Interference between initial- and final-state radiation

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1 Is there physics interest in the interference?

In this workshop we are looking at what can be learned from photon radiation off quarks at LEP. In such discussions one usually assumes that a large-angle isolated photon is always generated by 'final-state' radiation, i.e. the Feynman diagrams where the bremsstrahlung photon is actually emitted by the outgoing q or $\bar{q}$. At the Z resonance this assumptions is quite justified since (a) initial-state radiation (with the photon coming from the ingoing e^+e^- lines) is heavily suppressed, (b) such photons tend to come out at small angles, and (c) there is not much interesting physics in initial-state radiation (except for things as neutrino counting with which we do not deal here). It should not be forgotten, however, that there is also interference between initial- and final-state radiation, which (a) is only suppressed by about the square root of the amount of initial-state radiation suppression, (b) will be most important just when the photon is not particularly collinear with any fermion (hence isolated), and (c) may contain interesting physics. I have been asked to look into this.

The possible interest in the interference becomes clear if we review briefly the analysis of [1]. Forgetting for simplicity about mass effects and higher-order corrections, so that u and c quarks can be treated as identical, and d,s and b quarks likewise, the total hadronic width of the Z can be writtem as

$$\Gamma(Z \to q\bar{q}) = 2A_u + 3A_d \quad , \qquad (1)$$

where $A_{u,d}$ can be calculated and contain the electroweak coupling constants. Similarly, the radiative width is

$$\Gamma(Z \to q\bar{q}\gamma) = 2Q_u^2 A'_u + 3Q_d^2 A'_d \quad , \qquad (2)$$

with similar $A'_{u,d}$ that contain the same electroweak couplings in the same combinations as the $A_{u,d}$. Since the two widths depend on the A, A' in different linear combinations (because of the quark charges $Q_u = 2/3$ and $Q_d = -1/3$), a measurement of both allows one to extract information on the quark couplings to the Z. Similarly, *if* we can find an observable that depends on the interference between initial- and final-state radiation:

$$O_{\rm int} = 2Q_u A''_u + 3Q_d A''_d \ , \tag{3}$$

we may hope to repeat the same game and come up with yet another constraint on the quark electroweak couplings. Obviously, this can only help in the analysis, if even by a small amount.

Let us quickly look at previous experience. At PETRA, one has been able to measure (to some extent) the charge of a jet, but does not see a difference between a u jet and a $\bar{d}$ jet. The observation of a jet forward-backward asymmetry $A_{FB}^{\rm jet}$, moreover, gives direct evidence for interference since this asymmetry is zero at the Born level (at least, in the limit $m_Z \to \infty$). Also, in such an analysis the 'positive' jet is the q jet for u, but the $\bar{q}$ jet for d: since $A_{FB} > 0$ for u and $A_{FB} < 0$ for d, the effects from different flavours add up.

At LEP the situation would appear to be worse: we can measure the charge of a jet not much better than at PETRA, and there is already a forward-backward asymmetry at the Born level, which is positive since $A_{FB} \propto v_q a_q > 0$ for all flavours. We might therefore expect at least a partial cancellation of asymmetry effects between the different flavours.

In order to establish some physics interest in the interference we need: (a) to establish an *a-priori* measure for the observability of interference: a *quality factor*, and (b) to find an observable that is sensitive to the interference. This last point is the most difficult since no simple observable leaps to the mind, and it will need a lot of playing around with Monte Carlo simulations to find one: the quality factor, however, is easier to define and measure.

2 Quality factors for interference

General lore has it that the interference is small at the peak of the Z resonance, of order $\alpha\Gamma_Z/m_Z$. This is because of the finite Z lifetime which serves to separate the wavefunctions for emission by the initial and the final state. When tight cuts are imposed the interference would be back to its 'natural' value of a few per cent. It is not clear if the selection cuts on hard photons are tight in this sense. A look at figure 4 of reference [1] shows, depressingly, that once one adds the quark flavours the ratio of hard-photon cross sections $\sigma^{\rm int}/\sigma^{\rm tot}$ is very close to zero.

In our search for a quality factor we concentrate on the dynamics of the radiative matrix elements, in particular the Z propagator. Let us denote by x the photon energy in units of the beam energy, which we require to be larger than some value y. Then, the matrix element for initial-state radiation will look roughly like

$$M_i \sim \frac{1}{s(1-x) - m_Z^2 + im_Z\Gamma_Z} \quad , \tag{4}$$

and that for final-state radiation like

$$M_f \sim \frac{Q}{s - m_Z^2 + im_Z\Gamma_Z} \quad . \tag{5}$$

if we also assume an isolation cut that keeps the photon away from all the fermions, all other relevant invariant masses will be of the same order of magnitude so that these forms for $M_{i,f}$ may serve as a realistic model for the cross section. The interference has a chance to be sizeable if $| M_i | \sim | M_f |$: we define our quality factor F to be

$$F \equiv \frac{| M_f(x=y) |}{| M_i(x=y) |}. \tag{6}$$

A value of F close to unity can *in principle* be arranged by running at an energy above the Z peak and looking for sufficiently hard photons. Of course, the price to be paid is that the cros section will then be smaller than the peak cross section by a factor

$$S = \frac{m_Z^2\Gamma_Z^2}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2} \quad . \tag{7}$$

In table 1 I list F and S for various y and s values. I have used $m_Z = 91.2$ GeV and $\Gamma_Z = 2.5$ GeV. From these numbers I conclude that the interference as such is actually not tremendously small. Of course the quality factor improves with decreasing y: at this moment a value for y of 0.2 is more typical than 0.1, and for such an energy cut one has to move quite a bit away from the resonance to get to $F \sim 1$: it may not be reasonable to hope for a dedicated LEP run at, for instance $\sqrt{s} = 95$GeV just in order to see interference in the $u\bar{u}\gamma$ channel!

3 Searching for observables

In order to try and define an observable that is sensitive to the interference I used a simple Monte Carlo simulation to generate $q\bar{q}\gamma$ events using the following semi-realistic cuts. The invariants $m_{q\bar{q}}^2$, $m_{q\gamma}^2$, $m_{\bar{q}\gamma}^2$ must all be larger than $ys/2$ (each of the final particle energies larger than $(1-y)\sqrt{s}/2$), with $y = 0.10$ which is somewhat optimistic. The angle between the photon and the beams must be larger than some minimum which I took to be 30 degrees. Note that these cuts avoid most of the

Q_f	y	$\sqrt{s}$	F	S
-1/3	0.1	89	1.127	0.254
		m_Z	0.793	1
		91.8	1.005	0.796
		92.5	1.536	0.465
2/3	0.1	89	0.563	0.254
		m_Z	0.397	1
		92	0.562	0.692
		92.9	0.995	0.339
-1/3	0.2	89	0.683	0.254
		m_Z	0.408	1
		93.3	0.997	0.252
		94	1.365	0.159
2/3	0.2	89	0.341	0.254
		m_Z	0.204	1
		93	0.432	0.314
		95	1.007	0.093

Table 1: Quality and suppression factors for various energies.

peaking regions of the cross section, so that the only real structure in the matrix element is the $1/E_\gamma$ behaviour of the photon spectrum. For this reason a small dedicated Monte Carlo simulation is better than a full-fledged one such as **MUONMC** [2] which spends most of its times in the peaking regions. Having generated events I then computed the matrix elements both with and without interference (using the formulae in **MUONMC**). This will then give us any desired observable with and without interference so that its quality as a probe for the interference can be assessed.

I examined a few possible observables:

1. The total cross section σ. This is of course the most desirable from the statistics point of view.

2. The forward-backward cross section σ_{FB} where an event is added if the outgoing quark is in the e^+ hemisphere, and subtracted for the outgoing quark in the e^- hemisphere.

3. The 'cone' cross section σ_c where an event is added if the photon is inside the cone that fits between the outgoing quark and the e^+ beam (its opening angle is half the quark scattering angle), else subtracted. Since this is a specifically three-paticle correlation one might hope that this observable is more sensitive to interference effects.

	Q=2/3		Q=-1/3	
	$\sqrt{s} = m_Z$	$\sqrt{s}$ =92.9	$\sqrt{s} = m_Z$	$\sqrt{s} = 91.8$
σ	1.237 10^{-7}	5.22 10^{-9}	4.164 10^{-8}	3.773 10^{-8}
	0.018 10^{-7}	0.07 10^{-9}	0.055 10^{-8}	0.044 10^{-8}
	1.242 10^{-7}	4.73 10^{-8}	4.162 10^{-8}	3.46 10^{-8}
	0.018 10^{-7}	0.06 10^{-8}	0.055 10^{-8}	0.04 10^{-8}
σ_{FB}	-4.14 10^{-9}	3.75 10^{-9}	-3.83 10^{-10}	-4.45 10^{-9}
	3.28 10^{-9}	1.01 10^{-9}	10.83 10^{-10}	0.8 10^{-9}
	-1.69 10^{-9}	-2.97 10^{-9}	-1.61 10^{-9}	-1.59 10^{-8}
	3.29 10^{-9}	1.00 10^{-9}	1.08 10^{-9}	0.89 10^{-8}
σ_c	-1.19 10^{-7}	-5.08 10^{-8}	-4.00 10^{-8}	-3.20 10^{-8}
	0.02 10^{-7}	0.08 10^{-8}	0.61 10^{-8}	0.05 10^{-8}
	-1.20 10^{-7}	-4.54 10^{-8}	-3.99 10^{-8}	-3.31 10^{-8}
	0.02 10^{-7}	0.07 10^{-8}	0.06 10^{-8}	0.05 10^{-8}

Table 2: Results for three observables at different energies. For each observable the entries are in the first and second line the result and its error with interference included, the third and fourth line the result and its error witn no interference present.

I made runs both at the Z peak and at the 'optimal' energy, which has $F \sim 1$. In all cases two runs were made, one with and one without interference, using the same random numbers so as to minimize the effects of random fluctuations. In table 2 I present the resulting cross sections in GeV^{-2}.

4 Conclusions

After this admittedly brief examination I come to the following tentative conclusions. At the Z peak interference effects are always quite small, partly because of the lowish quality factors, but mainly because no good simple observable presents itself. At the optimal energies, the interference shows up as well (or as badly) in the total cross section σ as in the more complicated σ_{FB} and σ_c. Note that due to the very limited statistics, anything one does in the analysis by which events get subtracted (or, even worse, rejected) will deteriorate the statistical error substantially. Other alternative observables that I looked at, such as

$$
\begin{aligned}
\sigma_{FB}^{q\gamma} &= q_F\gamma_F - q_F\gamma_B - q_B\gamma_F + q_B\gamma_B \ , \\
\sigma_{\text{angle}}^{q\gamma} &= \sigma(\theta_\gamma < \theta_q) - \sigma(\theta_\gamma < \theta_q) \ ,
\end{aligned}
\tag{8}
$$

with obvious notation, do not behave better.

In all, interference between initial- and final-state bremsstrahlung is larger, on the amplitude level, than one might naively expect on the basis of current LEP folk-lore. On the other hand, no observable appears to be significantly more sensitive to it than the good old total cross section. The phenomenological interest in the interference may not be zero, but is at any rate quite small. With a high-luminosity option for LEP some additional information may be gleaned from it on (for instance) the quark couplings, but it may not be expected to be very significant.

References

[1] P. Mättig and W. Zeuner, Z. Phys. **C52**(1991)31, and these proceedings.

[2] R. Kleiss, in 'Z physics at LEP', G. Altarelli, R. Kleiss and C, Verzegnassi, eds., CERN 86-02, Vol. 3.