

**QUARK AND GLUON DISTRIBUTIONS AND  $\alpha_S$  FROM NUCLEON  
STRUCTURE FUNCTIONS AT LOW X**

New Muon Collaboration

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**Abstract**

The  $Q^2$  dependence of the structure functions  $F_2^p$  and  $F_2^d$  recently measured by the NMC is compared with the predictions of perturbative QCD at next-to-leading order. Good agreement is observed, leading to accurate determinations of the quark and gluon distributions in the range  $0.008 \leq x \leq 0.5$ . The strong coupling constant is measured from the low  $x$  data; the result agrees with previous determinations.

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## 1 Introduction

In a previous letter [1] the New Muon Collaboration (NMC, CERN-NA37) presented proton ( $F_2^p$ ) and deuteron ( $F_2^d$ ) structure functions obtained from simultaneous measurements of deep inelastic muon scattering on hydrogen and deuterium targets at incident muon energies of 90 and 280 GeV. The data cover a wide range in the Bjorken scaling variable  $x$  and in the square of the four-momentum transfer  $-Q^2$  ( $0.008 \leq x \leq 0.5$  and  $0.8 \leq Q^2 \leq 48 \text{ GeV}^2$ ). An important feature of these data is their extension with good accuracy to low values of  $x$ . In this letter we present the results of an analysis of the NMC data in terms of quantum chromodynamics (QCD).

In the QCD parton model the  $x$  and  $Q^2$  dependences of  $F_2$  are related to those of the quark and gluon distributions. The  $Q^2$  dependences of these distributions, due to the processes of gluon radiation and  $q\bar{q}$  pair creation, are obtained from the QCD evolution equations [2], using their  $x$  dependences at a given  $Q^2$  as inputs. Perturbative QCD does not predict the  $x$  dependences. In leading order (LO) the  $Q^2$  evolution of a linear combination  $F$  of quark ( $q$ ) and antiquark ( $\bar{q}$ ) distributions is given by:

$$\frac{\partial F(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_S(Q^2)}{2\pi} \left\{ \int_x^1 \frac{dy}{y} P_{qq}\left(\frac{x}{y}\right) F(y, Q^2) + \sum_{i=1}^f (c_i + \bar{c}_i) \int_x^1 \frac{dy}{y} P_{qg}\left(\frac{x}{y}\right) G(y, Q^2) \right\} \quad (1)$$

with

$$F(x, Q^2) = \sum_{i=1}^f \left\{ c_i q_i(x, Q^2) + \bar{c}_i \bar{q}_i(x, Q^2) \right\}. \quad (2)$$

Here the sum runs over the active flavours  $i = u, d, s, \dots$ ;  $\alpha_S$  is the strong coupling constant,  $P_{qq}$  and  $P_{qg}$  are QCD splitting functions and  $G$  is the gluon distribution. A similar evolution equation exists for  $G$ .

The function  $F$  can be expressed as a linear combination of a flavour non-singlet distribution, for which  $\sum(c_i + \bar{c}_i) = 0$ , and the singlet distribution which is the sum of all quark and antiquark distributions ( $c_i = \bar{c}_i$ ), see e.g. ref. [3]. Whereas the  $Q^2$  evolution of a non-singlet distribution does not depend on  $G$  (the second term on the right hand side of eq. (1) vanishes), the  $Q^2$  evolution of the singlet distribution is coupled to that of the gluon.

In addition to the logarithmic  $Q^2$  evolution predicted by perturbative QCD (eq.(1)), non-logarithmic contributions to the  $Q^2$  dependence of  $F_2$  come from target mass effects and the interaction of the struck quark with the spectator quarks (higher twist effects). At a given  $x$ , they both behave like power series in  $1/Q^2$  and may thus become important at low values of  $Q^2$ .

## 2 QCD Analysis

The essence of a QCD analysis of structure functions is the comparison of their  $Q^2$  dependence with the prediction of perturbative QCD. From parametrisations of the effective singlet quark, non-singlet quark and gluon distributions at a given scale  $Q_0^2$  –  $q^{SI}(x, Q_0^2)$ ,  $q^{NS}(x, Q_0^2)$  and  $G(x, Q_0^2)$  – and from the value of the strong coupling constant  $\alpha_S(Q_0^2)$ , one can compute structure functions at any  $x$  and  $Q^2$  using the QCD evolution equations. In a QCD fit to a set of structure function measurements the coefficients of these parametrisations and  $\alpha_S$  are adjusted to give the best agreement between the measured and the computed structure functions over the whole  $x$  and  $Q^2$  domain. In such a

procedure, the value of  $Q_0^2$  is arbitrary; in practice one often chooses a value typical of the data.

To perform this task, we have used a computer program developed from that of ref. [4]. This program performs a vectorized fully numerical integration of the evolution equations in next-to-leading order (NLO) in the  $\overline{\text{MS}}$  renormalisation and factorisation schemes [5]. In the program, the  $Q^2$  evolution (“running”) of the strong coupling constant was calculated from the NLO renormalisation group equation, with the flavour thresholds treated as described in ref. [6]. To calculate the  $Q^2$  evolution of the charmed sea quark distribution, which differs from that of quasi-massless quarks (u, d and s), the prescription of ref. [7] was adopted.

The fit was performed simultaneously on the measured values of  $F_2^p$ , which has both flavour singlet and non-singlet components, and of  $F_2^d = (F_2^p + F_2^n)/2$ , which is nearly a pure singlet structure function. This allows a reliable determination of both the singlet and non-singlet quark distributions to be made. We have checked that the small non-singlet component of  $F_2^d$ , proportional to  $q_s - q_c$ , changes the  $Q^2$  evolution of  $F_2$  negligibly. Thus, it proved more practical to determine the deuteron quark distribution  $q^d(x, Q_0^2)$ , instead of the singlet distribution  $q^{SI}(x, Q_0^2)$ , from the QCD fits. We assumed that the gluon distribution is the same in the proton and the deuteron; this is compatible with recent experimental results [8].

The value of  $Q_0^2$  was chosen to be 7 GeV<sup>2</sup> and the parametrisations used in the fit are:

$$xq^{NS}(x, Q_0^2) = Ax^\alpha(1-x)^\beta \quad (3)$$

$$xq^d(x, Q_0^2) = Bx^\gamma(1-x)^\delta(1+b_1v+b_2v^2) \text{ with } v = 0.1 - x \quad (4)$$

$$xG(x, Q_0^2) = C(1-x)^\eta(1+c_1w+c_2w^2+c_3w^3) \text{ with } w = 0.1 \ln(1 + e^{10-100x}) \quad (5)$$

The variable  $w$  in eq. (5) was chosen such that it differs from zero only for  $x < x_0 = 0.1$ , so that the behaviour of the gluon distribution at  $x > x_0$  has the usual form proportional to  $(1-x)^\eta$ . It was verified that these parametrisations are flexible enough to describe the distributions at  $Q^2$  values other than 7 GeV<sup>2</sup>; adding extra parameters or using other functional forms does not significantly improve the quality of the fit. In the fit the momentum sum rule was imposed; that is the singlet quark distribution and the gluon distribution were continued to  $x = 0$  and 1, and their integrals required to add up to unity:  $\int_0^1 x(q^{SI} + G)dx = 1$ . The sensitivity of the results to this assumption is discussed later.

The effect of higher twist contributions on the  $Q^2$  dependence of  $F_2$  cannot be calculated from theory. It was taken into account in the following way. The functions fitted to the data were parametrised as

$$F_2(x, Q^2) = F_2^{LT}(x, Q^2) \cdot \{1 + H(x)/Q^2\} , \quad (6)$$

where  $F_2^{LT}$  obeys the NLO QCD evolution equations and  $H(x)/Q^2$  is a phenomenological description of the twist-four contribution<sup>1)</sup>. The limited range in  $Q^2$  does not allow  $H(x)$  to be unambiguously determined from the present data. It was therefore kept fixed in the fit. Above  $x = 0.2$ ,  $H(x)$  was taken from ref. [9], averaged over the proton and the deuteron. At lower values of  $x$ ,  $H(x)$  was linearly extrapolated to  $x = 0$ . The value of

<sup>1)</sup> The function  $H(x)$  may also partly describe next-to-next-to-leading order QCD contributions or saturation effects in parton densities.

$H(x=0) = -0.13 \text{ GeV}^2$  gives the best agreement between the data and the result of the QCD fit. With this choice, higher twist contributions are moderate or small in the entire kinematic range of the data. The sensitivity to alternative extrapolations of  $H(x)$  at small  $x$  is discussed below.

Target mass corrections [10] were calculated from the measured structure functions and taken into account; they are small in the kinematic range of the data. Corrections for Fermi motion in the deuteron were estimated to be small and were not applied. Also shadowing in the deuteron was not taken into account.

We have excluded from the fit data with  $Q^2 < 1 \text{ GeV}^2$ . No further cuts on the data were made. In ref. [1], the measurement of  $F_2^p$  and  $F_2^d$  was obtained from the cross sections using a phenomenological parametrisation [11] for  $R$ , the ratio of longitudinally to transversely polarised virtual photon absorption cross sections. We have checked that using the QCD prediction for  $R$  [12] instead does not significantly affect the results of our analysis.

Values of the fitted parameters in eqs. (3-5) were obtained from a  $\chi^2$  minimisation procedure with the weights computed from statistical errors only. The relative normalisation  $\delta N$  between the 90 and 280 GeV data was a free parameter in the fits and the quantity  $(\delta N/\Delta N)^2$  was added to the  $\chi^2$ , where  $\Delta N = 2.1\%$  is the estimated uncertainty in this relative normalisation [1]. The treatment of other systematic errors is discussed in the next section.

### 3 Results on the quark and gluon distributions

We performed the QCD analysis in two parts. In the first one, described in this section, the value of  $\alpha_S$  was fixed to obtain the quark and gluon distributions with the best precision. We used the value of  $\alpha_S(Q_0^2 = 7 \text{ GeV}^2) = 0.240$ , which corresponds to  $\alpha_S(M_Z^2) = 0.113$ , the average of measurements from deep inelastic scattering [13], and which agrees with the recommended value of ref. [14].

In fig.1 the data are presented together with the result of the fit. The solid curves correspond to the QCD fit, including contributions from higher twist terms (see eq.(6)). This fit provides a good overall description of the data ( $\chi^2/\text{dof} = 333/239$ , statistical errors). The fitted relative normalisation of the 90 and 280 GeV data sets was found to be 1.018 ; in the fit (and also in fig. 1) the 90 GeV data were lowered by 0.7%, and the 280 GeV data raised by 1.1%. This is within the normalisation errors given in [1]. To illustrate the importance of higher twist effects,  $F_2^{LT}$  as defined in eq. (6) is also shown in fig. 1 (dashed curves). The fitted values of the parameters of  $xq^d(x, Q_0^2)$ ,  $xq^{NS}(x, Q_0^2)$  and  $xG(x, Q_0^2)$  in eqs. (3-5) are given in table 1.

Four kinds of error contribute to the uncertainty in the results of the fit.

- (i) The statistical error corresponding to a  $\chi^2$  increase of  $\chi^2/\text{dof}$ .
- (ii) The experimental systematic error obtained by repeating the fit with  $F_2$  offset according to each source of systematic error [1] in turn and adding the resulting deviations in quadrature. This procedure takes into account correctly the correlations for each source of systematic error, as described in the preprint version of ref. [1]. This error includes the effect of a  $\pm 2\%$  overall normalisation uncertainty.
- (iii) An error due to uncertainty in the continuation of the distributions into the unmeasured region,  $x = 0-0.008$ , which contains about 5% of the nucleon momentum. A 100% error was assigned to this estimate. The resulting uncertainty of the fits was determined by repeating them with the momentum sum constrained to 1.05 and 0.95.

- (iv) Errors due to the uncertainties in  $\alpha_S$  and higher twist effects. These are discussed below.

We quote as the error on the parametrisations of the quark and gluon distributions the quadratic sum of contributions (i) – (iii) only.

The non-singlet quark distribution  $xq^{NS}$  resulting from the fit is shown in fig. 2a at  $Q^2 = 7 \text{ GeV}^2$ . It corresponds to  $3(F_2^p - F_2^n)$  and can be used to estimate the Gottfried sum,  $\int_0^1 q^{NS}(x)/3 dx = 0.243 \pm 0.030$ , in agreement with the result of ref. [15]. As  $F_2^p - F_2^n$  is a non-singlet structure function, its  $Q^2$  evolution is determined by the strong coupling constant only. We have checked that the  $x$  dependence of the slopes  $d(F_2^p - F_2^n)/d \ln Q^2$  is consistent with the QCD prediction. The present data on  $F_2^p - F_2^n$  are not accurate enough however to significantly constrain  $\alpha_S$ .

The  $x$  dependence of the fitted deuteron quark distribution  $xq^d(x, Q_0^2)$  (corresponding to  $^{18/5}F_2^d$ ) and of the gluon distribution  $xG(x, Q_0^2)$  are shown in figs. 2b and 2c. The deuteron quark (gluon) distribution is extracted with an uncertainty of about  $\pm 5\%$  ( $\pm 20\%$ ) at  $x = 0.01$ . At higher  $x$  the uncertainty is smaller. The momentum fraction carried by the quarks was found to be  $0.55 \pm 0.02$  at  $Q^2 = 7 \text{ GeV}^2$ . At  $Q^2 = 1.25$  and  $48 \text{ GeV}^2$ , the extreme values of  $Q^2$  in our data, this fraction evolves to 0.60 and 0.52, respectively. The fitted gluon distribution, evolved to 1.25 and  $48 \text{ GeV}^2$  is shown in the insert of fig. 2c; at the lowest  $x$  it changes by an order of magnitude.

The sensitivity of the gluon distribution to the assumed value of  $\alpha_S$  is displayed in fig. 3a. The dashed curves are the results of fits with  $\alpha_S(7 \text{ GeV}^2)$  fixed to 0.215 and 0.265 which reflect the total error on  $\alpha_S$  from deep inelastic scattering experiments [13]. The uncertainty in  $\alpha_S$  has a negligible effect on the quark distribution. The sensitivity of the gluon distribution to higher twist effects is shown in fig. 3b. The dashed curve is the result of a fit with no higher twist terms. The effect of such terms on the quark distribution is much smaller. The sensitivities shown in figs. 3a and 3b represent upper limits and are comparable to the total experimental error.

The gluon distribution from this analysis is compared in fig. 4 to previous determinations from deep inelastic scattering data, from BCDMS and SLAC in NLO [9] and from CDHSW in LO [16]. The improvement in precision at low  $x$  is apparent.

We observe that the gluon distribution obtained in a similar kinematic range by NMC from an analysis of inelastic  $J/\psi$  production [17] agrees with the present result. Determinations of the gluon distribution from the observation of direct photons in hadron-hadron interactions [18], often obtained at larger  $x$  or  $Q^2$ , are also in agreement with the present result.

#### 4 Measurement of the strong coupling constant and test of QCD

In the second part of the analysis, we determined  $\alpha_S(Q_0^2)$  from the NMC data, by leaving it as a free parameter in the fit. This resulted in a value for the strong coupling constant

$$\alpha_S(7 \text{ GeV}^2) = 0.264 \pm 0.018 (\text{stat.}) \pm 0.070 (\text{syst.}) \pm 0.013 (\text{h.t.}) .$$

The systematic error includes sources of uncertainty listed under (ii-iii) in the previous section. The dominant sources of systematic error are the uncertainties in the spectrometer acceptance correction and on the energy calibration [1]. The ‘‘h.t.’’ error results from the uncertainty in the higher twist terms; it was obtained from fits where the function  $H(x)$  of eq. (6) was changed at  $x > 0.20$  within the errors given in [9], and at lower  $x$  such that  $H(x = 0)$  varied between 0 and  $-0.25 \text{ GeV}^2$ .

The present result corresponds to  $\alpha_S(M_Z^2) = 0.117^{+0.011}_{-0.016}$ . It is consistent with other measurements of  $\alpha_S$  [19], in particular with the present average from deep inelastic scattering,  $\alpha_S(M_Z^2) = 0.113 \pm 0.002$  (exp.) [13], used in the previous section for the determination of quark and gluon distributions. We have checked that applying different  $Q^2$  cuts on the data does not significantly change the value of  $\alpha_S$  obtained. Uncertainties of theoretical origin in  $\alpha_S$  are dominated by the arbitrariness of the choice of the renormalisation and factorisation scales; these have been studied in refs.[9, 20] and are small compared to our experimental error.

An imperfect representation of the  $x$  dependence of  $F_2$  in the fit may bias the result on  $\alpha_S$ . To check for such a bias, the fit was repeated with the gluon and quark distributions allowed to adjust to an optimal value in each bin of  $x$  separately. No statistically significant adjustments were found nor was the  $\chi^2$  substantially improved.

The agreement over the full  $x$  range of the  $Q^2$  dependences of the data with those predicted by QCD is the important illustration of their consistency. For that purpose, the average logarithmic slopes  $d \ln F_2 / d \ln Q^2$  were determined in each bin of  $x$  separately, both from the data and the QCD fit<sup>2)</sup>. These are shown in fig. 5; the points correspond to the  $Q^2$  evolution of the data and the solid curves to that of the fit. Good agreement is observed. In fig. 5, the dashed curves correspond to the  $Q^2$  evolution of  $F_2^{LT}$ , see eq. (6). The difference between the dashed and solid curves indicates that the higher twist contribution to the logarithmic slopes is moderate.

The dotted curves in fig. 5 indicate the  $Q^2$  evolution due to quarks only and the shaded areas between the dashed and dotted curves represent the contribution of gluons. It is clear that for most of the NMC data, the  $Q^2$  evolution is driven by the gluon distribution: thus the present analysis is sensitive to the product  $\alpha_S \times G(x)$ . As the gluon distribution can be constrained by the momentum sum rule,  $\alpha_S$  can be determined. In previous QCD analyses of deep inelastic scattering data,  $\alpha_S$  was mainly constrained by the  $Q^2$  evolution at high  $x$ , where the gluon distribution has a small influence. The present analysis extends the determination of  $\alpha_S$  and the test of QCD to the low  $x$  domain.

## 5 Conclusions

We have presented a next-to-leading order QCD analysis of the  $F_2$  structure functions of the proton and the deuteron recently obtained by the NMC. The  $Q^2$  evolution of  $F_2^p$  and  $F_2^d$  is in good agreement with perturbative QCD down to  $Q^2 = 1 \text{ GeV}^2$ , with only a moderate contribution from higher twist terms. The evaluation of the strong coupling constant  $\alpha_S$  at low  $x$  and  $Q^2$  agrees with previous determinations in deep inelastic scattering. We have obtained an accurate measurement of the quark and gluon distributions down to  $x = 0.008$ .

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<sup>2)</sup> In the latter case, an error equal to that of the measured  $F_2$  was assigned to the predicted values, in each bin of  $x$  and  $Q^2$ .

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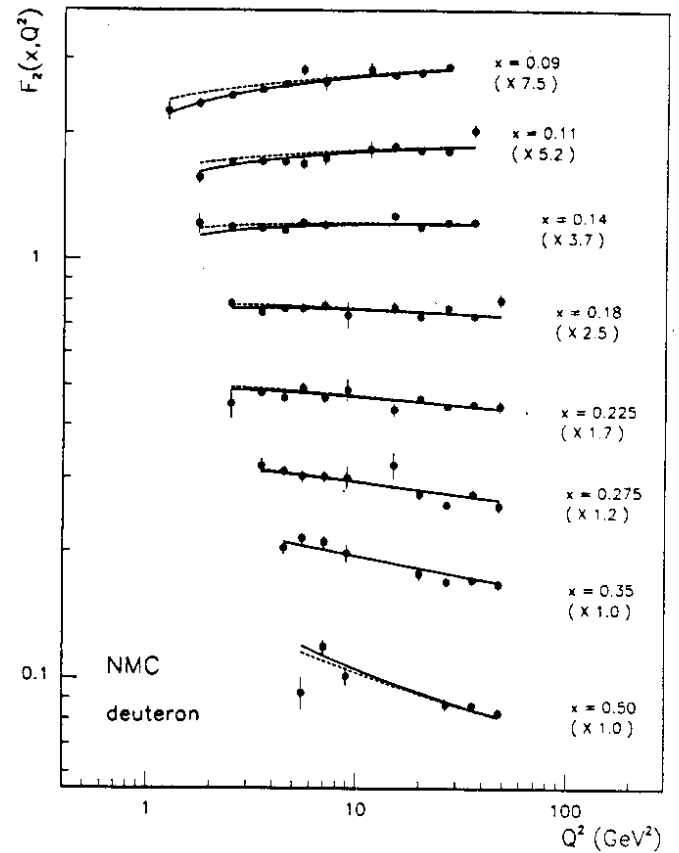
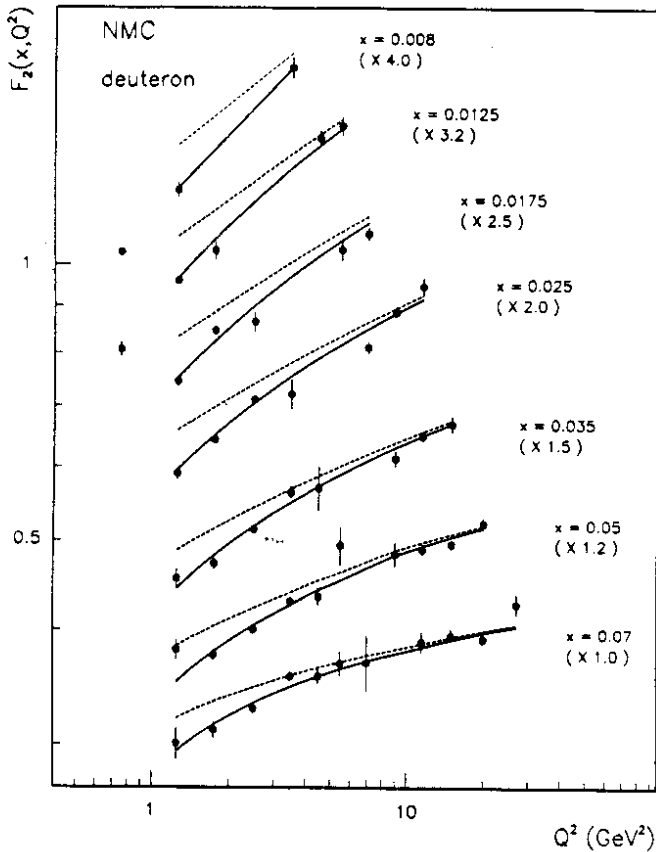
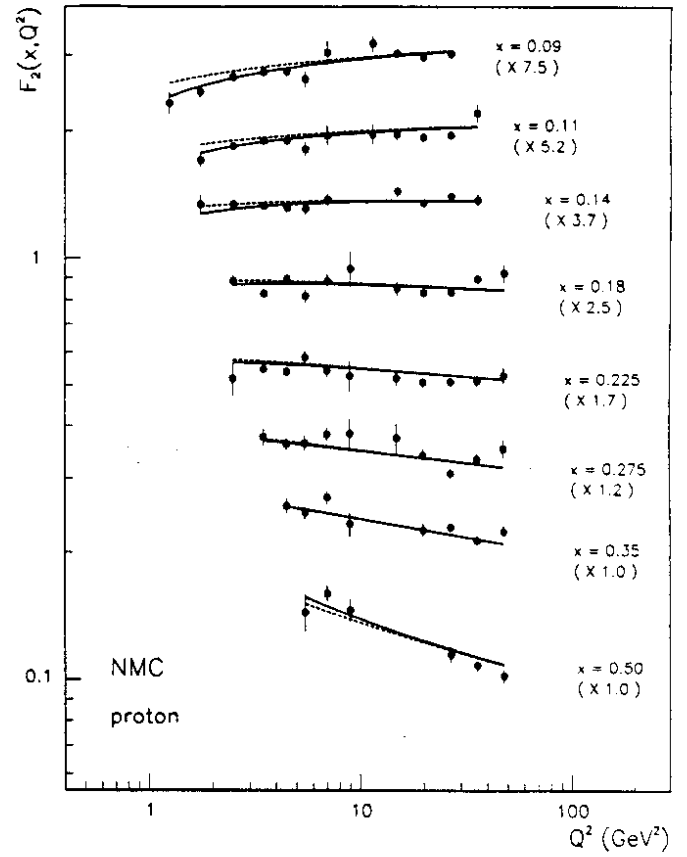
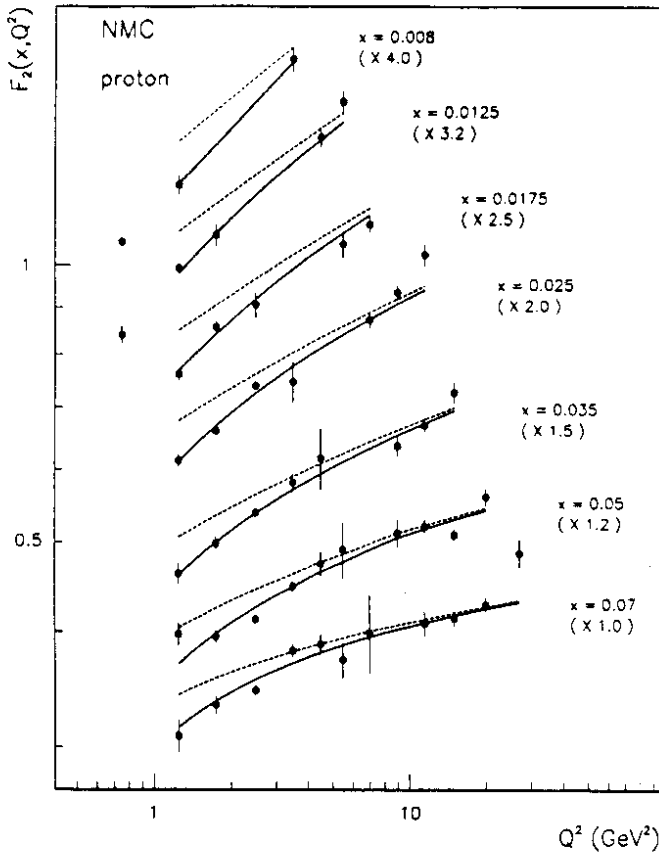
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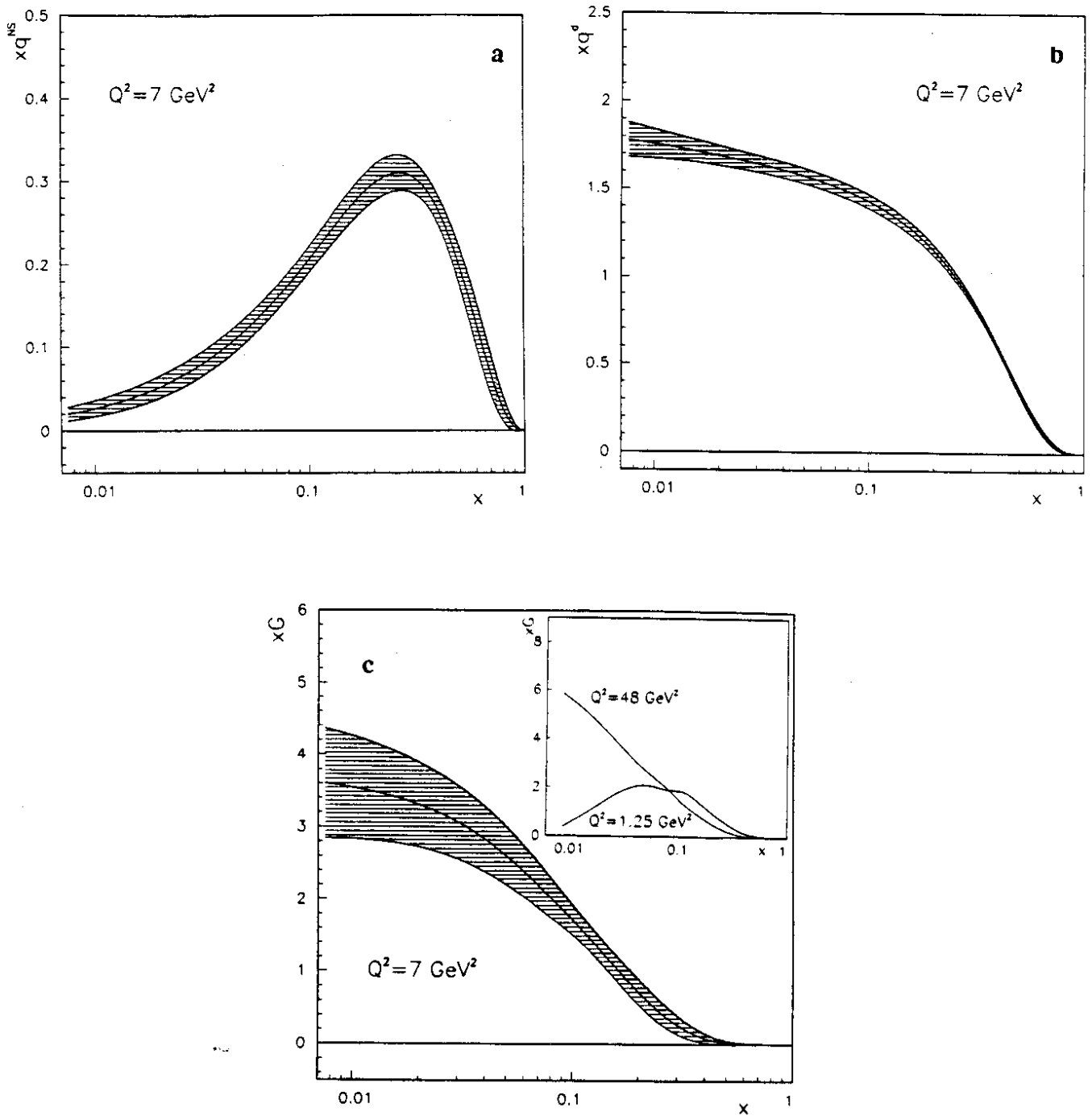
Table 1:

The values of the parameters of the fitted distributions  $xq^{NS}$ ,  $xq^d$  and  $xG$ , eqs. (3-5). Figures under 'Central value' correspond to the result of the fit with  $\alpha_s$  fixed to 0.240 at 7 GeV<sup>2</sup>. The columns 'Lower limit' ('Upper limit') correspond to a parametrisation of the lower (upper) bound of the error bands on the distributions shown in fig. 2. Note that there are twelve independent parameters because the singlet quark and gluon distributions are required to satisfy the momentum sum rule.

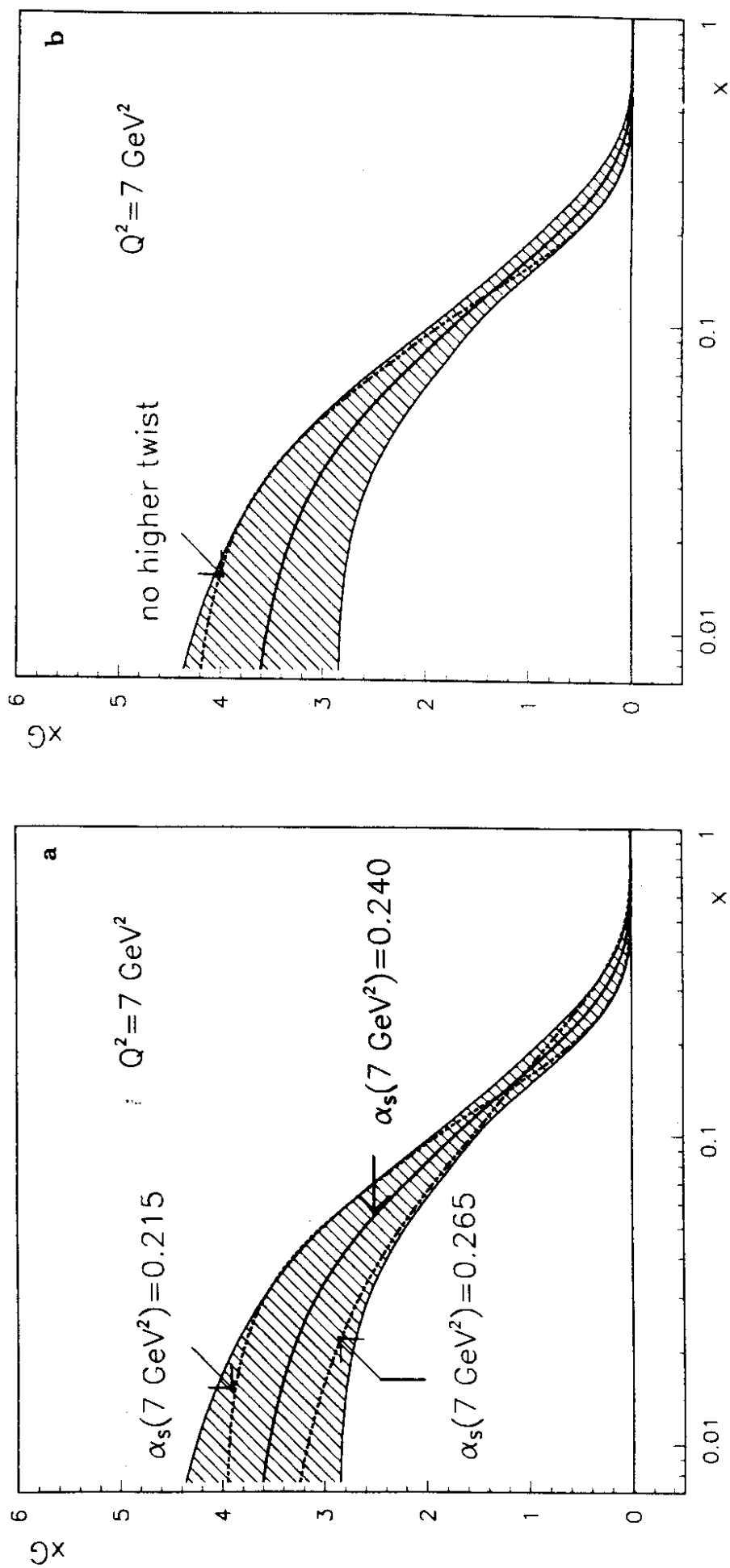
	Lower limit	Central value	Upper limit
A	2.249	2.774	3.700
$\alpha$	0.892	1.003	1.148
$\beta$	2.413	2.787	3.271
B	1.900	2.036	2.390
$\gamma$	-0.059	-0.037	0.022
$\delta$	3.725	4.159	4.675
$b_1$	-2.831	-3.086	-2.986
$b_2$	1.978	4.664	8.640
C	3.733	3.781	5.332
$\eta$	6.391	7.427	10.610
$c_1$	0.205	-0.105	-1.515
$c_2$	0.068	0.385	2.679
$c_3$	-0.029	-0.283	-1.669



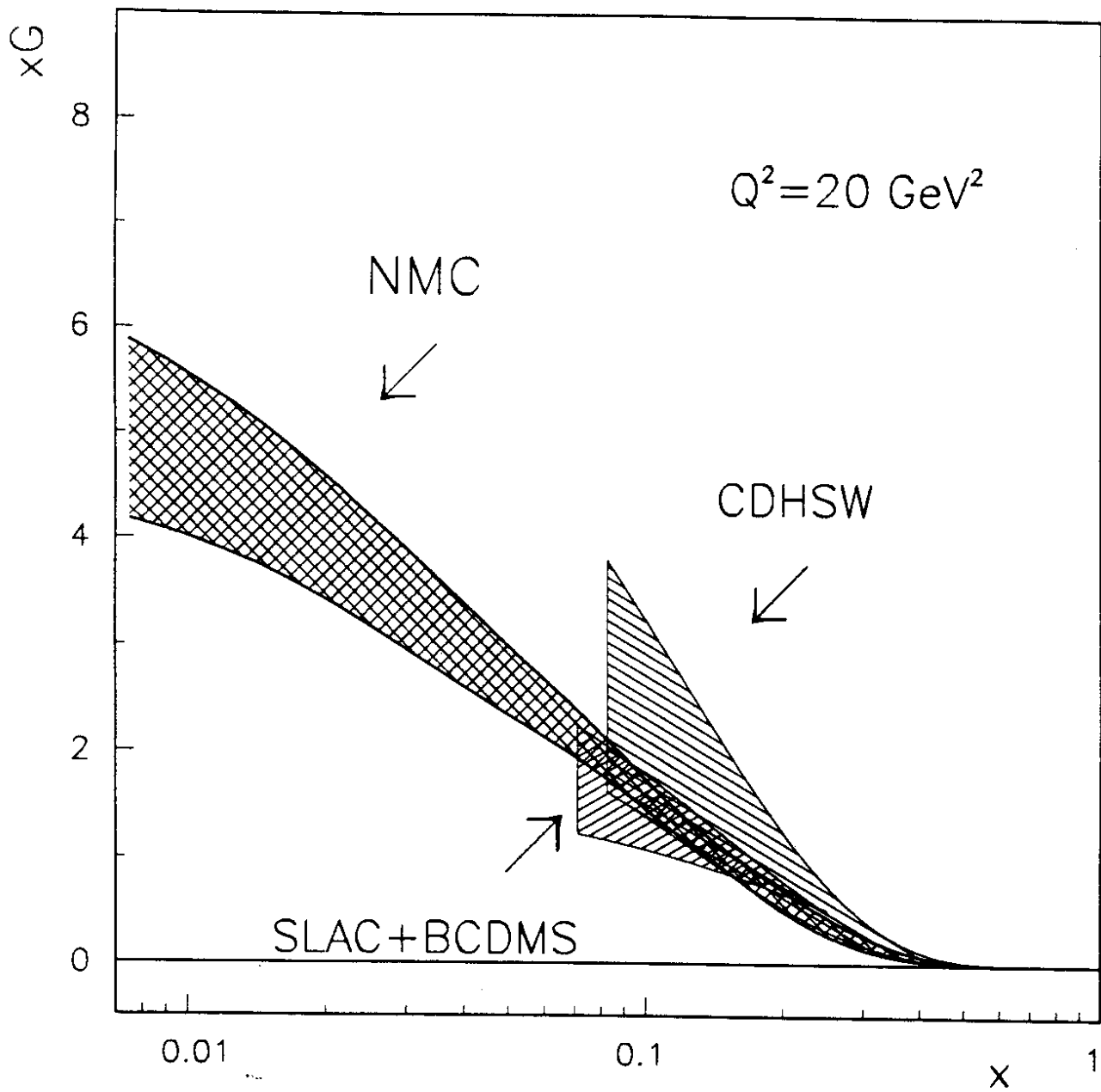
**Figure 1:** The structure functions  $F_2^p$  and  $F_2^d$  measured at incident muon energies of 90 and 280 GeV. The 90 (280) GeV data are renormalised by 0.993 (1.011). The errors shown are statistical only. The solid curves correspond to the result of the QCD fit described in section 3. The dashed curves are the result of the same fit with the higher twist contributions subtracted.



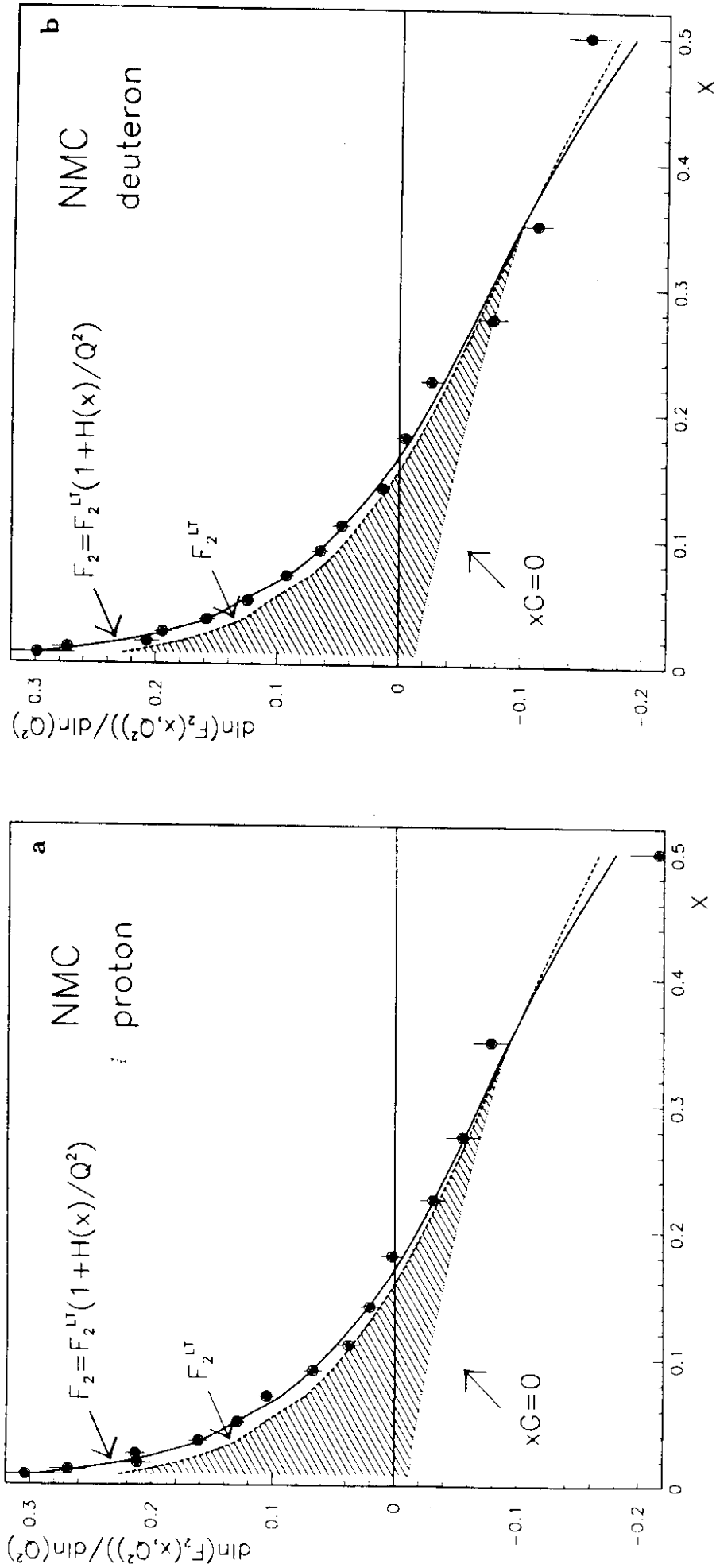
**Figure 2:** (a) The non-singlet quark distribution  $xq^{NS}$ , (b) the deuteron quark distribution  $xq^d$  and (c) the gluon distribution  $xG$ , all at a fixed value of  $Q^2 = 7 \text{ GeV}^2$ , obtained from this QCD analysis. The central curves correspond to the result of the fit described in section 3; the error is indicated by the bands around the curves. In the insert the gluon distribution from this analysis, evolved to  $Q^2 = 1.25 \text{ GeV}^2$  and  $48 \text{ GeV}^2$ , is shown. The wiggle is an artefact of the parametrisation used.



**Figure 3:** (a) The sensitivity of the gluon distribution to the assumed value of  $\alpha_s$ . The solid curve is the result of the QCD fit with  $\alpha_s$  fixed to 0.240 at 7 GeV<sup>2</sup>. The error band is identical to that of fig. 2c. The two dashed curves are the results of fits with  $\alpha_s$  fixed to 0.265 and 0.215. (b) The sensitivity of the gluon distribution to higher twist effects. The dashed curve corresponds to the gluon distribution obtained from a fit with all higher twist terms fixed to zero ( $H(x) \equiv 0$ ) and  $\alpha_s = 0.240$ . The solid curve and the error band are as in (a).



**Figure 4:** The gluon distribution from this analysis compared to two previous determinations in deep inelastic scattering, from the BCDMS and SLAC hydrogen and deuterium data in NLO [9] and from CDHSW iron data in LO [16].



**Figure 5:** The logarithmic slopes  $d \ln F_2 / d \ln Q^2$  of the data (circles) compared to those of the QCD fit of section 4 (solid curves) for (a) the proton and (b) the deuteron. The errors shown are statistical only. The dashed curves correspond to the QCD prediction without higher twist terms (see text). The dotted curves correspond to the  $Q^2$  evolution from quarks only.