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# Computations of Longitudinal Electron Dynamics in the Recirculating CW RF Accelerator-Recuperator for the High Average Power FEL

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## ABSTRACT

The use of optimal longitudinal phase-energy motions conditions for bunched electrons in a recirculating RF accelerator gives the possibility to increase the final electron peak current and, correspondingly, the FEL gain. The computer code RECFEL, developed for simulations of the longitudinal compression of electron bunches with high average current, essentially loading the CW RF cavities of the recirculator-recuperator, is briefly described and illustrated by some computational results.

The longitudinal phase-energy motion of electrons in a recirculating RF accelerator-recuperator (see Fig.1) can be optimized with a view to increase the final electron peak current by using the linear optics equations. An electron with the initial longitudinal phase deviation $\Delta\varphi_0$ from the equilibrium phase $\varphi_s$, having passed through RF cavities, has the energy deviation $\Delta E$ from the equilibrium energy $eU_{RF}cos(\varphi_s)$, in linear consideration, as

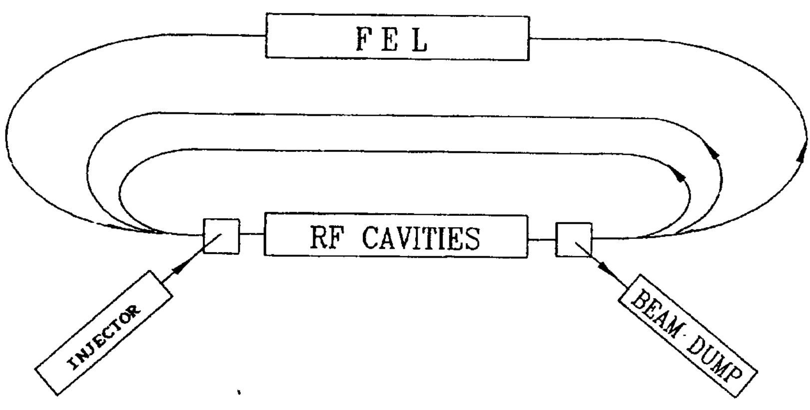



*Fig. 1.* The principal scheme of the RF recirculating accelerator-recuperator dedicated for FEL.

$$\Delta E=-eU_{RF}\sin(\varphi_s)\cdot\Delta\varphi_0 ,$$

where $U_{RF}$ is the total RF voltage amplitude and $e$ is the electron charge. So if we define the initial (before the passage through RF cavities) and final (after the passage) phase-energy deviations as two vectors $\begin{pmatrix}\Delta\varphi_0\\ \Delta E_0\end{pmatrix}$ and $\begin{pmatrix}\Delta\varphi\\ \Delta E\end{pmatrix}$, respectively, we may use the following transformation of thin lens matrix for such vector after the passage through RF cavities:

$$\begin{pmatrix}\Delta\varphi\\ \Delta E\end{pmatrix} = \begin{pmatrix}1 & 0\\ -1/F & 1\end{pmatrix}\begin{pmatrix}\Delta\varphi_0\\ \Delta E_0\end{pmatrix} ,$$

where $F = \dfrac{1}{eU_{RF}\sin(\varphi_s)}$.

Having passed through the beam transport magnetic system, whose longitudinal dispersion is $D$, the electron with the initial energy deviation $\Delta E_0$ from the equilibrium energy $E_s$ has the longitudinal phase deviation:

$$\Delta\varphi = \frac{D}{\lambda_{RF} E_s}\cdot\Delta E_0 ,$$

where $\lambda_{RF} = \dfrac{c}{2\pi f_{RF}}$, $f_{RF}$ is the frequency of RF cavities. Using here the transformation of drift space matrix, we can write down the above expression as follows:

$$\begin{pmatrix}\Delta\varphi\\ \Delta E\end{pmatrix} = \begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}\Delta\varphi_0\\ \Delta E_0\end{pmatrix} ,$$

where $L = \dfrac{D}{\lambda_{RF} E_s}$.

Now we see, that it is possible to consider the longitudinal phase-energy motion of the electrons of the bunch in the recirculating accelerator using matrix optics, where we have several thin lenses, separated by the drift spaces. In this case, the longitudinal phases deviation $\Delta\varphi$ is an analog of the coordinate deviation and the energy deviation $\Delta E$ is an analog of the angular deviation. If the initial electron bunch, injected into the recirculator, has a Gaussian longitudinal phase-energy distribution $P(\Delta\varphi,\Delta E)$, its linear evolution over the longitudinal coordinate $S$ along the beam path always can be written as

$$P(\Delta\varphi,.\Delta E) = \frac{1}{2\pi\varepsilon}\cdot exp\left[-\frac{\gamma\cdot\Delta\varphi^2+2\alpha\cdot\Delta E\cdot\Delta\varphi+\beta\cdot\Delta E^2}{2\varepsilon}\right] ,$$

where $\alpha(S)$, $\beta(S)$, $\gamma(S)$ are conventional parameters of the Twiss ellipse and $\varepsilon$ is the invariant emittance of the ellipse. So if the initial bunch has the longitudinal phase dispersion $\sigma_{\varphi i}$, the energy dispersion $\sigma_{Ei}$ and the Twiss parameter $\alpha_i$, we can express the following values:

$$\varepsilon = \frac{\sigma_{\varphi i}\cdot\sigma_{Ei}}{\sqrt{1+\alpha_i^2}} ; \qquad \beta_i = \frac{\sigma_{\varphi i}^2}{\varepsilon} = \frac{\sigma_{\varphi i}}{\sigma_{Ei}}\cdot\sqrt{1+\alpha_i^2} ; \qquad \gamma_i = \frac{1+\alpha_i^2}{\beta_i} .$$

As an example, we consider the recirculating RF accelerator-recuperator with three beam tracks for the high power FEL project of Japan Atomic Energy Researches Institute (JAERI) [1]. Its initial beam of electron energy $E_i$=15 MeV, injected into the recirculator, will have

$$\sigma_{Ei}/E_i = 4\cdot 10^{-3},\ \sigma_{Ei} = 0.06 \text{ MeV and } \sigma_{\varphi i} = 3^\circ = 0.052 \text{ rad}$$

(corresponding to the f.w.h.m. of about 40 psec). So, for the injected beam with $\alpha_i$=0, the corresponding longitudinal emittance and the longitudinal $\beta$-function are

$$\varepsilon = 3.1\cdot 10^{-3} \text{ rad}\cdot\text{MeV} \quad \text{and} \quad \beta_i = 0.87 \text{ rad/MeV} .$$

Now we have to estimate the needed value of the final $\beta$-function $\beta_f$ of the bunch, passing through the FEL undulator straight section. Let us take the final electron energy optimal dispersion $\sigma_{Ef}$, matched with the FEL undulator, from

$$\frac{\sigma_{Ef}}{E_f} = \frac{1}{4N_u} ,$$

where $N_u$ is the number of undulator periods and $E_f$ is the final electron energy. This dispersion value does not yet essentially decrease the ratio of the FEL gain per the beam peak current (by about two times). So the required value of the final $\beta$-function $\beta_f$ can be derived from

$$\sigma_{Ef} = \sqrt{\frac{\varepsilon}{\beta_f}\cdot(1+\alpha_f^2)} = \frac{E_f}{4N_u} .$$

Assuming $\alpha_f=0$ we obtain

$$\beta_f = \frac{16\ \varepsilon N_u^2}{E_f^2} ,$$

and for $E_f \approx 240$ MeV and $N_u=61.5$ we have

$$\beta_f = 3.3\cdot 10^{-3} \text{ rad/MeV} .$$

In linear approximation, this optimal increase of the final beam peak current, caused by the longitudinal focusing, can be estimated as

$$\sqrt{\frac{\beta_i}{\beta_f}} = \frac{E_f\sqrt{(1+\alpha_i^2)}}{4N_u\sigma_{Ei}} = 16$$

for the considered case, and the maximal corresponding increase of the FEL gain is expected to be about two times lower. Since the initial peak current is proportional to the ratio of the average electron current $\bar{I}$ per initial longitudinal phase dispersion (the bunch length) and the ratio of the maximized FEL gain per peak current is proportional to $\frac{N_u^2}{E_f}$, the maximal FEL gain value, achievable with the longitudinal focusing, is proportional to $\frac{\bar{I}N_u}{\varepsilon}$ with the natural dependence on the bunch longitudinal emittance $\varepsilon$. It is also remarkable that the maximal peak current value is reached only at the final maximal electron energy, so the emittance growth, caused by the effects of the high space charge density, is minimized.

Now let us estimate the required value $F$ of the focusing lenses and the drift spaces length $L$ between the lenses to obtain the needed value $\beta_f$ of the final $\beta$-function. It may be shown that in an optical system with the uniformly distributed focusing strength

$$K = \frac{d(1/F)}{dS} ,$$

the beam $\beta$-function oscillates along the longitudinal coordinate $S$ of the beam path between its extrema $\beta_{max}$ and $\beta_{min}$, spatially separated by the distance $\frac{\pi}{2}\cdot\beta_K$, where

$$\beta_K = 1/\sqrt{K}$$

is the so called the eigen $\beta$-function of this system, and these extrema are satisfy the equation

$$\sqrt{\beta_{max}\cdot\beta_{min}} = \beta_K .$$

For the system of the identical uniformly-distributed focusing lenses, we have

$$K = \frac{1}{FL} \quad \text{and} \quad \beta_K = \sqrt{FL} \ .$$

In the considered case of three accelerating passages through the RF cavities we have to satisfy also the following relation to obtain the bunch of minimum length at the FEL undulator straight section:

$$\frac{\pi}{2}\cdot\beta_K = \frac{7}{2}\cdot L \ .$$

Finally we have the system of two equations to solve:

$$\begin{cases} \sqrt{FL} = \sqrt{\beta_i \beta_f} \\ \frac{\pi}{2}\sqrt{FL} = \frac{7}{2}\cdot L \end{cases} ;\text{therefore} \quad \begin{aligned} F &= \frac{7}{\pi}\sqrt{\beta_i\beta_f} = \frac{28 \ N_u \sigma_{\varphi i}}{\pi E_f} = 0.12 \text{ rad/MeV} \\ L &= \frac{\pi}{7}\sqrt{\beta_i\beta_f} = \frac{4\pi N_u \sigma_{\varphi i}}{7 E_f} = 0.024 \text{ rad/MeV} \end{aligned}$$

So the optimal equilibrium phase of RF cavities passages should be

$$\varphi_s = \arcsin\frac{1}{FeU_{RF}} = 6.4^{\circ}$$

for $U_{RF}$=75 MV in our case, and the corresponding longitudinal dispersions of each of three magnetic systems of the recirculating beam tracks should be

$$D_n = L\lambda_{RF} E_{sn} \ ,$$

where $E_{sn}$ is the equilibrium electron energy of the corresponding beam track ($n$=1,2,3 is the track number).

The RECFEL computer code, based on the Turbo-Pascal compiler for PC AT computers, was developed to observe the longitudinal phase-energy motion of electrons and its interaction with the RF cavities of the CW recirculator-recuperator dedicated to the FEL. Each simulated electron bunch consists of a number of particles. The particle of charge $q$ induces RF voltage in the passed RF cavities with the same RF phase, and its amplitude is

$$V_q = -\pi q R_w f_{RF} ,$$

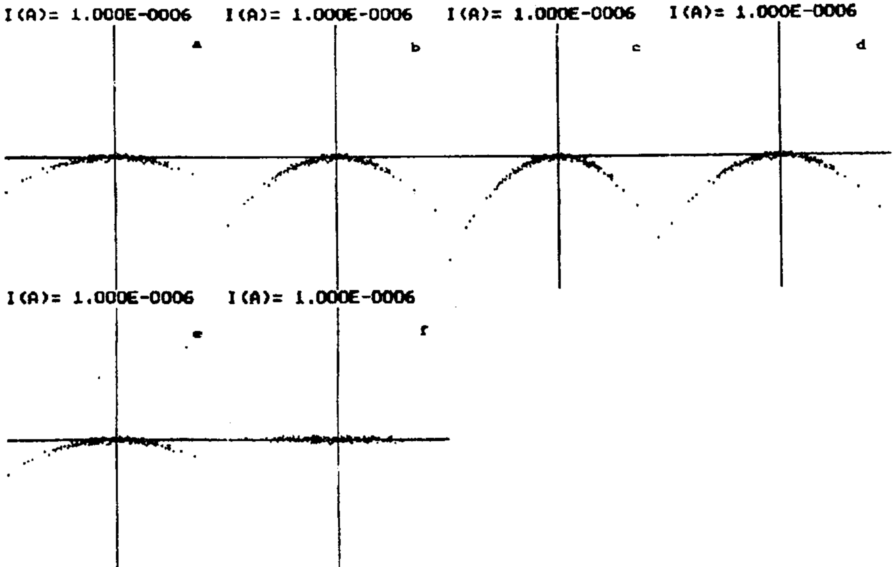
where $R_w$ is the total wave impedance of RF cavities. Further, this induced RF voltage, summarized over all particles in electron bunches, is oscillating with the eigen frequency of RF cavities, which can be unequal to the RF generator frequency, and it damps with the damping time of RF cavities

$$\tau_{RF} = \frac{Q_{RF}}{\pi f_{RF}} ,$$

where $Q_{RF}$ is the quality value of the loaded RF cavities ($Q_{RF}$=200000 for the JAERI superconducting RF cavities).

The total RF voltage of RF cavities, loaded by the electron beam, is always the vector sum of the RF voltage of the unloaded RF cavities and the RF voltage induced by electron bunches. The RECFEL computer code provides the storage of information on the longitudinal phase-energy deviations of each particle of all electron bunches, simultaneously existing at the recirculator tracks, and on the present status of the amplitude and the phase of the RF voltage, induced by the electron beam. The RECFEL code also simulates the feedback of supplying RF generators to stabilize the RF cavities total voltage amplitude and phase. The small signal FEL gain is calculated by averaging over the electron bunch particles near the longitudinal phase with maximum electron peak current.

On Fig.2 there are six RECFEL-computed diagrams of longitudinal phase-energy electron motion in the scales $\mp 10^{\circ}$ of the electron RF phase deviation and $\mp$4.5 MeV of the electron energy deviation, corresponding to three consecutive accelerating (a,b,c) and three consecutive decelerating (d,e,f) beam passages through the RF cavities of the JAERI recirculator-recuperator at low average beam current (1$\mu$A) for the case of isochronous magnetic optics of

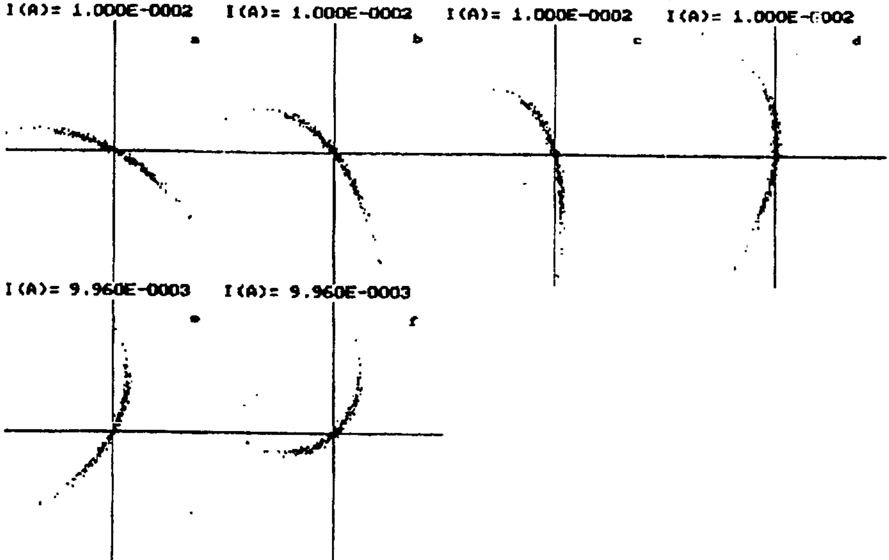



Longitudinal phase scale(degrees)= 1.000E+0001 Energy scale(eV)= 4.500E+0006
Injections energy(eV)= 1.500E+0007 Long.foc.coefficient(rad/eV)= 0.000E+0000
RF periods number=961 Electron beams average current(A)= 1.000E-0006
Beams power losses instantaneous(W)= 0.000E+0000 and averaged(W)= 0.000E+0000
RF voltage(V)= 7.50000E+0007 RF detuning phase(degrees)= 1.17614E-0007
Total RF power(W)= 9.38598E+0000 Fine fast RF cavity detuning=-1.51671E-0014
Injections phase(degrees)= 0.000E+0000

Phase dispersion(degrees)= 2.941E+0000 Relative energy dispersion= 1.883E-0003
Peak electron current(A)= 2.54E-0003 Eff.dispersion= 7.33E-0004 Gain= 1.35E-0005

Maximal power(W): RF= 9.386E+0000

*Fig. 2.* The RECFEL-computed diagrams of the longitudinal phase-energy bunch motion in the scales $\mp 10^{\circ}$ of the electron RF phase deviation and $\mp 4.5$ MeV of the electron energy deviation, corresponding to three consecutive accelerating (a,b,c) and three consecutive decelerating (d,e,f) passages of beam through the RF cavities of the JAERI recirculator-recuperator at low average beam current (1$\mu$A) for the case of isochronous magnetic optics of beam tracks ($\varphi_s = 0^{\circ}$).




Longitudinal phase scale(degrees)= 1.000E+0001 Energy scale(eV)= 4.500E+0006
Injections energy(eV)= 1.500E+0007 Long.foc.coefficient(rad/eV)= 1.700E-0008
RF periods number=205921 Electron beams average current(A)= 1.000E-0002
Beams power losses instantaneous(W)= 0.000E+0000 and averaged(W)= 2.689E+0003
RF voltage(V)= 7.50059E+0007 RF detuning phase(degrees)= 1.20467E-0003
Total RF power(W)= 4.06797E+0003 Fine fast RF cavity detuning= 2.05742E-0007
Injections phase(degrees)= 8.000E+0000

Phase dispersion(degrees)= 1.203E+0000 Relative energy dispersion= 5.056E-0003
Peak electron current(A)= 9.03E+0001 Eff.dispersion= 3.08E-0003 Gain= 3.59E-0001

Maximal power(W): RF= 2.969E+0004

*Fig. 3.* Diagrams, similar those in Fig.2, for the case of nonisochronous magnetic optics of beam tracks, using the longitudinal focusing with the dispersion coefficient $L$=0.017 rad/MeV and the beam passages RF phase $\varphi_s=8^o$ at a designed average beam current of 10mA of the considered project.

beam tracks (without the use of the longitudinal focusing, with $\varphi_s = 0^\circ$). For the third diagram (c) of the beam at the FEL undulator straight section, the calculated FEL gain is 1.35% per 1mA of the average beam current. On Fig.3 there are similar diagrams for the case of nonisochronous magnetic optics of beam tracks, using the longitudinal focusing with the dispersion coefficient $L$=0.017 rad/MeV and the beam passages RF phase $\varphi_s = 8^\circ$, numerically optimized for the maximum FEL gain, equal to 3.59 %/mA, which is somewhat lower than the value expected from the above linear estimations. Here we have already done simulations at a full average beam current of 10mA, designed for the JAERI FEL recirculator project. Initially, during 100000 RF time periods the electron current was increased slowly from zero value to 10 mA, and after additional 100000 RF time periods there are no any indications of the longitudinal beam instabilities.

Now, using the RECFEL code, we are studying the stability of the longitudinal phase-energy bunch motion of the high average current beam in the CW RF microtron-recuperator FEL project of the Budker Institute of Nuclear Physics [2].

In conclusion we have to acknowledge Dr. M.Sugimoto from the JAERI FEL Laboratory for his careful computer assistance in the RECFEL code development.

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