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CP VIOLATION

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ABSTRACT

An overview of the phenomenology of CP violation is presented. The Standard Model mechanism of CP violation and its main experimental tests, both in the kaon and bottom systems, are discussed.

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An overview of the phenomenology of CP violation is presented. The Standard Model mechanism of CP violation and its main experimental tests, both in the kaon and bottom systems, are discussed.

1 Introduction

Charge conjugation (C) and Parity (P) are drastically violated by the weak interactions; however, their product CP happens to be a good symmetry in nearly all observed phenomena. So far, only in the decay of neutral kaons a slight violation of the CP symmetry ($\sim 2\times 10^{-3}$) has been established. No observation of this phenomena has been made in any other system. Our understanding of CP violation is therefore very poor. We do not know yet whether CP violation is simply an accident proper to the neutral kaons, due to the fact that the K^0 oscillates into its antiparticle, or if it is a general property of weak interactions which could manifest in other weak decays.

In the three-generation Standard Model (SM), CP violation originates from the single phase naturally occurring in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1, 2]. The present experimental observations are in agreement with the SM expectations; nevertheless, the correctness of the CKM mechanism is far from being proved. We have no understanding of why nature has chosen the number and properties of fundamental fields just so that CP-violation may be possible. Like fermion masses and quark-mixing angles, the origin of the CKM phase lies in the more obscure part of the SM Lagrangian: the scalar sector. Obviously, CP violation could well be a sensitive probe for new physics beyond the SM.

The purpose of these lectures is to give an overview of the phenomenology of CP violation. The SM mechanism of CP violation is presented in Sect. 2. Sect. 3

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shows different ways of generating an observable CP-violating effect and summarizes the present experimental evidence in the kaon system; the SM predictions for these CP-violation observables are also discussed. The strong CP problem is very briefly mentioned in Sec. 4. Sects. 5 and 6 describe future tests of CP violation in rare K decays and with B mesons, respectively. The present tests on the unitarity of the CKM matrix and a few sumarizing comments are finally collected in Sect. 7. A more extensive discussion can be found in many reviews of CP violation, published recently [3–7].

2 The CKM Mechanism of CP Violation

CP violation requires the presence of complex phases. The only part of the SM Lagrangian containing complex couplings is the Yukawa sector, introduced to generate the femion masses:

$$\mathcal{L}_{Yukawa} = -\left(\overline{U}'_L\ \mathbf{m}\ U'_R\ +\ \overline{D}'_L\ \widetilde{\mathbf{m}}\ D'_R\ +\ h.c.\right)\left(1+\frac{\Phi_o}{v}\right) . \qquad (1)$$

Here Φ_o stands for the scalar Higgs field and v is its vacuum expectation value. The quark fields $U'_{L,R}$ and $D'_{L,R}$ are the 3-component vectors in flavour space for the up- and down-type quarks respectively,

$$U'_{L,R} = \left(\frac{1\mp\gamma_5}{2}\right)\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} , \qquad\qquad D'_{L,R} = \left(\frac{1\mp\gamma_5}{2}\right)\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} , \qquad (2)$$

and $\mathbf{m}$, $\widetilde{\mathbf{m}}$ are 3×3 mass matrices of arbitrary complex numbers, the elements of which are $m_{ij} = -\frac{v}{\sqrt{2}} Y_{ij}$ and $\widetilde{m}_{ij} = -\frac{v}{\sqrt{2}}\widetilde{Y}_{ij}$ where Y_{ij} and $\widetilde{Y}_{ij}$ are the Yukawa coupling constants with $i \equiv u, c$ or t and $j \equiv d, s$ or b.

In general, $\mathbf{m}$ and $\widetilde{\mathbf{m}}$ are not diagonal. The diagonalization of the quark-mass matrices, $\mathbf{m}_D = V_L \mathbf{m} V_R^\dagger$ and $\widetilde{\mathbf{m}}_D = \widetilde{V}_L \widetilde{\mathbf{m}} \widetilde{V}_R^\dagger$, defines the physical (mass-eigenstates) fermion fields $U_{L,R} = V_{L,R} U'_{L,R}$ and $D_{L,R} = \widetilde{V}_{L,R} D'_{L,R}$ where $V_{L,R}$ and $\widetilde{V}_{L,R}$ are unitary matrices.

The coupling of the physical quarks to the neutral Z preserves the observed absence of flavour-changing neutral currents (FCNC), while their coupling to the charged $W^\pm$ introduces the mixing between families. The charged-current couplings are:

$$\mathcal{L}_W = \frac{g}{\sqrt{2}}\left\{\overline{U}_L\gamma^\mu W_\mu^+ \mathbf{V} D_L\ +\ \overline{D}_L\gamma^\mu W_\mu^- \mathbf{V}^\dagger U_L\ \right\} , \qquad (3)$$

where $\mathbf{V} \equiv V_L \widetilde{V}_L^\dagger$ is an unitary 3×3 matrix called the quark-mixing or Cabibbo-

Kobayashi-Maskawa (CKM) matrix [1,2]:

$$\mathbf{V} \;=\; \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} . \tag{4}$$

Obviously this picture can be extended to more than three families and could be valid (with massive neutrinos) for the leptonic sector.

The Yukawa couplings Y_{ij} and $\widetilde{Y}_{ij}$ are completely arbitrary complex numbers; therefore, the resulting quark masses and CKM matrix elements cannot be predicted in any way. Our lack of understanding of the scalar sector translates into a proliferation of free parameters. A general unitary $n \times n$ matrix can be characterized by n^2 independent real parameters. Not all these parameters are, however, physical observables. The SM Lagrangian remains invariant under the following transformation:

$$U^i_{L,R} \longrightarrow e^{i\phi(u^i)} U^i_{L,R}, \quad D^j_{L,R} \longrightarrow e^{i\phi(d^j)} D^j_{L,R}, \quad V_{i,j} \longrightarrow e^{i\left(\phi(u^i)-\phi(d^j)\right)} V_{i,j}. \tag{5}$$

Thus, $(2n-1)$ phases, where n is the number of fermion families, can be reabsorbed by an appropriate redefinition of the quark fields. The most general CKM matrix contains then $(n-1)^2$ real parameters: $n(n-1)/2$ mixing angles and $(n-1)(n-2)/2$ phases. With more than two families, the elements of $\mathbf{V}$ can be complex numbers which allow the possibility for generating CP violation through the interference of two diagrams involving different matrix elements.

With $n = 3$, the CKM matrix is described by 3 angles and 1 phase. Different (but equivalent) representations can be found in the literature. The Particle Data Group [8] advocates the use of the following one as the "standard" CKM parametrization:

$$\mathbf{V} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix} . \tag{6}$$

Here $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$, with i and j being "generation" labels $(i, j = 1, 2, 3)$. The real angles θ_{12}, θ_{23} and θ_{13} can all be made to lie in the first quadrant, by an appropriate redefinition of quark field phases; then, $c_{ij} \geq 0$, $s_{ij} \geq 0$ and $0 \leq \delta_{13} \leq 2\pi$.

It is an empirical fact that the CKM matrix shows a hierarchical pattern, with the diagonal elements being very close to one, the ones connecting the two first generations having a size $\lambda \simeq \sin\theta_C \approx 0.22$, the mixing between the second and third families being of order λ^2, and the mixing between the first and third quark flavours having a much smaller size of about λ^3. It is then quite practical to use the so-called

Wolfenstein [9] approximate representation of V:

$$\mathbf{V} = \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right) . \qquad (7)$$

The values of A, ρ and η are poorly measured: $A = 0.82 \pm 0.10$ and $|\rho|, |\eta| < 0.5$. CP is violated if $\eta \neq 0$.

Since δ_{13} (η) is the oly possible source of CP violation, the SM predictions for CP-violating phenomena are quite constrained. Moreover, the CKM mechanism requires several necessary conditions in order to generate an observable CP-violation effect. With only two fermion generations, the quark-mixing mechanism cannot give rise to CP violation; therefore, for CP violation to occur in a particular process, all 3 generations are required to play an active role. In the kaon system, for instance, CP-violation effects can only appear at the one-loop level, where the top-quark is present. In addition, all CKM-matrix elements must be non-zero and the quarks of a given charge must be non-degenerate in mass. If any of these conditions were not satisfied, the CKM-phase could be rotated away by a redefinition of the quark fields. CP-violation effects are then necessarily proportional to the product of all CKM-angles, and should vanish in the limit where any two (equal-charge) quark-masses are taken to be equal. All these necessary conditions can be summarized in a very elegant way as a single requirement on the original quark-mass matrices $\mathbf{m}$ and $\widetilde{\mathbf{m}}$ [10]:

$$\text{CP violation} \iff \text{Im}\{\det[\mathbf{m}\mathbf{m}^\dagger, \widetilde{\mathbf{m}}\widetilde{\mathbf{m}}^\dagger]\} \neq 0 \qquad (8)$$

Without performing any detailed calculation, one can make the following general statements on the implications of the CKM mechanism of CP violation:

- Owing to unitarity, for any choice of i, j, k, l (between 1 and 3),

$$\text{Im}[V_{ij} V_{ik}^* V_{lk} V_{lj}^*] = \mathcal{J} \sum_{m,n=1}^{3} \epsilon_{ilm} \epsilon_{jkn} , \qquad (9)$$

$$\mathcal{J} = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta_{13} \approx A^2 \lambda^6 \eta < 10^{-4} . \qquad (10)$$

  Any CP-violation observable involves the product $\mathcal{J}$ [10]. Thus, violations of the CP symmetry are necessarily small.

- In order to have sizeable CP-violating asymmetries $[(\Gamma - \overline{\Gamma})/(\Gamma + \overline{\Gamma})]$, one should look for very suppressed decays, where the decay widths already involve small CKM matrix elements.

- In the SM, CP violation is a low-energy phenomena in the sense that any effect should dissapear when the quark-mass difference $m_c - m_u$ becomes negligible.

- B decays are the optimal place for CP-violation signals to show up. They involve small CKM elements and are the lowest-mass processes where the 3 generations play a direct (tree level) role.

3 Indirect and Direct CP Violation in the Kaon System

Any observable CP-violation effect is generated by the interference between different amplitudes contributing to the same physical transition. This interference can occur either through meson-antimeson mixing or via final-state interactions, or by a combination of both effects.

3.1 K^0-$\bar{K}^0$ Mixing

The strangeness (flavour) quantum number is not conserved by weak interactions. Thus a K^0 state can be transformed into its antiparticle $\bar{K}^0$ (and analogously for D^0 and B^0 mesons). Assuming CPT symmetry to hold, the 2×2 K^0-$\bar{K}^0$ mixing matrix can be written as

$$\mathcal{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}, \tag{11}$$

where the diagonal elements M and Γ are real parameters. If CP were conserved, M_{12} and Γ_{12} would also be real. The physical eigenstates of $\mathcal{M}$ are

$$|K_{S,L}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left[p\,|K^0\rangle \mp q\,|\bar{K}^0\rangle \right], \tag{12}$$

where

$$\frac{q}{p} \equiv \frac{1-\bar{\varepsilon}}{1+\bar{\varepsilon}} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \right)^{1/2}. \tag{13}$$

Clearly if M_{12} and Γ_{12} were real, then $q/p = 1$ and $|K_{S,L}\rangle$ would correspond to the CP-even (K_1) and CP-odd (K_2) states $|K_{1,2}\rangle \equiv \left(|K^0\rangle \mp |\bar{K}^0\rangle\right)/\sqrt{2}$ [we use the phase convention* $CP|K^0\rangle = -|\bar{K}^0\rangle$]. Note that if the K^0-$\bar{K}^0$ mixing violates CP, the two

*Since the flavour quantum number is conserved by strong interactions, there is some freedom in defining the phases of the flavour eigenstates. In general, one could use

$$|K^0_\zeta\rangle \equiv e^{-i\zeta}|K^0\rangle, \qquad |\bar{K}^0_\zeta\rangle \equiv e^{i\zeta}|\bar{K}^0\rangle,$$

which satisfy $CP|K^0_\zeta\rangle = -e^{-2i\zeta}|\bar{K}^0_\zeta\rangle$. Both basis are of course trivially related: $M^\zeta_{12} = e^{2i\zeta}M_{12}$, $\Gamma^\zeta_{12} = e^{2i\zeta}\Gamma_{12}$ and $(q/p)_\zeta = e^{-2i\zeta}(q/p)$. Thus, in general, $q/p \neq 1$ does not necessarily imply CP violation. CP is violated in the mixing matrix if $|q/p| \neq 1$, i.e. $\mathrm{Re}(\bar{\varepsilon}) \neq 0$ and $\langle K_L|K_S\rangle \neq 0$. Note that $\langle K_L|K_S\rangle_\zeta = \langle K_L|K_S\rangle$. Another phase-convention independent quantity is $\frac{q}{p}\frac{\bar{A}_f}{A_f}$, where $A_f \equiv A(K^0 \to f)$ and $\bar{A}_f \equiv A(\bar{K}^0 \to f)$, for any final state f.

mass eigenstates are no longer orthogonal:

$$\langle K_L | K_S \rangle = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} \approx 2\mathrm{Re}(\bar{\varepsilon}). \tag{14}$$

The departure of $|p/q|$ from unity can be measured, by looking to a CP-violating asymmetry in a flavour-specific decay, i.e. a decay into a final state which can only be reached from an initial K^0 (or $\bar{K}^0$) but not from both:

$$K^0 \to \pi^- l^+ \nu_l, \qquad \bar{K}^0 \to \pi^+ l^- \bar{\nu}_l. \tag{15}$$

In the SM, $|A(\bar{K}^0 \to \pi^+ l^- \bar{\nu}_l)| = |A(K^0 \to \pi^- l^+ \nu_l)|$; therefore,

$$\delta \equiv \frac{\Gamma(K_L \to \pi^- l^+ \nu_l) - \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)}{\Gamma(K_L \to \pi^- l^+ \nu_l) + \Gamma(K_L \to \pi^+ l^- \bar{\nu}_l)} = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = \frac{2\mathrm{Re}(\bar{\varepsilon})}{(1 + |\bar{\varepsilon}|^2)}. \tag{16}$$

The experimental measurement [8], $\delta = (3.27 \pm 0.12) \times 10^{-3}$, implies

$$\mathrm{Re}(\bar{\varepsilon}) = (1.63 \pm 0.06) \times 10^{-3}. \tag{17}$$

3.2 Direct CP Violation

If the flavour of the decaying meson P is known, any observed difference between the decay rate $\Gamma(P \to f)$ and its CP conjugate $\Gamma(\bar{P} \to \bar{f})$ would indicate that CP is directly violated in the decay amplitude. One could study, for instance, CP asymmetries in charged-kaon decays, such as $K^\pm \to \pi^\pm \pi^0$, where the charge of the final pions clearly identifies the flavour of the decaying kaon (these types of decays are often referred to as self-tagging modes). No positive signal has been reported up to date.

Since at least two interfering amplitudes are needed to generate a CP-violating effect, let us write the amplitudes for the transitions $P \to f$ and $\bar{P} \to \bar{f}$ as

$$A[P \to f] = M_1 \, e^{i\phi_1} \, e^{i\alpha_1} + M_2 \, e^{i\phi_2} \, e^{i\alpha_2}, \tag{18}$$

$$A[\bar{P} \to \bar{f}] = M_1 e^{-i\phi_1} e^{i\alpha_1} + M_2 e^{-i\phi_2} e^{i\alpha_2}, \tag{19}$$

where ϕ_1, ϕ_2 denote the weak phases, α_1, α_2 strong final-state phases (and/or strong phases between S- and P-wave contributions in the case of baryon decays), and M_1, M_2 the moduli of the matrix elements. The rate asymmetry is then given by

$$\frac{\Gamma[P \to f] - \Gamma[\bar{P} \to \bar{f}]}{\Gamma[P \to f] - \Gamma[\bar{P} \to \bar{f}]} = \frac{-2 M_1 M_2 \sin(\phi_1 - \phi_2) \sin(\alpha_1 - \alpha_2)}{|M_1|^2 + |M_2|^2 + 2 M_1 M_2 \cos(\phi_1 - \phi_2) \cos(\alpha_1 - \alpha_2)}. \tag{20}$$

Eq. (20) tells us that the following requirements are needed in order to generate a direct-CP asymmetry:

- Two (at least) interfering amplitudes.

- Two different weak phases [$\sin(\phi_1 - \phi_2) \neq 0$].
- Two different strong phases [$\sin(\alpha_1 - \alpha_2) \neq 0$].

Moreover, in order to get a sizeable asymmetry (rate difference / sum), the two amplitudes M_1 and M_2 should be of comparable size.

In the kaon system, direct CP violation has been searched for in decays of neutral kaons, where K^0-$\bar{K}^0$ mixing is also involved. Thus, both direct and indirect CP-violation effects need to be taken into account, simultaneously. Since the $\pi^+\pi^-$ and $2\pi^0$ states are even under CP, only the K_1 state could decay into 2π if CP were conserved; 3π's at least would then be required to allow a hadronic decay of the K_2. Therefore, owing to the phase-space suppression of the $K_L \to 3\pi$ decay mode, the $K_L \approx K_2 + \bar{\varepsilon} K_1$ state has a much longer lifetime than the $K_S \approx K_1 + \bar{\varepsilon} K_2$. Since CP is violated, the K_L does decay into 2π. The CP-violation signal is provided by the asymmetries:

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv |\eta_{+-}| e^{i\phi_{+-}} \approx \varepsilon + \frac{\varepsilon'}{1 + \omega/\sqrt{2}}, \tag{21}$$

$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv |\eta_{00}|\, e^{i\phi_{00}} \approx \varepsilon - \frac{2\varepsilon'}{1 - \sqrt{2}\omega}, \tag{22}$$

where

$$\varepsilon \equiv \bar{\varepsilon} + i\xi_0, \tag{23}$$

$$\varepsilon' \equiv \frac{i}{\sqrt{2}}\, \omega\, (\xi_2 - \xi_0), \tag{24}$$

$$\omega \equiv \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)}\, e^{i(\delta_2 - \delta_0)}. \tag{25}$$

A_I and δ_I are the decay-amplitudes and strong phase-shifts of isospin $I = 0, 2$ (these are the only two values allowed by Bose symmetry for the final 2π state),

$$A[K^0 \to (2\pi)_I] \equiv i A_I e^{i\delta_I}, \qquad A[\bar{K}^0 \to (2\pi)_I] \equiv -i A_I^* e^{i\delta_I}, \tag{26}$$

and

$$\xi_I \equiv \frac{\mathrm{Im}(A_I)}{\mathrm{Re}(A_I)}. \tag{27}$$

In Eqs. (21) and (22), terms quadratic in the small CP-violating quantities have been neglected.

The parameter ε is related to the indirect CP violation. Note that ε is a physical (measurable) phase-convention-independent quantity, while $\bar{\varepsilon}$ is not [$\varepsilon = \bar{\varepsilon}$ in the phase convention $\mathrm{Im}(A_0) = 0$; however, $\mathrm{Re}(\varepsilon) = \mathrm{Re}(\bar{\varepsilon})$ in any convention]. Direct CP violation is measured through ε', which is governed by the phase-difference between the two isospin amplitudes. The CP-conserving parameter ω gives the relative size between these two amplitudes; experimentally, one finds a very big enhancement of the $I = 0$ channel with respect to the $I = 2$ one, which is known as the $\Delta I = 1/2$ rule:

$$|\omega| \approx \frac{1}{22}, \qquad \delta_2 - \delta_0 = -45^\circ \pm 6^\circ. \tag{28}$$

The small size of $|\omega|$ implies a strong suppression of ε'.

From the eigenvector equations for K_S and K_L one can easily obtain the relation

$$\bar{\varepsilon} \approx e^{i\phi_{SW}} \frac{\mathrm{Im}(M_{12}) - \frac{i}{2}\mathrm{Im}(\Gamma_{12})}{\sqrt{\Delta M^2 + \frac{1}{4}\Delta\Gamma^2}}, \tag{29}$$

where [8] $\Delta M \equiv M(K_L) - M(K_S) = (3.522 \pm 0.016) \times 10^{-12}$ MeV, $\Delta\Gamma \equiv \Gamma(K_L) - \Gamma(K_S) \approx -\Gamma(K_S) = -(7.377 \pm 0.017) \times 10^{-12}$ MeV, and

$$\phi_{SW} \equiv \arctan\left(\frac{-2\Delta M}{\Delta\Gamma}\right) = 43.68^\circ \pm 0.15^\circ \tag{30}$$

is the so-called superweak phase. Since $\Delta\Gamma \approx -2\Delta M$, one has $\phi_{SW} \approx \pi/4$. Moreover, Γ_{12} is dominated by the $K^0 \to (2\pi)_{I=0}$ decay mode; therefore, $\mathrm{Im}(\Gamma_{12})/\mathrm{Re}(\Gamma_{12}) \approx -2\xi_0$. Using these relations, one gets the approximate result

$$\varepsilon \approx \frac{e^{i\pi/4}}{\sqrt{2}} \left\{\frac{\mathrm{Im}(M_{12})}{2\,\mathrm{Re}(M_{12})} + \xi_0\right\}. \tag{31}$$

Notice that $\delta_2 - \delta_0 + \pi/2 \approx \pi/4$, i.e.

$$\varepsilon' \approx \frac{e^{i\pi/4}}{\sqrt{2}} |\omega| \, (\xi_2 - \xi_0). \tag{32}$$

Thus, owing to the particular numerical values of the neutral-kaon-decay parameters, the phases of ε and ε' are nearly equal.

The experimental world-averages quoted by the Particle Data Group [8] are

$$|\eta_{+-}| = (2.268 \pm 0.023) \times 10^{-3}, \tag{33}$$
$$|\eta_{00}| = (2.253 \pm 0.024) \times 10^{-3}. \tag{34}$$

These two numbers are equal within errors, showing that indeed $|\varepsilon'| << |\varepsilon|$ as expected from the $|\omega|$ suppression. Moreover, from Eq. (17) and $\arg(\varepsilon) \approx \pi/4$, we have $|\varepsilon| \approx 2.3 \times 10^{-3}$, in good agreement with the value extracted from $K^0 \to 2\pi$.

The ratio ε'/ε can be determined through the relation

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) \approx \frac{1}{6}\left\{1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right\}. \tag{35}$$

Two different experiments [11,12] have recently reported a measurement of this quantity:

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \begin{cases} (23.0 \pm 6.5) \times 10^{-4} & \text{NA31 [11]}, \\ (7.4 \pm 5.9) \times 10^{-4} & \text{E731 [12]}. \end{cases} \tag{36}$$

The NA31 measurement provides evidence for a non-zero value of ε'/ε (i.e. direct CP violation), with a statistical significance of more than three standard deviations. However, this is not supported by the E731 result, which is compatible with $\varepsilon'/\varepsilon = 0$, thus with no direct CP violation. The probability for the two results being statistically compatible is only 7.6%.

Clearly, new experiments with a better sensitivity are required in order to resolve this discrepancy. A next generation of ε'/ε experiments is already under construction at CERN [13] and Fermilab [14]. Moreover, a dedicated ϕ factory (DAΦNE), providing large amounts of tagged K_S, K_L and $K^\pm$ ($\phi \to K\bar{K}$), is already being built at Frascati [15]. The goal of all these experiments is to reach sensitivities better than 10^{-4}. In the meantime, a much modest 10^{-3} sensitivity should be reached by the CPLEAR experiment [6], presently running at CERN.

3.3 Time Evolution

K^0-$\bar{K}^0$ mixing implies that a state which was originally produced as a K^0 or a $\bar{K}^0$ will not develop in time in a purely exponential fashion. The time-dependent amplitudes for the decay into a given final state f are given by:

$$A(K^0 \to f) \sim \left\{ A(K_S \to f)\, e^{-iM_S t}\, e^{-\Gamma_S t/2} + A(K_L \to f)\, e^{-iM_L t}\, e^{-\Gamma_L t/2} \right\}, \tag{37}$$

$$A(\bar{K}^0 \to f) \sim \left\{ A(K_S \to f)\, e^{-iM_S t}\, e^{-\Gamma_S t/2} - A(K_L \to f)\, e^{-iM_L t}\, e^{-\Gamma_L t/2} \right\}. \tag{38}$$

In terms of the ratio

$$\eta_f \equiv \frac{A(K_L \to f)}{A(K_S \to f)} \equiv |\eta_f| e^{i\phi_f}, \tag{39}$$

the time evolution of the decay rates can then be written as

$$\Gamma(K^0 \to f) \sim e^{-\Gamma_S t} + |\eta_f|^2\, e^{-\Gamma_L t} + 2|\eta_f|\, \cos(\phi_f - \Delta M t)\, e^{-(\Gamma_L+\Gamma_S)t/2}, \tag{40}$$

$$\Gamma(\bar{K}^0 \to f) \sim e^{-\Gamma_S t} + |\eta_f|^2\, e^{-\Gamma_L t} - 2|\eta_f|\, \cos(\phi_f - \Delta M t)\, e^{-(\Gamma_L+\Gamma_S)t/2}. \tag{41}$$

By measuring the decay rate as a function of time, the ratio η_f (both modulus and phase) and the mass-difference ΔM can be obtained. For the dominant 2π modes, the measured phases [8],

$$\phi_{+-} = (46.6 \pm 1.2)^\circ, \tag{42}$$

$$\phi_{00} = (46.6 \pm 2.0)^\circ, \tag{43}$$

are very close to $\pi/4$, as expected.

3.4 SM Predictions

The CKM mechanism generates CP-violation effects both in the $\Delta S = 2$ K^0-$\bar{K}^0$ transition (box-diagrams) and in the $\Delta S = 1$ decay amplitudes (penguin diagrams). Although a straightforward and well-defined technique, which makes use of the Operator Product Expansion, is available for a short-distance analysis of these interactions, the final quantitative predictions are obscured by the presence of hadronic matrix-elements of weak four-quark operators, which are governed by long-distance physics.

Figure 1: $\Delta S = 2$ box diagrams.

Figure 2: $\Delta S = 1$ penguin diagrams.

Only one such operator appears in the K^0-$\bar{K}^0$ mixing analysis. Including the short-distance QCD corrections, the box-diagram calculation of M_{12} yields

$$\begin{aligned} M_{12} &= \frac{G_F^2 M_W^2}{16\pi^2} \left\{\lambda_c^2\, \eta_1\, S(r_c) + \lambda_t^2\, \eta_2\, S(r_t) + 2\lambda_c\lambda_t\, \eta_3\, S(r_c, r_t)\right\} \\ &\quad \times \frac{1}{2M_K}\, \alpha_s(\mu^2)^{-2/9} \langle \bar{K}^0 | \, (\bar{s}\gamma^\mu(1-\gamma_5)d)\, (\bar{s}\gamma_\mu(1-\gamma_5)d)\, | K^0\rangle \, , \end{aligned} \tag{44}$$

where

$$\lambda_i \equiv V_{is}\, V_{id}^* \qquad\qquad (i = u, c, t), \tag{45}$$

and $S(r_i)$, $S(r_i, r_j)$ are functions of $r_i \equiv (m_i/M_W)^2$. Owing to the unitarity of the CKM matrix, $\lambda_u + \lambda_c + \lambda_t = 0$, and the contributions of the up, charm and top quarks to the box diagram add to zero in the limit of massless quarks (GIM mechanism [16]). The loop functions $S(r_i)$ and $S(r_i, r_j)$ are then very sensitive to the quark masses $[S(r_i) \approx r_i$, for $r_i << 1]$. For large m_t the second term in Eq. (44) dominates.

The factors η_i represent short-distance QCD corrections to the lowest-order box-diagram calculation ($\eta_i = 1$ in the absence of QCD effects): $\eta_1 \approx 0.85$, $\eta_2 \approx 0.57$ and $\eta_3 \approx 0.36$ [5]. In addition, one needs to compute the hadronic matrix element of the $\Delta S = 2$ four-quark operator in Eq. (44), which is usually parametrized in terms of the so-called B_K parameter:

$$\alpha_s(\mu^2)^{-2/9} \langle \bar{K}^0 | \, (\bar{s}\gamma^\mu(1-\gamma_5)d)\, (\bar{s}\gamma_\mu(1-\gamma_5)d)\, | K^0\rangle \equiv 2\left(1 + \frac{1}{3}\right) \left(\sqrt{2} f_K M_K\right)^2 B_K. \tag{46}$$

$B_K = 1$ corresponds to the factorization approximation, which consists in splitting the matrix element in a product of two currents, by inserting the vacuum in all possible ways $[\langle \bar{K}^0 | \, (\bar{s}\gamma^\mu(1-\gamma_5)d)\, |\emptyset\rangle \, \langle\emptyset | \, (\bar{s}\gamma_\mu(1-\gamma_5)d)\, | K^0\rangle]$. Clearly, this approximation can only be taken as an order-of-magnitude estimate, since it completely ignores the renormalization-group factor $\alpha_s(\mu^2)^{-2/9}$, where μ is an arbitrary renormalization

B_K	Method	Reference
0.33	Lowest-order Chiral Perturbation Theory	[17]
0.39 ± 0.10	QCD Sum Rules + Chiral Symmetry	[18]
$0.5 \pm 0.1 \pm 0.2$	QCD Sum Rules (3-point function)	[19]
0.4 ± 0.2	Effective Action	[20]
$0.38^{+0.10}_{-0.02}$	Estimate of $\mathcal{O}(p^4)$ Chiral corrections	[21]
3/4	Leading $1/N_c$	
0.70 ± 0.10	$1/N_c$ expansion	[22]
0.8 ± 0.2	Lattice	[23]

Table 1: Values of B_K obtained by various methods.

scale. The total product (and therefore B_K) in Eq. (46) is of course μ-independent, because the dependence on the renormalization scale is exactelly cancelled by the hadronic matrix element. Unfortunately, it is very difficult to make a calculation of this matrix element from first principles. Table 1 shows the values of B_K obtained by various methods. The present uncertainty associated with the size of the hadronic matrix element is reflected in the broad range of calculated B_K values.

In order to compute ε, $\mathrm{Im}(M_{12})$ is obtained from Eq. (44) while $\mathrm{Re}(M_{12})$, which is much more sensitive to long-distance effects, is taken from the measured neutral-kaon mass diference ΔM. The result depends on the unknown top mass and on the values of the CKM elements. Using the Wolfenstein parametrization (7), the experimental value of ε specifies a hyperbola in the (ρ, η) plane [5]:

$$\eta \left[(1-\rho) A^2 r_t^{0.76} + P_C\right] A^2 B_K = 0.50, \tag{47}$$

where $P_C \approx 2/3$ contains the cc and tc box-diagram contributions†.

The theoretical estimate of ε'/ε is much more involved, because ten four-quark operators need to be considered in the analysis and the presence of cancellations between different contributions tends to amplify the sensitivity to the not very well-controlled long-distance effects. A detailed discussion has been given in refs. [24]. For large values of the top-mass, the Z^0-penguin contributions strongly suppress the expected value of ε'/ε, making the final result very sensitive to m_t. In the presently favoured range of top masses, $m_t \sim 100 - 200\ GeV$, the theoretical estimates [24] give $\varepsilon'/\varepsilon \sim (2-27) \times 10^{-4}$, with large uncertainties. More theoretical work is needed in order to get firm predictions.

†The power-like dependence on r_t (and similar ones that will appear in the following sections) represents a numerical fit to the exact loop functions [5]. In the range 100 GeV $< m_t <$ 200 GeV, the exact result is reproduced to an accuracy better than 3%.

4 Strong CP Violation

CP violation could also originate from and additional term in the QCD Lagrangian,

$$\mathcal{L}_\theta = \theta_0 \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} \sum_a G_{\mu\nu}^{(a)} G_{\rho\sigma}^{(a)} \, , \qquad (48)$$

which violates P, T and CP. Although (48) is a total derivative, it can give rise to observable effects because of the non-perturbative structure of the QCD vacuum (see ref. [25] for a detailed discussion).

One could try to impose P and T conservation in strong interactions, i.e. $\theta_0 = 0$. However, owing to the axial anomaly of the $U(1)_A$ current, a non-zero value of θ_0 would be again generated when diagonalizing the quark-mass matrices $\mathbf{m}$ and $\widetilde{\mathbf{m}}$. A $U(1)_A$ rotation is needed to eliminate a global phase of the quark-mass matrices, but due to the existence of a quantum anomaly, the full theory is not invariant under this transformation; the phase can be shifted from the quark-mass matrices to $\mathcal{L}_\theta$, but it cannot be eliminated. In fact, the physical parameter (i.e. the one which remains invariant under the phase rotation) is not quite θ_0, but rather the combination

$$\theta \equiv \theta_0 + \arg\left\{\det\left(\mathbf{m}\right) \det\left(\widetilde{\mathbf{m}}\right)\right\}. \qquad (49)$$

A non-zero value of θ could lead to observable effects in flavour-conserving transitions. It may generate, in particular, a sizeable neutron electric dipole moment, which very refined experiments have constrained down to a very high precision [26]:

$$d_n^\gamma < 12 \times 10^{-26} \, e \, \mathrm{cm} \quad (95\% \text{ C.L.}). \qquad (50)$$

This provides a stringent upper limit on $|\theta|$. The more recent estimate of d_n^γ [25], done in the framework of Chiral Perturbation Theory (ChPT), gives

$$|\theta| < 5 \times 10^{-10} \, . \qquad (51)$$

The smallness of this number, makes the θ effect completely irrelevant for the phenomenology of CP violation in weak transitions. However, it leaves as an open question the reason for such a small quantity. The initial value of θ_0 and the messy phases present in the original Yukawa couplings should conspire to generate a huge cancellation giving rise to such a tiny value of θ! This is usually known as the "strong CP problem". The most plausible explanation is that the effective θ is probably just zero, because there is some additional symmetry which makes it unobservable (i.e. it can be finally rotated away anyhow) [27]. Unfortunately, a clear solution of the problem is still missing.

5 Rare K Decays

High precision experiments on rare kaon decays [28] offer the exciting possibility of unravelling new physics beyond the SM. Searching for forbidden flavour-changing processes ($K_L \to \mu e$, $K_L \to \pi^0 \mu e$, $K^+ \to \pi^+ \mu e$, ...) at the 10^{-10} level, one is actually exploring energy-scales above the 10 TeV region. The study of allowed (but highly suppressed) decay modes provides, at the same time, very interesting tests of the SM itself. Electromagnetic-induced non-leptonic weak transitions and higher-order weak processes are a useful tool to improve our understanding of the interplay among electromagnetic, weak and strong interactions. In addition, new signals of CP violation, which would help to elucidate the source of CP-violating phenomena, can be looked for.

5.1 $K_L \to \pi^0 \nu \bar{\nu}$

Long-distance effects play a negligible role in the decays $K \to \pi \nu \bar{\nu}$, which proceed through W-box and Z-penguin diagrams. The resulting amplitudes are proportional to the matrix element of the $\Delta S = 1$ vector current,

$$T(K^+ \to \pi^+ \nu \bar{\nu}) \sim \sum_{i=u,c,t} F(\lambda_i, r_i) \, (\bar{\nu}_L \gamma_\mu \nu_L) \, \langle \pi | (\bar{s} \gamma^\mu d) | K \rangle \, , \tag{52}$$

which is known from the K_{l3} decays.

The decay $K^+ \to \pi^+ \nu \bar{\nu}$ provides then a good test of the radiative structure of the SM, and could be used to extract clean information on the CKM factors. Summing over the three neutrino flavours, its branching ratio is expected to be around $(1-5) \times 10^{-11}$ [5], while the present experimental upper bound is $Br(K^+ \to \pi^+ \nu \bar{\nu}) < 5.2 \times 10^{-9}$ (90% C.L.) [28]. Experiments aiming to reach a sensitivity at the level of the SM prediction are already under way.

The CP-violating decay $K_L \to \pi^0 \nu \bar{\nu}$ has been suggested [29] as a good candidate to look for (nearly) pure direct CP-violating transitions. Its decay amplitude its related to the $K^+ \to \pi^+ \nu \bar{\nu}$ one by isospin:

$$T(K_L \to \pi^0 \nu \bar{\nu}) = \frac{1}{2} \Big\{ (1 + \bar{\varepsilon}) T(K^+ \to \pi^+ \nu \bar{\nu}) - (1 - \bar{\varepsilon}) T(K^- \to \pi^- \nu \bar{\nu}) \Big\} \, . \tag{53}$$

The contribution coming from indirect CP-violation via K^0-$\bar{K}^0$ mixing is predicted to be around 10^{-15} [29]. Direct CP-violation generates a much bigger contribution [5]:

$$Br(K_L \to \pi^0 \nu \bar{\nu}) \, \approx \, 0.82 \times 10^{-10} r_t^{1.18} A^4 \eta^2 \, . \tag{54}$$

The clean observation of just a single unambiguous event would indicate the existence of CP-violating $\Delta S = 1$ transitions. The possibility of detecting such a decay mode is, of course, a big experimental challenge. The present (90% C.L.) experimental limit is $Br(K_L \to \pi^0 \nu \bar{\nu}) < 2.2 \times 10^{-4}$. A much better sensitivity of about 10^{-7} is expected to be achieved in the near future [28].

5.2 $K_L \to \pi^0 e^+ e^-$

The $K_L \to \pi^0 e^+ e^-$ decay looks more promising. If CP were an exact symmetry, only the CP-even state K_1 could decay via one-photon emission, while the decay of the CP-odd state K_2 would proceed through a two-photon intermediate state and, therefore, its decay amplitude would be suppressed by an additional power of α. When CP violation is taken into account, however, an $\mathcal{O}(\alpha)$ $K_L \to \pi^0 e^+ e^-$ decay amplitude is induced, both through the small K_1 component of the K_L (ε effect) and through direct CP violation in the $K_2 \to \pi^0 e^+ e^-$ transition. The electromagnetic suppression of the CP-conserving amplitude then makes it plausible that this decay is dominated by the CP-violation contributions.

The branching ratio induced by the direct CP-violation amplitude is predicted [30] to be around 10^{-11}, the exact number depending on the values of m_t and the quark-mixing angles [5]:

$$Br(K_L \to \pi^0 e^+ e^-)\Big|_{\text{Direct CP}} \approx 0.23 \times 10^{-10} r_t^{1.18} A^4 \eta^2 \,. \tag{55}$$

The indirect CP-violating contribution is given by the $K_S \to \pi^0 e^+ e^-$ amplitude times the CP-mixing parameter ε. Using ChPT techniques, it is possible to relate [31] the K_S decay amplitude to the measured $K^+ \to \pi^+ e^+ e^-$ transition. Present data implies [32]

$$Br(K_L \to \pi^0 e^+ e^-)\Big|_{\text{Indirect CP}} \leq 1.6 \times 10^{-12}. \tag{56}$$

Therefore, the interesting direct CP-violating contribution is expected to be bigger than the indirect one. This is very different from the situation in $K \to \pi\pi$, where the contribution due to mixing completely dominates.

The present experimental upper bound [28] (90% C.L.)

$$Br(K_L \to \pi^0 e^+ e^-)\Big|_{\text{Exp}} < 5.5 \times 10^{-9}, \tag{57}$$

is still far away from the expected SM signal, but the prospects for getting the needed sensitivity of around 10^{-12} in the next few years are rather encouraging. In order to be able to interpret a future experimental measurement of this decay as a CP-violating signature, it is first necessary, however, to pin down the actual size of the two-photon-exchange CP-conserving amplitude.

The $K_L \to \pi^0 \gamma\gamma$ amplitude can be computed within ChPT [33]. One can then estimate the two-photon-exchange contribution to $K_L \to \pi^0 e^+ e^-$, by taking the absorptive part due to the two-photon discontinuity as an educated guess of the actual size of the complete amplitude [31, 34]. At the lowest non-trivial order in the momentum expansion, $\mathcal{O}(p^4)$, the $K_L \to \pi^0 e^+ e^-$ decay amplitude is strongly suppressed (it is proportional to m_e), owing to the helicity structure of the $K_L \to \pi^0 \gamma\gamma$ decay amplitude [31]:

$$Br(K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 e^+ e^-)\Big|_{\mathcal{O}(p^4)} \sim 5 \times 10^{-15}. \tag{58}$$

This helicity suppression is, however, no longer true at the next order in the chiral expansion, because an additional Lorentz structure is then allowed in the decay amplitude. An estimate of the dominant $\mathcal{O}(p^6)$ contribution can be done, by using the measured [35] photon spectrum in the decay $K_L \to \pi^0\gamma\gamma$. The most recent analysis [36] gives

$$Br(K_L \to \pi^0\gamma^*\gamma^* \to \pi^0 e^+e^-)\Big|_{\mathcal{O}(p^6)} \sim (0.3 - 1.8) \times 10^{-12} \,, \tag{59}$$

implying that this decay is in fact dominated by the CP-violation contribution.

5.3 Longitudinal Muon Polarization in $K_L \to \mu^+\mu^-$

The longitudinal muon polarization $\mathcal{P}_L$ in the decay $K_L \to \mu^+\mu^-$ is an interesting measure of CP violation. As for every CP-violating observable in the neutral kaon system, both indirect and direct CP-violation contributions need to be considered.

In the SM, the direct CP-violating amplitude is induced by Higgs exchange with an effective one-loop flavour-changing $\bar{s}dH$ coupling [37]. The present lower bound [38] on the Higgs mass $m_H > 63.5$ GeV (95% C.L.), implies [37, 39] a conservative upper limit $|\mathcal{P}_{L,\mathrm{Direct}}| < 10^{-4}$. A much larger value $\mathcal{P}_L \sim O(10^{-2})$ appears quite naturally in various extensions of the SM [40]. It is worth emphasizing that $\mathcal{P}_L$ is especially sensitive to the presence of light scalars with CP-violating Yukawa couplings. Thus, $\mathcal{P}_L$ seems to be a good signature to look for new physics beyond the SM; for this to be the case, however, it is very important to have a good quantitative understanding of the SM prediction to allow us to infer, from a measurement of $\mathcal{P}_L$, the existence of a new CP-violation mechanism.

The $K_1^0 \to \mu^+\mu^-$ decay amplitude can be unambiguously calculated in ChPT [41]. This allows us to make a reliable estimate‡ of the contribution to $\mathcal{P}_L$ due to K^0-$\bar{K}^0$ mixing [41]:

$$1.9 < |\mathcal{P}_{L,\varepsilon}| \times 10^3 \left(\frac{2 \times 10^{-6}}{Br(K_S \to \gamma\gamma)}\right)^{1/2} < 2.5. \tag{60}$$

Taking into account the present experimental errors [43] in $Br(K_S \to \gamma\gamma)$ [$Br = (2.4 \pm 1.2) \times 10^{-6}$] and the inherent theoretical uncertainties due to uncalculated higher-order corrections, one can conclude that experimental indications for $|\mathcal{P}_L| > 5 \times 10^{-3}$ would constitute clear evidence for additional mechanisms of CP violation beyond the SM.

6 B decays

Differences of rates that signal CP violation are proportional to the small product $A^2\lambda^6\eta$, but the corresponding asymmetries (difference / sum) are enhanced in B-decay

‡Taking only the absorptive parts of the $K_{1,2} \to \mu^+\mu^-$ amplitudes into account, a value $|\mathcal{P}_{L,\varepsilon}| \approx 7\times10^{-4}$ was estimated previously [42]. However, this is only one out of four contributions to $\mathcal{P}_L$ [41], which could all interfere constructively with unknown magnitudes.

relative to K-decay because the B-decay widths involve much smaller CKM elements ($|V_{cb}|^2$ or $|V_{ub}|^2$ $<<$ $|V_{us}|^2$). If the SM is correct, sizeable CP-violation asymmetries should be expected to show up in many decay modes of beauty particles [44].

6.1 Indirect CP Violation

The general formalism to describe mixing among the neutral B^0 and $\bar{B}^0$ mesons is completely analogous to the one used in the kaon sector. However, the physical mass-eigenstates $[CP|B^0\rangle = -|\bar{B}^0\rangle]$

$$|B_{\mp}\rangle \; = \; \frac{1}{\sqrt{|p|^2+|q|^2}} \left[p\,|B^0\rangle \mp q\,|\bar{B}^0\rangle\right] \tag{61}$$

have now a comparable lifetime, because many decay modes are common to both states and therefore the available phase space is similar.

The flavour-specific decays

$$B^0 \to l^+\nu_l X\,, \qquad\qquad \bar{B}^0 \to l^-\bar{\nu}_l X\,, \tag{62}$$

provide the most direct way to measure the amount of CP violation in the B^0-$\bar{B}^0$ mixing matrix. The asymmetry between the number of l^+l^+ and l^-l^- pairs produced in the processes $e^+e^- \to B^0\bar{B}^0 \to l^{\pm}l^{\pm}X$ is easily found to be

$$a_{SL} \equiv \frac{N(l^+l^+)-N(l^-l^-)}{N(l^+l^+)+N(l^-l^-)} = \frac{|p/q|^2-|q/p|^2}{|p/q|^2+|q/p|^2} \approx 4\mathrm{Re}(\bar{\varepsilon}_B). \tag{63}$$

Unfortunately, this $\Delta B = 2$ asymmetry is expected to be quite tiny in the SM, because $|\Delta\Gamma/\Delta M| \approx |\Gamma_{12}/M_{12}| << 1$ $[\Delta M \equiv M_{B_+} - M_{B_-},\, \Delta\Gamma \equiv \Gamma_{B_+} - \Gamma_{B_-}]$. This can be easily understood, by looking to the relevant box diagrams contributing to the B^0-$\bar{B}^0$ transition; the mass mixing is dominated by the top-quark graph, while the decay amplitudes get obviously its main contribution from the $b \to c$ transition. Thus,

$$\frac{\Gamma_{12}}{M_{12}} \approx \frac{3\pi}{2}\frac{m_b^2}{m_t^2}\frac{1}{E'(r_t)} << 1, \tag{64}$$

where [45] $E'(r_t)$ is a slowly decreasing function of r_t $[E'(0) = 1,\, E'(\infty) = 1/4]$. One has then [see Eq. (13)]

$$\left|\frac{q}{p}\right| \approx 1 + \frac{1}{2}\left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_{\Delta B=2}, \tag{65}$$

where

$$\phi_{\Delta B=2} \equiv \arg\left(\frac{M_{12}}{\Gamma_{12}}\right). \tag{66}$$

The factor $\sin\phi_{\Delta B=2}$ involves an additional GIM suppression,

$$\sin\phi_{\Delta B=2} \approx \frac{8}{3}\frac{m_c^2-m_u^2}{m_b^2}\,\mathrm{Im}\left(\frac{V_{cb}V_{cq}^*}{V_{tb}V_{tq}^*}\right), \tag{67}$$

implying a value of $|q/p|$ very close to 1. Here, $q \equiv d, s$ denote the corresponding CKM matrix elements for B_q^0 mesons. Therefore, one expects

$$a_{SL} \leq \begin{cases} 10^{-3} & (B_d^0), \\ 10^{-4} & (B_s^0). \end{cases} \tag{68}$$

The observation of an asymmetry a_{SL} at the percent level, would then be a clear indication of new physics beyond the SM.

6.2 Direct CP violation

Direct CP violation could be established by measuring a non-zero rate asymmetry in $B^\pm$ decays. One example is the decay $B^\pm \to K^\pm \rho^0$ which proceeds via a tree- and a penguin-diagram (Fig. 3), the weak couplings of which are given by $V_{ub}V_{us}^* \approx A\lambda^4(\rho - i\eta)$ and $V_{tb}V_{ts}^* \approx -A\lambda^2$, respectively[§]. Although the penguin contribution is of higher-order in the strong coupling, and suppressed by the loop factor $1/(16\pi^2)$, one could expect both amplitudes to be of comparable size, owing to the additional λ^2 suppression factor of the tree diagram. The needed strong-phase difference can be generated through the absorptive part of the penguin diagram, corresponding to on-shell intermediate particle rescattering [46]. Therefore, one could expect a sizeable asymmetry, provided the strong-phase difference is not too small. However, a very large number of $B^\pm$ is required, because the branching ratio is quite suppressed ($\sim 10^{-5}$). Other decay modes such as $B^\pm \to K^\pm K_S, K^\pm K^{*0}$ [47] involve the interference between penguin diagrams only and might show sizeable CP-violating asymmetries as well, but the corresponding branching fractions are expected to be even smaller than the previously discussed one.

Figure 3: Feynman diagrams contributing to $B^- \to K^- \rho^0$

The two interfering amplitudes can also be generated through other mechanisms. For instance, one can have an interplay between two different cascade processes [48,49] like $B^- \to D^0 X^- \to K_S Y X^-$ and $B^- \to \bar{D}^0 X^- \to K_S Y X^-$. Another possibility would be an interference between two tree-diagrams corresponding to two different decay mechanisms like direct decay (spectator) and weak annihilation [50]. Direct CP violation could also be studied in decays of bottom baryons [51], where it could show up as a rate asymmetry and in various decay parameters.

Note that, for all these flavour-specific decays, the necessary presence of strong phases makes very difficult to extract useful information on the CKM factors from

[§]Since $m_u, m_c << M_W$, we can neglect the small quark-mass corrections in the up and charm penguin contributions. These two diagrams then differ in their CKM factors only, and their sum is regulated by the same CKM factor than the top-quark loop, due to the unitarity of the CKM matrix.

their measured CP asymmetries. Nevertheless, the experimental observation of a non-zero CP-violating asymmetry in any of these decay modes would be a major milestone in our understanding of CP-violation phenomena, as it would clearly establish the existence of direct CP violation in the decay amplitudes.

6.3 Interplay Between Mixing and Direct CP Violation

There are quite a few non-leptonic final states which are reachable both from a B^0 and a $\bar{B}^0$. For these flavour non-specific decays the B^0 (or $\bar{B}^0$) can decay directly to the given final state f, or do it after the meson has been changed to its antiparticle via the mixing process; i.e. there are two different amplitudes, $A(B^0 \to f)$ and $A(B^0 \to \bar{B}^0 \to f)$, corresponding to two possible decay paths. CP-violating effects can then result from the interference of these two contributions.

The time evolution of a state which was originally produced as a B^0 or a $\bar{B}^0$ is given by

$$\begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} g_1(t) & \frac{q}{p} g_2(t) \\ \frac{p}{q} g_2(t) & g_1(t) \end{pmatrix} \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix} . \tag{69}$$

where

$$\begin{pmatrix} g_1(t) \\ g_2(t) \end{pmatrix} = e^{-iMt} e^{-\Gamma t/2} \begin{pmatrix} \cos\left[(\Delta M - \frac{i}{2}\Delta\Gamma)t/2\right] \\ -i \sin\left[(\Delta M - \frac{i}{2}\Delta\Gamma)t/2\right] \end{pmatrix} . \tag{70}$$

Since for B^0 mesons $|\Delta\Gamma/\Delta M| << 1$, we will neglect the tiny $\Delta\Gamma$ corrections in what follows.

The time-dependent decay probabilities for the decay of a neutral B meson created at the time $t_0 = 0$ as a pure B^0 ($\bar{B}^0$) into the final state f ($\bar{f} \equiv CP\, f$) are

$$\begin{aligned} \Gamma[B^0(t) \to f] \;&\propto\; \frac{1}{2} e^{-\Gamma t} |A_f|^2 \Big\{ [1 + |\bar{\rho}_f|^2] + [1 - |\bar{\rho}_f|^2] \cos(\Delta M t) \\ &\qquad - 2\mathrm{Im}\left(\frac{q}{p}\bar{\rho}_f\right) \sin(\Delta M t) \Big\} , \end{aligned} \tag{71}$$

$$\begin{aligned} \Gamma[\bar{B}^0(t) \to \bar{f}] \;&\propto\; \frac{1}{2} e^{-\Gamma t} |\bar{A}_{\bar{f}}|^2 \Big\{ [1 + |\rho_{\bar{f}}|^2] + [1 - |\rho_{\bar{f}}|^2] \cos(\Delta M t) \\ &\qquad - 2\mathrm{Im}\left(\frac{p}{q}\rho_{\bar{f}}\right) \sin(\Delta M t) \Big\} , \end{aligned} \tag{72}$$

where we have introduced the notation

$$\begin{array}{lll} A_f \equiv A[B^0 \to f], & \bar{A}_f \equiv -A[\bar{B}^0 \to f], & \bar{\rho}_f \equiv \bar{A}_f / A_f, \\ A_{\bar{f}} \equiv A[B^0 \to \bar{f}], & \bar{A}_{\bar{f}} \equiv -A[\bar{B}^0 \to \bar{f}], & \rho_{\bar{f}} \equiv A_{\bar{f}} / \bar{A}_{\bar{f}}. \end{array} \tag{73}$$

CP invariance demands the probabilities of CP conjugate processes to be identical. Thus, CP conservation requires $A_f = \bar{A}_{\bar{f}}$, $A_{\bar{f}} = \bar{A}_f$, $\bar{\rho}_f = \rho_{\bar{f}}$ and $\mathrm{Im}(\frac{q}{p}\bar{\rho}_f) = \mathrm{Im}(\frac{p}{q}\rho_{\bar{f}})$. Violation of any of the first three equalities would be a signal of direct CP violation.

The fourth equality tests CP violation generated by the interference of the direct decay $B^0 \to f$ and the mixing-induced decay $B^0 \to \bar{B}^0 \to f$.

Note that in order to be able to observe any CP-violating asymmetry, one needs to distinguish between B^0 and $\bar{B}^0$ decays. However, a final state f that is common to both B^0 and $\bar{B}^0$ decays cannot reveal by itself whether it came from a B^0 or a $\bar{B}^0$. Therefore, one needs independent information on the flavour identity of the decaying neutral B meson; this is referred to as "flavour tagging". Since beauty hadrons are always produced in pairs, one can use for instance the flavour-specific decays of one B to "tag" the flavour of the companion B.

An obvious example of final states f which can be reached both from the B^0 and the $\bar{B}^0$ are CP eigenstates, i.e. states such that $\bar{f} = \zeta_f f$ ($\zeta_f = \pm 1$). The ratios $\bar{\rho}_f$ and $\rho_{\bar{f}}$ depend in general on the underlying strong dynamics. However, for CP self-conjugate final states, all dependence on the strong interaction disappears [48,49] if only one weak amplitude contributes to the $B^0 \to f$ and $\bar{B}^0 \to f$ transitions. In this case, we can write the decay amplitude as $A_f = M e^{i\phi_D} e^{i\delta_s}$, where $M = M^*$, ϕ_D is the phase of the weak decay amplitude and δ_s is the strong phase associated with final-state interactions. It is easy to check that the ratios $\bar{\rho}_f$ and $\rho_{\bar{f}}$ are then given by ($A_{\bar{f}} = M \zeta_f e^{i\phi_D} e^{i\delta_s}$, $\bar{A}_f = M \zeta_f e^{-i\phi_D} e^{i\delta_s}$, $\bar{A}_{\bar{f}} = M e^{-i\phi_D} e^{i\delta_s}$)

$$\rho_{\bar{f}} = \bar{\rho}_f^* = \zeta_f e^{2i\phi_D}. \tag{74}$$

The unwanted effect of final-state interactions cancels out completely from these two ratios. Moreover, $\rho_{\bar{f}}$ and $\bar{\rho}_f$ simplify in this case to a single weak phase, associated with the underlying weak quark transition.

Since $|\rho_{\bar{f}}| = |\bar{\rho}_f| = 1$, the time-dependent decay probabilities given in Eqs. (71) and (72) become much simpler. In particular, there is no longer any dependence on $\cos(\Delta M t)$. Moreover, for B mesons $|\Gamma_{12}/M_{12}| << 1$, implying

$$\frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} \approx \frac{V_{tb}^* V_{tq}}{V_{tb} V_{tq}^*} \equiv e^{-2i\phi_M}. \tag{75}$$

Here $q \equiv d, s$ stands for B_d^0, B_s^0. In deriving this relation we have used the fact that M_{12} is dominated by the top contribution, due to the quadratic dependence with the mass of the quark running along the internal lines of the box diagram. Therefore, the mixing ratio q/p is also given by a known weak phase, and the coefficients of the sinusoidal terms in the time-dependent decay amplitudes are then fully known in terms of CKM mixing angles only:

$$\mathrm{Im}\left(\frac{p}{q}\rho_{\bar{f}}\right) \approx -\mathrm{Im}\left(\frac{q}{p}\bar{\rho}_f\right) \approx \zeta_f \sin\left[2(\phi_M + \phi_D)\right] \equiv \zeta_f \sin(2\Phi). \tag{76}$$

The time-dependent decay rates are finally given by

$$\Gamma[B^0(t) \to f] = \Gamma[B^0 \to f]\, e^{-\Gamma t} \{1 + \zeta_f \sin(2\Phi) \sin(\Delta M t)\}, \tag{77}$$
$$\Gamma[\bar{B}^0(t) \to \bar{f}] = \Gamma[\bar{B}^0 \to \bar{f}]\, e^{-\Gamma t} \{1 - \zeta_f \sin(2\Phi) \sin(\Delta M t)\}. \tag{78}$$

In this ideal case, the time-dependent CP-violating decay asymmetry

$$\frac{\Gamma[B^0(t) \to f] - \Gamma[\bar{B}^0(t) \to \bar{f}]}{\Gamma[B^0(t) \to f] + \Gamma[\bar{B}^0(t) \to \bar{f}]} = \zeta_f \sin(2\Phi) \sin(\Delta M t) \tag{79}$$

provides a direct and clean measurement of the CKM parameters [52]. Integrating over all decay times yields

$$\int_0^\infty dt\, \Gamma[\overset{(-)}{\mathrm{B}^o}(t) \to \overset{(-)}{f}] \propto 1 \mp \zeta_f \sin(2\Phi) \frac{x}{1+x^2}, \tag{80}$$

where $x \equiv \Delta M/\Gamma$. For B_d^0 mesons, $x_d = 0.70 \pm 0.07$ [53]; thus, the mixing term $x_d/(1+x_d^2)$ suppresses the observable asymmetry by a factor of about two. For B_s^0 mesons, one expects $x_s \sim x_d |V_{ts}|^2/|V_{td}|^2 \sim x_d/\{\lambda^2[(1-\rho)^2+\eta^2]\} >> x_d$, and therefore the large B_s^0-$\bar{B}_s^0$ mixing would lead to a huge dilution of the CP asymmetry. The measurement of the time-dependence is then a crucial requirement for observing CP-violating asymmetries with B_s^0 mesons.

In e^+e^- machines, running near the $B^0\bar{B}^0$ production threshold, there is an additional complication coming from the fact that the B meson used to "tag" the flavour is also a neutral one, and therefore both mesons oscillate. Moreover, the $B^0\bar{B}^0$ pair is produced in a coherent quantum state which is a C eigenstate (C-odd in $e^+e^- \to B^0\bar{B}^0$, C-even in $e^+e^- \to B^0\bar{B}^0\gamma$). Taking that into account, the observable time-dependent asymmetry takes the form

$$\frac{\Gamma[(B^0\bar{B}^0)_{C=\mp} \to f + (l^-\bar{\nu}_l X^+)] - \Gamma[(B^0\bar{B}^0)_{C=\mp} \to f + (l^+\nu_l X^-)]}{\Gamma[(B^0\bar{B}^0)_{C=\mp} \to f + (l^-\bar{\nu}_l X^+)] + \Gamma[(B^0\bar{B}^0)_{C=\mp} \to f + (l^+\nu_l X^-)]} = \zeta_f \sin(2\Phi) \sin[\Delta M(t \mp \bar{t})], \tag{81}$$

where the B flavour has been assumed to be "tagged" through the semileptonic decay, and t ($\bar{t}$) denotes the time of decay into f ($l^\pm$). Note that for $C = -1$ the asymmetry vanishes if t and $\bar{t}$ are treated symmetrically. A measurement of at least the sign of $\Delta t \equiv t - \bar{t}$ is necessary to detect CP violation in this case.

We have assumed up to now that there is only one amplitude contributing to the given decay process. Unfortunately, this is usually not the case. If several decay amplitudes with different weak and strong phases contribute, $|\bar{\rho}_f| \neq 1$, and the interference term will depend both on the CKM mixing parameters and on the strong dynamics embodied in the ratio $\bar{\rho}_f$.

The leading contributions to $\bar{b} \to \bar{q}'q'\bar{q}$ decay amplitudes are either "direct" (Fermi) or generated by gluon exchange ("penguin"). Although of higher order in the strong coupling constant, penguin amplitudes are logarithmically enhanced, due to the virtual W-loop, and are potentially competitive. Table 2 contains the CKM factors associated with the direct and penguin diagrams for different B-decay modes into CP-eigenstates. Also shown is the relevant angle Φ. In terms of CKM elements,

Decay	CKM factor (Direct)	CKM factor (Penguin)	Exclusive channels	Φ
$\bar{b} \to \bar{c}c\bar{s}$	$A\lambda^2$	$-A\lambda^2$	$B_d^0 \to J/\psi K_S, J/\psi K_L$	β
			$B_s^0 \to D_s^+ D_s^-, J/\psi\eta$	0
$\bar{b} \to \bar{s}s\bar{s}$	–	$-A\lambda^2$	$B_d^0 \to K_S\phi, K_L\phi$	β
$\bar{b} \to \bar{d}d\bar{s}$	–	$-A\lambda^2$	$B_s^0 \to K_S K_S, K_L K_L$	0
$\bar{b} \to \bar{c}c\bar{d}$	$-A\lambda^3$	$A\lambda^3(1-\rho-i\eta)$	$B_d^0 \to D^+D^-, J/\psi\pi^0$	$\approx \beta$
			$B_s^0 \to J/\psi K_S, J/\psi K_L$	0
$\bar{b} \to \bar{u}u\bar{d}$	$A\lambda^3(\rho+i\eta)$	$A\lambda^3(1-\rho-i\eta)$	$B_d^0 \to \pi^+\pi^-, \rho^0\pi^0, \omega\pi^0$	$\approx \beta+\gamma$
			$B_s^0 \to \rho^0 K_S, \omega K_S, \pi^0 K_S,$	$\approx \gamma$
			$\rho^0 K_L, \omega K_L, \pi^0 K_L$	
$\bar{b} \to \bar{s}s\bar{d}$	–	$A\lambda^3(1-\rho-i\eta)$	$B_d^0 \to K_S K_S, K_L K_L$	0
			$B_s^0 \to K_S\phi, K_L\phi$	$-\beta$

Table 2: CKM factors and relevant angle Φ for some B-decays into CP-eigenstates.

the angles α, β and γ are:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right], \tag{82}$$

Due to the unitarity of the CKM matrix, $\alpha+\beta+\gamma=\pi$.

The $\bar{b} \to \bar{c}c\bar{s}$ quark decays are theoretically unambiguous [54]: the direct and penguin amplitudes have the same weak phase $\Phi = \beta$ (0), for B_d^0 (B_s^0). Ditto for $\bar{b} \to \bar{s}s\bar{s}$ and $\bar{b} \to \bar{d}d\bar{s}$, where only the penguin mechanism is possible. The same is true for the Cabibbo-suppressed $\bar{b} \to \bar{s}s\bar{d}$ mode, which only gets contribution from the penguin diagram; the B_d^0 (B_s^0) phases are 0 ($-\beta$) in this case. The $\bar{b} \to \bar{c}c\bar{d}$ and $\bar{b} \to \bar{u}u\bar{d}$ decay modes are not so simple; the two decay mechanisms have the same Cabibbo suppression (λ^3) and different weak phases, but the penguin amplitudes are down by $(\alpha_s/6\pi)\ln(m_W/m_b) \approx 3\%$: these decay modes can be used as approximate measurements of the CKM factors. We have not considered doubly Cabibbo-suppressed decay amplitudes, such as $\bar{b} \to \bar{u}u\bar{s}$, for which penguin effects can be important and spoil the simple estimates based on the direct decay mechanism.

Presumably the most realistic channels for the measurement of the angles $\Phi = (\beta,\, \alpha,\, \gamma)$ are $B_d^0 \to J/\psi K_S$, $B_d^0 \to \pi^+\pi^-$ ($\beta+\gamma = \pi-\alpha$) and $B_s^0 \to \rho^0 K_S$, respectively. The first of these processes is no doubt the one with the cleanest signature and the most tractable background. The last process has the disadvantage of requiring a B_s^0 meson and, moreover, its branching ratio is expected to be very small because the "direct" decay amplitude is colour suppressed, leading presumably to a much larger penguin contamination; thus, the determination of γ, through this decay mode looks a quite formidable task.

The decay modes where $\Phi = 0$ are useless for making a determination of the CKM factors. However, they provide a very interesting test of the SM mechanism of CP-violation, because the prediction that no CP-asymmetry should be seen for these modes is very clean. Any detected CP-violating signal would be a clear indication of new physics.

Many other decay modes of B mesons can be used to get information on the CKM factors responsible for CP violation phenomena. A recent summary, including alternative ways of measuring γ, can be found in ref. [44].

7 Summary

The SM incorporates a mechanism to generate CP violation, through the single phase naturally occurring in the CKM matrix. So far, only one non-zero CP-violation effect has been clearly established: $\varepsilon \approx 2.3 \times 10^{-3} e^{i\pi/4}$. Therefore, we do not have yet an experimental test of the CKM mechanism. The value of ε is just fitted with the CKM phase; but we could also fit this measured parameter using other non-standard sources of CP violation. In fact, the present observations can still be explained with the old "superweak" mechanism [55], which associates CP violation with some unknown $\Delta S = 2$ interaction, i.e. with a K^0-$\bar{K}^0$ mixing effect.

Since all CP-violating effects are supossed to be generated by the same CKM phase, the SM predictions are quite constrained. Moreover, as shown in Sect. 2, the CKM mechanism implies very specific requirements for CP-violation phenomena to show up. The experimental verification of the SM predictions is obviously a very important challenge for future experiments, which could lead to big surprises.

In the SM, CP violation is associated with a charged-current interaction which changes the quark flavours in a very definite way: $u_i \to d_j W^+$, $d_j \to u_i W^-$. Therefore, CP should be directly violated in many ($\Delta S = 1$, $\Delta D = 1$, $\Delta B = 1$, $\ldots$) decay processes without any relation with meson-antimeson mixing. Although the quantitative predictions are often uncertain, owing to the not so-well understood long-distance strong-interaction dynamics, the experimental observation of a non-zero CP-violating asymmetry in any self-tagging decay mode would be a major achivement, as it would clearly establish the existence of direct CP violation in the decay amplitudes.

The SM mechanism of CP violation is based in the unitarity of the CKM matrix. Testing the unitarity relations of the CKM matrix elements is then a way to test the source of CP violation. Up to now, the only relation which has been precisely tested is the one associated with the first row of the CKM matrix:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \; = \; 0.9981 \pm 0.0027. \qquad (83)$$

The unitarity relation is very well satisfied in this case, providing a nice confirmation of the SM¶. However, only the moduli of the CKM matrix elements are involved in

¶In fact, this is a test of the SM radiative corrections, which are crucial for extracting V_{ud} with the quoted precision. If these corrections were neglected, unitarity would be violated by many σ's [56].

Eq. (83), while CP violation has to do with their phases.

More interesting are the off-diagonal unitarity conditions:

$$\begin{aligned} V_{ud}^* V_{us} &+ V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0\,, \\ V_{us}^* V_{ub} &+ V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0\,, \\ V_{ub}^* V_{ud} &+ V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0\,. \end{aligned} \tag{84}$$

These relations can be visualized by triangles in a complex plane [57]. Owing to Eq. (9), the three triangles have the same area $|\mathcal{J}|/2$. In the absence of CP violation, these triangles would degenerate into segments along the real axis.

In the first two triangles, one side is much shorter than the other two (the Cabibbo suppression factors of the three sides are λ, λ and λ^5 in the first triangle, and λ^4, λ^2 and λ^2 in the second one). This is the reason why CP effects are so small for K mesons (first triangle), and why certain asymmetries in B_s decays are predicted to be tiny (second triangle).

Figure 4: The unitarity triangle. Also shown are various topics in B physics that allow to measure its sides and angles [44].

The third triangle looks more interesting, since the three sides have a similar size of about λ^3. They are small, which means that the relevant b-decay branching ratios are small, but once enough B mesons would be produced, CP-violation asymmetries are going to be sizeable. This triangle is shown in Fig. 4, where it has been scaled by dividing its sides by $|V_{cb}^* V_{cd}|$. In the Wolfenstein parametrization (7), where $V_{cb}^* V_{cd}$ is real, this aligns one side of the triangle along the real axis and makes its length equal to 1; the coordinates of the 3 vertices are then $(0,0)$, $(1,0)$ and (ρ,η). The three angles of the triangle are just the angles α, β and γ in Eq. (82), which regulate the B-decay asymmetries. Note that, although the orientation of the triangle in the complex plane is phase-convention dependent, the triangle itself is a physical object: the length of the sides and/or the angles can be directly measured.

At present, we already have some information on this unitarity triangle. CP-conserving measurements can provide a determination of its sides. The measured [53] $\Gamma(b \to u)/\Gamma(b \to c)$ ratio fixes one side to be:

$$R_b \equiv \left|\frac{V_{ub}}{\lambda V_{cb}}\right| = \sqrt{\rho^2 + \eta^2} = 0.34 \pm 0.12. \tag{85}$$

The other side can be extracted from the observed [53] B_d^0-$\bar{B}_d^0$ mixing:

$$R_t \equiv \left|\frac{V_{td}}{\lambda V_{cb}}\right| = \sqrt{(1-\rho)^2 + \eta^2} = 0.97 \pm 0.43. \tag{86}$$

R_t depends quite sensitively on the non-perturbative parameter $B_B f_B^2$ [the B analogous of $B_K f_K^2$ in Eq. (46)] and on m_t. The number in Eq. (86) corresponds to the presently favoured values $\sqrt{B_B} f_B = (1.6 \pm 0.4) f_\pi$ and $m_t = 160 \pm 30$. In principle, the measurement of these two sides could make possible to establish that CP is violated (assuming unitarity), by showing that they indeed give rise to a triangle and not to a straight line. With the present experimental and theoretical errors, this is however not possible.

A third constraint is obtained from the measured value of the CP-violation parameter ε, which forces the (ρ, η) vertex to lie in the hyperbola (47). The value of the B_K parameter is the dominant source of uncertainty; taking the conservative estimate $1/3 < B_K < 1$, which is in agreement with all determinations in Table 1, $\eta > 0$, but the sign of ρ is still not fixed. The final allowed domain for the vertex (ρ, η), satisfying these three constraints, is quite large because of the present theoretical uncertainties.

The observation of CP-violating asymmetries with neutral B mesons, would allow to independently measure the three angles of the triangle, providing an overconstrained determination of the CKM matrix. As shown in the previous section, theoretical uncertainties can largely be avoided, for instance in decays into CP-selfconjugate states, so that CP-odd signals can be directly translated into clean measurements of these angles. If the measured sides and angles turn out to be consistent with a geometrical triangle, we would have a beautiful test of the CKM unitarity, providing strong support to the SM mechanism of CP violation. On the contrary, any deviation from a triangle shape would be a clear proof that new physics is needed in order to understand CP-violating phenomena.

CP violation is a broad and fascinating subject, which is closely related to the so-far untested scalar sector of the SM. New experimental facilities are needed for exploring CP-violating phenomena and test the SM predictions. Large surprises may well be discovered, probably giving the first hints of new physics and offering clues to the problems of fermion-mass generation, quark mixing and family replication.

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