## Kinematic Dependence of $J/\psi$ Polarization Measurements in Hadronic Collisions

Pietro Faccioli<sup>1</sup>, Carlos Lourenço<sup>2</sup>, João Seixas<sup>3</sup> and Hermine K. Wöhri<sup>1</sup>

1- LIP, Lisbon, Portugal

2- CERN, Geneva, Switzerland

3- IST and CFTP, Lisbon, Portugal

We emphasize the importance of performing polarization measurements in two "orthogonal" frames, such as the Collins-Soper and helicity frames. The present experimental results on  $J/\psi$  polarization, lacking in most cases such full information, need to be interpreted by means of assumptions. Assuming, for example, that the natural polarization axis is along the direction of the relative motion of the colliding partons, the seemingly contradictory  $J/\psi$  polarization results reported by E866, HERA-B and CDF consistently imply that directly produced  $J/\psi$ 's are longitudinally polarized at low momentum and transversely polarized at high momentum.

The existing measurements of  $J/\psi$  polarization in hadronic collisions represent one of the most difficult challenges currently faced by models of quarkonium production (see, for example, Refs. [1, 2] and references therein). The disagreement between experiment and theory is, however, only one aspect of the problem. The experimental knowledge itself looks contradictory in terms of "sign", magnitude and kinematic dependence, as illustrated in Fig. 1, which shows the data reported by CDF [2], HERA-B [3] and E866 [4].

Having a clear (data driven) description of the polarization measurements is also important to evaluate detector specific corrections needed to extract physics results from the data. Production cross sections, for instance, might significantly depend on the polarization scenario used in the calculation of acceptance corrections. The polarization measurements are undeniably complex and involve difficult experimental problems. There is, however, an additional cause for the blurred picture emerging from the comparison of the existing measurements: different experiments have often chosen different polarization frames to perform their analyses. The influence of such choices on the measured angular distribution of the de-

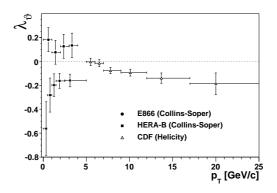


Figure 1:  $\lambda_{\vartheta}$  versus  $p_{\rm T}$ , as reported by E866, HERA-B and CDF (statistical and systematic errors added in quadrature).

cay leptons is generally underestimated. In fact, as detailed in Refs. [5, 6], different analyses of *the same* two-body angular decay distribution may give qualitatively and quantitatively different results depending on the definition of the polarization frame.

Several polarization frame definitions have been used in the past. In the helicity frame (H) the polar axis coincides with the flight direction of the  $J/\psi$  in the centre-of-mass frame of the colliding hadrons. A very different approach is implicit in the definition of the Collins-Soper [7] frame (CS), where the polar axis reflects, on average, the direction of the relative

velocity of the colliding partons. We denote by  $\vartheta$  the angle between the direction of the positive lepton and the chosen polar axis, and by  $\varphi$  the azimuthal angle, measured with respect to the plane formed by the momenta of the colliding hadrons in the  $J/\psi$  rest frame (the "production plane"). The angular decay distribution, symmetric with respect to the production plane and invariant under parity transformation [7, 8], is usually defined as:

$$W(\cos\vartheta,\varphi) \propto 1 + \lambda_{\vartheta} \cos^2\vartheta + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos\varphi + \lambda_{\varphi} \sin^2\vartheta \cos 2\varphi \quad . \tag{1}$$

If the  $J/\psi$  is observed in a given kinematic configuration, any two polarization frames differ only by a rotation around the axis perpendicular to the production plane (the "y axis"). The functional dependence of the decay distribution on the angles  $\vartheta$  and  $\varphi$  is invariant with respect to such a rotation, but the numerical values of  $\lambda_{\vartheta}$ ,  $\lambda_{\vartheta\varphi}$  and  $\lambda_{\varphi}$  change in a correlated way. In particular, a rotation by the angle  $\delta = 1/2 \arctan[2 \lambda_{\vartheta\varphi}/(\lambda_{\varphi} - \lambda_{\vartheta})]$  (or 45° when  $\lambda_{\varphi} = \lambda_{\vartheta}$ ) leads to a frame where  $\lambda_{\vartheta\varphi}$  is zero, i.e., a frame with axes along the principal symmetry axes of the polarized angular distribution. The experimental determination of  $\lambda_{\vartheta\varphi}$  can, therefore, provide a criterion for the choice of a particularly convenient reference frame for the description of the angular distribution.

While all three coefficients provide interesting and independent information, most available measurements of  $J/\psi$  polarization are restricted to  $\lambda_{\vartheta}$ . This limits the possible interpretations of the results and forces us to rely on model-dependent assumptions when comparing results obtained by experiments using different reference frames. Even the seemingly simple classification of "transverse" or "longitudinal" polarization, depending on the sign of  $\lambda_{\vartheta}$ , is, in fact, dependent on the reference frame. This is particularly evident when the decaying particle is produced with small longitudinal momentum, the CS and H polar axes becoming perpendicular to each other. In this case (assuming  $\lambda_{\varphi} = \lambda_{\vartheta\varphi} = 0$ , for simplicity), if in one frame we measure a value  $\lambda_{\vartheta}$ , the value measured in the second frame is smaller and of opposite sign,  $\lambda'_{\vartheta} = -\lambda_{\vartheta}/(2+\lambda_{\vartheta})$ , while an azimuthal anisotropy appears,  $\lambda'_{\varphi} = \lambda_{\vartheta}/(2+\lambda_{\vartheta})$ .

There is a further reason for performing the experimental analyses in more than one reference frame. The  $J/\psi$  acquires its polarization with respect to a "natural" polarization axis which is, a priori, unknown and not necessarily definable event-by-event in terms of observable quantities. In practice, a fine-grained scan of the multidimensional phase-space of the  $J/\psi$  production process is not possible, due to the limited sample of collected events, which forces the decay distribution to be measured as an average over a wide spectrum of kinematic configurations. This means that the orientation of the polar axis of the chosen frame with respect to the "natural axis" changes from event to event, depending on the momentum of the produced  $J/\psi$ . The resulting superposition of many distributions, equal in shape but randomly rotated with respect to one another, is "smeared" into a more spherically symmetric shape. As a consequence, the measured absolute values of  $\lambda_{\vartheta}$  and  $\lambda_{\varphi}$  are smaller than what would be measured in a fixed kinematic configuration and in the "natural frame". Therefore, independently of any prior theoretical expectation, the frame closest to the natural frame is the one providing the smallest  $\delta$  angle and the most significant  $|\lambda_{\vartheta}|$ .

The HERA-B experiment recently reported the three coefficients determining the  $J/\psi$  decay angular distribution, in three reference frames [3], providing a clear picture of how the shape of the distribution changes from frame to frame. Before discussing kinematical dependencies, we start by considering the values integrated in the phase space window covered by HERA-B: in the CS frame,  $\lambda_{\vartheta} = -0.31 \pm 0.05$  and  $\lambda_{\varphi} = -0.02 \pm 0.02$ ; in the H frame,  $\lambda_{\vartheta} = -0.11 \pm 0.05$  and  $\lambda_{\varphi} = -0.07 \pm 0.02$  (statistical and systematic errors added in quadrature). Furthermore,  $\delta$  has a much larger error in the H frame ( $10^{\circ} \pm 20^{\circ}$ ) than

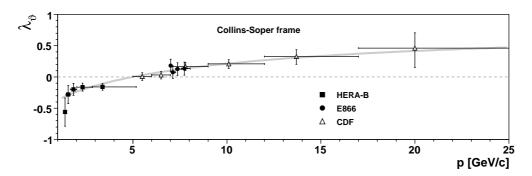


Figure 2:  $\lambda_{\vartheta}$  as a function of p, from E866, HERA-B and CDF data (statistical and systematic errors added in quadrature).

in the CS frame  $(3^{\circ} \pm 3^{\circ})$ , reflecting the poorer precision with which the "tilt" of a more spherically symmetric shape can be determined. With the largest  $|\lambda_{\vartheta}|$  and a  $\lambda_{\varphi}$  compatible with zero, the CS frame is shown by the HERA-B measurements to provide a simpler angular distribution than the H frame.

We now address the kinematical dependence of the  $J/\psi$  polarization. Figure 1 shows that, in the CS frame, E866 [4] observed a small  $J/\psi$  transverse polarization ( $\lambda_{\vartheta} \approx 0.1$ ) while the HERA-B pattern indicates longitudinal polarization, of decreasing magnitude with increasing  $p_{\rm T}$ . These are not conflicting observations, given the significantly different  $x_{\rm F}$  windows covered: the average  $J/\psi$  longitudinal momentum, in the centre of mass of the collision system, is 7 and -1.4 GeV/c for E866 and HERA-B, respectively. Indeed, Fig. 2 shows that the *total*  $J/\psi$  momentum (here calculated using average  $x_{\rm F}$  values) provides a good scaling between the two fixed-target data sets. As also shown in Fig. 1, CDF [2] reported that, above  $p_{\rm T} = 5 \text{ GeV}/c$ , the prompt- $J/\psi$  polarization is longitudinal in the H frame, with  $\lambda_{\vartheta}$  steadily decreasing with  $p_{\rm T}$ . To see how the CDF pattern compares to the fixed-target data sets, we need to convert the published values to the CS frame. We did this translation (using the relations presented above) assuming that  $\lambda_{\varphi} = 0$  in the CS frame, as suggested by the HERA-B measurements. The resulting pattern, seen in Fig. 2, is perfectly aligned with the HERA-B and E866 data points.

This smooth overlap of the three data sets suggests a simple polarization scenario, where the CS frame is taken to be a good approximation of the natural polarization frame ( $\lambda_{\varphi} = 0$ ,  $\lambda_{\vartheta\varphi} = 0$ ) and  $\lambda_{\vartheta}$  is a monotonically increasing function of the total J/ $\psi$  momentum. Before searching a suitable function, we remind that a significant fraction of the observed prompt J/ $\psi$  mesons results from  $\chi_c$  and  $\psi'$  feed-down decays [9]:  $f_{\rm fd} = 0.33 \pm 0.05$ . Irrespectively of the possible polarizations of these charmonium states, it is reasonable to assume that the strong kinematical smearing induced by the varying decay kinematics reduces the *observable* polarization of the indirectly produced J/ $\psi$  mesons to a negligible level. Therefore, the observed polarization should be essentially determined by the directly produced J/ $\psi$ 's. The curve in Fig. 2 represents a fit of all the data points using the simple parametrization

$$\lambda_{\vartheta} = (1 - f_{\rm fd}) \times \left[ 1 - 2^{1 - (p/p_0)^{\kappa}} \right] \quad , \tag{2}$$

where the polarization of the *directly produced*  $J/\psi$ 's changes from fully longitudinal at zero momentum to fully transverse at asymptotically high momentum. The fit gives  $p_0 =$ 

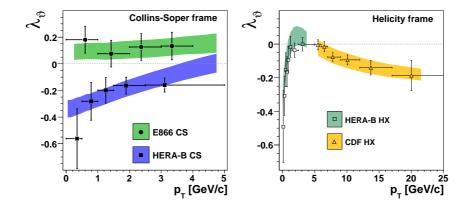


Figure 3:  $p_{\rm T}$  dependences of  $\lambda_{\vartheta}$  in the CS and H frames, according to Eq. 2 and as reported by E866, HERA-B and CDF.

 $5.0 \pm 0.4 \text{ GeV}/c$  and  $\kappa = 0.6 \pm 0.1$ , with  $\chi^2/\text{ndf} = 3.6/13$ .

Our simple parametrization provides a good description of the existing data sets, as can be seen in Fig. 3, where the widths of the bands correspond to  $\pm 1\sigma$  variations in the two fitted parameters as well as in the  $J/\psi$  feed-down fraction. The derivation of the  $\lambda_{\vartheta}$  pattern in the H frame (needed, in particular, to address the CDF case) incorporates the "kinematical smearing" induced by the decays and the differential acceptances of the experiments (using a simple Monte Carlo procedure). In the narrow rapidity window of CDF, where the maximum  $J/\psi$  longitudinal momentum (~4 GeV/c) is always smaller than the minimum  $p_{\rm T}$  (5 GeV/c), the H and CS frames are essentially orthogonal to each other. Therefore, the decrease of  $\lambda_{\vartheta}$  with  $p_{\rm T}$  seen in the H frame (Fig. 3, right) is equivalent to an increase in the CS frame (Fig. 4, left).

Assuming that the decay distribution has a purely polar anisotropy in the CS frame, with  $\lambda_{\vartheta}$  depending on momentum according to Eq. 2, CDF should observe a significant azimuthal anisotropy in the H frame, with a  $\lambda_{\varphi}$  pattern (Fig. 4, right) similar in magnitude but of opposite sign with respect to their  $\lambda_{\vartheta}(p_{\rm T})$  curve. By simply repeating the J/ $\psi$  polarization analysis using the CS frame and by reporting the azimuthal angular distribution, CDF can clarify whether the polarization of the J/ $\psi$  is, also at collider energies, induced along a direction close to the parton-parton interaction line.

Figure 4 also shows the calculated  $p_{\rm T}$  dependence of  $\lambda_{\vartheta}$ , in the CS frame, and of  $\lambda_{\varphi}$ , in the H frame, for the kinematical conditions of the PHENIX ( $\sqrt{s} = 200 \text{ GeV}$ ,  $|\eta| < 0.35$ ) and CMS ( $\sqrt{s} = 14 \text{ TeV}$ ,  $|\eta| < 2.4$ ) experiments. If Eq. 2 remains valid up to LHC energies, we should see  $\lambda_{\vartheta}$  saturating for  $p_{\rm T}$  values higher than those probed by CDF, with a magnitude determined by the fraction of directly produced J/ $\psi$  mesons.

We will now summarise our main messages. 1) To investigate the  $J/\psi$  polarization and understand its origin, it is essential to know both the polar and azimuthal distributions, and their kinematical dependencies, in at least two frames. The CS and H frames, exactly orthogonal to each other at mid-rapidity, represent a good minimal set of polarization frames. 2) The HERA-B measurements show a pure polar anisotropy in the CS frame while a mixture of polar and azimuthal anisotropies is seen in the H frame, indicating that the  $J/\psi$  decay angular distribution assumes its simplest shape when observed with respect to a polar axis

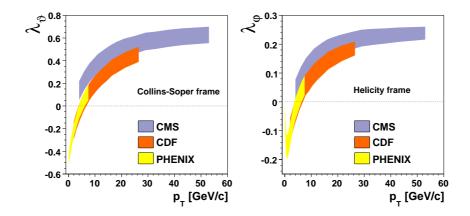


Figure 4:  $p_{\rm T}$  dependencies of  $\lambda_{\vartheta}$  in the CS frame and  $\lambda_{\varphi}$  in the H frame, calculated for the energy and rapidity windows of PHENIX, CDF and CMS.

that reflects the relative momentum of the colliding partons rather than the  $J/\psi$  momentum. 3) The seemingly contradictory patterns published by E866, HERA-B and CDF can be consistently reproduced assuming that the polarization (in the CS frame) of the directly produced  $J/\psi$  mesons changes gradually from fully longitudinal at zero momentum to fully transverse at very high p. 4) This suggests that the longitudinal polarization reported by CDF in the H frame is, in fact, the reflection of a transverse polarization (around twice as large) in the CS frame, increasing with  $p_T$ . Moreover, an azimuthal anisotropy of the decay distribution should exist in the H frame, with the same significance as the polar result. 5) Our polarization scenario predicts that the polar anisotropy of the prompt  $J/\psi$ sample will saturate, for  $p_T$  above ~ 25 GeV/c, at  $\lambda_{\vartheta} \approx 0.6$ -0.7, a value determined by the magnitude of the  $\psi'$  and  $\chi_c$  feed-down contributions, assumed to be of negligible observable polarization. This prediction, easily verifiable at the LHC, can be placed on more robust grounds once CDF reports the complete angular distribution in the CS frame or, at least, the azimuthal component in the H frame.

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