi_{\mathrm{adj}}}}, \quad (23)$$

where G is the feedback gain, τ_d is the overall loop delay, and ϕ_{adj} is the phase adjustment. The flat response is achieved for $1/G = (R/Q)\omega_{\mathrm{rf}}\tau_d$.

For $\tau_d = 700$ ns (the loop delay in the LHC [13]), the direct rf feedback significantly suppresses the growth rates of longitudinal coupled-bunch modes so they are below the synchrotron radiation damping rate (see Fig. 7). Additional

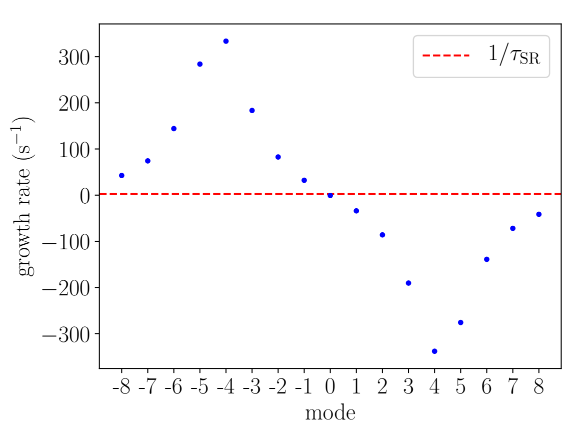

Figure 6: Growth rates of longitudinal coupled-bunch modes from Eq. (22) for the cavity detuning $\Delta\omega = \Delta\omega_{\mathrm{opt}}/\sin^2\phi_s = 13.1$ kHz.

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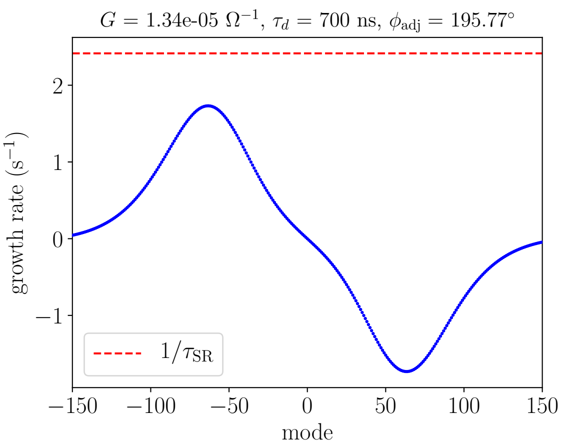

Figure 7: Growth rates of longitudinal coupled-bunch modes in the presence of the direct rf feedback calculated using Eqs. (22, 23) for the cavity detuning $\Delta\omega = \Delta\omega_{opt}/\sin^2\phi_s = 13.1$ kHz.

reduction of the impedance around multiples of the revolution harmonic using one-turn delay feedback could be evaluated in further studies.

PARTIAL COMPENSATION OF TRANSIENT BEAM LOADING

If the beam induced modulations are not acceptable, one can try to reduce the transients by the direct rf feedback. In this case the system of equations for the phase and the amplitude evolution of the cavity voltage can be obtained from Eq. (9) by substitution

$$I_g e^{i\phi_L} \to I_g e^{i\phi_L} - G\left[A(t)e^{i\phi(t)} - A_{ref}(t)e^{i\phi_{ref}(t)}\right], \quad (24)$$

where the amplitude $A_{ref}(t)$ and phase $\phi_{ref}(t)$ of the reference signal are obtained from the amplitude $A_0(t)$ and the phase $\phi_0(t)$ modulations calculated with the constant generator current and no rf feedback

$$A_{ref}(t)e^{i\phi_{ref}(t)} = V_{cav} + (1-k)\left[A_0(t)e^{i\phi_0(t)} - V_{cav}\right]. \quad (25)$$

Here, k is the compensation factor. The comparison of beam phase modulation with the direct rf feedback for $k = 0$ and $k = 0.3$ is shown in Fig. 8. For $k = 0.3$, one can see about 20% reduction of the beam phase modulation with about 5% increase of the instantaneous generator power $P = (R/Q)Q_L|I_g|^2/2$ (see Fig. 9). Further optimizations with different filling schemes and direct rf feedback parameters are needed to minimize transient beam loading.

CONCLUSIONS

In the present work the beam loading in FCC-ee high-current machine was analyzed. For the assumed regular filling schemes, the main contribution to the phase and amplitude modulation of the cavity voltage comes from the abort gap. The resulted peak-to-peak value of the bunch-by-bunch

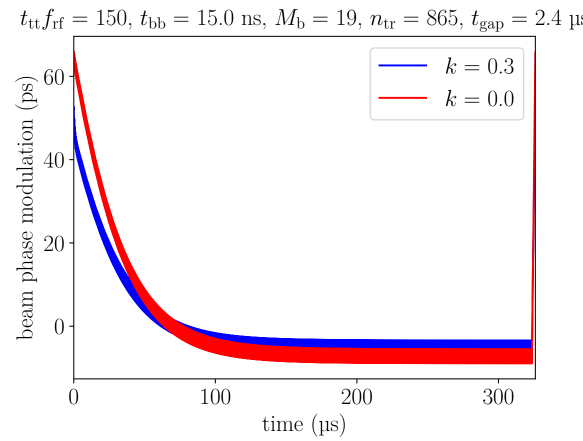

Figure 8: Comparison of beam phase modulation with the direct rf feedback for different compensation factors k.

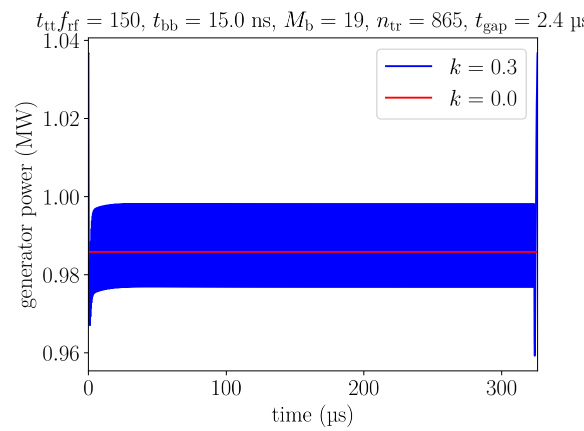

Figure 9: Instantaneous generator power with the direct rf feedback for calculations shown in Fig. 8.

phase modulation is larger than 70 ps for the abort gap longer than 2 µs. The larger train spacings lead to stronger modulation of cavity and beam parameters, which can be reduced by using filling schemes with the larger bunch spacings.

The growth rates of the first several longitudinal coupled-bunch modes are larger than the synchrotron radiation damping rate for the optimum detuning, that is about four times the revolution frequency. The direct rf feedback with the overall loop delay τ_d = 700 ns can stabilize the beam.

The direct feedback can also mitigate the transient beam loading with the cost of an additional generator power. For the discussed example, about 5% increase of the generator power is sufficient to reduce the bunch-by-bunch phase modulation by 20%.

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