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Radiative correction effects of a very heavy top

Riccardo Barbieri^{1,2,†} Matteo Beccaria^{1,2} Paolo Ciafaloni^{1,2}
Giuseppe Curci^{2,1} Andrea Viceré^{3,2}

(1) Dipartimento di Fisica, Università di Pisa

Piazza Torricelli 2, I-56100 Pisa, Italy

(2) I.N.F.N., sez. di Pisa

Via Livornese 582/a, I-56010 S. Piero a Grado (Pisa) Italy

(3) Scuola Normale Superiore

Piazza dei Cavalieri 7, I-56100 Pisa, Italy

Abstract

If the top is very heavy, $m_t \gg M_Z$, the dominant radiative correction effects in all electroweak precision tests can be exactly characterized in terms of two quantities, the ρ -parameter and the GIM violating $Z \rightarrow b\bar{b}$ coupling. These quantities can be computed using the Standard Model Lagrangian with vanishing gauge couplings. This is done here up to two loops for arbitrary values of the Higgs mass.

† Address from May 1, 1992: CERN, Geneva, Switzerland

1. The effects of virtual heavy top quark exchanges in electroweak precision tests are recognized as being very important. In the Standard Model of the electroweak interactions these effects are used to get a significant constraint on the top quark mass, m_t . Perhaps even more importantly, these effects are so large that, until the value of m_t will not be measured directly and independently, they will obscure the comparison of different models, including the SM itself, in their predictions of electroweak precision tests. We have especially in mind the effects that grow like m_t^2 at one loop level, which affect the electroweak ρ -parameter [1] (and all related quantities) and the GIM-violating coupling of the Z boson to a $b\bar{b}$ pair [2]. The main contribution of this work is the explicit calculation of the two loop m_t^4 -corrections to these quantities in the SM for arbitrary values of the Higgs mass, m_H . In the literature [3] one finds already the m_t^4 contributions to ρ for $m_H \ll m_t$, which we confirm.

The actual calculation of these corrections is greatly simplified by the observation that to obtain them it is enough to consider the lagrangian of the SM in the limit of vanishing coupling constants: the gauge bosons play the rôle of external sources and the relevant propagating fields are the top quark, the massless bottom quark, the Higgs field and the charged and neutral Goldstone bosons, ϕ^\pm, χ . We call this the Gaugeless Limit of the Standard Model.

2. A way to relate proper quantities computed in the reduced model to physical observables is to consider the Ward Identities satisfied by the charged weak current J_μ^\pm and by the usual combination of neutral currents, $J_\mu = J_\mu^3 - \sin^2 \theta_W J_\mu^{em}$, that couples to the Z boson. When the bare neutral component of the Higgs doublet acquires a vev v , both these currents get a piece proportional to the derivative of the Goldstone fields

$$\begin{aligned} J_\mu &= \hat{J}_\mu + \frac{v}{\sqrt{2}} \partial_\mu \chi \\ J_\mu^\pm &= \hat{J}_\mu^\pm \mp i v \partial_\mu \phi^\pm \end{aligned} \tag{1}$$

From the current conservation eq.s $\partial_\mu J_\mu = \partial_\mu J_\mu^\pm = 0$, it is easy to derive the following Ward Identities ($s^2 \equiv \sin^2 \theta_W$, $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$)

$$\begin{aligned}
(p - p')_\mu \Gamma_\mu(p, p') &= \frac{iv}{\sqrt{2}} \Gamma(p, p') + \\
&+ \frac{1}{2} \left[\left(1 - \frac{2}{3}s^2\right) (S^{-1}(p') P_L - P_R S^{-1}(p)) + \right. \\
&\quad \left. - \frac{2}{3}s^2 (S^{-1}(p') P_R - P_L S^{-1}(p)) \right] \\
q_\mu q_\nu \Pi_{\mu\nu}(q) &= \frac{v^2}{2} \Pi(q) \\
q_\mu q_\nu \Pi_{\mu\nu}^\pm(q) &= v^2 \Pi^\pm(q)
\end{aligned} \tag{2}$$

which involve the b -quark propagator $S(p)$; the self energies, $\Pi(q)$, $\Pi^\pm(q)$, of the neutral and charged Goldstones; the correlation functions $\Pi_{\mu\nu}(q)$, $\Pi_{\mu\nu}^\pm(q)$ of the currents \hat{J}_μ , \hat{J}_μ^\pm respectively; the vertex $\Gamma_\mu(p, p')$ between b -quarks of momentum p, p' and the current \hat{J}_μ ; the vertex $\Gamma(p, p')$ between b -quarks of momentum p, p' and the neutral Goldstone χ . Both the Π 's and the Γ 's are meant to be irreducible.

To get the physical observables we now define the following constants

$$\begin{aligned}
\Gamma_\mu(p, p') &\simeq -\frac{i}{2} \left[\left(1 - \frac{2}{3}s^2\right) Z_1 \gamma_\mu P_L - \frac{2}{3}s^2 \gamma_\mu P_R \right] \text{ at } p \simeq p' \\
\Gamma(p, p') &\simeq Z_1^\chi \frac{\not{p}' - \not{p}}{\sqrt{2}v} P_L \text{ at } p \simeq p' \\
S_b^{-1}(p) &\simeq i Z_2^b \not{p} P_L + i \not{p} P_R \text{ at } p^2 \simeq 0 \\
\Pi_{\mu\nu}(q) &\simeq \frac{v^2}{2} (Z - 1) \delta_{\mu\nu} \text{ at } q \simeq 0 \\
\Pi_{\mu\nu}^\pm(q) &\simeq v^2 (Z^\pm - 1) \delta_{\mu\nu} \text{ at } q \simeq 0 \\
\Pi(q) &\simeq (Z_2^\chi - 1) q^2 \text{ at } q^2 \simeq 0
\end{aligned} \tag{3}$$

$$\Pi^\pm(q) \simeq (Z_2^\varphi - 1)q^2 \text{ at } q^2 \simeq 0$$

They satisfy, from the Ward Identities (2), the relations

$$\left(1 - \frac{2}{3}s^2\right) Z_1 = Z_1^\chi + \left(1 - \frac{2}{3}s^2\right) Z_2^b; \quad Z = Z_2^\chi; \quad Z^\pm = Z_2^\varphi \quad (4)$$

By recalling now that, in the full SM Lagrangian, the W - and Z -bosons couple to the current J_μ^\pm, J_μ via

$$\Delta\mathcal{L} = \frac{g}{\sqrt{2}} (J_\mu^+ W_\mu^- + J_\mu^- W_\mu^+) + \frac{g}{c} J_\mu Z_\mu \quad (5)$$

these relations enable us to compute, to leading order in the $SU(2)$ gauge coupling g , the physical $Z_\mu \rightarrow b\bar{b}$ vertex

$$V_\mu = -i \frac{g}{2c} \left[\left(1 - \frac{2}{3}s^2 + \frac{Z_1^\chi}{Z_2^b}\right) \gamma_\mu P_L - \frac{2}{3}s^2 \gamma_\mu P_R \right] \quad (6)$$

and the vector boson masses

$$M_Z^2 = \frac{g^2 v^2}{2c^2} Z = \frac{g^2 v^2}{2c^2} Z_2^\chi \quad (7)$$

$$M_W^2 = \frac{g^2 v^2}{2} Z^\pm = \frac{g^2 v^2}{2} Z_2^\varphi \quad (8)$$

These are indeed the physical masses, as $g \rightarrow 0$, because the displacement between $q^2 = 0$ and the pole at $q^2 = M^2$ is irrelevant and because there is no wave function renormalization of the vector bosons in this limit. The ρ parameter is therefore [4]

$$\rho = \frac{M_W^2}{M_Z^2 c^2} = \frac{Z_2^\varphi}{Z_2^\chi} \quad (9)$$

Notice that the Ward Identities (2) are identically true, as they stand, also in the full theory, with the gauge couplings switched on, if one works in the

Background Gauge. Let us also remark that, if we had used an effective Lagrangian formalism, the constants Z_1^x , Z_2^x , Z_2^φ would have appeared in front of terms involving derivatives of the Goldstone bosons. Eq.s (6-8) could have then be simply obtained by proper covariantization of these derivatives. Since the W and the Z are treated as external sources, there is never the need to fix the gauge and break gauge invariance.

What remains to be done at this point is the re-expression of the various parameters appearing in (6) and (9) in favor of physically measurable quantities. As usual, g , v and c are traded for M_Z , eq. (7-8), the fine structure constant α and the Fermi constant G_μ as measured in μ -decay, which, in our approximation, are given by

$$\alpha = \frac{g^2}{4\pi s^2}, \quad G_\mu = \frac{\sqrt{2}g^2}{8M_W^2} = \frac{\sqrt{2}}{4v^2 Z_2^\varphi} \quad (10)$$

From eq.s (6-10), by retaining only the non vanishing corrections as $g \rightarrow 0$, the radiative effects in all electroweak precision tests can be “non-perturbatively” characterized in terms of the two quantities ρ and $\tau \equiv Z_1^x/Z_2^b$ appearing in the GIM-violating $Z \rightarrow b\bar{b}$ vertex. In particular, wherever s^2 appears, it must be replaced, from eq.s (7-10), with

$$s^2 = \frac{1}{2} \left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2 \rho}} \right) \quad (11)$$

As an example, in terms of ρ and τ , the width of the Z into a $b\bar{b}$ pair is given by

$$\Gamma(Z \rightarrow b\bar{b}) = \rho \frac{G_\mu M_Z^3}{8\pi\sqrt{2}} \sqrt{1 - \frac{4m_b^2}{M_Z^2}} \left[(g_{bV}^2 + g_{bA}^2) \left(1 + 2\frac{m_b^2}{M_Z^2} \right) - 6g_{bA}^2 \frac{m_b^2}{M_Z^2} \right]$$

$$g_{bV} = 1 - \frac{4}{3}s^2 + \tau, \quad g_{bA} = 1 + \tau \quad (12)$$

where the kinematical dependence on the b quark mass, m_b , is also introduced. Of course, hidden in ρ and τ there are the top Yukawa coupling g_t

and the quartic Higgs coupling λ , which must also be expressed in favor of the top and of the Higgs mass, defined as the positions of the poles of the corresponding propagators. From the top propagator

$$S_t^{-1}(p) \simeq i Z_{2L}^t \not{p} P_L + i Z_{2R}^t \not{p} P_R + g_t v B \quad \text{at } \not{p} \simeq m_t \quad (13)$$

one has

$$m_t = g_t v B / (Z_{2L}^t Z_{2R}^t)^{1/2} \quad (14)$$

whereas, up to the order we are working, $m_H^2 = 4\lambda v^2$

3. Fig 1 contains the relevant one loop diagrams contributing to the constants Z_2^X , Z_2^φ and Z_1^X which allow to determine the coefficients of the first term in an expansion in powers of $G_\mu m_t^2$ both of ρ and of the GIM violating $Z \rightarrow b\bar{b}$ vertex. The diagrams contributing to Z_2^X and Z_2^φ and only containing Higgs or Goldstone boson lines, but no quark lines, have not been drawn since they do not affect ρ or more precisely, the difference $Z_2^X - Z_2^\varphi$, in terms of which ρ can be identically written, using eq.s (9, 10), as

$$\frac{1}{\rho} - 1 = \frac{4G_\mu v^2}{\sqrt{2}} (Z_2^X - Z_2^\varphi) \quad (15)$$

In the literature [5], this is often called the irreducible part of the corrections to the ρ parameter. At order $(G_\mu m_t^2)^2$, other than the contributions of the genuine two loops irreducible diagrams, one has to compute the $G_\mu m_t^2$ corrections to the wave function renormalizations of the top and bottom quarks as specified in eq.s (6, 14).

In a decently compact way, the results of the calculations can be given in an analytic form in two different asymptotic regimes ($x = \frac{G_\mu m_t^2}{8\pi^2 \sqrt{2}}$, $N_c = 3$, $r = \frac{m_t^2}{m_H^2}$)

(a) $m_t \gg m_H$

$$\begin{aligned} \frac{1}{\rho} - 1 &= -N_c x \left[1 + x (19 - 2\pi^2) \right] \\ \frac{Z_1^X}{Z_2^b} &= -2x \left[1 + \frac{x}{3} (27 - \pi^2) \right] \end{aligned} \quad (16)$$

(b) $m_H \gg m_t$

$$\begin{aligned}
\frac{1}{\rho} - 1 = & \\
& -N_c x \left\{ 1 + x \left[\frac{49}{4} + \pi^2 + \frac{27}{2} \log r + \frac{3}{2} \log^2 r + \right. \right. \\
& \quad \left. \left. + \frac{r}{3} (2 - 12\pi^2 + 12 \log r - 27 \log^2 r) + \right. \right. \\
& \quad \left. \left. + \frac{r^2}{48} (1613 - 240\pi^2 - 1500 \log r - 720 \log^2 r) \right] \right\} \\
\frac{Z_1^x}{Z_2^b} = & \\
& -2x \left\{ 1 + \frac{x}{144} \left[311 + 4\pi^2 + 282 \log r + 90 \log^2 r + \right. \right. \\
& \quad \left. \left. -4r (40 + 6\pi^2 + 15 \log r + 18 \log^2 r) + \right. \right. \\
& \quad \left. \left. + \frac{3r^2}{100} (24209 - 6000\pi^2 - 45420 \log r - 18000 \log^2 r) \right] \right\} \\
& (17)
\end{aligned}$$

In the expression for $m_H \gg m_t$ we have made explicit some sub-asymptotic terms vanishing in the limit $r \rightarrow 0$ in such a way that the expansion can be trusted even for m_H close to m_t . For practical purposes, the combined use of eq.s (16, 17) allows to control numerically the m_t^4 terms for all values of m_H . The effects of these corrections, in view of the attainable experimental precision, start being significant for $m_t \gtrsim 200$ GeV. These corrections are being implemented in the ZFITTER code [6].

A detailed description of the calculation will be given elsewhere.

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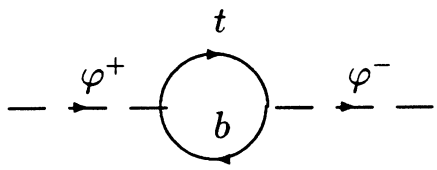
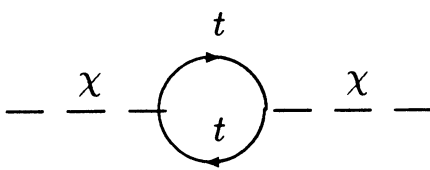
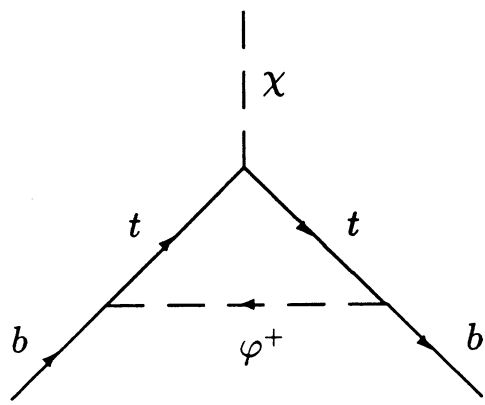


Figure 1: relevant one loop diagrams contributing to the constants Z_2^x , Z_2^φ and Z_1^x

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