TRI PP 94 19
SW 9423

TRI-PP-94-19
Apr 1994

# Chiral Meson Lagrangians from the QCD Based NJL Model Modified by Nonlocal Effects

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* (supported by Deutscher Akademischer Austauschdienst, DAAD)

## Abstract

Starting from a QCD inspired bilocal quark interaction we obtain a local effective meson lagrangian. In contrast to previous local (NJL-like) approaches, we include nonlocal corrections related to the finite meson size which we characterize by a small parameter. After bosonization using the heat-kernel method we predict the structure coefficients of the Gasser-Leutwyler $p^4$-lagrangian up to first order in this parameter. The modifications for the $L_i$ coefficients are typically of the order 15-20%, except for $L_5$, where we find a stronger nonlocal influence.



(submitted to Physical Review D)

## I. Introduction

Quantum chromodynamics (QCD) is generally accepted as the theory of strong interactions in terms of quarks and gluons, and is experimentally well established in the perturbative regime at high energies ($E \gg 1\,\mathrm{GeV}$). Since quarks and gluons have not been observed as free particles, it is assumed that they are confined into color-neutral hadrons. Many phenomenological approaches and effective models exist in the literature to describe hadrons and their interactions at low and medium energies ($E < 1\,\mathrm{GeV}$). However, up to now it is still impossible to derive a meson theory from QCD in a mathematical exact way, including the confinement of quarks and gluons. This problem is connected, in particular, with the unknown behavior of the QCD Green's functions at low energies. Nevertheless, there is some success related with effective bilocal approaches based on the application of functional methods to approximate forms of QCD (see [1]–[4] and references therein). In the local limit, these approaches reduce to the Nambu–Jona-Lasinio (NJL) model [5]. The bosonization of the NJL model leads to effective chiral meson lagrangians [6]–[12].

In this work we derive an effective chiral $p^4$-lagrangian of the type established by Gasser and Leutwyler [13] using the effective bilocal approach. We introduce a small parameter $\alpha$ characterizing the size of the nonlocal corrections. We predict the structure constants of the effective $p^4$-lagrangian to first order in $\alpha$. Of course, in the limit $\alpha \to 0$, our method reproduces the results of the NJL model. In Sec. 2 we briefly review the standard method of transforming the QCD lagrangian into an effective 4-quark lagrangian. In Sec. 3 we estimate the size of the $\bar{q}q$ system within a nonrelativistic Schrödinger approach for constituent quarks. We use the result for a quantitative estimate of the nonlocal effects and then describe a fixed-distance approximation. In Sec. 4 we present the bosonization in the more general dynamical bilocal approach. For that purpose we consider a separable ansatz for the bilocal collective fields. Solving the heat-kernel equation, in Sec. 5 we calculate the modified structure constants $L_i$ of the chiral $p^4$-lagrangian.

## II. From QCD to the Bilocal Effective Action

The starting point of our consideration is the generating functional of QCD in Minkowski space,

$$Z[\xi,\bar{\xi},J^a_\mu] = \int D\bar{q}DqDA\,\exp\Big(iS[\bar{q},q,A] + i\int d^4x(\bar{q}\xi + \bar{\xi}q + J^a_\mu A^{a\mu})\Big)\,, \quad (1)$$

where

$$S[\bar{q},q,A] = \int d^4x\Big[\bar{q}(i\hat{D} - m_0)q - \frac{1}{4}\sum_{a=1}^{8} G^a_{\mu\nu}G^{a\,\mu\nu}\Big]\,, \quad (2)$$

and $\xi$, $\bar{\xi}$, $J^a_\mu$ are the external sources associated with the fields $\bar{q}$, $q$, $A^a_\mu$; $q$ is the quark field; $m_0$ is the current quark mass matrix; $A^a_\mu$ represents a gluon with color index $a$, and $\lambda^a_C$ are $SU(3)_C$ matrices. The covariant derivative is defined as

$$D_\mu = \partial_\mu - ig\sum_{a=1}^{8}\frac{\lambda^a_C}{2}A^a_\mu\,, \quad (3)$$

and the gluon field-strength tensor is of the form

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu \ , \qquad (4)$$

where $g$ is the QCD coupling constant, and $f_{abc}$ are the $SU(3)$ structure constants. We use $\hat{D}$ for $\gamma_\mu D^\mu$. The Faddeev-Popov ghost fields and gauge fixing terms are included in the gluon measure.

Using standard techniques of path integration [1]–[3], after integration over the gluon fields, one finds

$$Z \ = \ \int D\bar{q} Lq \exp\Big[i \int d^4x \ \bar{q}(x)(i\hat{\partial} - m_0)q(x)\Big] \exp(iW[j]) \ , \qquad (5)$$

where we drop a normalization factor and do not consider external quark sources. In Eq. (5) $W[j]$ is given by

$$W[j] \ = \ \sum_{n=2}^{\infty} \frac{1}{n!} \int d^4x_1 ... d^4x_n D^{a_1 ... a_n}_{\mu_1 ... \mu_n}(x_1, ..., x_n) \prod_{i=1}^{n} j^{a_i}_{\mu_i}(x_i) \ , \qquad (6)$$

where

$$j^a_\mu(x) \ = \ \bar{q}(x)\gamma_\mu \frac{\lambda^a_C}{2} q(x)$$

is the flavor-singlet local quark current. The function $D^{a_1 ... a_n}_{\mu_1 ... \mu_n}$ is the $n$-point connected gluon Green's function containing all the information about the gluon dynamics. As long as the behavior of the gluon propagator and the running coupling constant at long distances are unknown, an analytical integration of Eq. (6) is impossible.

If one retains only the first term in the expansion of $W[j]$ of Eq. (6), neglecting triple and higher-order gluon vertices, the generating functional can be written in the effective form of truncated QCD,

$$Z \ = \ \int D\bar{q} Dq \exp\Big\{i\Big[\int d^4x \ \bar{q}(x)(i\hat{\partial} - m_0)q(x)\Big] + iS_{int}\Big\} \ , \qquad (7)$$

where

$$S_{int} \ = \ -i\frac{g^2}{2} \iint d^4x \, d^4y \ j^{a\mu}(x) \, D^{ab}_{\mu\nu}(x-y) j^{b\nu}(y) \ , \qquad (8)$$

is the part of the effective action corresponding to quark interaction via single-gluon exchange. In the Feynman gauge the nonperturbative gluon propagator is defined as

$$D^{ab}_{\mu\nu}(x) = g^{-2}\delta^{ab} g_{\mu\nu} D(x) \ . \qquad (9)$$

As the behavior of the Green's function $D(x)$ is unknown for large distances, a specific ansatz has to be used. For example, after a Fierz transformation, the local ansatz $D(x) \sim \delta^{(4)}(x)$ will lead to the NJL-type lagrangian of the effective four-quark interaction [5],

$$\mathcal{L}_{int} = 2G_1 \Big[\Big(\bar{q}\frac{\lambda^a}{2}q\Big)^2 + \Big(\bar{q}i\gamma_5\frac{\lambda^a}{2}q\Big)^2\Big] - 2G_2 \Big[\Big(\bar{q}\gamma_\mu\frac{\lambda^a}{2}q\Big)^2 + \Big(\bar{q}\gamma_5\gamma_\mu\frac{\lambda^a}{2}q\Big)^2\Big] \ , \qquad (10)$$

with some universal coupling constants $G_1$ and $G_2$. In the following we discuss an approach which goes beyond the usual local ansatz.



## III. Bilocal Fixed-Distance Approximation

Here we will outline the ideas which motivate the bilocal *fixed-distance* approach. One of the essential principles of a bilocal meson theory is the choice of the relativistic covariant form for the instantaneous four-quark interaction. The existence of this interaction type is the direct consequence of the reduced phase-space method of quantization of chromodynamics [14]. Assuming the dominant role of equal-time interactions in the formation of bound states, the Schwinger-Dyson equations of the bilocal meson theory reduce to the nonrelativistic Schrödinger approach corresponding to the description of $\bar{q}q$-pairs interacting via some effective gluonic potential.

If a meson is considered as a bound $\bar{q}q$-system analogous to the hydrogen atom in quantum mechanics, the nonrelativistic Schrödinger equation reads [15] ($\hbar = c = 1$)

$$-\frac{1}{2m}\nabla^2\Psi(\mathbf{r}) + [V(\mathbf{r}) - E]\Psi(\mathbf{r}) = 0 \ . \qquad (11)$$

In Eq. (11) $\Psi$ is the wave function of the internal motion, $m$ is the reduced constituent quark mass of a two-body system, $m = (m_1 m_2)/(m_1 + m_2)$, $\mathbf{r}$ is the relative coordinate, $V(\mathbf{r})$ is the interaction potential and $E$ is the eigenvalue of the hamiltonian. For a spherical potential the wave function $\Psi$ is usually written as a product of the radial function $R(r)$ with $r = |\mathbf{r}|$ and the spherical harmonics $Y_{lm}(\Theta, \phi)$: $\Psi(\mathbf{r}) = R(r) Y_{lm}(\Theta, \phi)$. The radial Schrödinger equation is of the form

$$-u''(r) = 2m\Big[E - V(r) - \frac{l(l+1)}{2mr^2}\Big]u(r) \ , \ u(r) = rR(r) \ , \qquad (12)$$

with the boundary condition $u(0) = 0$.

The results of a QCD lattice analysis [16] show that at large distances the effective quark-antiquark interaction can be approximated by a linear potential *. If one approximates the quark–antiquark potential by a linear potential (Coulomb and other corrections are neglected here),

$$V(r) = \sigma \cdot r \ , \qquad (13)$$

with $\sigma \approx 0.27 \ GeV^2$ for $m_u = m_d = \mu = 0.336 \ GeV$ [17], the characteristic distance between the quark and antiquark, $< r >$, can easily be determined using the virial theorem,

$$< r > \equiv h = \frac{2E_1}{3\sigma} \approx 0.68 \, fm \ , \qquad (14)$$

where $E_1 = 2.238(\sigma^2/2\mu)^{1/3}$ is the ground state energy. In the following we only consider the ground state ($l = 0$), i.e., neglect excited mesonic states such as $\pi^*$, $K^*$ etc.. After scaling Eq. (12) by introducing $\rho = r/h_0$ and $w(\rho) = u(r)$ one finds $w''(\rho) + (\epsilon - 2\rho)w(\rho) = 0$, with $\epsilon = 2m(2m\sigma)^{-\frac{2}{3}}E$. For large $\rho$ the solution behaves like the Airy function, $w(\rho) \sim Ai(\rho)$, decreasing exponentially to zero. The radial wave function $R(r)$ defined in Eq. (12) decreases even stronger. Thus one expects a small root-mean-square deviation $\Delta r$ for the distance between the constituent quarks.

*Screening effects by virtual $\bar{q}q$ pairs at very large distances are omitted here.

In order to obtain a qualitative estimate of the nonlocal corrections from the bilocal effective action, we make the following ansatz. We consider the case where the constituent quarks in the meson are localized at the scale $h$. This we do by including a delta function $\delta((x-y)^2-h^2)$ into the integrand $S_{int}$, Eq. (8),

$$S_{int} = -i\frac{\kappa^2}{2}\iint d^4x\, d^4y\, j^a_\mu(x)\, j^{a\mu}(y)\, D(x-y)\, \delta\left((x-y)^2-h^2\right) , \qquad (15)$$

where the correct dimension is obtained by introducing a constant $\kappa$ ( $([\kappa]=m^{-1})$. After shifting the argument $y$ by a Lorentz-invariant operator,

$$q(y) = \exp\left((y-x)_\mu \partial^\mu\right) q(x) ,$$

the effective action, Eq. (15), becomes

$$S_{int} = -i\frac{\kappa^2 D(h)}{2}\int d^4x\, j^a_\mu(x) K(h,x)\, j^{a\mu}(x) , \qquad (16)$$

where

$$K(h,x) = \int d^4y\, \exp((y-x)_\mu\partial^\mu)\, \delta((x-y)^2-h^2) . \qquad (17)$$

Performing the integration in polar coordinates, Eq. (17) can be expanded in the following way,

$$K(h,x) = \pi^2 h^2 \sum_{n=0}^{\infty} \frac{1}{(2n)!}\frac{\Gamma(n+\frac{1}{2})}{\Gamma(\frac{1}{2})\Gamma(n+2)} h^{2n}\Box^n = \pi^2 h^2\left[1+\frac{1}{8}\frac{\Box}{\Lambda^2}+O\left(\frac{\Box^2}{\Lambda^4}\right)\right] , \qquad (18)$$

where $\Lambda = h^{-1}$, and thus Eq. (16) can be written in the form

$$S_{int} = -i\frac{9G}{16}\int d^4x\left[j^a_\mu(x)\, j^{a\mu}(x) + \frac{1}{8\Lambda^2} j^a_\mu(x)(\Box j^{a\mu}(x))\right] + O\left(\frac{\Box^2}{\Lambda^4}\right) , \qquad (19)$$

with $G = \frac{8}{9}\pi^2\kappa^2 h^2 D(h)$.

After a Fierz transformation the action, Eq. (19), reads

$$S_{int} = i\frac{9G}{16}\int d^4x \left(\bar{q}(x)\frac{\mathcal{M}^\theta}{2} q(x)\, \bar{q}(x)\frac{\mathcal{M}^\theta}{2} q(x) + \frac{1}{8\Lambda^2}\bar{q}(x)\frac{\mathcal{M}^\theta}{2}\Box[q(x)\bar{q}(x)]\frac{\mathcal{M}^\theta}{2} q(x)\right) , \qquad (20)$$

where $\mathcal{M}^\theta$ are tensor products of Dirac, flavor and color matrices of the type

$$\left\{1, i\gamma_5, i\sqrt{\frac{1}{2}}\gamma^\mu, i\sqrt{\frac{1}{2}}\gamma_5\gamma^\mu\right\}^D \left\{\frac{1}{2}\lambda^a_F\right\}^F \left\{\frac{4}{3}1\right\}^C .$$

Here we consider the $SU(3)_F$ flavor group with flavor matrices $\lambda^a_F$, and we restrict ourselves to the color-singlet $\bar{q}q$ contributions. The first term in Eq. (20) leads to the effective four-quark interaction of the NJL model, Eq. (10), with $G_1 = 2G_2 = G/4$ while the second term, proportional to $1/\Lambda^2$, takes into account the finite-size effects of collective mesons.



## IV. Dynamical Bilocal Approach

Before considering the physical results of the bilocal fixed-distance approximation, let us study a more general approach using a *dynamical* bilocal model. Starting with the effective action, Eq. (8), and performing a Fierz transformation one finds

$$S_{int} = \frac{i}{2}\iint d^4x\, d^4y\, D(x-y)\,\bar{q}(x)\frac{\mathcal{M}^\theta}{2} q(y)\,\bar{q}(y)\frac{\mathcal{M}^\theta}{2} q(x) . \qquad (21)$$

Introducing scalar ($S$), pseudoscalar ($P$), vector ($V$) and axial-vector ($A$) bilocal collective meson fields [1]–[3] leads to an effective action which is bilinear in the quark fields,

$$S_{int} = \iint d^4x\, d^4y \left\{-\frac{9}{8D(x-y)}\mathrm{tr}\left[(\tilde{S}(x,y))^2 + (\tilde{P}(x,y))^2 + 2\left((\tilde{V}_\mu(x,y))^2 + (\tilde{A}_\mu(x,y))^2\right)\right] + \bar{q}(x)\tilde{\eta}(x,y) q(y)\right\} , \qquad (22)$$

with

$$\tilde{\eta}(x,y) = -\tilde{S}(x,y) - i\gamma^5\tilde{P}(x,y) + i\gamma^\mu\tilde{V}_\mu(x,y) + i\gamma^\mu\gamma^5\tilde{A}_\mu(x,y) , \qquad (23)$$

where

$$\tilde{S} = \tilde{S}^a\frac{\lambda^a}{2},\quad \tilde{P} = \tilde{P}^a\frac{\lambda^a}{2},\quad \tilde{V}_\mu = -i\tilde{V}^a_\mu\frac{\lambda^a}{2},\quad \tilde{A}_\mu = -i\tilde{A}^a_\mu\frac{\lambda^a}{2} , \qquad (24)$$

are the matrix-valued collective fields associated with the following quark bilinears,

$$\begin{aligned}
\tilde{S}^a(x,y) &= -\frac{8}{9}D(x-y)\bar{q}(y)\frac{\lambda^a}{2}q(x) ,\\
\tilde{P}^a(x,y) &= -\frac{8}{9}D(x-y)\bar{q}(y)i\gamma^5\frac{\lambda^a}{2}q(x) ,\\
\tilde{V}^a_\mu(x,y) &= -\frac{4}{9}D(x-y)\bar{q}(y)\gamma_\mu\frac{\lambda^a}{2}q(x) ,\\
\tilde{A}^a_\mu(x,y) &= -\frac{4}{9}D(x-y)\bar{q}(y)\gamma_\mu\gamma^5\frac{\lambda^a}{2}q(x) .
\end{aligned}$$

Following Ref. [4], we assume a strong localization of the bilocal fields and make the ansatz

$$\tilde{\eta}(x,y) \to \tilde{\eta}(z,t) = \eta(z)f(t) + \eta_\mu(z)t^\mu g(t) + \cdots , \qquad (25)$$

where $z=(x+y)/2$, $t=(y-x)/2$ are the global and relative coordinates, respectively. The function

$$\eta(z) = -S(z) - i\gamma^5 P(z) + i\gamma^\mu V_\mu(z) + i\gamma^\mu\gamma^5 A_\mu(z) , \qquad (26)$$

combines the local collective fields of the composite operators $\bar{q}(z)q(z)$, $\bar{q}(z)i\gamma^5 q(z)$, $\bar{q}(z)\gamma_\mu q(z)$ and $\bar{q}(z)\gamma_\mu\gamma^5 q(z)$, corresponding to the lowest meson excitations $0^{++}$, $0^{-+}$, $1^{--}$, $1^{++}$. The next order term of Eq. (25), proportional to $\eta_\mu$, can be identified with the excitations $1^{--}$, $1^{+-}$, $2^{++}$, $2^{--}$ [4]. The functions $f(t)$ and $g(t)$ rapidly decrease for $|t^2| \gg h^2$ and strongly localize the bilocal fields $\tilde{\eta}(x,y)$ to the effective size of the collective meson $h \equiv 1/\Lambda$.

Expanding $q(y)$ and $\bar{q}(x)$ in a Taylor series about $z$,

$$q(y) \;=\; q(z) + t^\mu \partial_\mu q(z) + O(t^2)\,,\; \bar{q}(x) \;=\; q(z) - t^\mu \partial_\mu q(z) + O(t^2)\,,$$

and using Eq. (25) we obtain

$$\iint d^4x\, d^4y\; \bar{q}(x)\tilde{\eta}(x,y)q(y) = 2\int d^4z\, \bar{q}(z)\eta(z)q(z) \int d^4t\, f(t) + 2\int d^4z \partial^\mu \bar{q}(z)\eta(z)\partial_\mu q(z) \int d^4t\, t^2 f(t) + (\text{excitation terms})\,.$$

Then, for the first generation of mesons corresponding to the $(0^{++}, 0^{-+}, 1^{--}, 1^{++})$ multiplets, the generating functional is

$$Z = \int \mathcal{D}\Phi\, \mathcal{D}\Phi^\dagger\, \mathcal{D}V\, \mathcal{D}A\, \exp\Big\{\int d^4z\Big[-\frac{1}{4G_1}\mathrm{tr}[\Phi(z)^\dagger\Phi(z)] - \frac{1}{4G_2}\mathrm{tr}(V_\mu^2(z) + A_\mu^2(z)) + \bar{q}(z)i\widehat{\mathrm{D}}q(z) - \frac{\alpha}{\Lambda^2}\partial^\mu \bar{q}(z)\,\eta(z)\,\partial_\mu q(z)\Big]\Big\}\,, \tag{27}$$

where $\widehat{\mathrm{D}}$ is the Dirac operator in the presence of local collective meson fields,

$$i\widehat{\mathrm{D}} = i(\hat{\partial} + \hat{V} + \hat{A}\gamma^5) - P_R(\Phi + m_0) - P_L(\Phi^\dagger + m_0) = [i(\hat{\partial} + \hat{A}^{(+)}) - (\Phi + m_0)]P_R + [i(\hat{\partial} + \hat{A}^{(-)}) - (\Phi^\dagger + m_0)]P_L\,. \tag{28}$$

Here $\Phi = S + iP$, $\hat{V} = V_\mu\gamma^\mu$, $\hat{A} = A_\mu\gamma^\mu$; $P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$ are chiral right/left projectors; $\hat{A}^{(\pm)} = \hat{V} \pm \hat{A}$ are right and left combinations of fields. The parameter $\alpha$ is defined as

$$\frac{\alpha}{\Lambda^2} \;=\; \frac{1}{2}\int d^4t\, t^2 f(t)\,, \tag{29}$$

where $f(t)$ is normalized as $2\int d^4t f(t) = 1$.

The coupling constants $G_1$ and $G_2$ are defined by

$$\frac{1}{G_1} \;=\; \frac{1}{2G_2} \;=\; \frac{9}{8}\int d^4t\, \frac{f^2(t)}{D(2t)}\,. \tag{30}$$

Note that in such an approximation the ratio of $G_2$ and $G_1$ is 1/2, whereas phenomenology predicts $G_2/G_1 \sim 4$. This problem can, in principle, be solved by introducing different functions $f_\sigma(t)$ $(\sigma = 0, 1, \cdots)$ into Eqs. (25) and (26), corresponding to different localizations of spin-0 and spin-1 mesons. However, in our approximation we neglect such a spin dependence.

The first three terms of the action of Eq. (27) are identical with the corresponding terms arising from the linearization of the four-quark local interaction of the extended NJL model described by the lagrangian

$$\mathcal{L}_{NJL} \;=\; \bar{q}(i\hat{\partial} - m_0)q + \mathcal{L}_{int}\,,$$



with $\mathcal{L}_{int}$ given by Eq. (10). Performing a partial integration and dropping the surface term, the last term in Eq. (27) can be rewritten in the form

$$\int d^4z \partial^\mu \bar{q}(z)\,\eta(z)\,\partial_\mu q(z) \;=\; -\int d^4z \bar{q}(z)[\partial^\mu \eta(z)\,\partial_\mu + \eta(z)\,\partial^2]q(z)\,. \tag{31}$$

Of course, we do not know the explicit form of the function $f(t)$. However, we can now use the fixed-distance approximation of Eq. (15) to estimate the parameter $\alpha$. Indeed, the second term in Eq. (20) can be transformed into the form

$$S^2_{int} \;=\; \frac{i}{16\Lambda^2}\int d^4x \bar{q}(x)[\partial^\mu\eta(x)\,\partial_\mu + \eta(x)\,\partial^2]q(x) + (\text{excitation terms})\,, \tag{32}$$

where $\eta(x)$ is the combination of the local collective fields, Eq. (26), which now are defined by the following quark bilinears,

$$S^a(x) = -G\,\bar{q}(x)\frac{\lambda^a}{2}q(x),\quad P^a(x) = -G\,\bar{q}(x)i\gamma_5\frac{\lambda^a}{2}q(x)\,,$$

$$V^a_\mu(x) = -\frac{G}{2}\,\bar{q}(x)\gamma_\mu\frac{\lambda^a}{2}q(x),\quad A^a_\mu(x) = -\frac{G}{2}\,\bar{q}(x)\gamma_\mu\gamma_5\frac{\lambda^a}{2}q(x).$$

Comparing Eqs. (31) and (32) we can fix the value $\alpha$, $\alpha = 1/16$, corresponding to the naive bilocal fixed-distance approximation. Using the values $\Lambda = 0.28\,\mathrm{GeV}$ and $\mu = 0.336\,\mathrm{GeV}$, the ratio is estimated to be of the order

$$\frac{\alpha\mu^2}{\Lambda^2} \approx 0.09\,. \tag{33}$$

This value will be used as a small parameter for further numerical estimates of nonlocal effects.

After integration over the quark fields, the full action arising from the generating functional, Eq. (27), is

$$S(\Phi,\Phi^\dagger,V,A) = \int d^4z\Big[-\frac{1}{4G_1}\mathrm{tr}(\Phi^\dagger\Phi) - \frac{1}{4G_2}\mathrm{tr}(V_\mu^2 + A_\mu^2)\Big] - i\,\mathrm{Tr}'[\log(i\widehat{\widehat{\mathrm{D}}})]\,. \tag{34}$$

Here the second term is the quark determinant of the Dirac operator $i\widehat{\widehat{\mathrm{D}}}$ which is extended to the case of nonlocality. It is obtained from the usual operator, Eq. (28), by the following replacement,

$$A^{(\pm)}_\mu \to A^{(\pm)}_\mu\Big(1 + \frac{\alpha}{\Lambda^2}\partial^2\Big) + \frac{\alpha}{\Lambda^2}\big(\partial_\nu A^{(\pm)}_\mu\big)\,\partial^\nu\,,$$

$$\Phi \to \Phi\Big(1 + \frac{\alpha}{\Lambda^2}\partial^2\Big) + \frac{\alpha}{\Lambda^2}\big(\partial_\nu\Phi\big)\,\partial^\nu\,. \tag{35}$$

The "trace" $\mathrm{Tr}'$ is to be understood as a space-time integration and a "normal" trace with respect to Dirac, color and flavor matrices,

$$\mathrm{Tr}' = \int d^4x \mathrm{tr}'\,,\quad \mathrm{tr}' = \mathrm{tr}_\gamma \cdot \mathrm{tr}_C \cdot \mathrm{tr}_F\,.$$

In the following we will only consider the non-anomalous part of the effective meson action which corresponds to the modulus of the quark determinant †. The modulus of the quark determinant can be calculated using the heat-kernel technique with proper-time regularization [18]–[20],

$$\log|\det i\widehat{\widetilde{\mathbf{D}}}| = -\frac{1}{2}\mathrm{Tr}'\log(\widehat{\widetilde{\mathbf{D}}}^{\dagger}\widehat{\widetilde{\mathbf{D}}}) = -\frac{1}{2}\int_{1/\tilde{\Lambda}^2}^{\infty} d\tau \frac{1}{\tau}\mathrm{Tr}'\exp(-\widehat{\widetilde{\mathbf{D}}}^{\dagger}\widehat{\widetilde{\mathbf{D}}}\tau)\,, \qquad (36)$$

with $\tilde{\Lambda}$ as the cutoff parameter of intrinsic regularization. The operator $\widehat{\widetilde{\mathbf{D}}}^{\dagger}\widehat{\widetilde{\mathbf{D}}}$ can be written as

$$\widehat{\widetilde{\mathbf{D}}}^{\dagger}\widehat{\widetilde{\mathbf{D}}} = \beta\partial^2 + \mu^2 + 2\Gamma_\mu\partial^\mu + \Gamma_\mu^2 + a(x)$$
$$+\frac{\alpha}{\Lambda^2}\Big[b(x) + Q_\alpha(x)\partial^\alpha + c(x)\partial^2 + 2(\Gamma_\mu\partial^2 + \partial_\alpha\Gamma_\mu\partial^\alpha)\partial^\mu\Big] + O\Big(\frac{\alpha^2}{\Lambda^4}\Big),$$

where $\beta = 1{+}2\alpha\mu^2/\Lambda^2$ and $\mu$ is a new mass scale. It arises as a nonvaninishing vacuum expectation value of the scalar field $S$, and corresponds to the constituent quark mass. The combinations $a(x), b(x), c(x)$ and $Q_\alpha(x)$ do not contain any differential operator acting on the quark fields and they are defined as

$$a(x) = i\gamma^\mu(P_R D_\mu\Phi + P_L\overline{D}_\mu\Phi^\dagger) + P_R\mathcal{M} + P_L\overline{\mathcal{M}} + \frac{1}{4}[\gamma^\mu,\gamma^\nu]\Gamma_{\mu\nu}\,,$$
$$b(x) = i\gamma^\mu\Big[P_R(A_\mu^{(-)}\partial^2\Phi + \partial_\alpha A_\mu^{(-)}\partial^\alpha\Phi - \Phi\partial^2 A_\mu^{(+)} - \partial_\alpha\Phi\partial^\alpha A_\mu^{(+)})$$
$$+P_L(A_\mu^{(+)}\partial^2\Phi^\dagger + \partial_\alpha A_\mu^{(+)}\partial^\alpha\Phi^\dagger - \Phi^\dagger\partial^2 A_\mu^{(-)} - \partial_\alpha\Phi^\dagger\partial^\alpha A_\mu^{(-)})\Big]$$
$$+P_R(\Phi^\dagger\partial^2\Phi + \partial_\alpha\Phi^\dagger\partial^\alpha\Phi) + P_L(\Phi\partial^2\Phi^\dagger + \partial_\alpha\Phi\partial^\alpha\Phi^\dagger) + \Gamma_\mu\partial^2\Gamma^\mu$$
$$+(\partial_\alpha\Gamma_\mu)^2 + \frac{1}{4}[\gamma^\mu,\gamma^\nu]\Big([\partial_\alpha\Gamma_\mu,\partial^\alpha\Gamma_\nu] + \Gamma_\mu\partial^2\Gamma_\nu - \Gamma_\nu\partial^2\Gamma_\mu\Big)\,,$$
$$c(x) = a(x) + i\gamma^\mu\Big[P_R(A_\mu^{(-)}\Phi - \Phi A_\mu^{(+)}) + P_L(A_\mu^{(+)}\Phi^\dagger - \Phi^\dagger A_\mu^{(-)})\Big]$$
$$+P_R\mathcal{M} + P_L\overline{\mathcal{M}} + 2\Gamma_\mu^2 + \frac{1}{4}[\gamma^\mu,\gamma^\nu][\Gamma_\mu,\Gamma_\nu]\,,$$
$$Q_\alpha(x) = 3\Gamma_\mu\partial_\alpha\Gamma^\mu + \partial_\alpha\Gamma_\mu\Gamma^\mu + \partial_\alpha a(x)$$
$$+2i\gamma^\mu\Big[P_R(A_\mu^{(-)}\partial_\alpha\Phi - \Phi\partial_\alpha A_\mu^{(+)}) + P_L(A_\mu^{(+)}\partial_\alpha\Phi^\dagger - \Phi^\dagger\partial_\alpha A_\mu^{(-)})\Big]$$
$$+2(P_R\Phi^\dagger\partial_\alpha\Phi + P_L\Phi\partial_\alpha\Phi^\dagger) + \frac{1}{2}[\gamma^\mu,\gamma^\nu](\Gamma_\mu\partial_\alpha\Gamma_\nu - \Gamma_\nu\partial_\alpha\Gamma_\mu)\,.$$

Here, $\Gamma_{\mu\nu} = \partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu + [\Gamma_\mu,\Gamma_\nu]$, $\Gamma_\mu = P_L A_\mu^{(+)} + P_R A_\mu^{(-)}$, $\mathcal{M} = \Phi^\dagger\Phi - \mu^2$, $\overline{\mathcal{M}} = \Phi\Phi^\dagger - \mu^2$. Furthermore,

$$D_\mu * = \partial_\mu * + (A_\mu^{(-)} * - * A_\mu^{(+)}), \quad \overline{D}_\mu * = \partial_\mu * + (A_\mu^{(+)} * - * A_\mu^{(-)})$$

are the covariant derivatives; $d_\mu = \partial_\mu + \Gamma_\mu$; the differential operator $\partial_\mu$ acts only on $x$.

† The imaginary part of the quark determinant is related to the anomalous part of the action.



## V. The Effective $p^4$-Lagrangian Including Nonlocal Corrections

To obtain the modulus of the quark determinant using the heat-kernel method we expand

$$< x \,|\, \exp(-\widehat{\widetilde{\mathbf{D}}}^{\dagger}\widehat{\widetilde{\mathbf{D}}}\tau) \,|\, y >$$

around its "free" part,

$$K_0(x,y;\tau) = < x \,|\, \exp(-(\beta\Box + \mu^2)\tau) \,|\, y > = \frac{1}{(4\pi\beta\tau)^2} e^{-\mu^2\tau + (x-y)^2/(4\beta\tau)}\,,$$

in powers of the proper-time $\tau$ with the so-called Seeley-deWitt coefficients $h_k(x,y)$,

$$< x \,|\, \exp(-\widehat{\widetilde{\mathbf{D}}}^{\dagger}\widehat{\widetilde{\mathbf{D}}}\tau) \,|\, y > = K_0(x,y;\tau)\sum_{k=0}^{\infty} h_k(x,y)\cdot\tau^k.$$

After integrating over $\tau$ in Eq. (36) one obtains

$$\frac{1}{2}\log(\det\widehat{\mathbf{D}}^{\dagger}\widehat{\mathbf{D}}) = -\frac{1}{2}\frac{\mu^4}{(4\pi\beta)^2}\sum_{k=0}^{\infty}\frac{\Gamma(k-2,\mu^2/\tilde{\Lambda}^2)}{\mu^{2k}}\mathrm{Tr}' h_k\,,$$

where

$$\Gamma(n,x) = \int_x^\infty dt\, e^{-t} t^{n-1}$$

is the incomplete gamma function.

The heat-kernel coefficients $h_k(x) = h_k(x,\, y = x)$ are obtained from the recursive relation.

$$2\frac{\alpha}{\Lambda^2}\Gamma_\mu t^\mu t^2 h_{n+3}(x,y)$$
$$+\frac{\alpha}{\Lambda^2}\Big[t^2(2\Gamma_\mu^2 + c(x)) + 2\Gamma_\mu(3t^\mu + 2t^\mu t_\alpha\partial^\alpha + t^2\partial^\mu) + 2\partial_\alpha\Gamma_\mu t^\mu t^\alpha\Big]h_{n+2}(x,y)$$
$$+\Big[n + 1 + 2t_\mu d^\mu + \frac{\alpha}{2\Lambda^2}\Big(+4c(x)(1 + t_\alpha\partial^\alpha) - 8\mu^2 t_\alpha d^\alpha + 2Q_\alpha(x)t^\alpha$$
$$+2\partial_\alpha\Gamma_\mu(t^\mu\partial^\alpha + t^\alpha\partial^\mu) + 4\Gamma_\mu(2\partial^\mu + 2t_\alpha\partial^\alpha\partial^\mu + t^\mu\partial^2)\Big)\Big]h_{n+1}(x,y)$$
$$+\Big[a(x) + d_\mu d^\mu + \frac{\alpha}{\Lambda^2}\Big(b(x) + Q_\alpha(x)\partial^\alpha + 2(\Gamma_\mu\partial^2 + \partial_\alpha\Gamma_\mu\partial^\alpha)\partial^\mu$$
$$+c(x)\partial^2\Big)\Big]h_n(x,y) = 0\,,$$

where the differential operator $\partial_\mu$ acts on $x$. For $V_\mu = A_\mu = 0$, the recursive relation for the heat-kernel coefficients reduces to

$$\frac{\alpha}{\Lambda^2}t^2\tilde{c}(x)h_{n+2}(x,y)$$
$$+\Big\{n + 1 + 2t_\mu\partial^\mu + \frac{\alpha}{\Lambda^2}\Big[2(\tilde{a}(x) + \tilde{c}(x))(1 + t_\nu\partial^\nu) - 4\mu^2 t_\mu\partial^\mu$$
$$+t_\mu(\partial^\mu\tilde{a}(x) + 2\tilde{Q}^\mu(x))\Big]\Big\}h_{n+1}(x,y)$$
$$+\Big\{\tilde{a}(x) + \partial^2 + \frac{\alpha}{\Lambda^2}\Big[\tilde{b}(x) + (\partial_\mu\tilde{a}(x) + 2\tilde{Q}^\mu(x))\partial^\mu$$
$$+(\tilde{a}(x) + \tilde{c}(x))\partial^2\Big)\Big]\Big\}h_n(x,y) = 0\,, \qquad (37)$$

where

$$\tilde{a}(x) = i\gamma^\mu(P_R\partial_\mu\Phi + P_L\partial_\mu\Phi^\dagger) + P_R\mathcal{M} + P_L\overline{\mathcal{M}},$$
$$\tilde{b}(x) = P_R(\Phi^\dagger\partial^2\Phi + \partial_\mu\Phi^\dagger\partial^\mu\Phi) + P_L(\Phi\partial^2\Phi^\dagger + \partial_\mu\Phi\partial^\mu\Phi^\dagger),$$
$$\tilde{c}(x) = P_R\mathcal{M} + P_L\overline{\mathcal{M}}, \quad \tilde{Q}_\mu(x) = P_R\Phi^\dagger\partial_\mu\Phi + P_L\Phi\partial_\mu\Phi^\dagger .$$

The expressions for the heat-coefficients $h_0, ..., h_3$ are obtained from Eq. (37) using the computer algebra system REDUCE and the recursive procedure described in [20],

$$\begin{aligned}
h_0(x) &= 1,\\
\mathrm{tr}'[h_1(x)] &= -\mathrm{tr}'\Big[\tilde{a} + \frac{\alpha}{\Lambda^2}\Big(\tilde{b} - \tilde{a}(\tilde{a}+\tilde{c})\Big)\Big] + O\Big(\frac{\alpha^2}{\Lambda^4}\Big),\\
\mathrm{tr}'[h_2(x)] &= \mathrm{tr}'\Big[\frac{1}{2}\tilde{a}^2 + \frac{\alpha}{\Lambda^2}\Big(\tilde{a}\tilde{b} - \frac{2}{3}\tilde{a}^2(\tilde{a}+\tilde{c}) + \frac{5}{12}(\partial_\mu\tilde{a})^2 - \frac{1}{12}\partial_\mu\tilde{a}\,\partial^\mu\tilde{c}\\
&\qquad -\tilde{a}\,\partial^\mu\tilde{Q}_\mu\Big)\Big] + O\Big(\frac{\alpha^2}{\Lambda^4}\Big),\\
\mathrm{tr}'[h_3(x)] &= -\mathrm{tr}'\Big\{\frac{1}{6}\tilde{a}^3 - \frac{1}{12}(\partial_\mu\tilde{a})^2 + \frac{\alpha}{\Lambda^2}\Big[\frac{1}{2}\tilde{a}^2\tilde{b} - \frac{1}{4}\tilde{a}^3(\tilde{a}+\tilde{c})\\
&\quad -\tilde{a}^2\Big(\frac{3}{10}\partial^2\tilde{a} + \frac{1}{2}\partial^\mu\tilde{Q}_\mu + \frac{2}{3}\partial^2\tilde{c}\Big)\\
&\quad -\tilde{a}\Big(\frac{5}{6}(\partial_\mu\tilde{a})^2 + \frac{1}{15}\partial^2\tilde{a}\,\tilde{c} + \frac{2}{5}\partial_\mu\tilde{a}\,\partial^\mu\tilde{c} + \frac{11}{30}\partial_\mu\tilde{c}\,\partial^\mu\tilde{a}\\
&\quad +\frac{1}{20}(\partial^2\partial^2\tilde{c} + \tilde{c}\partial^2\tilde{a}) + \frac{1}{6}(\partial^2\partial^\mu\tilde{Q}_\mu - \tilde{Q}_\mu\partial^\mu\tilde{a} + \partial^\mu\tilde{a}\,\tilde{Q}_\mu - \partial^2\tilde{b})\Big)\\
&\quad -\frac{1}{15}\Big(\partial^2\tilde{c}\,\partial^2\tilde{a} - \tilde{c}(\partial_\mu\tilde{a})^2 - (\partial^2\tilde{a})^2\Big) - \frac{1}{18}\mu^2(\partial_\mu\tilde{a})^2\Big]\Big\}\\
&\quad +O\Big(\frac{\alpha^2}{\Lambda^4}\Big).
\end{aligned} \tag{38}$$

First we present the heat-coefficients containing nonlocal corrections to the minimal (i.e. nonvanishing when $V_\mu = A_\nu = 0$) part of the effective meson lagrangian including $p^2$- and $p^4$-interactions. The minimal part has the general form

$$\begin{aligned}
\mathcal{L}_{eff}^{min} &= \frac{F_0^2}{4}\mathrm{tr}(\partial_\mu U\,\partial^\mu U^\dagger) + \frac{F_0^2}{4}\mathrm{tr}(MU + U^\dagger M)\\
&+\Big(L_1 - \frac{1}{2}L_2\Big)\,(\mathrm{tr}\partial_\mu U\,\partial^\mu U^\dagger)^2\\
&+L_2\mathrm{tr}\Big(\frac{1}{2}[\partial_\mu U, \partial_\nu U^\dagger]^2 + 3(\partial_\mu U\,\partial^\mu U^\dagger)^2\Big)\\
&+L_3\mathrm{tr}((\partial_\mu U\partial^\mu U^\dagger)^2) + L_4\mathrm{tr}(\partial_\mu U\,\partial^\mu U^\dagger)\,\mathrm{tr}M(U+U^\dagger)\\
&+L_5\mathrm{tr}\partial_\mu U\,\partial^\mu U^\dagger(MU + U^\dagger M),
\end{aligned} \tag{39}$$

where the dimensionless structure constants $L_i$ were introduced by Gasser and Leutwyler in Ref. [13]. Since we restrict ourselves to terms of first order in $m_0$, we have omitted the corresponding higher-order terms from Eq. (39). Here we have introduced the notations

$$U = \exp\Big(\frac{i\sqrt{2}}{F_0}\varphi(x)\Big), \quad \varphi(x) = \varphi^a(x)\frac{\lambda^a}{2},$$

where $\varphi(x)$ is the pseudoscalar meson matrix and $F_0$ is the bare $\pi$ decay constant.

Our calculation predicts the following expression for $F_0$,

$$F_0^2 = \frac{N_c\mu^2}{4\pi^2}\Big[y - \frac{4\pi^2 <\bar{q}q>}{\mu^3 N_c}\frac{\alpha\mu^2}{\Lambda^2}\Big], \tag{40}$$

where $y = \Gamma(0, \mu^2/\tilde{\Lambda}^2)$. In Eq. (40) the first term is the standard prediction of the local limit and the second corresponds to the nonlocal correction. For the meson mass matrix $M = diag(\chi_u^2, \chi_d^2, ..., \chi_n^2)$ we obtain

$$\chi_i^2 = \frac{N_c\mu m_i^0}{2\pi^2 F_0^2}\Big(\tilde{\Lambda}^2 e^{-\mu^2/\tilde{\Lambda}^2} - \mu^2 y\Big) = -\frac{2m_i^0 <\bar{q}q>}{F_0^2}. \tag{41}$$

Moreover, the coefficients $L_i$ are given by $L_1 - L_2/2 = L_4 = 0$ and

$$\begin{aligned}
L_2 &= \frac{N_c}{16\pi^2}\frac{1}{12}\Big(1 + 2\frac{\alpha\mu^2}{\Lambda^2}\Big),\\
L_3 &= -\frac{N_c}{16\pi^2}\frac{1}{6}\Big(1 + 5(1-y)\frac{\alpha\mu^2}{\Lambda^2}\Big),\\
L_5 &= \frac{N_c}{16\pi^2}x\Big[y - 1 - \frac{28}{3}\frac{\alpha\mu^2}{\Lambda^2}\Big],
\end{aligned} \tag{42}$$

where $x = -\mu F_0^2/(2 <\bar{q}q>)$. We made use of the approximation $\Gamma(k, \mu^2/\tilde{\Lambda}^2) \approx \Gamma(k)$ for $k \geq 1$ and $\mu^2/\tilde{\Lambda}^2 \ll 1$.

In the local limit, the part of the heat coefficient $h_4$ contributing to the $p^4$ langrangian is given by

$$\begin{aligned}
\mathrm{tr}'[h_4(x)^{(p^4)}] &= -\mathrm{tr}'\Big[\frac{1}{24}\tilde{a}^4 + \frac{1}{12}\Big(\tilde{a}^2\partial^2\tilde{a} + \tilde{a}(\partial_\mu\tilde{a})^2\Big)\\
&\quad +\frac{1}{720}\Big(7(\partial^2\tilde{a})^2 - (\partial_\mu\partial_\nu\tilde{a})^2\Big)\Big].
\end{aligned}$$

The contributions of $h_4$ linear in $\alpha$ only appear at order $p^6$ which are not considered here.

In a completely analogous way we estimate the nonlocal contributions to the non-minimal part of the effective meson lagrangian at $O(p^4)$. Here we restrict ourselves to the consideration of the structure coefficients $L_9$ and $L_{10}$ corresponding to the terms

$$\begin{aligned}
\mathcal{L}_{eff}^{(nmin)} &= L_9\mathrm{tr}\Big(F_{\mu\nu}^{(+)}D^\mu U\,\overline{D}^\nu U^\dagger + F_{\mu\nu}^{(-)}\overline{D}^\mu U^\dagger\,D^\nu U\Big)\\
&\quad -L_{10}\mathrm{tr}\Big(U^\dagger F_{\mu\nu}^{(+)} U F^{(-)\mu\nu}\Big),
\end{aligned}$$

with

$$F_{\mu\nu}^{(\pm)} = \partial_\mu A_\nu^{(\pm)} - \partial_\nu A_\mu^{(\pm)} + [A_\mu^{(\pm)}, A_\nu^{(\pm)}].$$

We obtain

$$L_9 = \frac{N_c}{16\pi^2}\frac{1}{3}\left(1 + \frac{21y - 26}{6}\frac{\alpha\mu^2}{\Lambda^2}\right),$$
$$L_{10} = -\frac{N_c}{16\pi^2}\frac{1}{6}\left(1 + \frac{15y - 10}{3}\frac{\alpha\mu^2}{\Lambda^2}\right). \quad (43)$$

In Eq. (40) we use $<\bar{q}q> = -(0.25\,\mathrm{GeV})^3$ and $F_0 = 92\,MeV$ to fix the value of $y$. In comparison with the local limit it changes from $y_{loc} \sim 1$ to $y_{nonloc} \approx 0.5$. It is worth to be mentioned here that the mass matrix is not affected by nonlocal corrections. The nonlocal corrections to the structure constants $L_2$, $L_3$, $L_9$ and $L_{10}$ (see Eqs. (42) and (43)) are estimated to be not larger than 15-20% in comparison with their values in the local limit. The structure coefficient $L_5$ is most sensitive to nonlocal corrections. As a result, the splitting of the decay constants $F_\pi$ and $F_K$ [13] seems to be strongly influenced by nonlocal effects.

## VI. Conclusion

We have studied the bosonization of an effective QCD-inspired quark interaction, including nonlocal effects. First we have considered a "fixed-distance" approximation in order to obtain a quantitative estimate of the size of the nonlocal corrections. The result was used as an input into a more general dynamical separation ansatz which then allowed to predict the modifications of the structure constants of the chiral meson lagrangian. We found that the nonlocal corrections to the $L_i$ coefficients were typically of the order 15-20%, except for $L_5$ which is stronly modified by nonlocal effects. Thus, the local NJL model has turned out to be a reasonable approximation for bosonization of low-energy quark interactions and deriving the effective meson lagrangian at $O(p^4)$-level. Of course, our analysis could be extended to any order in the momentum expansion. Given the fact that nonlocal corrections to the coefficients $L_i$ are of the order 20 %, it is to be expected that these corrections are of the same order of magnitude as local $p^6$ contributions.

The uncertainties arising from nonlocality make it difficult to distinguish between next to leading order effects of the momentum expansion and nonlocal contributions to the leading order. As an example where this is not the case we suggest to investigate the processes $\eta \to \pi^0\gamma\gamma$ and $\gamma\gamma \to \pi^0\pi^0$ where the nonzero *Born contributions* to the amplitudes appear only at $O(p^6)$-level.

## ACKNOWLEDGEMENTS

The authors are grateful to V.N.Pervushin and M.K.Volkov for useful discussions and helpful comments. One of the authors (A.A.Bel'kov) is grateful for the hospitality extended to him at TRIUMF, Vancouver. This work was supported in part by a grant of from the Natural Sciences and Engeneering Research Council of Canada (S.Scherer) and by the Russian Grant Center for Fundamental Researches (A.A.Bel'kov and A.V.Lanyov).



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