Generalized Numerical Inversion

A Neural Network Approach to Jet Calibrations

Aviv Cukierman, Ben Nachman, and Sana Ketabchi, on behalf of the ATLAS Collaboration

https://cds.cern.ch/record/2630972

Introduction

In ATLAS, collimated sprays of hadrons initiated by quarks or gluons are reconstructed as *jets*. The reconstructed energy of jets is not exactly the same as the truth-level energy, leading to a non-trivial *response.* This response needs to be corrected for in a jet calibration, while also taking into account various jet features which may have an effect on the response. Importantly, this calibration should be independent of the underlying truth distribution of jets, in order to have an unbiased calibration.

Generalized Numerical Inversion

The current procedure in ATLAS for the jet calibration uses *numerical inversion* (NI), correcting for each feature θ in sequence:

$$\boxed{f_\theta(x) \equiv \langle p_T^{\mathrm{reco}} | p_T^{\mathrm{true}} = x, \theta \rangle}$$

$$\boxed{p_T^{\mathrm{reco}} \mapsto \hat{p}_T^{\mathrm{reco}} = f_{\theta_n}^{-1}\left(\cdots f_{\theta_2}^{-1}\left(f_{\theta_1}^{-1}\left(p_T^{\mathrm{reco}}\right)\right)\cdots\right)}$$

Generalized numerical inversion generalizes NI, by learning the dependence of the response on each feature simultaneously:

1. Learn a neural network approximation $L(x,\theta)$ to the function $f_\theta(x) = \langle p_T^{\mathrm{reco}} | p_T^{\mathrm{true}} = x, \theta\rangle$. Note that $L(x,\theta) : \mathbb{R}^{n+1} \to \mathbb{R}$.
2. Learn a neural network $C(L(x,\theta),\theta)$ that tries to predict x given θ and $L(x,\theta)$. This is an approximation to the family of functions $f_\theta^{-1}(x)$. Note that learning the inverse this way is technically simple since L is single-valued.
3. Calibrate with $p_T^{\mathrm{reco}} \mapsto \hat{p}_T^{\mathrm{reco}} = C(p_T^{\mathrm{reco}},\theta)$. The calibration non-closure is defined as the deviation of $\langle \hat{p}_T^{\mathrm{reco}}/p_T^{\mathrm{true}} | p_T^{\mathrm{true}} = x, \theta\rangle$ from 1.

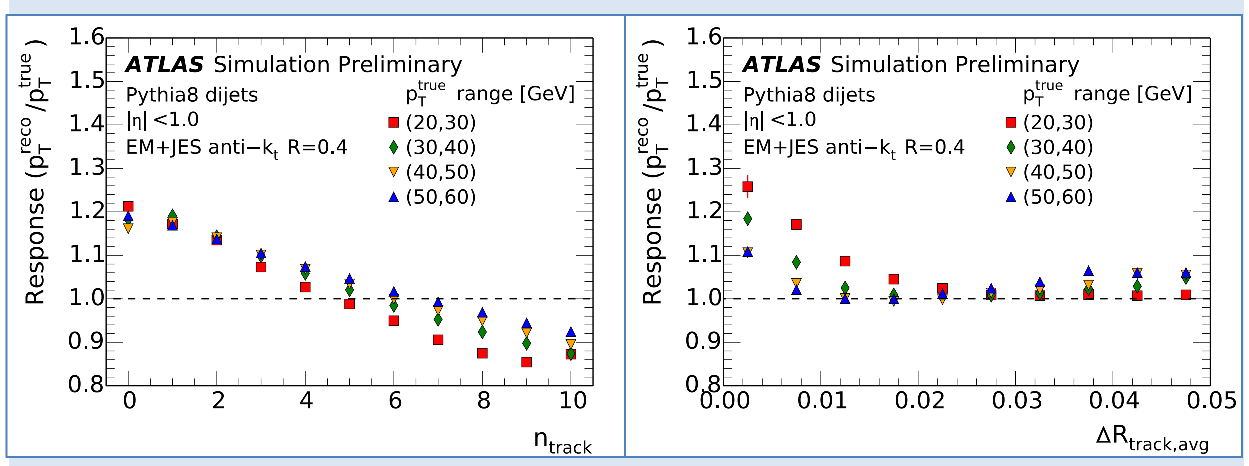
The new idea allows for a multivariate regression approach to the jet calibration, which should allow for improved performance.

Results

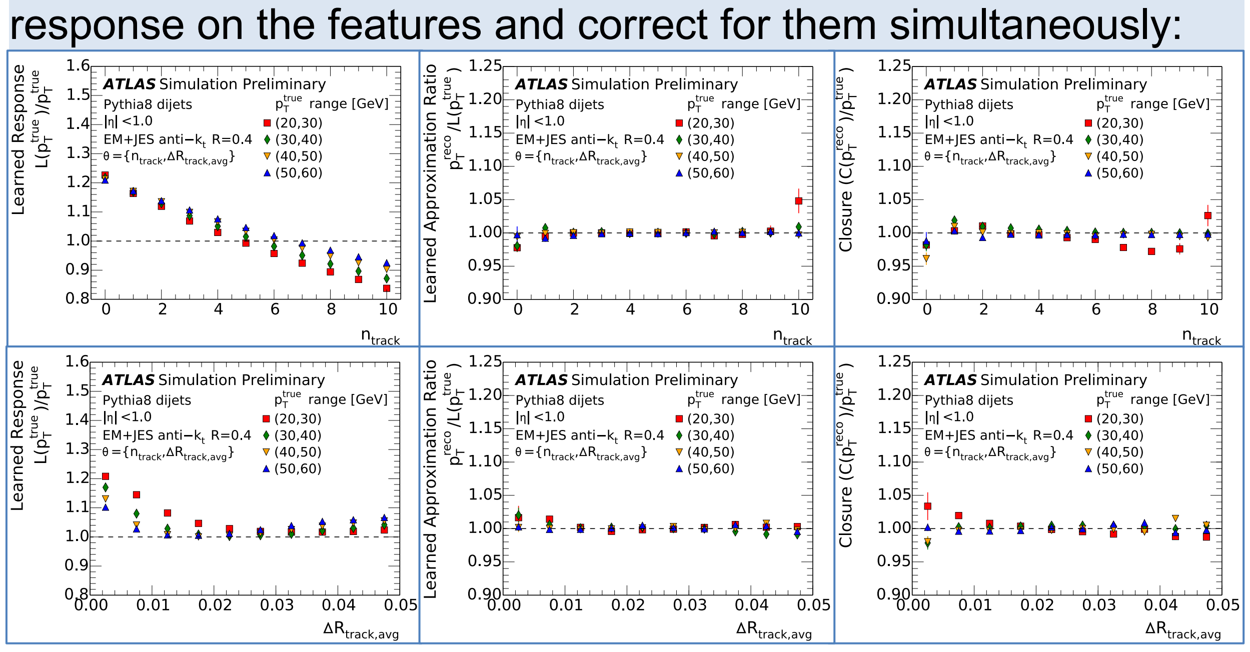
We examine a calibration depending on n_{track} and $\Delta R_{\mathrm{track,avg}}$:

$$\Delta R_{\mathrm{track,avg}} \equiv \begin{cases} \frac{1}{n_{\mathrm{track}}+1} \sum_{\mathrm{tracks}} \left(p_{\mathrm{T,track}} / \sum_{\mathrm{tracks'}} p_{\mathrm{T,track'}}\right) \times \Delta R_{\mathrm{track,jet}} & \text{if } n_{\mathrm{track}} > 0 \\ -1 & \text{if } n_{\mathrm{track}} = 0 \end{cases}$$

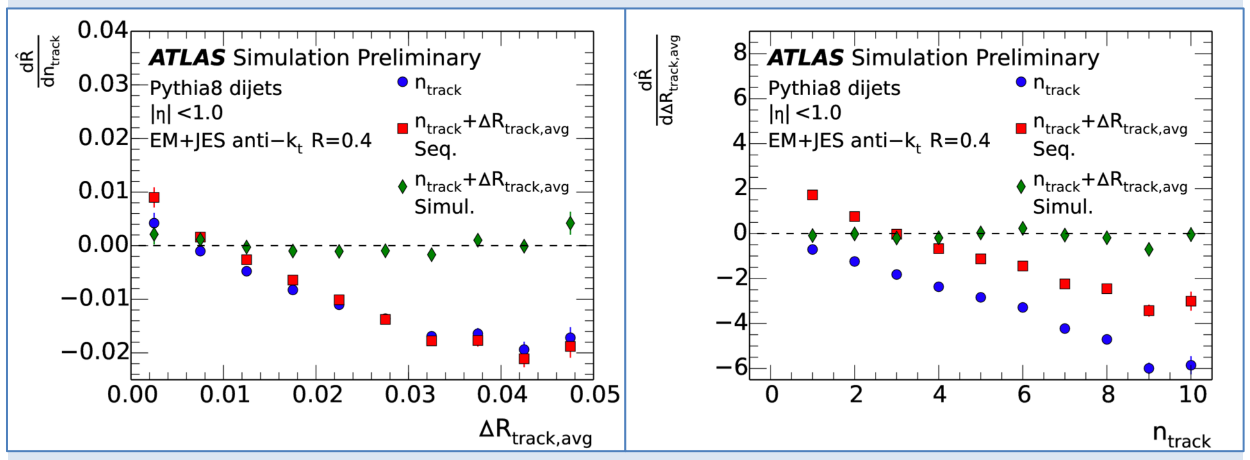
Before the calibration, the response depends strongly on these two features:



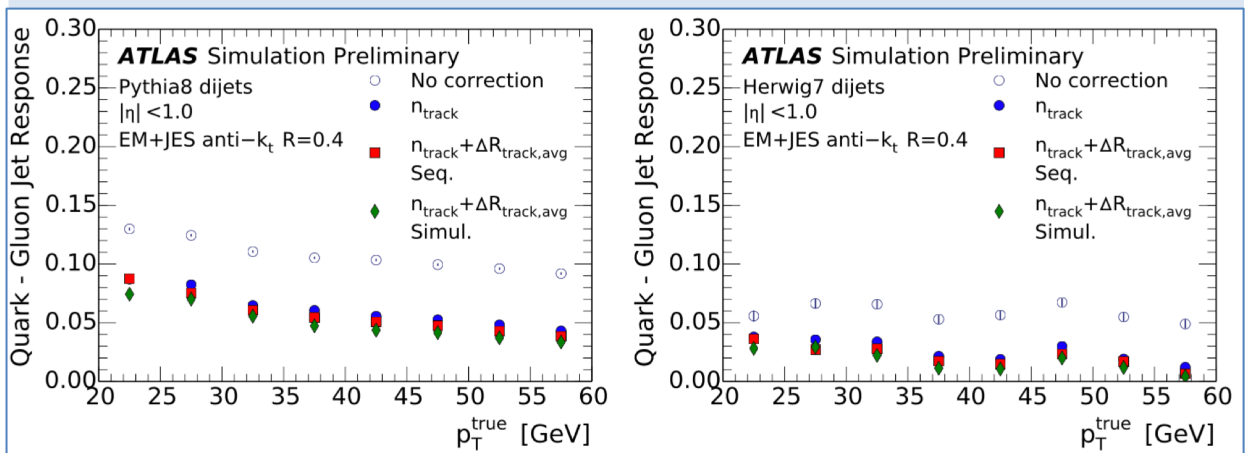
Generalized numerical inversion can learn the dependence of the response on the features and correct for them simultaneously:



The simultaneous calibration is able to correct for residual response dependencies that a sequential method is unable to account for:

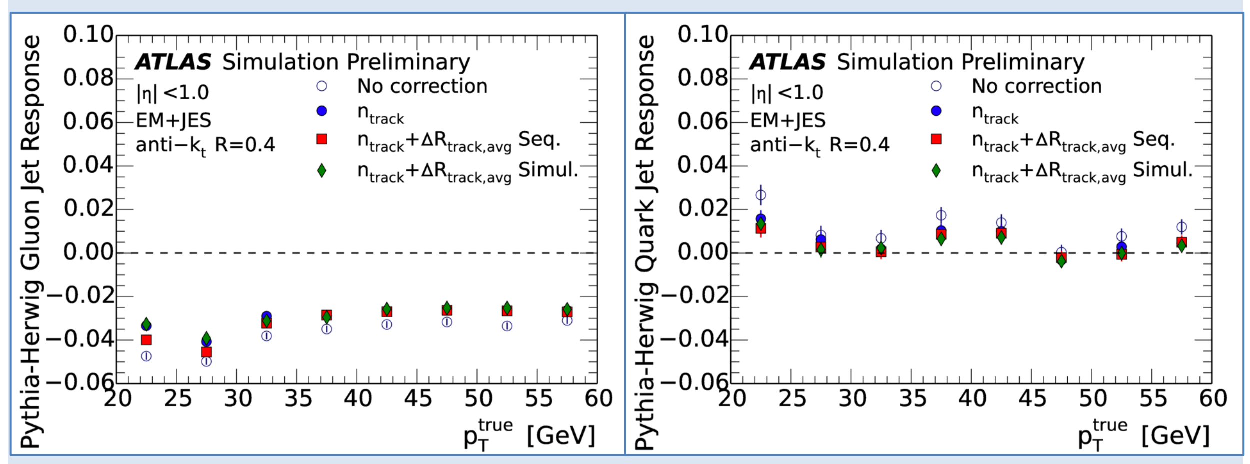


By taking into account multiple variables, the simultaneous calibration can reduce the difference in response between quark- and gluon-initiated jets, and this effect is robust to model differences:

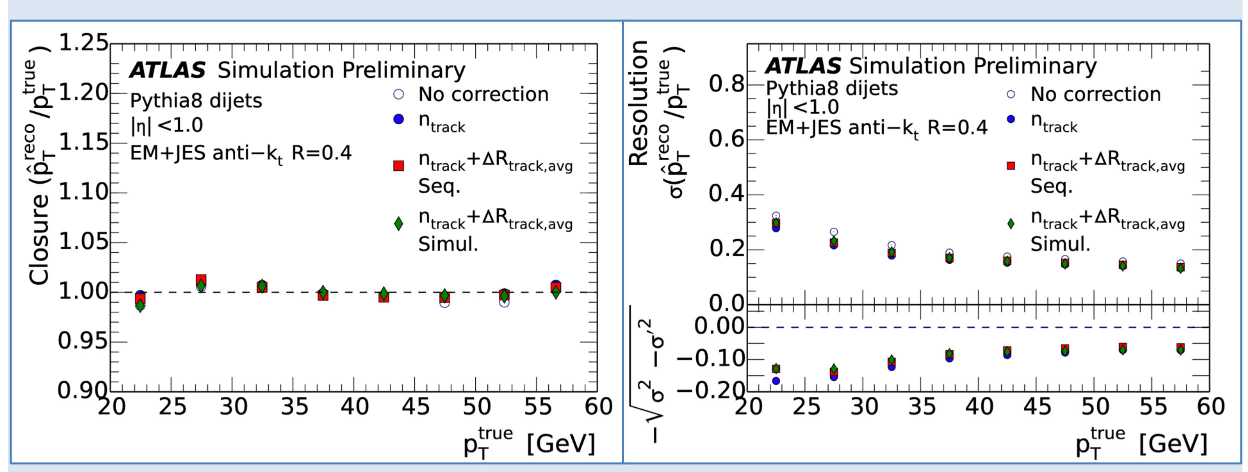


(a) Pythia 8 (b) Herwig 7

The simultaneous calibration can also reduce the difference in response between jets from different generators:



The response closes overall for both the simultaneous and the sequential calibrations and the resolution is reduced when taking into account the external variables:



Conclusions

Generalized numerical inversion is a new technique which allows for a regression-based jet calibration, avoids binning effects, and remains independent of the underlying jet distribution. The new method allows to correct for features simultaneously, which can correct the response dependence in all regions of the parameter space, reduce the difference between quark and gluon jets, and reduce model dependence. This method was demonstrated with neural networks, but more complicated architectures with more features should also be able to take advantage of this technique.