"A Renormalization Group Analysis of the NCG constraints $m_{top} = 2m_W$, $m_{Higgs}=3.14\ m_W$"

E. Alvarez, J.M. Gracia-Bondía and C.P. Martín

Departamento de Física Teórica

Facultad de Ciencias

Universidad de Zaragoza

CAESARAVG ✶ SIGILLV · SCHOLAE · SCIETIAR · STVD · GEN · CIVIT

A Renormalization Group Analysis
of the NCG constraints $m_{top} = 2m_W$, $m_{Higgs} = 3.14 m_W$

E. Álvarez, J.M. Gracia-Bondía [1] and C.P. Martín

Departamento de Física Teórica, CXI

Universidad Autónoma

28049 Madrid, Spain

We study the evolution under the renormalization group of the restrictions on the parameters of the standard model coming from Non-Commutative Geometry, namely $m_{top} = 2 m_W$ and $m_{Higgs} = 3.14 m_W$. We adopt the point of view that these relations are to be interpreted as *tree level* constraints, and, as such, can be implemented in a mass independent renormalization scheme only at a given energy scale μ_0. We show that the physical predictions on the top and Higgs masses depend weakly on μ_0.

---

1. Non-Commutative Geometry constraints on the parameters of the Standard Model

There has been recent interest in the field theoretical [1] applications of Connes' generalization of ordinary geometry [2], termed by him Non-Commutative geometry (NCG fron now on). Connes' approach amounts to a new gauge principle incorporating the Higgs sector together with the pure gauge terms. By the same token, the Dirac operator incorporates the matrix of Yukawa couplings between the chiral fermions and the Higgs field. Apparently, the Connes-Lott derivation of the standard model of the strong and electroweak interactions yields fewer arbitrary parameters than the Glashow-Weinberg-Salam formulation of it. There is some argument about this point [3] but in the last version [4], Connes claims that there are two, and only two, such relationships, namely:

$$m_{top} = 2m_W \tag{1.1}$$

$$m_{higgs} = 3.14 m_W \tag{1.2}$$

Note that the quotient of these relations can be obtained independently by considering the fermionic part of the lagrangian in NCG [5].

Our point of view (lacking a more fundamental option) is that once obtained, the NCG lagrangian is an ordinary quantum field theoretical one, in which some constraints are imposed at the tree level. Using standard techniques ([6], [7]) we showed analytically in a recent letter [8] that (at least in a toy model) those constraints could not be implemented in a renormalization-group invariant way; physically this means that if we impose them at one scale, say μ_0, then the RG evolution would violate them.

Our aim in the present letter is to address this same issue for the standard model itself (we shall find the same result as we did before in a simplified situation) as well as to draw some quantitative estimates on the amount of evolution of the relations themselves, as compared to some physically relevant quantity.

We shall present a certain amount of evidence that this evolution is "small" (in a well-defined sense), but we shall not venture any further hypothesis on the physical origin of this fact.

---

[1] And Departamento de Física Teórica, Universidad de Zaragoza, 50009 Zaragoza, Spain

2. The evolution of the NCG constraints under the renormalization group in the one-loop approximation

In spite of the non-existence of a fully self-consistent and systematic set of beta-functions (in particular, with respect to the treatment of the γ_5 à la 't Hooft and Veltman), we have borrowed the expressions of the one-loop beta functions (in the $\overline{MS}$ scheme) from the literature: [9]. Neglecting all leptonic Yukawa couplings, and all the quark Yukawas but the top, the corresponding beta functions are:

$$4\pi\beta_t = \alpha_t(9\alpha_t - 16\alpha_3 - \frac{9}{2}\alpha_2 - \frac{17}{10}\alpha_1)$$ (2.1)

$$4\pi\beta_3 = -14\alpha_3^2$$ (2.2)

$$4\pi\beta_2 = -\frac{19}{3}\alpha_2^2$$ (2.3)

$$4\pi\beta_1 = \frac{41}{5}\alpha_1^2$$ (2.4)

$$4\pi\beta_h = 6\alpha_h^2 + 12\alpha_h\alpha_t - \frac{9}{5}\alpha_h\alpha_1 - 9\alpha_h\alpha_2 + \frac{27}{50}\alpha_1^2 + \frac{9}{5}\alpha_1\alpha_2 + \frac{9}{2}\alpha_2^2 - 24\alpha_t^2$$ (2.5)

Where β_h is the beta function of the quartic Higgs coupling constant, $\alpha_h = \frac{\lambda}{4\pi}$; β_t the corresponding one for the top Yukawa coupling, $\alpha_t = \frac{g_t^2}{4\pi}$, and $\beta_i(i=1,2,3)$ are the beta functions corresponding to the gauge coupling constants of U(1) and U(2) (with an SU(5) normalization) and SU(3). Our Weinberg angle is defined through $\alpha = \alpha_2 sin^2\theta_w = \frac{3}{5}\alpha_1 cos^2\theta_w$. The physical masses are given in terms of the vacuum expectation value of the neutral component of the Higgs field, v, $m_W = \frac{g_2 v}{2}$; $m_H^2 = \frac{\lambda v^2}{2}$; $m_t = \frac{g_t v}{\sqrt{2}}$ (neglecting the Kobayashi-Maskawa phases).

The relations of NCG boil down in our notations to

$$\alpha_t - 2\alpha_2 = 0$$ (2.6)

$$\alpha_h - 4.93\alpha_2 = 0$$ (2.7)

which are physically equivalent to predictions on the masses of the top and Higgs particles:

$$m_t = 2 m_W$$ (2.8)

$$m_H = 3.14 m_W$$ (2.9)

As we have already said, from our point of view, those are tree level constraints. If we want to implement them in a mass-independent renormalization scheme, we have to choose

a particular scale μ_0 at which we assume them to be valid, and study afterwards the RG flow from these initial conditions.

To be specific, what we did was the following: we chose initial conditions at a point μ_0 corresponding to[[10]]

$$\alpha_3(\mu = m_Z) = 0.12$$ (2.10)

$$\alpha_2(\mu = m_Z) = 0.034$$ (2.11)

$$\alpha_1(\mu = m_Z) = 0.017$$ (2.12)

(in our approximation, these coupling constants form a closed set under the renormalization group) and, furthermore, we got the other two initial conditions from imposing the Connes-Lott constraints at μ_0:

$$\alpha_t(\mu_0) = 2\alpha_2(\mu_0)$$ (2.13)

$$\alpha_h(\mu_0) = 4.93\alpha_2(\mu_0)$$ (2.14)

We have displayed in the table the dependence on the physical predictions on the scale μ_0 at which one chooses to impose the Connes-Lott relations as initial conditions.

-- As is apparent from the table, this dependence never gets much bigger than 10%, even if one is willing to extrapolate the standard model as such as far as 10^8 GeV, which is the approximate location of the Landau pole for α_h with our set of initial conditions.

μ_0	m_{top}	m_{higgs}
10	138	283
92	160	252
200	166	244
300	169	241
400	171	239
500	172	237
600	173	236
700	174	235
800	175	234
900	176	233
10^3	176	233
10^4	186	223
10^5	192	217
10^6	197	215
10^7	199	213

It is sometimes held the viewpoint [11] that there are actually four relationships instead of only two, depending besides on an unknown parameter x, to wit:

$$\alpha_3 = \frac{1}{2}\left(\frac{4 - 2x}{1 - x}\right)^{1/2}\alpha_2$$

$$\sin^2\theta_W = \frac{3(1 - x/2)}{8 - 2x}$$

$$m_W = \frac{1}{2}\left(\frac{1 - x}{1 - x/2}\right)^{1/2}m_t$$

$$m_H = (3 - \frac{3}{2}\frac{3x^2 - 8x + 5}{5x^2 - 17x + 14})^{1/2}m_t$$

We just would like to point out here that these relationships are inconsistent. From the (known) evolution of α_2 and α_3 (we remind the reader that the initial conditions for the closed subset α_1, α_2 and α_3 are completely fixed by the experiment, and we can not change them) one determines the unique point (for each x) at which the first relationship can consistently be imposed. For $x = 0$ this gives $\mu_0 = 2 \times 10^{17} GeV.^2$

We should now impose all other relations at this value of μ_0 and get a *unique* prediction for m_t and m_H, but there are two problems: first of all, μ_0 is well above the Landau ghost for the Higgs self-coupling, so that one can not impose consistently initial conditions on it. Besides, even if this were not the case, the value one predicts for Weinberg's angle is incompatible with the experiment. This is actually the main reason why we have stuck all along the paper to the original Connes' interpretation of considering two constraints only.

3. Conclusions

Is all this physically significant?

Perhaps, at least if the top mass turns out to be experimentally close to 160 GeV.

The figures are small in absolute value (we are always talking of dimensionless quantities).

Still, does this mean anything physically?

Let us take the conservative point of view that if we impose a random constraint on the physical couplings, say,

$$\lambda \equiv \sum c_i \alpha_i = 0 \qquad (3.1)$$

2 The only other value of x for which there is a solution is $x \sim 1$, for which $\mu_0 \sim 10 GeV$, and which predicts huge values for the top and Higgs masses

5

at a given scale, μ_0, the natural expectation is to find that at another scale, μ, λ will not be zero, but instead, it will be of the order of

$$\lambda(\mu) \sim max_i[c_i(\alpha_i(\mu) - \alpha_i(\mu_0))] \qquad (3.2)$$

We would be tempted to say that when the corresponding quotient

$$q \equiv \frac{\lambda(\mu)}{max_i[c_i(\alpha_i(\mu) - \alpha_i(\mu_0))]} \qquad (3.3)$$

is smaller than one, the relation between coupling constants is "better than random" (whatever explanation it has, it is obvious that it is an statistical fact that there are roughly equal numbers of "good" than "bad" relations in the "space" of all possible relations; still, one would be inclined to look for a physical explanation only if this quantity is appreciably smaller than one).

In our case q turns ot to be

$$q(top) = 0.7 \qquad (3.4)$$

$$q(higgs) = 0.4 \qquad (3.5)$$

- if μ_0 ranges from m_Z up to 1 TeV.

A further point is the following: in a celebrated paper, Veltman [12] worked out the relations one needed to have among the parameters of the standard model if one wanted to have cancellation of quadratic divergences. We have explicitly checked that Connes relations are not compatible with Veltman's for any value of the parameter x introduced in ref. [11].

References

[1] A.H. Chamseddine,G. Felder and J. Fröhlich Nucl. Phys. B395 (1993) 672.

[2] A. Connes. Pub. Math. IHES, 62, (1985), 257

[3] R.Coquereaux.J. Geom. Phys. 11,(1993),307

[4] A. Connes, "Non-Commutative Geometry", IHES preprint (1993).

[5] J.C.Varilly and J.M.Gracia-Bondia,J.Geom. Phys. 12 (1993) 223

[6] J.C.Collins,"Renormalization" , (Cambridge University Press)

[7] W.Zimmermann,Comm.Math.Phys., 97,(1985),142

[8] E.Álvarez,J.M.Gracia-Bondía and C.P. Martín,Phys.Lett. B306 (1993) 55

[9] M.E. Machacek and M.T. Vaughin, Nucl. Phys. B222 (1983) 83;B236 (1984) 221; B249 (1985) 70

[10] P.Langacker, "Precision tests of the Standard Model, Philadelphia preprint, UPR-0555T (1993)

[11] D. Kastler and T. Schücker, Theor. Math. Phys. 92 (1092) 522

[12] M. Veltman, Acta Phys. Pol.B12 (1981) 437 ;

J. Kubo,K. Sibold and W. Zimmermann, Phys. Lett B220 (1989) 191