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On the p^4–corrections to $K \to 3\pi$ decay amplitudes in nonlinear and linear chiral models

A.A. Bel'kov[1], G. Bohm[2], A.V. Lanyov[1], A. Schaale[3]

[1] Particle Physics Laboratory, Joint Institute for Nuclear Research, Head Post Office, P.O. BOX 79, 101000 Moscow, Russia

[2] DESY-IfH Zeuthen, Postfach 15735 Zeuthen, Germany

[3] TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3 (supported by Deutscher Akademischer Austauschdienst, DAAD)

Abstract

The calculations of isotopic amplitudes and their results for the direct CP–violating charge asymmetry in $K^{\pm} \to 3\pi$ decays within the nonlinear and linear (σ–model) chiral Lagrangian approach are compared with each other. It is shown, that the latter, taking into account intermediate scalar resonances, does not reproduce the p^4–corrections of the nonlinear approach introduced by Gasser and Leutwyler, being saturated mainly by vector resonance exchange. The resulting differences concerning the CP violation effect are traced in some detail.

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The purpose of this short note is to clarify further the model–dependence of various predictions concerning the manifestation of direct CP violation in the charge asymmetry of $K^{\pm} \to 3\pi$ decays. Estimates for this charge asymmetry have been given in the soft pion limit [1], resulting in rather small effects. Only after taking into account higher orders of chiral perturbation theory, a large value for $|\Delta g|$ in relation to $\mathrm{Re}(\varepsilon'/\varepsilon)$ has been derived in [2], which has been met with some criticism [3–6] (see also the discussion on the Joint Lepton Photon and Europhysics Conference, Geneva 1991 [7]). After a reformulation of the bosonization prescription [8], and a more detailed investigation of the origin for the enhancement, a (slightly corrected, see below) new result for $|\Delta g|/\mathrm{Re}(\varepsilon'/\varepsilon)$ has been given in [9].

The effects of CP-violation to be observed appear from interfering amplitudes with different quantum numbers. In $K^{\pm} \to 3\pi$ decays (as opposed to $K^{\pm} \to 2\pi$, where only two amplitudes with isospin changes $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ interfere) there is possible an additional contribution from the interference of two different amplitudes both with $|\Delta I| = 1/2$. In the soft–pion limit this additional contribution becomes zero, and only interferences of amplitudes with $|\Delta I| = 1/2$ and 3/2 can contribute to the charge asymmetry Δg in this limit. However taking into account p^4-corrections and rescattering of mesons strongly modifies the soft-pion amplitudes and leads to a large value for this contribution, increasing the charge CP-asymmetry in $K^{\pm} \to 3\pi$ compared to old estimates in the soft-pion approximation [1].

In view of the great importance of possible direct CP-violation effects other than those intensively investigated in $K^0_{L,S}$ decays, it is certainly worthwhile to compare the prediction with those found in other models and to trace possible differences. In recent papers [10], Shabalin investigated the charge asymmetry in $K^{\pm} \to 3\pi$ decays in the framework of a linear σ–model, using the same ansatz for the weak interaction Lagrangian on quark level [11] as used in our papers [2,9]. The result of [10] differs from ours [9] by a factor $10 \div 20$. The origin of this discrepancy can be traced, by a straight forward comparison of both calculations, to the different treatment of the higher order corrections in both models, despite their practical equivalence with respect to the description of other data on K-decays, and rough numerical agreement in many intermediate parameters. We shall not enter a discussion of the absolute size of CP–violating effects, which have been investigated in detail by [12], restricting ourselves to a consideration of the relation between the charge asymmetry in $K^{\pm} \to 3\pi$ and $\mathrm{Re}(\varepsilon'/\varepsilon)$ as measured in $K^0_{L,S} \to 2\pi$ experiments.

The effective Lagrangian describing nonleptonic weak interactions with strangeness change $|\Delta S| = 1$ is given on the quark level by [11,13]:

$$\mathcal{L}_w^{nl} = \tilde{G} \sum_{i=1}^{6} c_i \, \mathcal{O}_i \, . \tag{1}$$

Here $\tilde{G} = \sqrt{2}\, G_F \sin\theta_C \cos\theta_C$ is the weak coupling constant; c_i are Wilson coefficient functions; $\mathcal{O}_i$ are the four-quark operators consisting of products of left- and/or right-handed quark currents. In the present paper we will use the operators $\mathcal{O}_i$ in the representation of Shifman–Vainshtein–Zakharov [11]. The bosonized Lagrangian of nonleptonic four–quark weak interactions (1) and the corresponding meson currents can be obtained by the functional method using the generating functional for Green functions of quark currents introduced in [14] and [8]. In such an approach the quark determinant, which leads to the effective Lagrangian of meson strong interaction, generates also the meson currents and scalar densities entering in the bosonized version of the nonleptonic weak Lagrangian.

The corresponding nonlinear effective meson strong Lagrangian, including p^2– and higher order derivatives terms, can be presented in the following general form

$$\mathcal{L}_{eff}^{(nlin)} = -\frac{F_0^2}{4}\,\mathrm{tr}\,(L_\mu L^\mu) + \frac{F_0^2}{4}\,\mathrm{tr}\,[M(U+U^+)]$$
$$+\left(L_1 - \frac{1}{2}L_2\right)(\,\mathrm{tr}\,L_\mu L^\mu)^2 + L_2\,\mathrm{tr}\left(\frac{1}{2}[L_\mu, L_\nu]^2 + 3(L_\mu L^\mu)^2\right) + L_3\,\mathrm{tr}\,((L_\mu L^\mu)^2)$$
$$+L_4\,\mathrm{tr}\,(D_\mu U\,\overline{D}^\mu U^+)\,\mathrm{tr}\,M(U+U^+) + L_5\,\mathrm{tr}\,D_\mu U\,\overline{D}^\mu U^+(MU+U^+M) + \ldots \quad (2)$$

where the dimensionless structure constants L_i were introduced by Gasser and Leutwyler in ref.[15]. Here we write down only the p^2– and p^4–terms relevant for the description of nonleptonic kaon decays restricting ourselves to terms up to order m_0 in the quark mass. $F_0 = 89$ MeV is the bare π decay constant; $L_\mu = D_\mu U\,U^+$, where U is a pseudoscalar meson matrix;

$$D_\mu * = \partial_\mu * + (A_\mu^{(-)} * - * A_\mu^{(+)}), \quad \overline{D}_\mu * = \partial_\mu * + (A_\mu^{(+)} * - * A_\mu^{(-)}) \quad (3)$$

are the covariant derivatives; $A_\mu^{(\pm)} = V_\mu \pm A_\mu$ with V_μ, A_μ being the external vector and axial–vector fields. The meson mass matrix $M = diag(\chi_u^2, \chi_d^2, \chi_s^2)$ with the parameters $\chi_i^2 = -2m_0^i <\overline{q}q> F_0^{-2}$, where $<\overline{q}q>$ is the quark condensate, can be fixed by the spectrum of pseudoscalar mesons. The coefficients L_i are given by $L_1 - \frac{1}{2}L_2 = L_4 = 0$ and

$$L_2 = \frac{N_c}{16\pi^2}\frac{1}{12}, \quad L_3 = -\frac{N_c}{16\pi^2}\frac{1}{6}, \quad L_5 = \frac{N_c}{16\pi^2}x(y-1), \quad (4)$$

where $y = 4\pi^2 F_0^2/(N_c\mu^2)$ and $x = -\mu F_0^2/(2<\overline{q}q>)$, $\mu = 380$ MeV is the averaged constituent quark mass.

The pseudoscalar meson matrix U arises in a nonlinear parameterization of chiral symmetry from the following representation of the combination of the external scalar (S) and pseudoscalar (P) fields:

$$\Phi = S + iP = \Omega\,\Sigma\,\Omega\,.$$

Here $\Sigma(x)$ is the matrix of scalar fields belonging to the diagonal flavour group while the matrix $\Omega(x)$ represents the pseudoscalar degrees of freedom φ living in the coset flavour space $U(3)_L\times U(3)_R/U_V(3)$, which can be parameterized by the unitary matrix

$$\Omega(x) = \exp\left(\frac{i}{\sqrt{2}F_0}\varphi(x)\right), \quad \varphi(x) = \varphi^a(x)\frac{\lambda^a}{2},$$

where λ^a are the generators of the $SU(3)$ flavour group. Assuming approximate flavour symmetry of the condensate ($\Sigma \approx \mu 1$) one obtaines $\Phi = \mu\Omega^2 = \mu U$ with $U = \Omega^2$.

The bosonized $(V \mp A)$ meson currents, corresponding to the quark currents $\overline{q}\frac{1\mp\gamma_5}{2}\gamma_\mu\frac{\lambda^a}{2}q$, can be obtained by varying the quark determinant with redefined vector and axial–vector fields

$$V_\mu \to V_\mu - i(\eta_{L\mu} + \eta_{R\mu}), \quad A_\mu \to A_\mu - i(\eta_{L\mu} - \eta_{R\mu}),$$

over the external sources $\eta_{L,R\mu} = \eta_{L,R\mu}^a\frac{\lambda^a}{2}$ coupling to the corresponding quark currents [8] (the fields V_μ, A_μ enter in the covariant derivatives). In this way the effective Lagrangian (2) generates the bosonized $(V-A)$ meson current of the form

$$J_{L\mu}^{(nlin)a} = i\frac{F_0^2}{4}\,\mathrm{tr}\left(\lambda^a L_\mu\right)$$
$$-i\,\mathrm{tr}\left\{\lambda^a\left[2L_2 L_\nu L_\mu L^\nu + (2L_2 + L_3)\{L_\mu, L_\nu L^\nu\}\right.\right.$$
$$\left.\left.-\frac{1}{2}L_5 U^+\{(MU + U^+M), L_\mu\}U\right]\right\}. \quad (5)$$

This current determines the meson matrix elements of $|\Delta I| = 1/2\,(\mathcal{O}_{1,2,3})$ and $|\Delta I| = 3/2\,(\mathcal{O}_4)$ non–penguin four–quark operators consisting of products of left–handed quark currents. The first term in (5) is generated by the kinetic term of the Lagrangian (2) while all other terms originate from its p^4–part.

The $(S-P)$ meson current corresponding to the bosonized scalar density and generated by the Lagrangian (2) can be obtained by variation of the quark determinant with redefined scalar and pseudoscalar fields

$$S \to S - (\eta_L + \eta_R), \quad P \to P - i(\eta_L - \eta_R), \quad (6)$$

where $\eta_{L,R} = \eta_{L,R}^a\frac{\lambda^a}{2}$ are the external sources coupling to the the quark densities $\overline{q}\frac{1\mp\gamma_5}{2}\frac{\lambda^a}{2}q$. The corresponding $(S-P)$ meson current has the form

$$J_L^{(nlin)a} = \frac{F_0^2}{8\mu}\,\mathrm{tr}\,(\lambda^a\partial^2 U) + \frac{F_0^2}{4}\mu R\,\mathrm{tr}\,(\lambda^a U)$$
$$-\frac{1}{\mu}\,\mathrm{tr}\left\{\lambda^a\left[L_2\partial_\mu(L_\nu L^\mu L^\nu) + (2L_2 + L_3)\partial_\mu(L_\nu L^\nu L^\mu)\right.\right.$$
$$\left.\left.-\frac{1}{2}L_5\left(\partial_\mu((MU + U^+M)L^\mu U) + 2\mu^2 R L_\mu L^\mu\right)\right]\right\}, \quad (7)$$

where $R = <\overline{q}q>/(\mu F_0^2)$. Here, the first and second terms are generated by the kinetic and mass terms of the Lagrangian (2), respectively, while all other terms originate from its p^4–part.

In papers [6,10] the $K \to 3\pi$ decays amplitudes were calculated within the linear σ–model with broken $U(3)_L \times U(3)_R$ symmetry. The effective Lagrangian of meson strong interactions used in ref.[10] is of the form

$$\mathcal{L}_{eff}^{(lin)} = \frac{1}{2}\,\mathrm{tr}\,(\partial_\mu\Phi\,\partial^\mu\Phi^+) + \frac{F_0}{2\sqrt{2}}\,\mathrm{tr}\,[(\Phi + \Phi^+)\widetilde{M}]$$
$$-c\,\mathrm{tr}\,(\Phi\,\Phi^+ - A^2\lambda_0^2)^2 - c\xi\left(\mathrm{tr}\,(\Phi\,\Phi^+ - A^2\lambda_0^2)\right)^2, \quad (8)$$

where $\Phi = \hat{\sigma} + i\hat{\pi}$; $\hat{\sigma}$ and $\hat{\pi}$ are matrices of nonets of scalar and pseudoscalar mesons; $\lambda_0 = \frac{1}{\sqrt{3}}\mathbf{1}$; $\widetilde{M}$ is the mass matrix, which gets in the approximation $m_u^0 = m_d^0$ the form

$$\widetilde{M} = \frac{1}{\sqrt{3}}(2m_K^2 + m_\pi^2)\lambda_0 - \frac{2}{\sqrt{3}}(m_K^2 - m_\pi^2)\lambda_8 .$$

The parameter c can be expressed through the masses m_π and m_K and the $\pi, K \to \mu\nu$ decay constants $F_{\pi,K}$:

$$c = \frac{m_K^2 - m_\pi^2}{4F_\pi^2(F_K/F_\pi - 1)(2F_K/F_\pi - 1)};$$

the constant A is connected to the quark condensate, and the value of the parameter $\xi = -0.225$ is fixed from the K_{e4} decay form factors. The $(V-A)$ and $(S-P)$ meson currents originating from kinetic and mass terms respectively of the effective Lagrangian (8) and used in [10] are

$$J_{L\mu}^{(lin)a} = i\,\mathrm{tr}\,(\lambda^a \partial_\mu \Phi\, \Phi^+)\,, \tag{9}$$

$$J_L^{(lin)a} = \frac{\sqrt{2}F_0 m_\pi^2}{m_u + m_d}\,\mathrm{tr}\,(\lambda^a \Phi)\,. \tag{10}$$

The penguin diagrams give a contribution to the effective weak interaction proportional to the $|\Delta I| = 1/2$ operator *

$$\mathcal{O}_5 = \bar{d}_L \gamma_\mu \lambda_c^a s_L \left(\sum_{q=u,d,s} \bar{q}_R \gamma^\mu \lambda_c^a q_R \right) \xrightarrow{Fierz} -4 \sum_q \bar{d}_L q_R \cdot \bar{q}_R s_L .$$

We can find all the meson matrix elements of $\bar{q}_L q_R'$, for example, using a modified version of the QCD Lagrangian in which the quarks are coupled to external $U(3)_L \times U(3)_R$ gauge fields, and the quark mass term is replaced by

$$\sum_q m_q^0 \bar{q}\, q \to \sum_{q,q'} \kappa_{qq'}(x) \bar{q}_L\, q_R' + h.c.\,,$$

where $\kappa(x)$ is an arbitrary space-time dependent 3×3 matrix of external fields (see the detailed discussion in [16]). In this approach the quark mass m_q^0 is replaced by the scalar source $\kappa_{qq'}(x)$ for the quark density. The meson matrix element of the operator $\mathcal{O}_5$ is found by replacing the quark density $J_L^{(q)} = \bar{q}_L\, q_R'$ by the meson scalar current $J_L^{(m)} = \frac{1}{4} < \bar{q}\, q > U_{q'q}^+$ generated from the chiral symmetry breaking part of the meson Lagrangian. Then

$$< \mathcal{O}_5 >_{m\,s} = -\frac{1}{4} < \bar{q}\, q >^2 \sum_q U_{sq} U_{qd}^+ = 0$$

*The contribution of the operator $\mathcal{O}_6$ is small and is therefore neglected .

because $UU^+ = 1$, therefore $(UU^+)_{sd} = 0$. This poses a problem for the naive penguin treatment in the chiral Lagrangian language: on quark level in the simple vacuum incertion approximation the meson matrix element $\mathcal{O}_5$ does not disappear.

To solve this problem, in ref.[16] the new additional symmetry breaking term $\sim tr\, m_0 D^2 U$ was added to the chiral symmetry breaking part of the effective meson Lagrangian (2). This new term leads to nonzero meson matrix elements of the penguin operator $\mathcal{O}_5$ due to the appearence of the additional contribution to the scalar density $\sim tr\, \partial^2 U$. This additional contribution automatically arises in [8] from the kinetic term of the effective Lagrangian (2) via the replacement $\mu U^+ \to \mu U^+ - 2\eta_L(x)$, corresponding to the redefinition of the scalar and pseudoscalar external fields (6). The term $\sim tr\, \partial^2 U$ in the bosonized scalar density was used in [17,19] and in our calculations [2,9]. Concerning the problem of the chiral bosonization of penquin operators in the linear σ-model, one should pay attention to the fact that in this case $(\Phi\Phi^+)_{sd} \neq 0$ and the $(S-P)$ current (10), generated by the mass term of Lagrangian (8), already ensures the nonzero value of the meson matrix element $\mathcal{O}_5$. Nevertheless, it is obvious that the contribution $\sim tr\, \partial^2 \Phi$ to the $(S-P)$ current must arise also in the same way from the kinetic part of Lagrangian (8) after redefinition of $\hat{\sigma}$ and $\hat{\pi}$ fields. However, the corresponding contribution was not considered in ref.[10].

The $K^+ \to 3\pi$ decay amplitudes can be parametrized using isospin relations as [20]

$$\begin{aligned} T_{K^+\to\pi^+\pi^+\pi^-} &= 2(\mathcal{A}_{11} + \mathcal{A}_{13}) - Y(\mathcal{B}_{11} + \mathcal{B}_{13} - \mathcal{B}_{23}) + O(Y^2), \\ T_{K^+\to\pi^0\pi^0\pi^+} &= (\mathcal{A}_{11} + \mathcal{A}_{13}) + Y(\mathcal{B}_{11} + \mathcal{B}_{13} + \mathcal{B}_{23}) + O(Y^2), \end{aligned} \tag{11}$$

where $Y = (s_3 - s_0)/m_\pi^2$ is the Dalitz variable and $s_i = (k - p_i)^2$, $s_0 = m_K^2/3 + m_\pi^2$; k, p_i are four-momenta of the kaon and ith pion ($i = 3$ belongs to the odd pion). The Dalitz-plot distribution can be written as a power series expansion of the amplitude squared, $|T|^2$, in terms of the corresponding kinematical variables Y and X

$$|T|^2 \propto 1 + gY + \ldots$$

The isotopic amplitudes $\mathcal{A}_{IJ}$, $\mathcal{B}_{IJ}$ of $K \to 3\pi$ decays have two indices: I, the isospin of the final state, and J, the doubled value of isospin change between the initial and final states. It is customary in anology to the 2π-system to introduce strong phase shifts α_1, β_1 and β_2 corresponding to the relevant isospin states $I = 1_s$ (symmetric), $I = 1_m$ (mixed symmetric), $I = 2$ by writing

$$\mathcal{A}_{11} + \mathcal{A}_{13} = (a_{11} + a_{13})\,e^{i\alpha_1}, \qquad \mathcal{B}_{11} + \mathcal{B}_{13} = (b_{11} + b_{13})\,e^{i\beta_1}, \qquad \mathcal{B}_{23} = b_{23}\,e^{i\beta_2}.$$

We shall use this representation here only in order to display more cleary the relationships between the main contributions to the direct CP-violation effect and for the comparision with calculations in other papers. Because the strong Hamiltonian is not necessarily diagonal with respect to the $I = 1_s, I = 1_m$ isospin states and, if isospin breaking is included, even $I = 1$ and $I = 2$ states get mixed, leading to the necessity of introducing more phases, the exact calculations of $\Delta g(K^\pm \to 3\pi)$ have to be done using the complex quantities $\mathcal{A}_{IJ}$, $\mathcal{B}_{IJ}$ given below by (12) directly, without introducing the strong phases α_1, $\beta_{1,2}$ explicitly.

Let us next introduce the contributions of the four-quark operators $\mathcal{O}_i$ to the isotopic amplitudes $\mathcal{A}^{(i)}_{IJ}$ and $\mathcal{B}^{(i)}_{IJ}$ by the relations

$$\mathcal{A}_{IJ} = -\sum_{i=1}^{5} \xi_i \left(\tilde{G}\,\frac{m_K^2 - m_\pi^2}{12}\right) \mathcal{A}^{(i)}_{IJ}, \qquad \mathcal{B}_{IJ} = -\sum_{i=1}^{5} \xi_i \left(\tilde{G}\,\frac{m_\pi^2}{4}\right) \mathcal{B}^{(i)}_{IJ}. \qquad (12)$$

Here ξ_i are parameters related to the Wilson coefficients c_i of (1) as

$$\xi_1 = c_1\left(1 - \frac{1}{N_c}\right), \quad \xi_{2,3,4} = c_{2,3,4}\left(1 + \frac{1}{N_c}\right), \quad \xi_5 = c_5 + \frac{1}{N_c} c_6, \qquad (13)$$

where the color factor $1/N_c$ originates from the Fierz-transformed contribution to the nonleptonic weak effective chiral Lagrangian [8]. The normalization factors for the amplitudes $\mathcal{A}^{(i)}_{IJ}$ and $\mathcal{B}^{(i)}_{IJ}$ in (12) were choosen in such way that in the "soft pion" limit, corresponding to the p^2-order of the chiral Lagrangian approach, one obtains

$$\mathcal{A}^{(1)}_{11} = \mathcal{B}^{(1)}_{11} = -\mathcal{A}^{(2,3)}_{11} = -\mathcal{B}^{(2,3)}_{11} = -1.$$

Then the other nonvanishing amplitudes in the soft pion limit are

$$\mathcal{A}^{(4)}_{13} = 1, \quad \mathcal{B}^{(4)}_{13} = -\frac{1}{4}\,\frac{5m_K^2 - 14m_\pi^2}{m_K^2 - m_\pi^2}, \quad \mathcal{B}^{(4)}_{23} = \frac{9}{4}\,\frac{3m_K^2 - 2m_\pi^2}{m_K^2 - m_\pi^2}.$$

The charge asymmetry of the slope parameters $\Delta g(K^\pm \to 3\pi)$ can be expressed by the formula

$$\Delta g\left(K^\pm \to \left\{\begin{matrix}\pi^\pm\pi^\pm\pi^\mp \\ \pi^0\pi^0\pi^\pm\end{matrix}\right\}\right) = \frac{\mathrm{Im}\,F_1\,\sin(\alpha_1 - \beta_1) \pm \mathrm{Im}\,F_2\,\sin(\alpha_1 - \beta_2)}{\mathrm{Re}\,F_1\,\cos(\alpha_1 - \beta_1) \pm \mathrm{Re}\,F_2\,\cos(\alpha_1 - \beta_2)}, \qquad (14)$$

where $F_1 = (a^*_{11} + a^*_{13})(b_{11} + b_{13})$ and $F_2 = -(a^*_{11} + a^*_{13})b_{23}$. † It is convenient to present the terms in the numerator of the right-hand side of eq.(14) for $\Delta g(K^\pm \to 3\pi)$ in a more visual form

$$\begin{aligned}
\mathrm{Im}\,F_1 &= \Delta^{(1/2,1/2)} + \Delta^{(1/2,3/2)}, \\
\Delta^{(1/2,1/2)} &= \mathrm{Re}\,a_{11}\ \mathrm{Im}\,b_{11} - \mathrm{Im}\,a_{11}\ \mathrm{Re}\,b_{11} \\
&= \mathrm{Im}\,\xi_5\left[\mathrm{Re}\,\mathcal{B}^{(5)}_{11}(\xi_{123}\,\mathrm{Re}\,\mathcal{A}^{(1)}_{11} + \xi_5\,\mathrm{Re}\,\mathcal{A}^{(5)}_{11}) - \mathrm{Re}\,\mathcal{A}^{(5)}_{11}(\xi_{123}\,\mathrm{Re}\,\mathcal{B}^{(1)}_{11} + \xi_5\,\mathrm{Re}\,\mathcal{B}^{(5)}_{11})\right], \\
\Delta^{(1/2,3/2)} &= \mathrm{Re}\,a_{13}\ \mathrm{Im}\,b_{11} - \mathrm{Im}\,a_{11}\ \mathrm{Re}\,b_{13} \\
&= \xi_4\,\mathrm{Im}\,\xi_5(\,\mathrm{Re}\,\mathcal{A}^{(4)}_{13}\,\mathrm{Re}\,\mathcal{B}^{(5)}_{11} - \mathrm{Re}\,\mathcal{A}^{(5)}_{11}\,\mathrm{Re}\,\mathcal{B}^{(4)}_{13}); \\
\mathrm{Im}\,F_2 &= \mathrm{Im}\,a_{11}\ \mathrm{Re}\,b_{23} \equiv \Delta'^{(1/2,3/2)} = \xi_4\,\mathrm{Im}\,\xi_5\,\mathrm{Re}\,\mathcal{A}^{(5)}_{11}\,\mathrm{Re}\,\mathcal{B}^{(4)}_{23}.
\end{aligned} \qquad (15)$$

Here $\Delta^{(1/2,1/2)}$ describes the contribution of the interference of isotopic amplitudes a_{11} and b_{11} for transitions with $|\Delta I| = 1/2$, and $\Delta^{(1/2,3/2)}$, $\Delta'^{(1/2,3/2)}$ are the contributions from interferences of amplitudes a_{IJ} and b_{IJ} with $|\Delta I| = 1/2$ and $3/2$. In writing

†In deriving charge asymmetries, one has to keep in mind, that charge conjugation does reverse the phases of ξ_i but not those of $A^{(i)}_I$, $\mathcal{A}^{(i)}_{IJ}$, $\mathcal{B}^{(i)}_{IJ}$.

eq.(15) we assume that direct CP-violation arises only due to the imaginary parts of the isotopic amplitudes with $|\Delta I| = 1/2$ generated by the imaginary part of the Wilson coefficient c_5 of the penguin operator $\mathcal{O}_5$. The fact that the relation $\mathcal{A}^{(1)}_{11} = \mathcal{B}^{(1)}_{11} = -\mathcal{A}^{(2,3)}_{11} = -\mathcal{B}^{(2,3)}_{11}$ is fulfiled always if there is no isotopic symmetry breaking was also used. In this case the contribution of the nonpenguin operators with $|\Delta I| = 1/2$ to nonleptonic kaon decays can be joint to a term, proportional to the combination $\xi_{123} = (-\xi_1 + \xi_2 + \xi_3)$.

In order to separate the contributions belonging to the dominating combination $(-\xi_1 + \xi_2 + \xi_3)$ and to ξ_4, ξ_5 respectively, we used in [9] the experimental data on $K \to 2\pi$, $K \to 3\pi$ decays and obtained the following values:

$$\xi_{123} = 6.96 \pm 0.48, \qquad \xi_4 = 0.516 \pm 0.025, \qquad \xi_5 = -0.183 \pm 0.022. \qquad (16)$$

As the analysis of the coefficients c_i in leading-log approximation of QCD has shown, the main contribution to direct CP-violation comes from the (gluonic) penguin diagram. The imaginary part of the coefficient c_5, responsible for the direct CP-violation, can be fixed as [9]

$$\mathrm{Im}\,c_5 = 0.053^{+0.015}_{-0.011}|\varepsilon'/\varepsilon|.$$

The results of our calculations of $K \to 3\pi$ decay isotopic amplitudes under successive inclusion of p^4-corrections are presented in Table 1a ‡. The role of the p^4-contributions for the enhancement of CP-violation due to interference of $|\Delta I| = 1/2$ amplitudes (the quantity $\Delta^{(1/2,1/2)}$ in the expression for $\mathrm{Im}\,F_1$ (15)) can be demonstrated by means of a simplified order of magnitude estimate of the effect. In particular, taking only the p^4-corrections, arising from the $L_{2,3}$-terms of Lagrangian (2) and the currents (5, 7) (the Skyrme and non-Skyrme interactions), we find for a_{11} and b_{11}:

$$a_{11} = \xi_{123}\Big(1 + \frac{m_K^2 - 3m_\pi^2}{12F_0^2\pi^2}\Big) + \xi_5 \cdot 4R\Big(1 - \frac{m_K^2 - 3m_\pi^2}{12F_0^2\pi^2}\Big),$$
$$b_{11} = \xi_{123}\Big(1 - \frac{m_K^2 + 3m_\pi^2}{12F_0^2\pi^2}\Big) + \xi_5 \cdot 4R\Big(1 + \frac{m_K^2 + 3m_\pi^2}{12F_0^2\pi^2}\Big).$$

Therefrom it is clear, that in the soft pion limit, with disappearence of the p^4-contributions $\sim 1/(12F_0^2\pi^2)$, the contribution from interference of $|\Delta I| = 1/2$ amplitudes equals zero. With p^4-corrections taken into account, we find for this interference term the simple expression

$$\Delta^{(1/2,1/2)} = \mathrm{Im}\,\xi_5\ \mathrm{Re}\,\xi_{123}\,4R\frac{m_K^2}{3F_0^2\pi^2} \approx 149.1 \cdot \mathrm{Im}\,c_5$$

in good agreement with Table 1a.

The Lagrangian (8) and currents (9), (10) were used in [6] for the calculation of $K \to 3\pi$ amplitudes in the soft pion limit. In the last publication [10] there was an attempt to include in the linear σ-model also the p^4-corrections via diagrams

‡The numerical values for the same isotopic amplitudes after additional inclusion of $(\pi^0 - \eta - \eta')$-mixing and one-loop corrections were given in [9].

with intermediate scalar resonances. For the further discussion and comparison with the predictions of the nonlinear Lagrangian approach it is convenient to present the results of the calculations of $K \to 3\pi$ decay isotopic amplitudes of ref.[10] in a numerical form similar to Table 1a. The corresponding isotopic amplitudes in the soft pion limit and the results of successive inclusion of p^4–corrections generated in the linear σ model by scalar resonance exchange are given in Table 1b. To present numerically the contributions of the penguin operator $\mathcal{O}_5$ we use the fact that the parameter $\beta = 2m_\pi^4/[(m_u + m_d)^2(m_{\sigma_\pi}^2 - m_\pi^2)] = 8.15$, introduced in ref.[10], can be assumed to be equal in our notations to the parameter $-R \approx 5.6$ defining the contribution of the bosonized scalar density (7). §

The quantities $\Delta^{(1/2,1/2)}$, $\Delta^{(1/2,3/2)}$ and $\Delta'^{(1/2,3/2)}$, corresponding to the definitions of ref.[10], can be estimated after the replacement $\xi_i \to c_i$ in eqs.(15). Using the phenomenological relations for the Wilson coefficients of ref.[10]

$$(c_1 - c_2 - c_3 - c_4) = -3.2\,, \quad c_4 = 0.328\,, \quad (c_1 - c_2 - c_3 + 4\beta c_5) = -10.13 \quad (17)$$

one can fix the parameters $(c_1 - c_2 - c_3)$, c_4 and $4\beta c_5$ and obtaines for $\Delta^{(1/2,1/2)}$, $\Delta^{(1/2,3/2)}$ and $\Delta'^{(1/2,3/2)}$ the values which are also given in Table 1a.

The comparison of Tables 1a and 1b shows that the p^4 contributions owing to the L_i–terms of the nonlinear Lagrangian (2) and currents (5), (7) both quantitatively and qualitatively differ from p^4 corrections generated within linear σ–model by scalar resonance exchange. In the nonlinear Lagrangian approach the p^4–corrections increase the amplitudes $\mathcal{A}_{11}^{(1)}$, $\mathcal{A}_{13}^{(4)}$, $\mathcal{B}_{11}^{(5)}$, $\mathcal{B}_{23}^{(4)}$ and decrease at the same time the amplitudes $\mathcal{A}_{11}^{(5)}$, $\mathcal{B}_{11}^{(1)}$, $\mathcal{B}_{13}^{(4)}$ in their absolute values. On the other hand, the scalar resonance exchange increases the absolute values of the amplitudes $\mathcal{A}_{11}^{(1)}$, $\mathcal{A}_{13}^{(4)}$, $\mathcal{B}_{11}^{(1,5)}$, $\mathcal{B}_{13}^{(4)}$ and decreases the absolute values of the amplitudes $\mathcal{A}_{11}^{(5)}$, $\mathcal{B}_{23}^{(4)}$. Besides that, after taking into account the scalar resonance exchange, the interferences $\Delta^{(1/2,1/2)}$ and $\Delta'^{(1/2,3/2)}$ prove to be respectively by factors 4 and 2 less than in the nonlinear model. In consequence, not only the effect of the enhancement of direct CP violation by the interference $\Delta^{(1/2,1/2)}$ is found to be suppressed in the estimates of [10] but also the contribution of the interference $\Delta'^{(1/2,3/2)}$, nonvanishing in soft pion limit, is suppressed by a factor 2.

The relative contribution of penguin and non-penguin operators in the linear σ–model [10] is determined by the ratio $4\beta c_5/(c_1 - c_2 - c_3) = 2.5$, showing, that about 80% of the observed amplitudes of $|\Delta I| = 1/2$ transitions in nonleptonic kaon decays is attributed to the contribution of the penguin operator $\mathcal{O}_5$. On the other hand the parameters ξ_i (16), fixed from the analysis of $K \to 2\pi$ and $K \to 3\pi$ experimental data, lead in the nonlinear chiral Lagrangian approach to the ratio $4R\xi_5/\xi_{123} = 0.58$. This ratio agrees with the results of our previous phenomenological analysis [2], where it was shown that the contribution of the penguin operator is less 40% of the experimentally measured amplitudes of $|\Delta I| = 1/2$ transitions. These estimates confirm the results of a consistent analysis of the $|\Delta I| = 1/2$ rule by Buras *et al.* [17,18] which was done in the nonlinear chiral Lagrangian approach with Wilson coefficients calculated in leading log approximation of QCD, where the contribution of penguin

§The (irrelevant) numerical discrepancy between R and β disappears, if $(m_u + m_d)/2 = 6$ MeV resp. 5 MeV are taken for β resp. R (in the text 5 MeV is used).

operators to $K \to 2\pi$ decays was estimated to be smaller then nonpenguin contribution within a wide range of the renormalization scale μ (see also the analysis of Wilson coefficients beyond leading logarithms in ref.[12]). If one uses the parameters ξ_i (16), instead of c_i (17) to estimate the interference $\Delta^{(1/2,1/2)}$ in the linear σ–model, taking into account scalar resonance exchange, the value $\Delta^{(1/2,1/2)} = 108.5\,\mathrm{Im}\,c_5$ will be obtained. This fact demonstrates that the estimates of the interference $\Delta^{(1/2,1/2)}$ are very sensitive not only to the difference of the dynamical behavior of penguin and non–penguin amplitudes at $O(p^4)$ level but also to their relative contributions to $|\Delta I| = 1/2$ transitions. In the case, when $|\Delta I| = 1/2$ transitions are dominated by the contributions of the penguin operator, the interference $\Delta^{(1/2,1/2)}$ becomes largely suppressed.

In this way it appears, that in the framework of [10] the interference term $\Delta^{(1/2,1/2)}$, which is mainly responsible for the enhancement of the charge asymmetry, is lower by a factor ~ 4 (as compared to [9]) already in Born approximation. The reason for this discrepancy has been discussed in some detail above. There is an additional enhancement of $\Delta^{(1/2,1/2)}$ by a factor 3 from contributions of meson loops to the real parts of isotopic amplitudes (see [9]). (In [10] only the absorptive parts of meson loops have been calculated). As a result, our value for the charge asymmetry Δg in $K^\pm \to 3\pi$ decays [9] should be about 12 times larger then that estimated by [10], whereby the discrepancy can be explained by the fact, that in the latter case by using a linear σ–model other corrections of order p^4 are considered, and by a more complete treatment of loop corrections in the first case. ¶

We leave aside a detailed discussion of the (strong) phase differences between isotopic amplitudes, which appear after the calculation of meson loops. As they are determined by the ratios of imaginary and real parts of amplitudes, a modification of the latter may be important and should be taken into account. In our case the phases have been extracted from direct calculations of one loop diagrams, using superpropagator regularization. Results of analogous calculations for $K \to 2\pi$ decays [21] are in agreement with Kambor *et al.* [22], where for the regularization of UV–divergences the usual method of introducing counter terms into the Lagrangian was used. We should mention, that in the latter paper also sizeable imaginary parts for the amplitudes β (see Table 2, *loc. cit.*) are found by the loop calculation, but the resulting phases are then suppressed by the choice of counterterms, making their perturbative approach – as the authors themselves remark – somewhat problematic. Of course it would be interesting to fix them directly from experimental data, but this was not possible until now neither in our fit nor in other work.

Concerning the linear σ–model in general, it was demonstrated by Gasser and Leutwyler [15], that it has the correct chiral structure, but a wrong phenomenology at the next-to-leading order in the chiral expansion for any value of the scalar resonance mass (see also the criticism of the linear σ model by Meissner [23] and lectures by Ecker [24] and Pich [25]). The p^4 corrections generated by scalar resonance exchange in the linear σ model are not equivalent to the p^4–corrections related to the L_i terms of the nonlinear Lagrangian (2) and the currents (5), (7). It was shown by

¶The difference of the estimates of ref.[9] with respect to the one given earlier [2] (less than a factor 2) is due to the effect of (π^0, η, η') mixing, formely not taken into account completely. On the other hand this suppression is compensated by a larger mass of the t quark $m_t \geq 100$ GeV by the additional (relative) enhancement effect arising from contributions of the electroweak penguin operator $\mathcal{O}_8$.

Ecker *et al.* [26] and by Donoghue *et al.* [27] that the structure constants L_i of the effective chiral Lagrangian for strong interactions of order p^4 are largely saturated by vector resonance exchange at order p^2. The most general analysis has been carried out in ref.[26], where all possible chiral couplings to the pseudoscalar mesons linear in the resonance fields were constructed to lowest order in the chiral expansion. In particular, the coupling constants $L_{2,3}$ are completely dominated by vector resonance exchange while scalar resonances contribute only to L_3, and this contribution does not exceed 20%. The derivation of the Skyrme–type p^4–interaction from the integrated out vector and axial–vector resonances has been given also by Igarashi [28] from the hidden–local symmetry Lagrangian of Bando *et al.* [29]. The resonance contributions to the pseudoscalar weak Lagrangian and the modification of its structure after integrating out the heavy meson exchanges were discussed recently by Ecker *et al.* [30] and by Isidori and Pugliese [31]. Thus, the attempt of ref.[10] to reproduce the p^4- contributions of the nonlinear Lagrangian (2) and the currents (5), (7) within the linear σ-model by taking into account intermediate scalar resonances seems to be not justified also for other phenomenological reasons.

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Table 1: Comparison of the $K \to 3\pi$ isotopic amplitudes, calculated in non-linear and linear chiral Lagrangian approaches (Born approximation)

a) Nonlinear chiral Lagrangian approach [9]

	Soft-pion limit			Inclusion of p^4-corrections		
	$\mathcal{O}_1$	$\mathcal{O}_4$	$\mathcal{O}_5$	$\mathcal{O}_1$	$\mathcal{O}_4$	$\mathcal{O}_5$
Re $\mathcal{A}_{11}$	-1.00		-22.42	-1.22		-19.22
Re $\mathcal{B}_{11}$	-1.00		-22.42	-0.70		-30.45
Re $\mathcal{A}_{13}$		1.00			1.22	
Re $\mathcal{B}_{13}$		-1.07			-0.91	
Re $\mathcal{B}_{23}$		6.93			7.39	
$\Delta^{(1/2,1/2)}$		0			165.4 Im c_5	
$\Delta^{(1/2,3/2)}$		22.5 Im c_5			26.4 Im c_5	
$\Delta'^{(1/2,3/2)}$		74.9 Im c_5			73.2 Im c_5	

b) Linear σ model [10]

	Soft-pion limit			Inclusion of p^4-corrections		
	$\mathcal{O}_1$	$\mathcal{O}_4$	$\mathcal{O}_5$	$\mathcal{O}_1$	$\mathcal{O}_4$	$\mathcal{O}_5$
Re $\mathcal{A}_{11}$	-1.00		-22.42	-1.80		-19.28
Re $\mathcal{B}_{11}$	-1.00		-22.42	-1.63		-26.19
Re $\mathcal{A}_{13}$		1.00			1.80	
Re $\mathcal{B}_{13}$		-1.25			-1.78	
Re $\mathcal{B}_{23}$		6.75			6.22	
$\Delta^{(1/2,1/2)}$		0			45.6 Im c_5	
$\Delta^{(1/2,3/2)}$		15.2 Im c_5			26.7 Im c_5	
$\Delta'^{(1/2,3/2)}$		49.6 Im c_5			39.3 Im c_5	