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Factorization Hypothesis for B Nonleptonic Decays within the Heavy Quark Effective Theory

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ABSTRACT

The heavy quark effective theory is applied to B-meson nonleptonic decays $\bar{B}^0 \to D^{(*)+}\pi$, $D^{(*)+}\rho$ at the level of $O(\overline{\Lambda}/m_c)$ ($\overline{\Lambda} \simeq O(\Lambda_{\rm QCD})$) corrections. It is shown that, with several parameters determined via semileptonic decays $\bar{B}^0 \to D^{(*)+}\ell\bar{\nu}_\ell$, these nonleptonic decays are well described under the factorization hypothesis, which can be interpreted as a new test of this hypothesis beyond the lowest order approximation.

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A new technique has been developed over the past few years in studies of hadrons containing a single heavy quark (Q): the heavy quark effective theory (hereafter HQET) [1].[♮1] We can thereby express various matrix elements of those hadrons with a small number of form factors. In particular, we need to know only one function, i.e., the so-called Isgur-Wise function in the limit $m_Q \to \infty$, which drastically simplifies treatments of various heavy hadron decays.

Several authors have made analyses (mainly of B semileptonic decays) in order to test the validity of this new procedure [3-7], and some results in favor of it have so far been obtained. The purpose of this article is not to make a further test of the HQET, but to study B meson nonleptonic decays by using this method as a tool of computations. It must be premature to conclude that the HQET has been established, but we believe it quite meaningful to explore such a possibility. We take here a view that the HQET technique is applicable to the b and c quarks safely, though in Refs. [5,7] has been gotten an indication that it works even for the strange quark.

Concretely, we shall perform a test of the factorization hypothesis

$$\langle BC|\,\bar{q}\Gamma_\mu q'\bar{q}''\Gamma^\mu q'''\,|A\rangle = \langle B|\,\bar{q}\Gamma_\mu q'\,|A\rangle\,\langle C|\,\bar{q}''\Gamma^\mu q'''\,|0\rangle \qquad (1)$$

$$(\ \Gamma_\mu \equiv \gamma_\mu(1-\gamma_5)\ )$$

in $\bar{B}^0 \to D\pi,\ D^*\pi,\ D\rho$ and $D^*\rho$. This is of course an approximation, and will receive corrections of $O(\overline{\Lambda}/m_Q)$ [8] ($\overline{\Lambda}$'s definition is given later, but $\overline{\Lambda} \simeq O(\Lambda_{\rm QCD})$) or of $O(1/N_{color})$ in the $1/N_{color}$ expansion [9]. However we have so far no theoretically reliable way to know the size of these corrections. Our strategy is as usual: If the above decays are nicely described in terms of the factorized matrix elements, it could be interpreted as a support to this hypothesis (in other words, an indication that the corrections are small).

Those nonleptonic decays have been investigated at the leading order of the HQET in Refs.[3,10,11]. What is important in the next step of such analyses is to

♮1 For other articles, see two excellent reviews [2] and references cited therein.

calculate the matrix elements up to $O(\overline{\Lambda}/m_Q)$ terms in the HQET. Otherwise, we cannot judge whether the corrections to the factorization are really small or these corrections and the above $O(\overline{\Lambda}/m_Q)$ corrections cancel to each other, even if we get a result in good agreement with data.

Let us explain several form factors in our analysis. The transition matrix elements between B (with velocity v_μ) and D, D^* (with velocity v'_μ) are expressed as

$$\langle D(v')|V^\mu|B(v)\rangle = \sqrt{m_D m_B}\{A(y)(v^\mu + v'^\mu) + B(y)(v^\mu - v'^\mu)\}, \qquad (2a)$$

$$\langle D^*(v')|V^\mu|B(v)\rangle = i\sqrt{m_{D^*} m_B}C(y)\epsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu v'_\alpha v_\beta, \qquad (2b)$$

$$\begin{aligned}&\langle D^*(v')|A^\mu|B(v)\rangle \\ &\quad = \sqrt{m_{D^*} m_B}\{D(y)\varepsilon^{*\mu} + E(y)(\varepsilon^*\!\cdot v)v^\mu + F(y)(\varepsilon^*\!\cdot v)v'^\mu\}. \qquad (2c)\end{aligned}$$

Here $y \equiv v\cdot v'$, ε^* is the polarization vector of D^*, and $A(y) \sim F(y)$ are given by

$$A(y) = \xi(y)\Big[\ 1 + \frac{\overline{\Lambda}}{m_c}\{\ \chi_1(y) - 2(y-1)\chi_2(y) + 6\chi_3(y)\ \}\ \Big], \qquad (3a)$$

$$B(y) = -\frac{\overline{\Lambda}}{m_c}\xi(y)\{\ (y+1)\xi_+(y) - \frac{1}{2}(y-2)\ \}, \qquad (3b)$$

$$C(y) = \xi(y)\Big[\ 1 + \frac{\overline{\Lambda}}{m_c}\{\ \chi_1(y) - 2\chi_3(y) + \frac{1}{2}\ \}\ \Big], \qquad (3c)$$

$$D(y) = \xi(y)\Big[\ y + 1 + \frac{\overline{\Lambda}}{m_c}\{\ (y+1)(\chi_1(y) - 2\chi_3(y)) + \frac{1}{2}(y-1)\ \}\ \Big], \qquad (3d)$$

$$E(y) = -\frac{\overline{\Lambda}}{m_c}\xi(y)\{\ 2\chi_2(y) + \xi_+(y) - \frac{1}{2}\ \}, \qquad (3e)$$

$$F(y) = -\xi(y)\Big[\ 1 + \frac{\overline{\Lambda}}{m_c}\{\ \chi_1(y) - 2\chi_2(y) - 2\chi_3(y) + \xi_+(y)\ \}\ \Big], \qquad (3f)$$

$$(\ \overline{\Lambda} \equiv m_B - m_b = m_D - m_c\ )$$

up to $O(\bar{\Lambda}/m_c)$ corrections [12].[♮2] Here $\xi(y)$ is the Isgur-Wise function, and $\chi_{1,2,3}(y)$, $\xi_+(y)$ are also form factors. (We take a little different definition for $\chi_{1,2,3}(y)$ and $\xi_+(y)$ from [12] so that these functions become dimensionless as $\xi(y)$.)

We cannot know their forms theoretically at present, but they are quite universal and applicable to other processes once they are determined phenomenologically. That is why the HQET approach is quite useful in actual analyses. Taking account of constraints $\xi(1) = 1$ and $\chi_{1,3}(1) = 0$ derived from a symmetry consideration [2,12], we assume here the following forms for $\xi(y)$

$$\xi(y) = 1 + a_{\rm IW}(1-y) \quad \text{(linear type)} \tag{4a}$$
$$= 1/\{1 - b_{\rm IW}(1-y)\} \quad \text{(pole type)} \tag{4b}$$
$$= \exp\{c_{\rm IW}(1-y)\} \quad \text{(exponential type)} \tag{4c}$$

and for $\chi_{1,2,3}(y)$, $\xi_+(y)$

$$\chi_1(y) = \chi_1^0(y-1), \quad \chi_2(y) = \chi_2^0, \quad \chi_3(y) = \chi_3^0(y-1), \quad \xi_+(y) = \xi_+^0, \tag{5}$$

where $a_{\rm IW}$, $b_{\rm IW}$, $c_{\rm IW}$, $\chi_{1,2,3}^0$ and ξ_+^0 are constants as Ito et al. did [7]. These forms of $\chi_{1,2,3}$ and ξ_+ correspond to keeping only the first terms in their expansion around $y = 1$. (Note $|y-1| \lesssim 0.6$ in the processes studied below.)

We start with determining parameters via a χ^2 fit using the data of $\bar{B}^0 \to D^{(*)}\ell\bar{\nu}_\ell$. The corresponding differential widths are given in terms of the above functions as [6]

$$\frac{d\Gamma}{dy}(\bar{B}^0 \to D^+\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} c_{\rm QCD}^2(\mu)|V_{cb}|^2 m_B^2 m_D^3 \sqrt{y^2-1} \times (y^2-1)\Big\{(1+r)A(y) - (1-r)B(y)\Big\}^2, \tag{6}$$

♮2 We do not take into account the $O(\bar{\Lambda}/m_b)$ terms since their sizes are expected to be the same order as those of the $O(\bar{\Lambda}^2/m_c^2)$ terms.

$$\frac{d\Gamma}{dy}(\bar{B}^0 \to D_T^{*+}\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} c_{\rm QCD}^2(\mu)|V_{cb}|^2 m_B^2 m_{D^*}^3 \sqrt{y^2-1} \times 2(1 - 2yr^* + r^{*2})\Big\{D^2(y) + (y^2-1)C^2(y)\Big\}, \tag{7}$$

$$\frac{d\Gamma}{dy}(\bar{B}^0 \to D_L^{*+}\ell\bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} c_{\rm QCD}^2(\mu)|V_{cb}|^2 m_B^2 m_{D^*}^3 \sqrt{y^2-1} \times \Big[(y-r^*)D(y) + (y^2-1)\{F(y) + r^*E(y)\} \Big]^2, \tag{8}$$

($r \equiv m_D/m_B$ and $r^* \equiv m_{D^*}/m_B$)

where D_T^{*+} and D_L^{*+} represent a transversely and longitudinally polarized D^{*+} meson respectively, and V_{cb} is the cb element of the Kobayashi-Maskawa (KM) mixing matrix. Concerning a QCD correction factor $c_{\rm QCD}(\mu)$, we are using the leading-logarithmic one [13], and set the energy scale parameter μ as m_c:

$$c_{\rm QCD}(\mu = m_c) = \left(\frac{\alpha_{\rm QCD}(m_b)}{\alpha_{\rm QCD}(m_c)}\right)^{-\frac{6}{25}}. \tag{9}$$

$\alpha_{\rm QCD}(m_{b,c})$ are computed from $\alpha_{\rm QCD}(M_Z) = 0.12$ [14]. There is an argument on the QCD effects that the full one-loop corrections need to be included in precise analyses [6], but the resultant uncertainty (and that from the choice of μ as well) will be diminished by treating $|V_{cb}|$ as a free parameter in the χ^2 fit. That is, we expect that a large part of the ambiguity is absorbed into $|V_{cb}|$.

We use the following data in the fit:

$$Br(\bar{B}^0 \to D^+\ell\bar{\nu}_\ell) = 1.8 \pm 0.5 \ (\%)$$
$$Br(\bar{B}^0 \to D^{*+}\ell\bar{\nu}_\ell) = 4.9 \pm 0.8 \ (\%)$$

which are given in [15],

$$\Gamma(\bar{B}^0 \to D_L^{*+}\ell\bar{\nu}_\ell)/\Gamma(\bar{B} \to D_T^{*+}\ell\bar{\nu}_\ell) = 1.105 \pm 0.26$$

in [16], and $dBr(\bar{B}^0 \to D^{*+}\ell\bar{\nu}_\ell)/dq^2$ for q^2 =1, 3, 5, 7, 9 and 11 GeV^2 in [10].

The results are

1) for the linear type Isgur-Wise function

$$\chi_1^0 = -1.655,\ \ \chi_2^0 = -0.482,\ \ \chi_3^0 = 0.253,\ \ \xi_+^0 = -0.562,$$
$$|V_{cb}| = 0.0361,\ \ a_{\rm IW} = 0.649,\ \ (\ \chi^2_{\rm min} = 3.384\ )\quad \text{for } \overline{\Lambda} = 0.3\ \text{GeV} \quad (10a)$$
$$\chi_1^0 = -1.262,\ \ \chi_2^0 = -0.661,\ \ \chi_3^0 = 0.204,\ \ \xi_+^0 = -0.664,$$
$$|V_{cb}| = 0.0361,\ \ a_{\rm IW} = 0.662,\ \ (\ \chi^2_{\rm min} = 3.383\ )\quad \text{for } \overline{\Lambda} = 0.4\ \text{GeV} \quad (10b)$$
$$\chi_1^0 = -1.052,\ \ \chi_2^0 = -0.548,\ \ \chi_3^0 = 0.160,\ \ \xi_+^0 = -0.528,$$
$$|V_{cb}| = 0.0361,\ \ a_{\rm IW} = 0.675,\ \ (\ \chi^2_{\rm min} = 3.383\ )\quad \text{for } \overline{\Lambda} = 0.5\ \text{GeV} \quad (10c)$$

2) for the pole type Isgur-Wise function

$$\chi_1^0 = -2.860,\ \ \chi_2^0 = -0.177,\ \ \chi_3^0 = 0.172,\ \ \xi_+^0 = -0.238,$$
$$|V_{cb}| = 0.0362,\ \ b_{\rm IW} = 0.468,\ \ (\ \chi^2_{\rm min} = 3.389\ )\quad \text{for } \overline{\Lambda} = 0.3\ \text{GeV} \quad (11a)$$
$$\chi_1^0 = -2.264,\ \ \chi_2^0 = -0.171,\ \ \chi_3^0 = 0.123,\ \ \xi_+^0 = -0.184,$$
$$|V_{cb}| = 0.0362,\ \ b_{\rm IW} = 0.461,\ \ (\ \chi^2_{\rm min} = 3.389\ )\quad \text{for } \overline{\Lambda} = 0.4\ \text{GeV} \quad (11b)$$
$$\chi_1^0 = -1.908,\ \ \chi_2^0 = -0.155,\ \ \chi_3^0 = 0.093,\ \ \xi_+^0 = -0.141,$$
$$|V_{cb}| = 0.0362,\ \ b_{\rm IW} = 0.454,\ \ (\ \chi^2_{\rm min} = 3.389\ )\quad \text{for } \overline{\Lambda} = 0.5\ \text{GeV} \quad (11c)$$

3) for the exponential type Isgur-Wise function

$$\chi_1^0 = 6.109,\ \ \chi_2^0 = 0.684,\ \ \chi_3^0 = 0.529,\ \ \xi_+^0 = 0.196,$$
$$|V_{cb}| = 0.0355,\ \ c_{\rm IW} = 1.895,\ \ (\ \chi^2_{\rm min} = 3.371\ )\quad \text{for } \overline{\Lambda} = 0.3\ \text{GeV} \quad (12a)$$
$$\chi_1^0 = 4.517,\ \ \chi_2^0 = 0.413,\ \ \chi_3^0 = 0.398,\ \ \xi_+^0 = 0.082,$$
$$|V_{cb}| = 0.0355,\ \ c_{\rm IW} = 1.899,\ \ (\ \chi^2_{\rm min} = 3.371\ )\quad \text{for } \overline{\Lambda} = 0.4\ \text{GeV} \quad (12b)$$
$$\chi_1^0 = 3.533,\ \ \chi_2^0 = 0.395,\ \ \chi_3^0 = 0.305,\ \ \xi_+^0 = 0.144,$$
$$|V_{cb}| = 0.0355,\ \ c_{\rm IW} = 1.902,\ \ (\ \chi^2_{\rm min} = 3.370\ )\quad \text{for } \overline{\Lambda} = 0.5\ \text{GeV} \quad (12c)$$

where we have used τ_B =1.29 ps [15] and $m_c = 1.5$ GeV. On the size of $\overline{\Lambda}$, we took three different values since $m_D - m_c \simeq$0.37 GeV for this m_c while $m_B - m_b \simeq$0.58

GeV if we take m_b =4.7 GeV (see, e.g., Ref. [17] for these $m_{c,b}$). Furthermore, one might claim that we should use m_{D^*} instead of m_D in the definition of $\overline{\Lambda}$ when D^* is produced, but the difference is a part of higher order terms which we do not take into account in the present analysis.

We see that $a_{\rm IW}$, $b_{\rm IW}$, $c_{\rm IW}$ and $|V_{cb}|$ are stable for variation of $\overline{\Lambda}$. On the other hand, $\chi^0_{1,2,3}$ and ξ_+^0 change to a certain extent and therefore it is difficult to draw a definite conclusion on them at this stage. In fact, different solutions exist for the same $\chi^2_{\rm min}$, and we gave the values corresponding to an initial condition $\chi^0_{1,2,3} = \xi_+^0 = 0$ as an example. More important is whether their sizes are $O(1)$ or not (much bigger). If they are, e.g., $O(10)$, it means the expansion is rather bad. From this point of view, the above solutions are all reasonable.

Concerning $|V_{cb}|$, Neubert has proposed a model-independent procedure to determine it in [6] (see also [18]). However, it is not our purpose to confirm the result there ($|V_{cb}| = 0.045(\pm 0.007)$), and we treated $|V_{cb}|$ as a free parameter as already mentioned. Therefore, our results on $|V_{cb}|$ are not model-independent in Neubert's sense. They are smaller than the above value and rather close to $|V_{cb}| = 0.037(\pm 0.004)$ derived through a different technique by Ball [19] who questions the reliability of Neubert's method.

Now we are in a position to study nonleptonic decays. The total widths of $\bar{B}^0 \to D^{(*)+}\pi^-(\rho^-)$ are calculated as

$$\begin{aligned}\Gamma(\bar{B}^0 \to D^+\pi^-) =& \frac{1}{16\pi}G_F^2 c_{\rm QCD}^2(\mu)|V_{cb}|^2|V_{ud}|^2 f_\pi^2 m_B m_D |p_D| \\ & \times \{A^2(y)(1-r)^2(y+1)^2 - 2A(y)B(y)(1-r^2)(y^2-1)\},\end{aligned} \quad (13)$$

$$\begin{aligned}\Gamma(\bar{B}^0 \to D^{*+}\pi^-) =& \frac{1}{16\pi}G_F^2 c_{\rm QCD}^2(\mu)|V_{cb}|^2|V_{ud}|^2 f_\pi^2 m_B m_{D^*} |p_{D^*}|(y^2-1) \\ & \times \{D^2(y) + (y-r^*)^2F^2(y) + 2(1-r^*y)D(y)E(y) \\ & + 2(1-r^*y)(y-r^*)E(y)F(y) + 2(y-r^*)D(y)F(y)\},\end{aligned} \quad (14)$$

$$\Gamma(\bar{B}^0 \to D^+\rho^-) = \frac{1}{16\pi}G_F^2 c_{\rm QCD}^2(\mu)|V_{cb}|^2|V_{ud}|^2 f_\rho^2 m_B m_D |p_D|(y^2-1)$$

$$\times \{(1+r)^2 A^2(y) - 2(1-r^2)A(y)B(y)\}, \qquad (15)$$

$$\Gamma(\bar{B}^0 \to D^{*+}\rho^-) = \frac{1}{16\pi} G_F^2 c_{QCD}^2(\mu)|V_{cb}|^2|V_{ud}|^2 f_\rho^2 m_B m_{D^*}|p_{D^*}|$$
$$\times \Big[\{ 2r_\rho^2 + (y-r^*)^2 \} D^2(y) + (y^2-1)^2 F^2(y)$$
$$- 2r^*(y^2-1)(r^*-y)D(y)E(y) + 2r^*(y^2-1)^2 E(y)F(y)$$
$$- 2(y^2-1)(r^*-y)D(y)F(y) + 2r_\rho^2(y^2-1)C^2(y) \Big]. \qquad (16)$$

($B^2(y)$ and $E^2(y)$ were dropped since they give only $O(\overline{\Lambda}^2/m_c^2)$ contribution.) Here r, r^* are defined just after Eq.(8), $r_\rho \equiv m_\rho/m_B$, and $|p_{D^{(*)}}|$ is the size of D's or D^*'s momentum in the B rest frame:

$$|p_{D^{(*)}}| = \frac{1}{2m_B}\sqrt{\{(m_B + m_{D^{(*)}})^2 - m_{\pi(\rho)}^2\}\{(m_B - m_{D^{(*)}})^2 - m_{\pi(\rho)}^2\}}. \qquad (17)$$

The QCD leading-log correction factor for $\mu = m_c$, which is again our choice here, is [8]

$$c_{QCD}(\mu = m_c) = \Big\{ \frac{1}{3}\Big(\frac{\alpha_{QCD}(M_W)}{\alpha_{QCD}(m_b)}\Big)^{-\frac{12}{23}} \Big(\frac{\alpha_{QCD}(m_b)}{\alpha_{QCD}(E)}\Big)^{-\frac{12}{25}}$$
$$+ \frac{2}{3}\Big(\frac{\alpha_{QCD}(M_W)}{\alpha_{QCD}(m_b)}\Big)^{\frac{6}{23}} \Big(\frac{\alpha_{QCD}(m_b)}{\alpha_{QCD}(E)}\Big)^{-\frac{3}{25}} \Big\} \Big(\frac{\alpha_{QCD}(E)}{\alpha_{QCD}(m_c)}\Big)^{-\frac{6}{25}}, \qquad (18)$$

where E is the total energy of the light quarks becoming together to form π or ρ.

Then, those widths are computed with the parameters in Eqs.(10a)-(12c). Concerning the π and ρ decay constants, f_π and f_ρ, and the KM matrix element between u and d, V_{ud}, we take f_π =132 MeV, f_ρ =208 MeV and $|V_{ud}|$ =0.9744. The numerical results are given in Table 1 with the corresponding data [15]. Considering the size of the experimental errors shown there, the agreement between the predictions and the data is fairly good.

Table 1

Next, we explore another possibility: whether we can describe the semileptonic and nonleptonic decays simultaneously with a better precision:

The results of a χ^2 fit using both of these decay modes are

1) for the linear type Isgur-Wise function

$$\chi_1^0 = -2.284,\ \ \chi_2^0 = -0.077,\ \ \chi_3^0 = 0.249,\ \ \xi_+^0 = -0.293,$$
$$|V_{cb}| = 0.0358,\ \ a_{IW} = 0.511,\ \ (\ \chi^2_{min} = 3.594\ )\quad \text{for } \overline{\Lambda} = 0.3 \text{ GeV} \qquad (19a)$$
$$\chi_1^0 = -1.833,\ \ \chi_2^0 = -0.162,\ \ \chi_3^0 = 0.183,\ \ \xi_+^0 = -0.281,$$
$$|V_{cb}| = 0.0358,\ \ a_{IW} = 0.506,\ \ (\ \chi^2_{min} = 3.586\ )\quad \text{for } \overline{\Lambda} = 0.4 \text{ GeV} \qquad (19b)$$
$$\chi_1^0 = -1.562,\ \ \chi_2^0 = -0.213,\ \ \chi_3^0 = 0.144,\ \ \xi_+^0 = -0.275,$$
$$|V_{cb}| = 0.0358,\ \ a_{IW} = 0.500,\ \ (\ \chi^2_{min} = 3.579\ )\quad \text{for } \overline{\Lambda} = 0.5 \text{ GeV} \qquad (19c)$$

2) for the pole type Isgur-Wise function

$$\chi_1^0 = -2.172,\ \ \chi_2^0 = 0.078,\ \ \chi_3^0 = 0.202,\ \ \xi_+^0 = -0.094,$$
$$|V_{cb}| = 0.0367,\ \ b_{IW} = 0.765,\ \ (\ \chi^2_{min} = 3.561\ )\quad \text{for } \overline{\Lambda} = 0.3 \text{ GeV} \qquad (20a)$$
$$\chi_1^0 = -1.767,\ \ \chi_2^0 = -0.052,\ \ \chi_3^0 = 0.149,\ \ \xi_+^0 = -0.140,$$
$$|V_{cb}| = 0.0366,\ \ b_{IW} = 0.743,\ \ (\ \chi^2_{min} = 3.556\ )\quad \text{for } \overline{\Lambda} = 0.4 \text{ GeV} \qquad (20b)$$
$$\chi_1^0 = -1.524,\ \ \chi_2^0 = -0.130,\ \ \chi_3^0 = 0.117,\ \ \xi_+^0 = -0.167,$$
$$|V_{cb}| = 0.0366,\ \ b_{IW} = 0.722,\ \ (\ \chi^2_{min} = 3.551\ )\quad \text{for } \overline{\Lambda} = 0.5 \text{ GeV} \qquad (20c)$$

3) for the exponential type Isgur-Wise function

$$\chi_1^0 = -1.675,\ \ \chi_2^0 = 0.042,\ \ \chi_3^0 = 0.238,\ \ \xi_+^0 = -0.166,$$
$$|V_{cb}| = 0.0364,\ \ c_{IW} = 0.776,\ \ (\ \chi^2_{min} = 3.565\ )\quad \text{for } \overline{\Lambda} = 0.3 \text{ GeV} \qquad (21a)$$
$$\chi_1^0 = -1.419,\ \ \chi_2^0 = -0.076,\ \ \chi_3^0 = 0.174,\ \ \xi_+^0 = -0.189,$$
$$|V_{cb}| = 0.0364,\ \ c_{IW} = 0.755,\ \ (\ \chi^2_{min} = 3.560\ )\quad \text{for } \overline{\Lambda} = 0.4 \text{ GeV} \qquad (21b)$$
$$\chi_1^0 = -1.263,\ \ \chi_2^0 = -0.146,\ \ \chi_3^0 = 0.136,\ \ \xi_+^0 = -0.203,$$
$$|V_{cb}| = 0.0364,\ \ c_{IW} = 0.734.\ \ (\ \chi^2_{min} = 3.555\ )\quad \text{for } \overline{\Lambda} = 0.5 \text{ GeV} \qquad (21c)$$

Here, $\chi^0_{1,2,3}$ and ξ_+^0 are again those obtained for the initial condition $\chi^0_{1,2,3} = \xi_+^0$=0.

The corresponding values of the ratio Γ_L/Γ_T and the various branching ratios for the above parameter values are in Table 2,

Table 2

and a comparison of $dBr(\bar{B}^0 \to D^{*+}\ell\bar{\nu}_\ell)/dq^2$ with the data is made in Fig.1.

Fig.1

Some discrepancies are seen between Eqs.(12a)-(12c) and Eqs.(21a)-(21c). However, it will not be an indication that the exponential type Isgur-Wise function does not work since both of the solutions produce results in good agreement with the data.

Now, what could we say from these results? We have no mind to go so far as to assert that the factorization has been justified. However, we will be allowed to conclude, at least at the same level as the validity of the HQET in B semileptonic decays, that the factorization works very well. This is consistent with the preceding analyses at the leading order [3,10,11], and a new support to this hypothesis at the level of the $O(\overline{\Lambda}/m_c)$ corrections.

In summary we have analyzed $\bar{B}^0 \to D^{(*)+}\pi$, $D^{(*)+}\rho$ in the framework of the heavy quark effective theory including the $O(\overline{\Lambda}/m_c)$ terms. Using the data of $\bar{B}^0 \to D^{(*)+}\ell\bar{\nu}_\ell$ as inputs to determine several unknown parameters, we performed a test of the factorization hypothesis. We also carried out a χ^2 fit using the semileptonic and nonleptonic data together. Although our analysis includes several assumptions inevitably, i.e., the forms of ξ, ξ_+ and $\chi_{1,2,3}$ functions and therefore not perfect, we obtained some interesting indication that the factorization holds to a good approximation in B nonleptonic decays. It is quite consistent with the previous works without the $O(\overline{\Lambda}/m_c)$ corrections. Concerning the validity of the HQET itself, on the other hand, it was not our purpose in this paper to test it, but we found that there is a "stable" parameter region in the unknown form factors

which describes nicely various processes. It is also a check of HQET's reliability.

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	$\overline{\Lambda}$	lin. type	pole type	exp. type	data
$\bar{B} \to D\pi$	0.3	0.330	0.321	0.281	0.32±0.07
	0.4	0.343	0.314	0.281	
	0.5	0.337	0.305	0.276	
$\bar{B} \to D^*\pi$	0.3	0.281	0.283	0.274	0.32±0.07
	0.4	0.281	0.284	0.274	
	0.5	0.282	0.284	0.274	
$\bar{B} \to D\rho$	0.3	0.789	0.768	0.688	0.90±0.60
	0.4	0.816	0.755	0.688	
	0.5	0.802	0.738	0.677	
$\bar{B} \to D^*\rho$	0.3	0.785	0.788	0.773	0.80±0.40
	0.4	0.786	0.789	0.773	
	0.5	0.787	0.790	0.774	

Table 1.

Branching ratios $Br(\bar{B}^0 \to D^+\pi)$, $Br(\bar{B}^0 \to D^{*+}\pi)$, $Br(\bar{B}^0 \to D^+\rho)$ and $Br(\bar{B}^0 \to D^{*+}\rho)$ in % for the linear (lin.) type, the pole type and the exponential (exp.) type Isgur-Wise functions with parameters determined from the data of the semileptonic B decays for $\overline{\Lambda} = 0.3$, 0.4 and 0.5 GeV.

	$\overline{\Lambda}$	lin. type	pole type	exp. type	data
$\bar{B} \to D\ell\bar{\nu}$	0.3	1.806	1.806	1.806	1.8±0.5
	0.4	1.806	1.806	1.806	
	0.5	1.806	1.806	1.806	
$\bar{B} \to D^*\ell\bar{\nu}$	0.3	4.747	4.757	4.755	4.9±0.8
	0.4	4.747	4.757	4.755	
	0.5	4.748	4.756	4.755	
Γ_L/Γ_T	0.3	1.151	1.131	1.136	1.105±0.26
	0.4	1.150	1.130	1.135	
	0.5	1.149	1.130	1.134	
$\bar{B} \to D\pi$	0.3	0.323	0.323	0.324	0.32±0.07
	0.4	0.323	0.323	0.324	
	0.5	0.323	0.323	0.324	
$\bar{B} \to D^*\pi$	0.3	0.299	0.303	0.301	0.32±0.07
	0.4	0.299	0.303	0.302	
	0.5	0.300	0.304	0.302	
$\bar{B} \to D\rho$	0.3	0.774	0.770	0.771	0.90±0.60
	0.4	0.774	0.771	0.771	
	0.5	0.774	0.771	0.771	
$\bar{B} \to D^*\rho$	0.3	0.824	0.832	0.829	0.80±0.40
	0.4	0.824	0.832	0.829	
	0.5	0.824	0.832	0.829	

Table 2.

The branching ratios (in %) of $\bar{B}^0 \to D^+\ell\bar{\nu}_\ell$, $D^{*+}\ell\bar{\nu}_\ell$, $D^+\pi$, $D^{*+}\pi$, $D^+\rho$, $D^{*+}\rho$, and the ratio $\Gamma_L/\Gamma_T \equiv \Gamma(\bar{B}^0 \to D_L^{*+}\ell\bar{\nu}_\ell)/\Gamma(\bar{B}^0 \to D_T^{*+}\ell\bar{\nu}_\ell)$ for the linear (lin.) type, the pole type and the exponential (exp.) type Isgur-Wise functions with parameters determined from these data and $dBr(\bar{B}^0 \to D^{*+}\ell\bar{\nu}_\ell)/dq^2$ for $\overline{\Lambda} = 0.3$, 0.4 and 0.5 GeV.

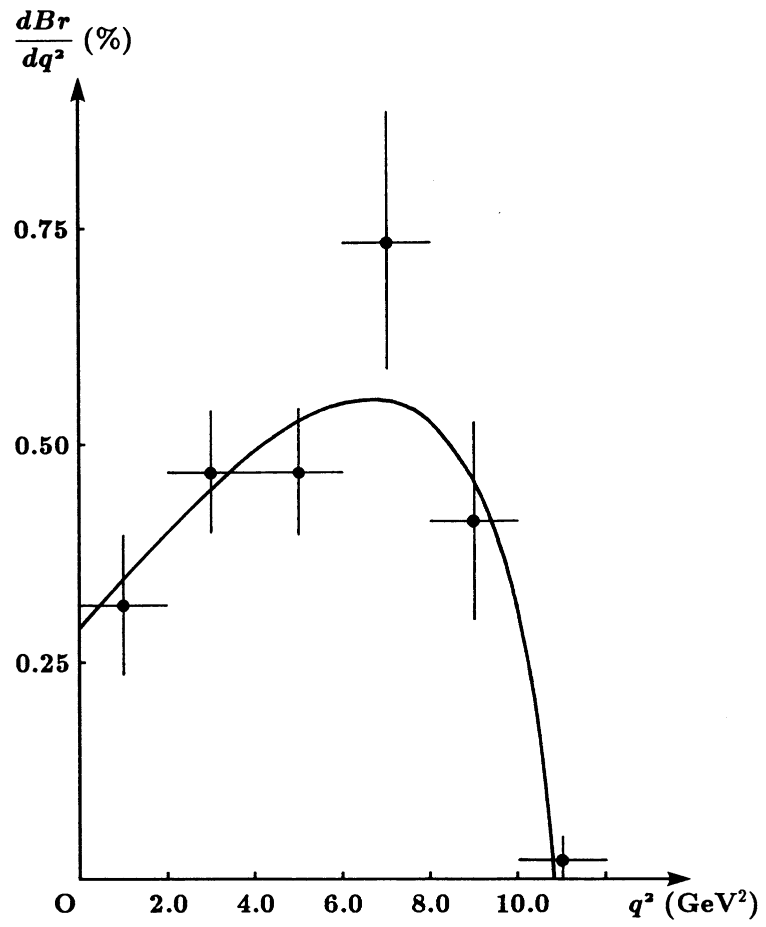

Fig.1

Comparison of the calculated $dBr(\bar{B}^0 \to D^{*+}\ell\bar{\nu}_\ell)/dq^2$ with the corresponding data. The curve is for the linear type Isgur-Wise function and $\overline{\Lambda}$ =0.4 GeV as an example, but similar curves are obtained for the other cases as well.