



Radiative D^* Decay Using Heavy Quark and Chiral Symmetry

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Abstract

The implications of chiral $SU(3)_L \times SU(3)_R$ symmetry and heavy quark symmetry for the radiative decays $D^{*0} \rightarrow D^0 \gamma$, $D^{*+} \rightarrow D^+ \gamma$, and $D_s^* \rightarrow D_s \gamma$ are discussed. Particular attention is paid to $SU(3)$ violating contributions of order $m_q^{1/2}$. Experimental data on these radiative decays provide constraints on the $D^* D \pi$ coupling.

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Recent CLEO data [1] (see Table 1) have brought the D^{*0} and D^{*+} branching ratios into agreement with expectations based on the constituent quark model [2]. In this letter, the rates for D^* decay are described in a model independent framework which incorporates the constraints on the decay amplitudes imposed by the heavy quark and chiral $SU(3)_L \times SU(3)_R$ symmetries of QCD.

Table 1: D^* Branching Ratios (%)

Decay Mode	Branching Ratio
$D^{*0} \rightarrow D^0 \pi^0$	$63.6 \pm 2.3 \pm 3.3$
$D^{*0} \rightarrow D^0 \gamma$	$36.4 \pm 2.3 \pm 3.3$
$D^{*+} \rightarrow D^0 \pi^+$	$68.1 \pm 1.0 \pm 1.3$
$D^{*+} \rightarrow D^+ \pi^0$	$30.8 \pm 0.4 \pm 0.8$
$D^{*+} \rightarrow D^+ \gamma$	$1.1 \pm 1.4 \pm 1.6$

At low momentum the strong interactions of the D and D^* mesons are described by the chiral Lagrange density [3]

$$\mathcal{L} = -i \text{Tr} \bar{H}_a \not{v} \partial^\mu H_a + \frac{1}{2} \text{Tr} \bar{H}_a H_b \not{v}_\mu [\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger]_{ba} + \frac{1}{2} g \text{Tr} \bar{H}_a H_b \gamma_\mu \gamma_5 [\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger]_{ba} + \dots \quad (1)$$

where the ellipsis denotes operators suppressed by factors of $1/m_Q$ and operators with more derivatives or factors of the light quark mass matrix. In Eq. (1), v^μ is the four velocity of the heavy meson. The field ξ is written in terms of the octet of pseudo-Nambu-Goldstone bosons

$$\xi = \exp(i\mathcal{M}/f), \quad (2)$$

where

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}. \quad (3)$$

At tree level f can be set equal to f_π , f_K or f_η . Our normalization convention has $f_\pi \simeq 132$ MeV. Under chiral $SU(3)_L \times SU(3)_R$ transformations,

$$\xi \rightarrow L\xi U^\dagger = U\xi R^\dagger, \quad (4)$$

where $L \in SU(3)_L$ and $R \in SU(3)_R$, and U is defined implicitly by Eq. (4). H_a is a 4×4 matrix that contains the D and D^* fields:

$$H_a = \frac{1}{2} (1 + \not{v}) [D_a^{*\mu} \gamma_\mu - D_a \gamma_5], \quad (5)$$

$$\bar{H}_a = \gamma^0 H_a^\dagger \gamma^0.$$

The index a represents light quark flavor, where $(D_1, D_2, D_3) = (D^0, D^+, D_s)$ and $(D_1^*, D_2^*, D_3^*) = (D^{*0}, D^{*+}, D_s^*)$. Under $SU(2)_c$ heavy quark spin symmetry and chiral $SU(3)_L \times SU(3)_R$ symmetry, H_a transforms as

$$H_a \rightarrow S(HU^\dagger)_a, \quad (6)$$

where $S \in SU(2)_v$. The $D^* D \pi$ coupling constant g is responsible for the $D^* \rightarrow D \pi$ decays. At tree level,

$$\Gamma(D^{*+} \rightarrow D^0 \pi^+) = \frac{g^2}{6\pi f_\pi^2} |\vec{p}^*|^3. \quad (7)$$

The decay width for $D^{*+} \rightarrow D^+ \pi^0$ is a factor of two smaller by isospin symmetry. The experimental upper limit [4] on the D^{*+} width of 131 keV when combined with the $D^{*+} \rightarrow D^+ \pi^0$ and $D^{*+} \rightarrow D^0 \pi^+$ branching ratios in Table 1 leads to the limit $g^2 \lesssim 0.5$.

The axial vector current obtained from the Lagrangian (1) is

$$\bar{q}_a T_{ab}^A \gamma_\nu \gamma_5 q_b = -g \text{Tr} \bar{H}_a H_b \gamma_\nu \gamma_5 T_{ba}^A + \dots \quad (8)$$

In Eq. (8) the ellipsis represents terms containing one or more Goldstone boson fields and T^A is a flavor $SU(3)$ generator. Treating the quark fields in Eq. (8) as constituent quarks and using the nonrelativistic quark model to estimate the D^* matrix element of the l.h.s. of Eq. (8) gives $g = 1$. (A similar estimate of the pion-nucleon coupling gives $g_A = 5/3$.) In the chiral quark model [5] there is a constituent quark-pion coupling gives the measured pion-nucleon coupling to determine the constituent quark pion coupling gives $g \simeq 0.8$. Thus various constituent quark model estimates lead to the expectation that g is near unity. In this paper, however, we wish to adopt a model independent approach to radiative D^* decay. From the point of view of chiral perturbation theory g is a free parameter and its value must be determined from experiment.

The $D_a^* \rightarrow D_a \gamma$ matrix element has the form

$$\mathcal{M}(D_a^* \rightarrow D_a \gamma) = e \mu_a \epsilon^{\mu\alpha\beta\lambda} \epsilon_\mu^*(\gamma) v_\alpha k_\beta \epsilon_\lambda(D_a^*), \quad (9)$$

where $e\mu_a/2$ is the transition magnetic moment, k is the photon momentum, $\epsilon(\gamma)$ is the polarization of the photon and $\epsilon(D^*)$ is the polarization of the D^* . The resulting decay rate is

$$\Gamma(D_a^* \rightarrow D_a \gamma) = \frac{\alpha}{3} |\mu_a|^2 |\vec{k}|^3. \quad (10)$$

The $D_a^* \rightarrow D_a \gamma$ matrix element gets contributions from the photon coupling to the light quark part of the electromagnetic current, $\frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s$, and the photon coupling to the heavy charm quark part of the electromagnetic current, $\frac{2}{3} \bar{c} \gamma_\mu c$. The part of μ_a that comes from the charm quark piece of the electromagnetic current, $\mu^{(h)}$, is determined by heavy quark symmetry. In the effective heavy quark theory [6], the Lagrange density for strong and electromagnetic interactions of the charm quark is

$$\begin{aligned} \mathcal{L} = & \bar{h}_v^{(c)} (i v \cdot D) h_v^{(c)} + \frac{1}{2m_c} \bar{h}_v^{(c)} (i D)^2 h_v^{(c)} \\ & - \frac{g_s}{2m_c} \bar{h}_v^{(c)} \sigma^{\mu\nu} T^a h_v^{(c)} G_{\mu\nu}^a - \frac{e}{3m_c} \bar{h}_v^{(c)} \sigma^{\mu\nu} h_v^{(c)} F_{\mu\nu} + \dots \end{aligned} \quad (11)$$

In Eq. (11), D_μ is the covariant derivative

$$D_\mu = \partial_\mu + ig_s A_\mu^a T^a + \frac{2}{3} ie A_\mu, \quad (12)$$

where g_s is the strong coupling and e is the electromagnetic coupling. The ellipsis denotes terms with more factors of $1/m_c$. It is to be understood that the operators and couplings in Eq. (11) are evaluated at a subtraction point $\mu = m_c$, and that corrections of order $\alpha_s(m_c)$ have been neglected. The last term in Eq. (11) is responsible for a D^* to D transition matrix element $\mu^{(h)}$. By heavy quark symmetry [7],

$$\mu^{(h)} = \frac{2}{3m_c}, \quad (13)$$

where $\mu^{(h)}$ is independent of the light quark flavor. Perturbative $\alpha_s(m_c)$ corrections to the above are computable, while corrections suppressed by a power of $1/m_c$ are related to those which occur in semileptonic $\bar{B} \rightarrow D^* e \bar{\nu}_e$ decays [8]. At order $1/m_c^2$, Eq. (13) becomes $\mu^{(h)} = (2/3m_c) [1 - 4\xi_+(1)/m_c]$, where ξ_+ is defined in Ref. [8].

The part of μ_a that comes from the photon coupling to the light quark piece of the electromagnetic current, $\mu_a^{(l)}$, is not fixed by heavy quark symmetry. The light quark piece of the electromagnetic current transforms as an octet under $SU(3)$ flavor symmetry

Since there is only one way to combine an 8, 3 and $\bar{3}$ into a singlet, in the limit of $SU(3)$ symmetry, the $\mu_a^{(l)}$ are expressible in terms of a single reduced matrix element,

$$\mu_a^{(l)} = Q_a \beta, \quad (14)$$

where β is an unknown constant and Q_a denotes the light quark charges $Q_1 = 2/3, Q_2 = -1/3, Q_3 = -1/3$. In the nonrelativistic constituent quark model $\beta \simeq 3 \text{ GeV}^{-1}$. Note that Eq. (14) includes effects suppressed by powers of $1/m_c$, since it follows from using only $SU(3)$ symmetry.

The leading $SU(3)$ -violating contribution to the transition amplitudes has a nonanalytic dependence on m_q of the form $m_q^{1/2}$ which arises from the one-loop Feynman diagrams shown in fig. 1. The strange quark mass, m_s , is not very small, and so the corrections to Eq. (14) from $SU(3)$ violation may be comparable to $\mu^{(h)}$, which is suppressed by $1/m_c$ relative to $\mu^{(l)}$. Including the leading $SU(3)$ violations, $\mu_a^{(l)}$ becomes

$$\begin{aligned} \mu_1^{(l)} &= \frac{2}{3} \beta - \frac{g^2 m_K}{4\pi f_K^2} - \frac{g^2 m_\pi}{4\pi f_\pi^2}, \\ \mu_2^{(l)} &= -\frac{1}{3} \beta + \frac{g^2 m_\pi}{4\pi f_\pi^2}, \\ \mu_3^{(l)} &= -\frac{1}{3} \beta + \frac{g^2 m_K}{4\pi f_K^2}. \end{aligned} \quad (15)$$

The difference between using $f = f_\pi$ and $f = f_K$ in Eq. (15) is a higher order effect. We have chosen to use $f = f_K \simeq 1.22 f_\pi$ for loops involving kaons and $f = f_\pi$ for loops involving pions. For $m_K \neq m_\pi$, the one loop contribution to $\mu_1^{(l)}$, $\mu_2^{(l)}$ and $\mu_3^{(l)}$ is not in the ratio 2 : -1 : -1 and hence violates $SU(3)$. It is easy to understand why the one-loop correction proportional to m_K is different for the $D^{*0} \rightarrow D^0 \gamma$ and $D^{*+} \rightarrow D^+ \gamma$ decays. Strong interactions can change a D^{*0} into a virtual $K^- D_s^*$ pair, while the D^{*+} changes into a virtual $K^0 D_s^*$ pair. In the latter case the virtual kaon is neutral and doesn't couple to the photon. Thus there is no $m_s^{1/2}$ correction to $\mu_2^{(l)}$. The most important correction to Eq. (15) comes from $SU(3)$ violating terms of order m_s . These terms are analytic in the strange quark mass, and are not determined by the lowest order Lagrangian.

Using

$$\mu_a = \mu_a^{(l)} + \mu^{(h)}, \quad (16)$$

with $\mu_a^{(l)}$ and $\mu^{(h)}$ given by Eqs. (15) and (13) respectively, determines the rates for $D^{*0} \rightarrow D^0 \gamma$, $D^{*+} \rightarrow D^+ \gamma$ and $D_s^* \rightarrow D_s \gamma$ in terms of β and g . Combining this with Eq. (7) and

using the measured value of $\text{BR}(D^{*0} \rightarrow D^0 \gamma) / \text{BR}(D^{*0} \rightarrow D^0 \pi^0)$ gives g as a function of the branching ratio for $D^{*+} \rightarrow D^+ \gamma$. This in fact gives four different solutions for g^2 ; we eliminated three of these by imposing the constraints $g < 1$ (as required by Ref. [4]) and $\mu_a^{(c)} > \mu^{(h)}$ i.e., the light quark transition moment is greater than that of the heavy quark. The result is shown in fig. 2. (We have taken $m_c = 1.7 \text{ GeV}$.) Note that the favored values for g are smaller than what is expected on the basis of the nonrelativistic constituent quark model. Since $1/m_c$ effects have been included in the radiative D^* decays, the value of g extracted in this way is an "effective" value of g that includes $1/m_c$ corrections. From Eq. (7) and our values of g we can compute the total width of the D^{*+} as a function of $\text{BR}(D^{*+} \rightarrow D^+ \gamma)$; this is plotted in fig. 3.

The $SU(3)$ violation plays an important role in our analysis. Fig. 4 shows the absolute values of the relative contributions to μ_1 of $\mu^{(h)}$ (dashed-dotted line), β (dotted line) and the one-loop nonanalytic contribution to $\mu_1^{(c)}$ (solid line). The values have all been multiplied by $3/2$, so that the dotted line is normalized to β . Note that values of β near the non-relativistic constituent quark model expectation of $\approx 3 \text{ GeV}^{-1}$ favor a small $D^{*+} \rightarrow D^+ \gamma$ branching ratio, and hence smaller values of g . In fig. 5 the value of g that follows from neglecting $SU(3)$ violation (i.e. using Eq. (14) for $\mu_a^{(c)}$) is shown. Larger values of g are favored when $SU(3)$ violation is neglected.

Nonanalytic dependence on m_s similar to what we have found in radiative D^* decay occurs in the $D_s - D^+$ mass difference. Including effects up to order $m_s^{3/2}$ [9]

$$m_{D_s} - m_{D^+} = C m_s - \frac{3g^2}{64\pi f_\pi^2} (2m_K^2 + m_\eta^2), \quad (17)$$

where we have set $m_u = m_d = 0$ and C is an unknown constant. Experimentally, $m_{D_s} - m_{D^+} \approx 100 \text{ MeV}$. The magnitude of the nonanalytic part is about 50% of the mass difference for $g = 0.5$. This gives us some confidence that the expansion is well behaved for at least some of the range of g 's in fig. 2.

The analysis in this paper allows us to predict the $D_s^* \rightarrow D_s \gamma$ rate as a function of the $D^{*+} \rightarrow D^+ \gamma$ branching ratio. However, for $D_s^* \rightarrow D_s \gamma$ there is a strong cancellor between $\mu_3^{(c)}$ and $\mu^{(h)}$, resulting in a very small D_s^* width. (Note that $D_s^* \rightarrow D_s \pi$ is forbidden by isospin.) In this situation, $SU(3)$ violating terms of order m_s may be very important.

Since heavy quark symmetry ensures that g and β are the same in the b and c systems (up to corrections of order $1/m_c$), the results of this paper can be used to predict the widths for radiative B^* decay. Neglecting effects of order $1/m_b$ and $1/m_c$, Eq. (10) becomes

$$\Gamma(B_s^* \rightarrow B_c \gamma) = \frac{\alpha}{3} |\mu_a^{(c)}|^2 |k|^3 \quad (18)$$

where $\mu_a^{(c)}$ is given by Eq. (15). An analysis of the radiative decays of charmed baryons using the same methods is possible. Unfortunately, at the present time there is no experimental information on radiative charmed baryon decays.

Work similar to that presented in this paper has also been done by Cho and Georgi [10]. We are grateful to them for communicating their results to us prior to publication. This work was supported in part by the Department of Energy under grant number DOE-FG03-90ER40546 and contract number DEAC-03-81ER40050, and by a National Science Foundation Presidential Young Investigator award number PHY-8958081. MJS acknowledges the support of a Superconducting Supercollider National Fellowship from the Texas National Research Laboratory Commission under grant FCFY9219.

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Figure Captions

- Fig. 1. Diagrams giving the leading non-analytic contributions to $\mu_a^{(4)}$.
- Fig. 2. The coupling constant g as a function of $\text{BR}(D^{*+} \rightarrow D^+\gamma)$ including leading $SU(3)$ -breaking effects. The shaded region indicates the uncertainty due to the 1σ variations in $\text{BR}(D^{*0} \rightarrow D^0\pi^0)$ and $\text{BR}(D^{*0} \rightarrow D^0\gamma)$. The arrows indicate the 90% confidence level limits on $\text{BR}(D^{*+} \rightarrow D^+\gamma)$ and the D^{*+} width.
- Fig. 3. Width of the D^{*+} as a function of $\text{BR}(D^{*+} \rightarrow D^+\gamma)$ including leading $SU(3)$ -breaking effects. The shaded region indicates the uncertainty due to the 1σ variations in $\text{BR}(D^{*0} \rightarrow D^0\pi^0)$ and $\text{BR}(D^{*0} \rightarrow D^0\gamma)$. The arrows indicate the 90% confidence level limits on $\text{BR}(D^{*+} \rightarrow D^+\gamma)$ and the D^{*+} width.
- Fig. 4. Relative contributions to μ_1 of $\mu^{(4)}$ (dashed-dotted line), β (dotted line), and the one-loop nonanalytic $m_q^{1/2}$ term (solid line) to the matrix element for $D^{*0} \rightarrow D^0\gamma$.
- Fig. 5. The coupling constant g as a function of $\text{BR}(D^{*+} \rightarrow D^+\gamma)$ ignoring $SU(3)$ violation. The shaded region indicates the uncertainty due to the 1σ variations in $\text{BR}(D^{*0} \rightarrow D^0\pi^0)$ and $\text{BR}(D^{*0} \rightarrow D^0\gamma)$. The arrows indicate the 90% confidence level limits on $\text{BR}(D^{*+} \rightarrow D^+\gamma)$ and the D^{*+} width.

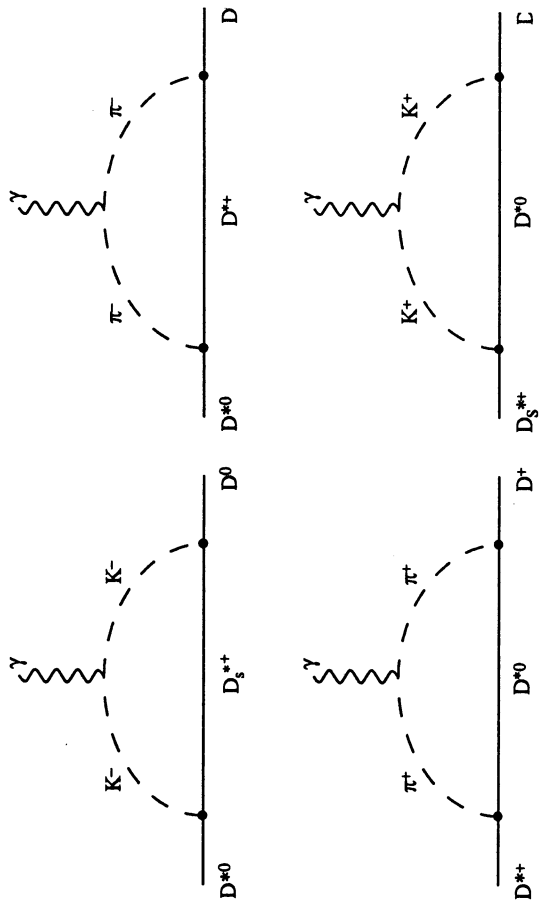


Figure 1

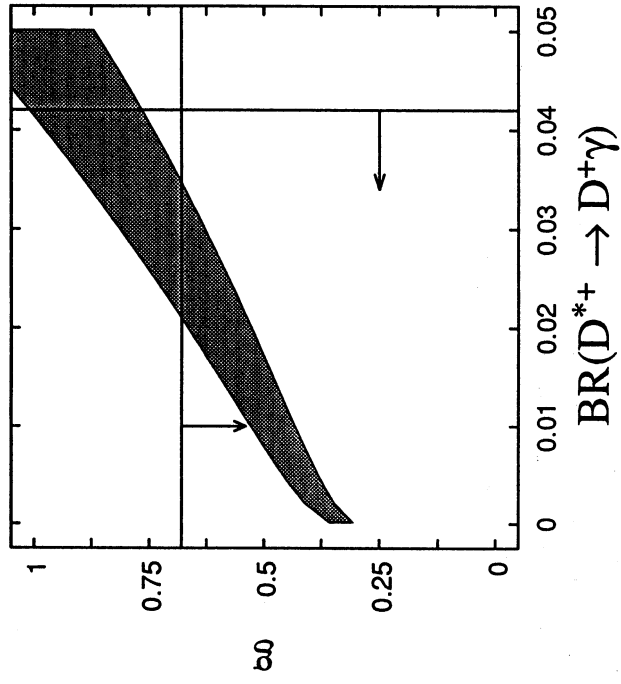


Figure 2

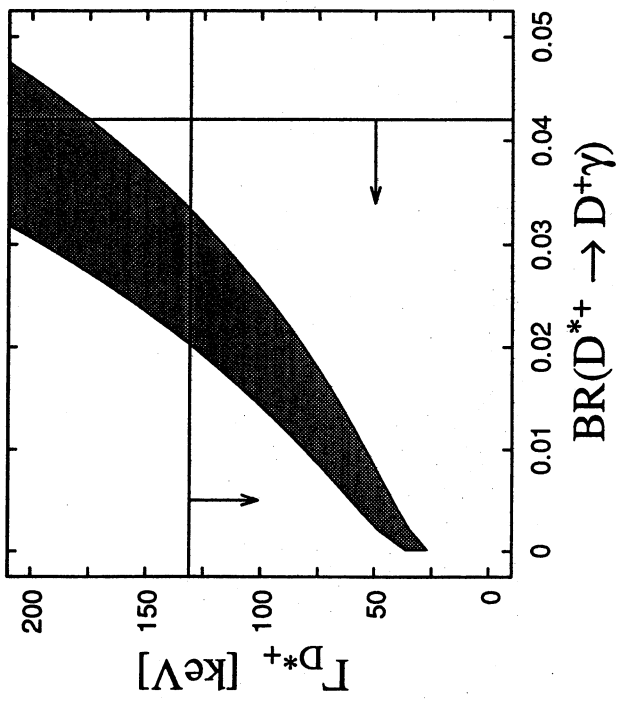


Figure 3

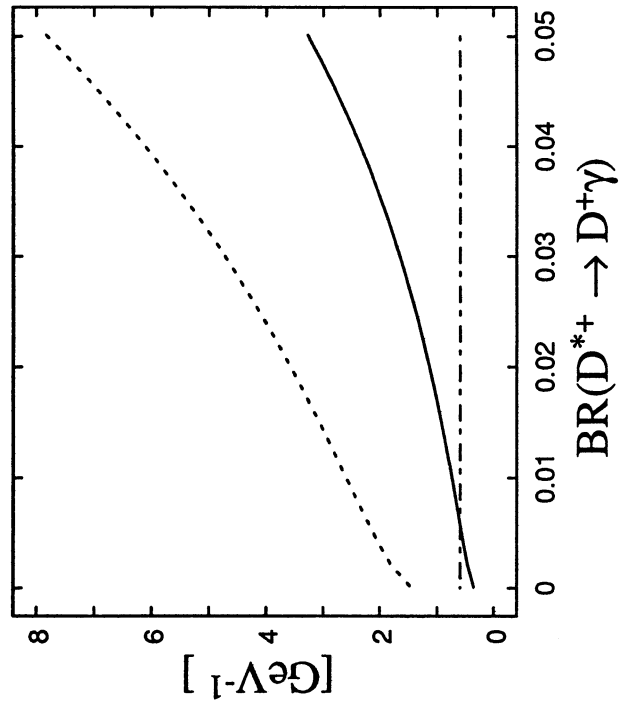


Figure 4

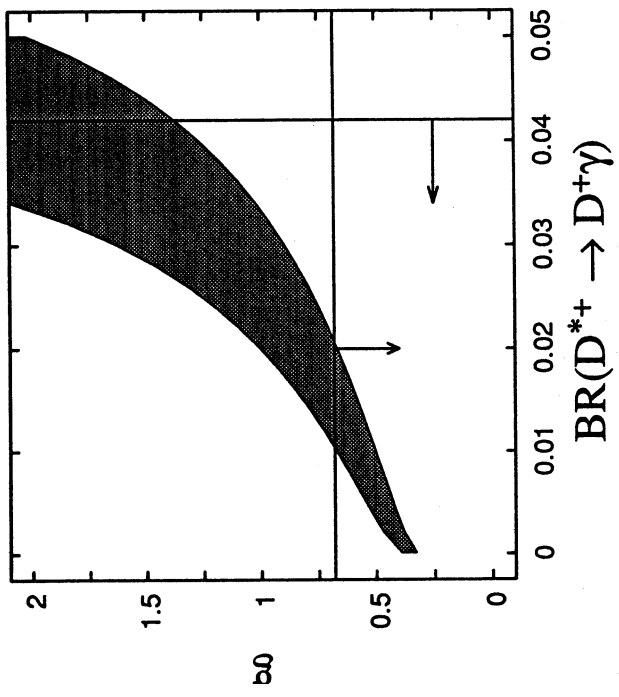


Figure 5