CERN–TH. 5664/90

STATUS AND PERSPECTIVES OF K DECAY PHYSICS

R. Battiston
Dipartimento di Fisica, Università di Perugia, Perugia, Italy
Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, Italy

D. Cocolicchio, G. L. Fogli†
CERN, CH-1211 Geneva 23, Switzerland

and

N. Paver
Dipartimento di Fisica Teorica, Università di Trieste, Trieste, Italy
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Italy

ABSTRACT

The current status of K decay physics is reviewed. Together with an analysis of the different decay modes, emphasizing their significance as tests of the standard model and as probes for new physics, a discussion of the experimental capabilities of the various existing facilities is presented. Also surveyed are the perspectives offered to such studies by future plans of high luminosity machines.

---

†On leave of absence from *Dipartimento di Fisica, Università di Bari, Bari, Italy*
Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy

CERN–TH. 5664/90
May 1990

I. Introduction

The study of K decays marks some of the most relevant steps in the understanding of particle physics. Related to the K physics are, in fact:

i) The parity violation [1], of which the $K \to 2\pi, 3\pi$ puzzle represented one of the first indications [2].

ii) The CP violation [3], discovered in the analysis of the $K\overline{K}$ system twenty-five years ago [4]. At present the $K\overline{K}$ system still remains the only known manifestation of CP violation and a laboratory to study it. From this point of view, also the introduction of the third generation of quarks and leptons [5] originally found its primary motivation in K physics, being related to the attempt of justifying the CP violation in the standard model in a natural way.

iii) The GIM mechanism [6], which implies that neutral currents naturally conserve flavour at the tree level, and that flavour-changing neutral currents are naturally suppressed at one-loop level. Based on the postulated existence of the charm quark, it originally found in the $K_L \to \mu^+\mu^-$ one of its cleanest motivations.

Even though the present interest of particle physics seems to be shifted to a large extent towards the physics which can be performed at the energies recently reached with the new colliders (typically the Z^0 physics studied presently at LEP/SLC) and towards the studies of heavy flavours, there are, however, good reasons to assert that K physics still maintains unaltered its interest and should well be the source of important results. Indeed, fundamental features such as e.g. the $\Delta I = 1/2$ rule are still difficult to account for from the theoretical point of view, and the origin of P and CP violations is not well identified as yet. Moreover the so-called "new physics", i.e. those manifestations which can be interpreted as clear indications of new effects beyond the standard model, could manifest itself also in low energy processes such as K decays (of course, through experiments with improved systematics and high statistics).

We can thus make in the following a list of the aspects on which K decays provide particularly important and detailed information:

a) A better understanding of the nature of CP violation. K decays offer a unique way to study old and/or new sources of CP violation, a point which is of crucial importance.

b) Clean and specific tests of the standard model. Specifically:

  - sensitivity to the still unknown mass of the top quark;
  - precision measurements (number of generations, universality, radiative corrections);
  - specific models (vector and axial form factors, short and long distance effects, chiral perturbation theory).

c) Possible existence of light neutral particles such as Higgs, hyperphotons, familons, axions, etc., even though the recent LEP experiments [7] appreciably reduce the margins of this kind of searches, in particular as far as light Higgs bosons are concerned.

d) Possible glimpses on the so-called "new physics". Here many possibilities and alternatives are open. Among the most interesting ones:

  - New alternative or complementary sources of CP violation, besides that related, in the standard model, to the phase in the Cabibbo-Kobayashi-Maskawa matrix, which can be easily accommodated with a simple extension of the standard model. An example is the spontaneous CP violation induced by multi-Higgs systems [8].
  - Indications of a fourth generation: essentially ruled out at the level of a light fourth neutrino by the recent results of the LEP/SLC experiments [9], there still remains the - admittedly less appealing - possibility of a fourth generation characterized by a heavy neutrino.
  - New kinds of interactions, mediated by quanta not "envisaged" in the

standard model, such as leptoquarks, superheavy Higgs, etc. Here the point is that there exist specific processes that, being forbidden in the standard model, are a clear evidence of "new physics" once observed. This is the case of all the processes related to a lepton flavour violation, typically for example $K_L \to \mu e$.

The aim of this paper is to present in a synthetic way a scheme of the present status of K decays, to describe the perspectives potentially offered by a future analysis of some of them and to indicate the experimental facilities at which those processes could be studied, with some emphasis for the new possibilities open by a ϕ factory.

The paper is organized as follows. In Sect. II we consider the problems posed by K physics from the experimental point of view: the different experimental facilities proposed to study K decays are compared in terms of their characteristics, such as available statistics, detector performances, background conditions, etc., and the present situation is briefly discussed, together with the perspectives offered in a rather near future. In the following sections the different processes are analysed, by grouping them into different categories, distinguished on the basis of their "rareness". We are aware that this introduces an element of arbitrariness. Nevertheless this facilitates the approach to the large number of processes to be discussed. Accordingly, in Sect. III the "common decays", $K \to 2\pi$, $K \to 3\pi$, $K_{\ell 2}$, $K_{\ell 3}$, $K_{\ell 4}$ are reviewed, while in Sect. IV we consider under the name "not so rare decays" the radiative leptonic and non-leptonic transitions $K \to (\ell\nu)\gamma$, $K \to (\ell\nu)(\ell\bar{\ell})$, $K_{2\gamma}$, $K \to (\ell\bar{\ell})\gamma$, $K \to \pi\gamma\gamma$, $K \to \pi\pi\gamma$. In Sect. V we consider the "rare decays" $K^{\pm} \to \pi^{\pm}(\ell^+\ell^-)$, $K^{\pm} \to \pi^{\pm}(\nu\bar{\nu})$, $K \to \mu^+\mu^-$, $K \to e^+e^-$. These are still accessible at the existing high intensity K beams and, of course, at the future kaon factories. In Sect. VI the "very rare" decays $K_L \to \pi^0(\ell^+\ell^-)$ and $K_L \to \pi^0(\nu\bar{\nu})$ are described, very rare because both GIM suppressed and proceeding, as far as the K_L is concerned, through CP violating amplitudes. In Sect. VII we consider the "not expected or forbidden decays" $K \to \mu e$, $K \to \pi\mu e$, whose observation would be an unambiguous signal of "new physics". Finally, in Sect. VIII we give some concluding remarks.

We have no hope to be exhaustive, because the material to be covered is huge.

Certainly, some topics reviewed in the present report would require a more accurate and specific analysis. For example, we consider only marginally the well-known problem of CP violation in the $K\overline{K}$ system and in $K \to 2\pi$, whose phenomenological and theoretical properties would need a more detailed description. Concerning the weak decays $K \to 3\pi$ we will give only a brief account of the perspectives offered by them as a possible test of CP violation.

II. Experimental considerations on K physics

Kaons are normally hadroproduced from high intensity proton beams colliding on fixed targets (CERN, FNAL, KEK, BNL, future kaon factories) or in low energy $p\bar{p}$ collisions (CERN/LEAR). With the advent of ϕ factories other clean and intense K sources will become available.

In the case of high energy hadroproduction, the kaon energy may range from as low as (few) GeV (used in high rate $K^+ \to \pi^+(\nu\bar{\nu})$ or $K_L \to \mu e$ experiments and muon polarization measurement in $K_{\mu 3}$ decay) to 100 GeV or more (suitable for experiments measuring $\mathrm{Re}(\epsilon'/\epsilon)$, $K_L \to \pi^0 e^+ e^-$ and $K_S \to 3\pi$). The beam intensity may be extremely high, reaching few 10^9 K_L s^{-1} as in the case of the upgrade proposed at the Tevatron [10], or even more, as it is foreseen at the future kaon factories [11]. These fluxes are much larger than what is expected at a ϕ factory or at LEAR (10^3 to $10^4 K$ s^{-1}); however, what matters is the rate of K decaying in the detector fiducial volume, for example in the case of high energy K_L (and $K^{\pm}$) beams, the experimental acceptance is normally very low (few %) due to their long decay path. Moreover, pure K_S beams are difficult to produce at fixed target sources. Conversely, at low energy a large fraction of the produced kaons decay in the detector fiducial volume.

In addition, experiments performed at intense K beams have to face severe backgrounds (muons, pions, neutrons, photons) accompanying the kaons, and to tolerate ex-

TABLE I. Recent progress in rare kaon decay. Some results are preliminary, while others are published.

decay mode	experiment	result	sensitivity	comments
$K^+ \to \pi^+$+nothing	BNL E787	$< 3 \times 10^{-8}$	10^{-10}	detector works well
$K^+ \to \pi^+\mu^+e^-$	BNL E777	$< 2.1 \times 10^{-10}$	1.5×10^{-10}	limited by beam halo
$K_L \to \mu^\pm e^\mp$	BNL E780	$< 1.9 \times 10^{-9}$		limited by beam halo
$K_L \to e^+e^-$		$< 1.2 \times 10^{-9}$		
$K_L \to \pi^0 e^+e^-$		$< 3.2 \times 10^{-7}$		will pursue ($\sim 10^{-10}$) in E845
$K_L \to \mu^\pm e^\mp$	BNL E791	$< 2.2 \times 10^{-10}$	$\sim 2 \times 10^{-11}$	limited by accidental from K_L decays
$K_L \to e^+e^-$		$< 3.2 \times 10^{-10}$		
$K_L \to \mu^\pm e^\mp$	KEK E137	$< 4 \times 10^{-10}$	$\sim 2 \times 10^{-11}$	will pursue in E162
$K_L \to e^+e^-$		$< 5.4 \times 10^{-10}$		$K_L \to \pi^0 e^+e^-(\sim 10^{-10})$
$K_S \to \pi^+\pi^-\pi^0$	FNAL E621	$< 1.5 \times 10^{-7}$	$\sim 3 \times 10^{-9}$	expected rate $\sim 1.2 \times 10^{-9}$
$K_L \to \pi^0 e^+e^-$	FNAL E731	$< 4.2 \times 10^{-8}$	$\sim 1 \times 10^{-8}$	
	CERN NA31	$< 4 \times 10^{-8}$		
$K_L \to \pi^0 e^+e^-$	KEK-162		10^{-10}	
$K_L \to \pi^0 e^+e^-$	FNAL P799		$10^{-10} - 10^{-11}$	

tremely high trigger rates. For this reason, in addition to experiments optimized for the study of direct CP violation in $K \to 2\pi$ decays, each experiment normally concentrates on only one (few) rare or forbidden reaction(s), like: $K^+ \to \pi^+$ + nothing, $\pi^+\mu^+e^-$, $K_L \to \mu e, e^+e^-, \pi^0 e^+e^-$ and $K_S \to \pi^+\pi^-\pi^0$. As summarized in Table I (updated from ref. [12]), the limits obtained on the above reactions are already in the 10^{-8} to 10^{-10}

TABLE II. Characteristics of the various kaon facilities.

experiment	status	physics	statistics	notes
CP-LEAR	ready soon	* $\not{CP}$ in K decays * CPT test	* expected $\sigma(\epsilon'/\epsilon)$=few 10^{-3}	* $p\bar{p}$ interaction at rest * statistics too small for $\sigma(\epsilon'/\epsilon) < 10^{-3}$ * other particles in the final states
ϕ factory	1993-94 (?) (to be approved and built)	* ϵ'/ϵ and other $\not{CP}$ in K decays * rare K_S decays * CPT test	* 10^{10} ϕ/year at $2.5 \times 10^{32}\text{cm}^{-2}\text{s}^{-1}$ * few 10^9 expected $K^\pm$ decays * 10^8–10^9 expected $K_{L,S}$ decays	* e^+e^- * very clean exp. environment * tagged ultrapure $K_{S,L}$, $K^\pm$ beams * $\sigma(\epsilon'/\epsilon) = 3 \cdot 10^{-4}$ with $\geq 10^{10}\phi$
NA31/E731	analysing '88-'89 data	* ϵ'/ϵ * CPT test	* few 10^6 observed $K_S \to 2\pi$ decays * few 10^5 observed $K_L \to 2\pi$ decays * $\sigma(\epsilon'/\epsilon) \simeq 10^{-3}$	* disagreement (2σ) on ϵ'/ϵ * new planned measurements
high intensity kaon beams	running + upgrades	* K_L, $K^\pm$ decays very rare or forbidden * $K_S \to \pi^+\pi^-\pi^0$	* 10^8–10^{11} observed decays	* dedicated exps. * some exps. are background limited
kaon factories	> 1995 planned	* K_L, K^+ decays very rare or forbidden	* 10^{11}–10^{13} expected decays	* dedicated exps. * background limitations

range and will reach in the next years the 10^{-11} to 10^{-13} level with the upgrade of the existing facilities and the construction of the proposed kaon factories.

On the other hand, low energy $K\overline{K}$ pairs produced in $p\overline{p}$ (K^-K^0, $K^+\overline{K}^0$) and e^+e^- (K^+K^-, K_LK_S) annihilations are more suited to study a broader set of CP violating neutral kaon decays, and in particular rare K_S decays, because of their unrivalled background conditions, K tagging capabilities and redundance of kinematical constraints. The two experimental approaches seem then complementary for the study of neutral kaons, with the exception of few issues such as the challenging measurement of Re(ϵ'/ϵ), where they are likely to be in direct competition. For what $K^\pm$ is concerned, beam facilities are rather competitive with respect to $p\overline{p}$ or e^+e^- machines; in fact, in addition to higher available statistics, in this case the beam backgrounds are expected to be much lower than for the neutral case and precise knowledge of the $K^\pm$ momentum is possible. An important measurement that could be performed is then the search for direct CP violating asymmetries in $K^\pm \rightarrow 3\pi$ decays (see later). Table II summarizes the characteristics of the various K sources.

Whichever is the topic studied (better understanding of CP and T violation, test of the standard model, search for "new physics"), the precision reached in a K experiment is always determined by the interplay of i) available statistics; ii) detector performances (acceptance, particle identification, calorimetry, tracking, systematics, etc.); iii) background conditions. In the following, on the basis of these items, we present some general considerations concerning the expected impact on various open K issues during the next few years and at the various facilities.

i) *Available statistics*

The number of K_L and K^+ decays already detected at high intensity K beams by experiments looking for rare decays is roughly of the same order of magnitude (few times 10^9) of what is expected in a year at a ϕ factory running at a luminosity of 2.5×10^{32} cm^{-2}s^{-1}. The expected upgrades of both experiments and beam lines should

allow an increase of the beam intensities by one order of magnitude. In addition, the advent of kaon factories around mid '90 should allow a gain of two more orders of magnitude, reaching the 10^{-13} sensitivity on selected K^+ and K_L decays, such as $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $K^+ \rightarrow \pi^+\mu^+\mu^-$, $K^+ \rightarrow \pi^+\mu^\pm e^\mp$ or $K_L \rightarrow \pi^0 e^+e^-$, $K_L \rightarrow e^+e^-$ and $K_L \rightarrow \mu e$. For what concerns K_S, it is more difficult to obtain pure and intense K_S beams, because their short lifetime limits the separation from the accompanying background. However, $K_S \rightarrow \pi^+\pi^-\pi^0$ is pursued by experiment E621 at Fermilab, where it is expected to improve the existing limit (BR $< 1.5 \times 10^{-7}$) by about two orders of magnitude before the advent of kaon factories (see Table I). If good quality, symmetric, high intensity K^+/K^- beams will be made available, there will be space for very significant sensitivity improvement in the search for $K^\pm \rightarrow 3\pi$ CP violating asymmetries.

On the other side, to be competitive on the measurement of Re(ϵ'/ϵ), a ϕ factory should be able to produce at least $\sim 10^{10}$ ϕ/year (see Table III). About 74% of ϕ's decay into pure $K\overline{K}$ pairs (with a very small radiative $K\overline{K}\gamma$ contribution well under control

TABLE III. Expected kaon fluxes at a ϕ factory for two different luminosities L.

	$L = 1 \times 10^{32}$ cm^{-2} s^{-1}	$L = 2.5 \times 10^{32}$ cm^{-2} s^{-1}
ϕ/year	4.1×10^9	1.0×10^{10}
$\phi \rightarrow K_L^0K_S^0$	1.4×10^9	3.5×10^9
$\phi \rightarrow K^+K^-$	2.0×10^9	5.0×10^9
tagged K_S^0	2.1×10^8	5.2×10^8
tagged K_L^0	9.6×10^8	2.4×10^9

[13]). Both charged and neutral K's are produced democratically (C-odd K^+K^- and K_LK_S pairs) and, thanks to their very low energy, a large fraction (100% of K_S, $\sim$ 25% of K_L and $\sim$ 70% of $K^\pm$) decays in the detector fiducial volume (assumed 1 m of radius). Concerning other experiments studying direct CP violation in $K \to 2\pi$ decays, this sample should allow a better statistical precision ($\simeq 3 \times 10^{-4}$ [14]) than at CP-LEAR (expected $\simeq 2 \times 10^{-3}$ [15]), and it would be comparable in size with the samples collected by the two most precise experiments, NA31 at CERN [16] and E731 at Fermilab [17].

Using this statistics, at a ϕ factory it would be possible to study other very interesting CP violating decays, by looking for the appropriate asymmetries in channels like $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$, $K^\pm \to \pi^0\pi^0\pi^\pm$, $K^\pm \to \pi^\pm\pi^0\gamma$, $K_{L,S} \to \pi^+\pi^-\gamma$, $K_L \to \pi^\pm l^\mp \nu$ and $K_L \to \pi^+\pi^-\pi^0$. In fact, the "natural" detector for this facility is a 4π experiment having very uniform acceptance over the whole phase space, allowing for precise study of Dalitz plots. In addition, all kind of kaons are produced at the same time and are monochromatic in energy: these facts should help in reducing systematic effects.

The situation seems also particularly favorable for what concerns K_S decays. In fact, for the first time it would be possible to obtain extremely pure (tagged) K_S beams, dramatically improving the knowledge of K_S branching ratios (most of them not yet measured, as for the decays $K_S \to \pi^0\nu\overline{\nu}$, $e^+e^-\gamma$, $\mu^+\mu^-\gamma$, $\pi^0e^+e^-$, $\pi^0\mu^+\mu^-$, $\pi^\pm\mu^\mp\nu_\mu(\overline{\nu}_\mu)$, $\pi^\pm$ $e^\mp\nu_e(\overline{\nu}_e)$, etc.) down to the 10^{-9} level (if the contamination coming from $\phi \to K\overline{K}\gamma$ is not unexpectedly large). As an example, by selecting events where the K_L decays into $\pi^\pm l^\mp \nu$ it should be possible to substantially improve the existing limit on the CP violating reaction $K_S \to 3\pi^0$, by looking for 6 low energy photons in the detector (we assume here good electromagnetic calorimetry over the full solid angle and capable of efficiently detecting photons down to energies of (few) ten(s) MeV). Another difficult search that could be performed concerns the decay $K_S \to \pi^0\gamma\gamma$, where it may be possible to detect a few events.

At CP-LEAR, the goal is to collect in the next few years $\sim 10^{13}$ $p\overline{p}$ annihilations at rest to study the (C-even) $K^+\pi^-K^0$ and $K^-\pi^+\overline{K}^0$ decays (the fraction $p\overline{p} \to K^\pm\pi^\mp K^0(\overline{K}^0)$ being of the order $\simeq 4 \times 10^{-3}$). Concerning direct CP violation in $K \to 2\pi$ decays, although the statistical error corresponding to this sample ($\sigma(\epsilon'/\epsilon) \simeq 2 \times 10^{-3}$) will be larger than for NA31 and E731, the systematic errors are expected to be completely different, motivating this experimental approach. In addition, this sample should allow a measurement of CP violation in the $K_S \to 3\pi$ channel (through the parameters η_{+-0} and η_{000}, discussed in the next Section). Finally, as a source of pure K^0 and $\overline{K}^0$ final states, LEAR represents a unique chance to study order ϵ CP violations in $K^0(\overline{K}^0) \to \gamma\gamma$ decays.

ii) *Detector performances*

To accurately measure the different K decays one needs detectors capable to identify $\pi^\pm, e^\pm, \mu^\pm$ and γ and to measure their energy and momentum.

At very low energy (ϕ factories, LEAR, stopped K^+ beams), good π/μ separation and efficient γ detection become difficult. For instance, in $K^+ \to \pi^+\nu\overline{\nu}$ experiments K^+ are stopped in the target, and then the detectable reaction signature consists of a single pion of non discrete energy occurring at a very low rate ($\sim 10^{-10}$) in the presence of a large background of muons (from $K^+ \to \mu^+\nu$ and $K^+ \to \mu^+\nu\gamma$). In this case the π/μ rejection factor ($\sim 10^{-6}$), given by energy and momentum-range measurement, is definitely not sufficient [18] (see Fig. 1, taken from ref. [19]), and it is necessary to follow the decay chain $\pi^+ \to \mu^+ \to e^+$. This method allows an additional few 10^{-5} rejection power but requires a continuous reading of the detector by means of high frequency (500 MHz) transient digitizers [19].

At higher energy ($p_K \sim O(10$ GeV/c$)$), like in experiments searching for $K_L \to \mu^\pm e^\mp$ or $K^+ \to \pi^+\mu^\pm e^\mp$, the leading potential background arises from the decay $K_L \to \pi^\pm e^\mp \nu_e$ followed by $\pi^\pm \to \mu^\pm\nu_\mu$ in flight. In order to suppress this background, high precision tracking of the charged decay products in a magnetic field is performed, to determine their momenta and trajectories and to reconstruct the K_L mass. To detect momentum change in flight a redundant muon energy determination is performed, measuring the momentum twice. Additional μ/π rejection is obtained also in this case by means of a range measurement, as it is shown schematically in Fig. 2 in the case of

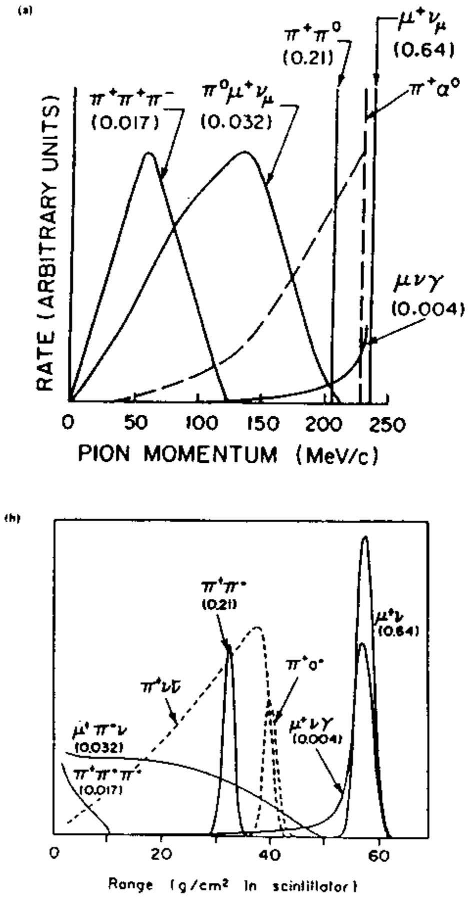

Fig. 1 - Spectra of K^+ decay modes. Dashed lines: $K^+ \to \pi^+\nu\bar{\nu}$ and $K^+ \to \pi x$. Solid lines: other modes. Branching ratios are shown in parentheses. The vertical scales are arbitrary and different for each decay. (a) Momentum spectra, and (b) range spectra in scintillator with finite resolution (taken from ref. [19]).

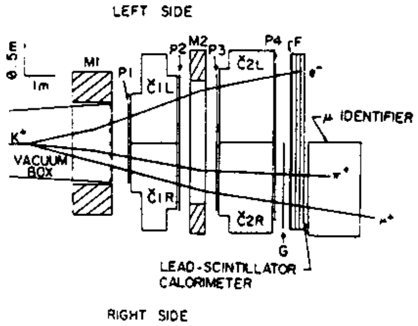

Fig. 2 - Schematic of the apparatus of BNL E777, used in the search for $K^+ \to \pi^+\mu e$ (from ref. [20]).

the experiment E777 at Brookhaven [20].

As far as photon detection is concerned, at low energy it is rather difficult to reliably detect them. In fact, in addition to the probability of not converting in the calorimeter, assuming $L/L_{rad} \simeq 15$

$$I_\gamma = e^{-\frac{7}{9}L/L_{rad}} \simeq 10^{-5} \quad , \tag{1}$$

shower fluctuations may cause additional inefficiencies. As an example, with a typical calorimeter (15 L_{rad} long, 1mm Pb/5mm scintillator sandwich [19], [21]), at $E_\gamma = 20$ MeV the visible energy results

$$E_{vis} = 0.44E_\gamma \pm 0.03\sqrt{E_\gamma} \simeq 8.8 \pm 4.2 \text{ MeV} \quad , \tag{2}$$

which implies, for $E_{thres} = 4$ MeV, a probability of $\simeq 15\%$ of not detecting the converted photon. Finally, a small γ fraction interacts directly with the nuclei instead of

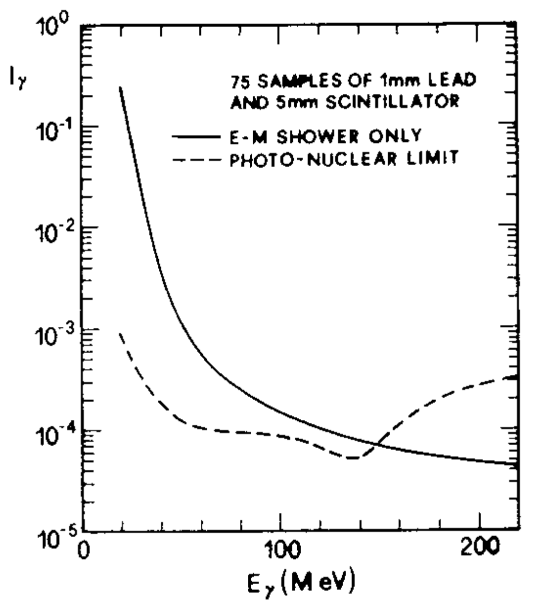

Fig. 3 - Photon detection inefficiency with lead scintillator calorimeter in the 20-220 MeV range (from ref. [22]).

undergoing an electromagnetic shower, so a large part of the energy goes into neutral reaction products and the photon is lost. Fig. 3 (taken from ref. [22]) shows the behaviour of the probability for a γ of not being detected as a function of its energy, in the range 20 – 220 MeV relevant for ϕ factories, LEAR and stopped K^+ beams. More sophisticated detectors using liquid Xenon or Krypton considered in [14] are expected to improve substantially the e.m. calorimetry at low energy.

The ability of detecting all photons in the event is essential to tag or veto various decays. For instance, when searching for $K^+ \to \pi^+ \nu\bar{\nu}$ decays, one has to reject $K^+ \to \mu^+ \nu\gamma$ or $K^+ \to \pi^+\pi^0$ decays by detecting all low energy photon(s) present in the events. Similarly, to search for the $K_S \to 3\pi^0$ decays at the ϕ resonance, one needs a very good acceptance on rather low energy photons.

At higher energies, the photon detection is easier because of the boost. Also the electromagnetic energy resolution, improving as $E^{-\frac{1}{2}}$, tends to favour the use of high energy K beams. On the other hand, high energy π^0's decay into photon pairs that often overlap, requiring good calorimetric resolution to be separated. Conversely, at low energy facilities, where the γ detection is more difficult, opening angles are large.

One should also recall that at higher energy, additional techniques of particles identification become available, like Čerenkov radiation detection and transition radiation detection (both useful for instance in π/e separation). However, as we will see in discussing the next point, although high energy experiments clearly have a better set of experimental techniques for particle identification, it is also true that their background conditions are much worse than at LEAR or at the future ϕ factories, because of the contamination of K^+ and K_L beams.

At a ϕ factory one expects that a 4π detector characterized by an excellent electromagnetic calorimetry, good momentum measurement, vertex reconstruction and $\pi/\mu/e$ separation, should be able to accurately measure all kinds of $K_{L,S}$, $K^\pm$ decays with rather good statistics, as it has been discussed above. If, from a purely statistics point of view, beam facilities look in perspective superior, in particular for what concerns K_L and $K^\pm$, at a ϕ factory the focus should be on precision measurements and on decay mode systematics. One could also expect to improve the existing limits on $K^\pm \to \pi^\pm\gamma\gamma$ by detecting tens or hundreds of events. Also in the case of the measurements of $K_{\ell 4}$ form factors or the measurements of final states with many neutral particles like $K^\pm \to \pi^0\mu^\pm\nu\gamma$, $\pi^\pm\gamma\gamma\gamma$, $l^\pm\nu\nu\nu$ and $K_{L,S} \to \pi^0\nu\nu$, this facility looks competitive.

It is also useful to recall that, in order to reach branching ratios of the order of 10^{-10} or less, which is the goal of most experiments at K sources, great care is needed to handle the large amount of data coming from the detector: trigger and data acquisition are clearly a key element in all these experiments.

When performing high precision measurements, as in the case of CP violating effects, systematic errors play a very important rôle. This matter is clearly rather complicated, but, without entering into details, it seems reasonable to expect that experiments performed at high intensity K beams will suffer from different kinds of systematics than experiments performed at $p\bar{p}$ or e^+e^- machines. Given similar statistical errors, this fact may help in the comparison of the various results.

iii) *Background conditions*

The background problems are clearly more severe at hadroproduced K beams. For instance in neutral kaons experiments one has to routinely tolerate neutron fluxes one order of magnitude larger than the number of K's decaying in the apparatus [10]. The beam design and the associated backgrounds are indeed basic elements when planning experiments at these facilities. In some case the sensitivity of the existing detectors is background limited, but in the last years the continuum upgrades of the sources and of the experimental performances has been very impressive and an additional boost will come when kaon factories will be operational. However, higher sensitivity is often attained designing very specialized experiments adapted to very few decay modes.

On the other hand, one of the strongest points in favor of a ϕ factory is the cleanliness of the final K state in comparison to the more complex final state in $p\bar{p}$ interactions or to the background accompanying high energy, high intensity K beams. Even though at a high luminosity e^+e^- machine we may expect a considerable flux of low energy photons and beam-gas interactions, it should be possible to eliminate this background by proper masking and by making use of the existing kinematical constraints. As an example, by making use of $K_{L,S}$ different decay lengths and selecting properly $K_L(K_S)$ decay containing two charged particles and no photons (i.e. $K_L \to \pi^\pm l^\mp \nu$, BR = 65.6%, and $K_S \to \pi^+\pi^-$, BR = 68.6%), it would be possible to "tag" $K_S(K_L)$ decays containing many photons with negligible contamination.

We then tentatively conclude that, for what concerns the study of some K issues, like for instance the K_S decays or K decays containing many neutral particles (ν, γ) in the final states, a ϕ factory equipped with a good 4π detector could be superior both to LEAR and to high intensity K beams.

In the following sections, various aspects of K physics still open (except $\mathrm{Re}(\epsilon'/\epsilon)$) are reviewed, keeping in mind the kind of experimental constraints discussed above. It

turns out that some of them will be better studied at high energy K beams (very rare and forbidden K decays), others at LEAR (CP violation in $K^0(\overline{K}^0)$ decays) and others at a ϕ factory (K_S decays, CP violating asymmetries in Dalitz plots).

Let us conclude with a comment on the challenging measurement of $\mathrm{Re}(\epsilon'/\epsilon)$, where all experimental approaches seem worth pursuing, because, while the statistical significance of the samples collected at the various machine seems comparable, the systematics are expected to be quite different.

III. Common decays: $K \to 2\pi$, $K \to 3\pi$, $K_{\ell 2}$, $K_{\ell 3}$, $K_{\ell 4}$

We consider here the purely hadronic decays

$$K \to 2\pi \quad , \qquad K \to 3\pi \quad , \tag{3}$$

together with the relatively common semileptonic decays

$$K_{\ell 2}(K \to \ell\nu_\ell) \quad , \quad K_{\ell 3}(K \to \pi\ell\nu_\ell) \quad , \quad K_{\ell 4}(K \to \pi\pi\ell\nu_\ell) \quad . \tag{4}$$

All these decays represent important sources of information either because of the appearance of symmetry violation effects (typically the CP violation in the $K \to 2\pi$ decays), or for the possibility of performing precision tests.

The purely hadronic decays $K \to 2\pi$ play a very important rôle in K decay physics. The decay $K_L \to 2\pi$, with typical branching ratio of the order of 10^{-3}, is until now the unique manifestation of "indirect" CP violation, measured by the parameter ϵ and induced by the state mixing. Furthermore, the comparative analysis of the two processes $K_L \to 2\pi$ and $K_S \to 2\pi$, through the simultaneous estimate of the two familiar amplitude ratios

$$\eta_{+-} = \frac{<\pi^+\pi^-|H_w|K_L>}{<\pi^+\pi^-|H_w|K_S>} = \epsilon + \epsilon' \quad , \quad \eta_{00} = \frac{<\pi^0\pi^0|H_w|K_L>}{<\pi^0\pi^0|H_w|K_S>} = \epsilon - 2\epsilon' \tag{5}$$

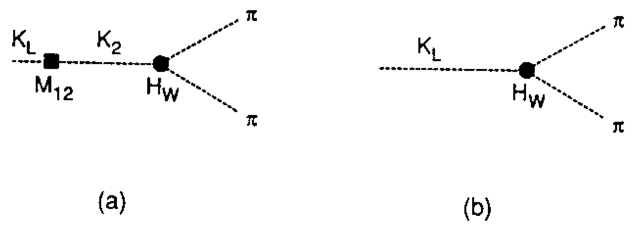

Fig. 4 - The decay $K_L \to 2\pi$ can manifest: (a) mass mixing and (b) direct $\Delta S = 1$ CP violation effects.

allows a measurement of the ratio ϵ'/ϵ, i.e. a test of the direct CP violation (see Fig. 4). A substantial effort has been recently performed from the experimental side, but with a rather controversial result, because there is a certain disagreement between the two most recent and precise determinations, those of the NA31 collaboration at CERN [16] and of the E731 experiment of the Chicago–Fermilab collaboration [17]:

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \begin{cases} (3.3 \pm 1.1) \times 10^{-3} & [16] \\ -(0.5 \pm 1.5) \times 10^{-3} & [17] \end{cases} \qquad (6)$$

We do not discuss here the $K \to 2\pi$ problem and its connection with the CP violation in the $K\overline{K}$ system, since the general formalism and its parametrization are considered in an exhaustive way in several reviews (for instance in [23] and more recently in [24]). We only give in Fig. 5 an account of the graphs that in the standard model are supposed to contribute to the kaon CP violation. A brief comment on their relevance is reported in the figure caption.

Quite recently, the analysis of ϵ'/ϵ in the standard model has been performed [25] by adding the Z^0 penguin contribution and considering a rather large value for the top-quark mass. It is somehow disappointing that the ratio ϵ'/ϵ is a decreasing function of m_t, so that the possibility of observing this effect as a check of the standard model

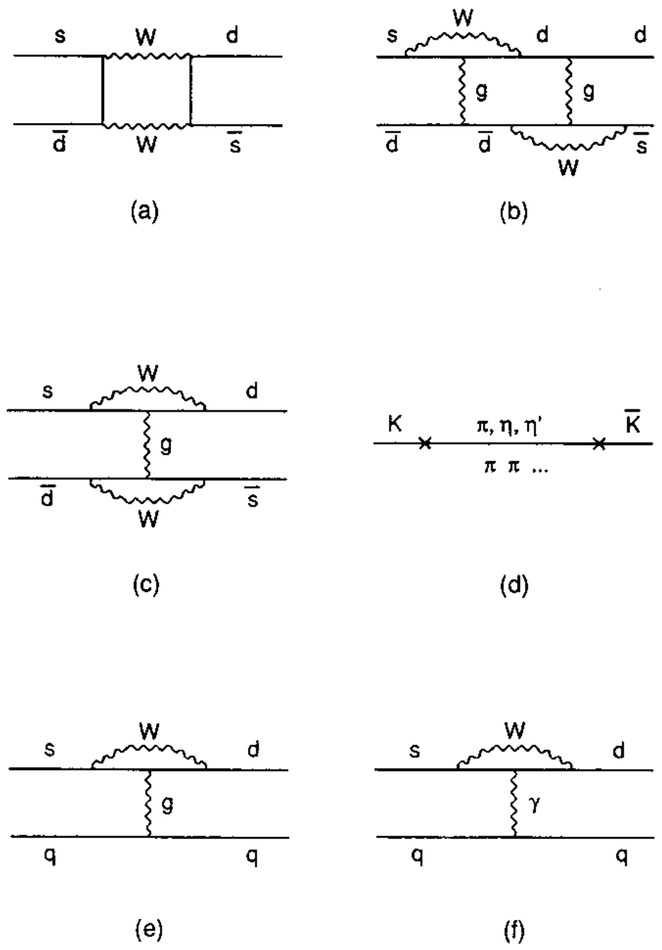

Fig. 5 - Diagrams contributing to CP violation in the $K\overline{K}$ system. Diagrams (a), (b), (c) and (d) are indirect (mass mixing) effects, while (e) and (f) are direct transitions. The so-called double penguin (b) and siamese penguin (c) give contributions smaller than the box diagram (a) because of the lack of heavy quark loops.

itself seems to be reduced.

In particular, the problem of determining $\mathrm{Re}(\epsilon'/\epsilon)$ at a ϕ factory from an analysis of the decay $\phi \to (K\overline{K})_{C=-1}$ has been faced in [26], [14], [27] and [13]: in the latter reference the possibility of performing the measurement also in presence of a possible contamination coming from the alternative decay $\phi \to \gamma S^*$ with $S^* \to (K\overline{K})_{C=+1}$ [28] has been considered too.

While the analysis of the $K \to 2\pi$ decays certainly is of central interest in K physics, a potentially important rôle is covered also by the $K \to 3\pi$ processes. The possibility of finding an evidence of CP nonconservation in $K \to 3\pi$ transitions could enable us to obtain further information, useful to distinguish between the various models of CP violation. The cleanest way to observe CP violation would be the search for the CP-forbidden decay $K_S \to 3\pi^0$, as $\mathrm{CP}(3\pi^0) = -1$. This search appears extremely difficult at high energy K^0 beams, mainly because of background induced by the uncertainty on the decay vertex position. It would become easier at a ϕ factory, provided one had an efficient detection system to reveal low energy photons. In fact, tagging the K_S and selecting $K_L \to \pi\mu\nu_\mu$, $\pi e \nu_e$ decays, one should search for events having 6 photons in the detector; it should not be too difficult to significantly improve the existing limit $\mathrm{BR}(K_S \to 3\pi^0) < 4 \times 10^{-5}$ [29]. However, from just mass mixing one would naively expect the branching ratio of $K_S \to 3\pi^0$ to be at most of the order of 10^{-9}. It follows that the statistics expected at a ϕ factory may not be sufficient to detect such a decay.

Actually, it should be interesting to analyse theoretically the possibility of some dynamical enhancement of the amplitude. Besides the well-known phenomenological analysis of Devlin and Dickey [30], there is a considerable amount of theoretical work on the $K \to 3\pi$ processes [31]. From the paper [32] to the most recent papers such as [33], [34], [35], the problem has been faced of connecting the new parameters characteristic of the direct CP violation of the $K \to 3\pi$ decay to the CP-violating parameters of $K \to 2\pi$. In particular, the amplitude ratio for $K_S \to 3\pi$ has been studied:

$$\eta_{+-0} = \frac{< \pi^+\pi^-\pi^0 | H_w^{(-)} | K_S >}{< \pi^+\pi^-\pi^0 | H_w^{(+)} | K_L >} = \epsilon + \epsilon'_{+-0} \qquad , \qquad (7)$$

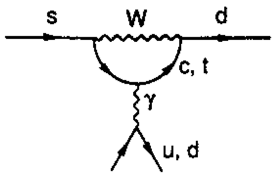

Fig. 6 - Electromagnetic penguin diagram, contributing to CP violation.

where $H_w^{(\pm)}$ are the CP–even and the CP–odd parts of the weak non–leptonic Hamiltonian. Within the standard model and using current algebra and the simplest linear parametrization of the $K \to 3\pi$ amplitudes, $\epsilon'_{+-0} = -2\epsilon'$ was derived [32]. A considerable modification of this result could follow from taking into account also electromagnetic penguin diagrams (see Fig. 6) and isospin violation contributions, as well as the possible effects of expanding the $K \to 3\pi$ amplitude at higher order in the pions momenta. Indeed the larger value $\epsilon'_{+-0} \simeq \epsilon/10$ has been obtained in this way in [33] (a result which does not seem unanimous, however, see ref. [34]). The other relevant question is the existence and the magnitude of the CP–conserving $K_S \to \pi^+\pi^-\pi^0$ amplitude, which is expected to be suppressed by an angular momentum (model dependent) factor. As a matter of fact, the indication seems to be that there should be a dominant (in spite of this suppression factor) CP–conserving, $\Delta I = \frac{3}{2}$ amplitude [34], [36], leading to $\mathrm{BR}(K_S \to \pi^+\pi^-\pi^0) \simeq (1-3) \times 10^{-7}$ (to be compared with the experimental upper limit of 1.5×10^{-7}), while the CP–violation transition would be an effect of order ϵ and thus $\sim 10^{-9}$. This gives the sensitivity required to measure CP violation in $K_S \to 3\pi$, which appears to be a hard measurement.

Of course, from the estimates [33] mentioned above, deviations of η_{+-0} from ϵ are even harder to be seen, but also a measurement of η_{+-0} to order ϵ could be significant:

for example, a theory of CP violation purely parity-conserving could generate ϵ in the $K\overline{K}$ system and could have η_{+-0} different from zero (and also as large as several times ϵ, at least in principle), while ϵ' could be strictly zero, as being related to a parity-violating process [33].

The existing limit on $K_S \to \pi^+\pi^-\pi^0$ should soon be substantially improved by experiment FNAL E621; the expected sensitivity is in fact $\simeq 3 \times 10^{-9}$, close to the predicted rate $\simeq 1.2 \times 10^{-9}$ [12], but a proposed upgrade of the same experiment should allow a test of this theoretical prediction.

Also the charged $K^\pm \to 3\pi$ are of great interest, because the enhancement effects of direct CP violation mentioned above might lead to charge asymmetries (both in the rates Γ's and in the linear slopes g's), in the perspective of a high statistics experiment [37]:

$$\begin{array}{ll}
\text{theory} & \text{experiment} \\
|\Delta\Gamma(K^\pm \to \pi^\pm\pi^\pm\pi^\mp)| \sim 0.39 \times 10^{-4} & \Delta\Gamma = \ \ (0.35 \pm 0.6) \times 10^{-3} \\
|\Delta\Gamma(K^\pm \to \pi^0\pi^0\pi^\pm)| \ \sim 0.11 \times 10^{-3} & \Delta\Gamma = -(0.15 \pm 2.7) \times 10^{-3} \quad . \\
|\Delta g(K^\pm \to \pi^\pm\pi^\pm\pi^\mp)| \ \simeq 0.14 \times 10^{-2} & \Delta g = -(0.75 \pm 0.5) \times 10^{-2} \\
|\Delta g(K^\pm \to \pi^0\pi^0\pi^\pm)| \ \simeq 0.14 \times 10^{-2} & \quad \text{not available}
\end{array} \qquad (8)$$

From the numbers above it seems that the search for CP asymmetries, in particular in the slopes of $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$, should be pursued at future high statistics experiments. This kind of measurements cannot be excluded for an experiment running at a ϕ factory, where K^+'s and K^-'s are produced in pairs and populate in the same way the detector fiducial region. This seems to allow for keeping the experimental systematics under control: with 10^{10} ϕ one expects to easily improve the existing experimental limits on the asymmetries by an order of magnitude, reaching the level of the theoretical predictions reported in eq. (8). Much larger statistics, however, might be collected at intense K^+/K^- beams. In this case, it is important to carefully design beams and apparatus to minimize any asymmetry in the detection efficiency for the two charged kaons.

We consider now the most common semileptonic decays. From $K_{\ell 2}$ ($K \to \mu\nu_\mu$, $K \to e\nu_e$) decay rates several precision measurements are possible. Let us mention in particular:

- the test of e–μ universality and V–A charged currents, inclusive of radiative correction effects, by improving the measurement of the ratio

$$R_K = \frac{\Gamma(K \to e\nu_e)}{\Gamma(K \to \mu\nu_\mu)} = (2.42 \pm 0.11) \times 10^{-5} \qquad ; \qquad (9)$$

- the constraints on non-(V–A) couplings in the strange sector.

As a byproduct of the analysis of these decays it was possible to include the search for effects of heavy neutrinos (ν_H), whose mixing was expected to influence the experimental ratio $(K \to e\nu)/(K \to \mu\nu)$, and whose presence would induce [38] low energy peaks in the μ momentum distribution in the reaction $K \to \mu\nu_H$. At present the possibility of relatively light neutrinos is, however, excluded by the recent results of LEP experiments [9].

$K_{\ell 3}$ decays ($K \to \pi\mu\nu_\mu$, $K \to \pi e\nu_e$) are relevant in testing the CKM matrix element $|V_{us}|$. The best determination of $|V_{us}|$ comes in fact from an analysis of $K_{\ell 3}$ decays performed in [39]: it is found

$$|V_{us}| = 0.2196 \pm 0.0014 \pm 0.0018 \qquad , \qquad (10)$$

where the first error comes from the experiment and the second combines the uncertainties due to SU(3) breaking and the isospin violation with those coming from radiative correction effects. On the other hand V_{us} enters, together with V_{ud} and V_{ub}, in the fundamental test of the unitarity of the CKM matrix in the usual scheme of three generations. At present [40]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9979 \pm 0.0021 \qquad . \qquad (11)$$

Since V_{ud} is expected to be improved, a better estimate of V_{us} seems of relevance.

Another important aspect is the measurement of form factors, which can severely test the models used to realize the non-perturbative, long-distance QCD [41]. Of great importance in this context is the Callan-Treiman relation [42], which represents a prediction from first principles of the slope of the scalar form factor. Not all the data are in agreement with this relation, and in general one observes discrepancies among $K^+_{\mu 3}$ and $K^0_{\mu 3}$, which should be resolved in a next generation of experiments. Also, and correlated with the above problem, there is the measurement of the longitudinal muon polarization: the various experimental data show appreciable disagreement among each other, and future improvements should be welcome also in this field.

Very importantly, the charge asymmetry in $K_L \to \pi \ell \nu$ decay provides an alternative measurement of the indirect CP violation parameter ϵ. The experimental result, averaged over $\ell = e, \mu$, is $\delta \simeq 2\mathrm{Re}\,\epsilon = (3.30 \pm 0.12) \times 10^{-3}$ [31], where in expressing the asymmetry δ as a function of $\mathrm{Re}\,\epsilon$ the small fraction of decays violating the $\Delta S = \Delta Q$ rule has been neglected.

An interesting aspect concerning $K_{\mu 3}$ decays could be the measurement of the muon transverse polarization $\sigma_\mu \cdot (p_\pi \times p_\mu)$. Here a finite value would indicate a violation of the time-reversal invariance [31], [43], up to higher order Coulomb corrections. The existing experimental limit on the T-violating component of the μ polarization $P_N = (-1.85 \pm 3.60) \times 10^{-3}$, derived [44] from both $K^+_{\mu 3}$ and $K^0_{\mu 3}$ decays, has been obtained with a rather small number of events ($\simeq 2 \times 10^7$), and the uncertainties are almost wholly statistical. The standard model would predict zero muon polarization, apart from electromagnetic corrections which are negligible for the $K^+_{\mu 3}$. Therefore this kind of measurement could be helpful to constrain alternative ideas on CP violation [45]. From a statistical point of view, with the available and future kaon sources, there is clearly the possibility to improve the accuracy by at least one order of magnitude. However, such an experiment would require a highly segmented polarimeter to measure the μ polarization through its $e\nu_e\nu_\mu$ decays. While the feasibility of this kind of detector has already been demonstrated at (low energy) K beams, the possibility to operate a 4π detector with μ polarimetry at a ϕ factory has not been analysed yet. In the affirmative case, this would be an important tool to study T-violating processes in K decays involving μ's, and in any case an interesting experiment *per se*. Thus, in conclusion, the $K_{\ell 3}$ decay still represents a valuable laboratory where to test current fundamental ideas underlying the theory of strong interactions, as well as to look for signatures of standard model extensions.

We close by stressing the interest of the $K_{\ell 4}$ decays $K \to \pi\pi e \nu_e$ and $K \to \pi\pi\mu\nu_\mu$, which occur at the level of $(2-6) \times 10^{-5}$, depending on the specific mode. They give important information on the $\pi\pi$ low-energy phase shifts [46], and in general the measurement of the vector and axial-vector form factors represents a test of the prediction of the various theoretical approaches, in particular those based on effective chiral lagrangians [47]. Indeed, only a very limited number of events (a few tens) for the K_L mode has been collected until now. Probably there is the possibility to fulfill this gap at a ϕ factory. Also in the case of the $K_{\ell 4}$ decays T-odd correlations can be searched for [43], [45].

IV. Not so rare decays: $K \to (\ell\nu)\gamma$, $K \to (\ell\nu)(\ell\bar{\ell})$, $K_{2\gamma}$, $K \to (\ell\bar{\ell})\gamma$, $K^\pm \to \pi^\pm\gamma\gamma$, $K \to \pi\pi\gamma$

This sector involves a conspicuous number of radiative leptonic and non-leptonic decays, whose branching ratios make still possible precision measurements. On the other hand, a theoretical understanding of these decays requires an accurate description of the hadronic structure and an estimate of the strong interaction effects.

A measurement of vector and axial-vector structure dependent form factors is possible from the analysis of the upper range of the photon spectrum in the decay $K \to (\ell\nu)\gamma$ [48], where for $\ell = e$ the interference with the bremsstrahlung is small. An information on these form factors is needed as it represents a portion of the $O(\alpha)$ corrections to the ratio R_K of eq. (9). Also, it has a distinguished rôle in the context of form factor calculations based on current algebra and PCAC low energy theorems, now implemented via chiral symmetry lagrangians [49], which are briefly introduced below. Present determinations of $K_{\ell 2\gamma}$ are very poor, and the same is true for the virtual photon process $K \to e\nu e^+e^-$, which depends on a third, additional form factor. The $K_{\ell 3\gamma}$ process is very appealing theoretically, as the structure dependent part can be predicted model independently [50]. It seems that the verification of this prediction could be a challenging (but rewarding) experimental problem.

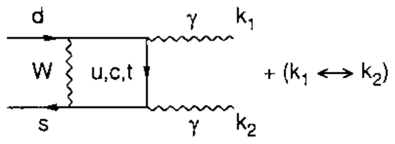

Fig. 7 - The short distance contribution to the $K_L \to \gamma\gamma$ decay.

The processes $K^0 \to \gamma\gamma$, $K^0 \to \gamma\ell^+\ell^-$ $(\ell = e, \mu)$, $K \to \pi\gamma\gamma$ and $K \to \pi\pi\gamma$ introduce the class of flavour–changing neutral current transitions with photons in the final state, suppressed in the standard model by the GIM mechanism, and with branching ratios in the range below 10^{-4}. These are described by an electroweak transition hamiltonian H_{eff} in terms of quark (and gluon) fields, where the dominance of short–distance effects is assumed. The example relevant to the decay $K_L \to \gamma\gamma$ is shown in Fig. 7. It is then necessary to estimate the hadronic matrix elements of such a H_{eff}. In addition to that, long–distance contributions are also expected, arising from non–perturbative strong interaction corrections to the quark diagrams at a low mass scale. They are entirely determined by hadronic degrees of freedom, thus by details of the confinement physics. An example of this kind of effect is represented, for $K_L \to \gamma\gamma$, by the polar diagrams of Fig. 8.

Besides probing the standard model, these decays are very useful in order to test the various dynamical approaches followed in estimating the hadronic matrix elements.

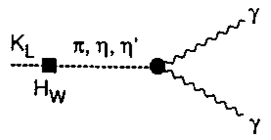

Fig. 8 - The pole diagrams for the $K_L \to \gamma\gamma$ decay.

In ref. [51], where these decays were first considered in the context of the standard model, simple quark model estimates were used. The framework which recently has become most popular is represented by the so-called "chiral perturbation theory", where effective chiral lagrangians are used as computational tools. The basic idea is represented by an assumed $SU(3)_L \times SU(3)_R$ chiral symmetry of strong interactions, valid in the zero mass limit of the lightest quarks (u, d and s). A spontaneous symmetry breaking to $SU(3)_V$ is supposed to occur in that limit, giving rise to an octet of massless Goldstone bosons, which are associated to the usual pseudoscalar octet of the lowest lying mesons, i.e. the pions, the kaons and the eta. It is possible to implement this idea in the framework of QCD, and to integrate out the gluon and quark degrees of freedom of the basic QCD lagrangian to obtain an effective (non linear) weak interaction non-leptonic lagrangian in terms of pseudoscalar meson fields, the photon and the lepton fields, with the desired symmetry properties mentioned above [49]. In particular such a lagrangian contains the soft pion results of current algebra and PCAC.

In this lagrangian approach, finite pseudoscalar meson masses and momenta are accounted for by respectively expanding amplitudes in the quark masses and by considering meson loops and the addition of higher derivative interactions. The general form of the one-loop effective action describing $\Delta S = 1$ non leptonic weak transition, and accounting for the orders (m^2p^2, p^4, m^4), has been worked out in ref. [52]. Attempts to account for some $O(p^6)$, and to incorporate vector meson resonances and the notion of vector meson dominance in the chiral lagrangian formalism, have appeared recently [53], [54]. The advantage of this scheme, which is supposed to reliably describe effects of QCD at low energy (i.e. at long distance), is that one can predict by means of the effective lagrangian a multitude of transition amplitudes involving pions and kaons (and the eta) in terms of a reduced number of constants, which characterize the higher order corrections mentioned above, and are fit from few experimental data. For this reason it has been extensively applied to kaon semileptonic and non-leptonic decays.

Indeed, specific treatments of the different decays within the chiral lagrangian approach, with refinements of the computational schemes, can be found in the literature. Examples are the analysis of the non-leptonic radiative K decays with at most one pion in the final state ($K \to \gamma\gamma$, $K \to (\ell\bar{\ell})\gamma$, $K \to \pi\gamma\gamma$, $K \to \pi\ell^+\ell^-$), performed in ref. [55], that of the semi-leptonic processes ($K_{\ell 3}$, $K_{\ell 3\gamma}$, $K_{\ell 4}$) of ref. [41], or the estimate of the branching ratio of the decay $K \to \gamma\gamma$ [56], [57], [58].

In Table IV, updated from ref. [59], are reported the predictions derived within the chiral perturbation theory in ref. [55] for the branching ratios of the radiative decays considered above, together with those obtained for more rare decays which will be considered in the following. To fit the coefficients of higher order terms in the chiral lagrangian, the measured rate of the decay $K^+ \to \pi^+ e^+ e^-$ has been used as an input. In the same Table we report, for comparison, the values originally obtained in [51]. The considerable agreement between theory and experiment [60] in the estimate of the decay $K_S \to \gamma\gamma$ is considered as an important success of the chiral lagrangian approach.

Briefly commenting the most important aspects, $K_S \to \gamma\gamma$ has only long-distance contributions, so it is determined by meson loops, an example of which is reported in Fig. 9. The same is true for $K_S \to \gamma\ell^+\ell^-$, where one photon becomes virtual. Conversely, $K_L \to \gamma\gamma$ (and $K_L \to \gamma\ell^+\ell^-$) has both short- and long-distance contributions.

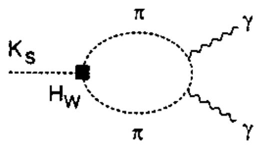

Fig. 9 - Chiral loop mechanism for $K_S \to \gamma\gamma$.

TABLE IV. Branching ratios of some rare kaon decays.

mode	theory	experiment
$K_S \to \gamma\gamma$	2×10^{-6} [1.4×10^{-6}]	$(2.4 \pm 1.2) \times 10^{-6}$
$K_S \to \pi^0\gamma\gamma$	3.3×10^{-8} [$10^{-8} - 10^{-9}$]	———
$K_S \to e^+e^-\gamma$	3.2×10^{-8}	———
$K_S \to \mu^+\mu^-\gamma$	7.5×10^{-10}	———
$K_S \to \pi^0 e^+e^-$	$5 \times 10^{-9} - 5 \times 10^{-10}$ [10^{-8}]	$< 4.5 \times 10^{-5}$
$K_S \to \pi^0\mu^+\mu^-$	$1 \times 10^{-9} - 1 \times 10^{-10}$	———
$K_L \to \gamma\gamma$	[10^{-4}]	$(4.9 \pm 0.4) \times 10^{-4}$
$K_L \to \pi^0\gamma\gamma$	6.8×10^{-7} [$< 10^{-7}$]	$(2.1 \pm 0.6) \times 10^{-6}$
$K_L \to e^+e^-\gamma$	9.1×10^{-6}	$(1.7 \pm 0.9) \times 10^{-5}$
$K_L \to \mu^+\mu^-\gamma$	2.3×10^{-7}	$(2.8 \pm 2.8) \times 10^{-7}$
$K_L \to \pi^0 e^+e^-$	$10^{-11} - 10^{-12}$	$< 4.2 \times 10^{-8}$
$K_L \to \pi^0\mu^+\mu^-$	$10^{-11} - 10^{-12}$	$< 1.2 \times 10^{-6}$
$K^\pm \to \pi^\pm e^+e^-$	input[10^{-6}]	$(2.6 \pm 0.5) \times 10^{-7}$
$K^\pm \to \pi^\pm\mu^+\mu^-$	6.1×10^{-8}	$< 2.3 \times 10^{-7}$
$K^+ \to \pi^+\gamma\gamma$	$\geq 4 \times 10^{-7}$ [$10^{-6} - 10^{-7}$]	$< 1 \times 10^{-6}$

Indeed the latter, represented in Fig. 8, should be the dominant one, and is quite sensitive to the details of $SU(3)$ symmetry breaking and $\eta - \eta'$ mixing [58]. It seems interesting to measure the Dalitz plot of this process, to distinguish various possible theoretical predictions. This analysis has been recently performed in [61]. On the contrary, no experimental upper limit exists for the $K_S \to \gamma \ell^+ \ell^-$. Limited to the e^+e^- mode, this seems in the reach of a ϕ factory. Also of interest, in the context of CP violation and of the high statistics measurements of the kaon decays, should be the asymmetry (τ being the proper time):

$$A(\tau) = \frac{\Gamma(K^0 \to \gamma\gamma) - \Gamma(\overline{K}^0 \to \gamma\gamma)}{\Gamma(K^0 \to \gamma\gamma) + \Gamma(\overline{K}^0 \to \gamma\gamma)} \quad , \qquad (12)$$

for which various estimates exist [62]. The two photons can be in two different CP states corresponding to the two states of polarization. Therefore, both $K_S \to \gamma\gamma$ and $K_L \to \gamma\gamma$ are allowed. Anyway, in presence of CP violation, interference between K_S and K_L amplitudes in an originally pure $K^0(\overline{K}^0)$ beam should occur, and could be detected in the measurement of the decay rate asymmetry.

This measurement seems ideal for a facility such as LEAR, where pure (tagged) $K^0(\overline{K}^0)$ beams are available by selecting $K^\pm \pi^\mp K^0(\overline{K}^0)$ events with the $K^\pm$ decaying semileptonically. With a sample of $\simeq 10^{13}$ $p\overline{p}$ interactions at rest (expected for CP-LEAR during the next few years) an attempt to measure this asymmetry seems feasible, at least from a statistical point of view. In fact, from the branching ratios for $K_S \to \gamma\gamma$ and $K_L \to \gamma\gamma$ measured by NA31 [60], and using 50% as detection efficiency, the expected statistical error is $\leq 10^{-3}$, i.e. of the same order of the ϵ effect.

Regarding the $K^\pm \to \pi^\pm \gamma\gamma$ decays, chiral perturbation theory is not able to predict the absolute rate, which depends on an unknown parameter. However it predicts in general the lower bound reported in Table IV. Actually, with the (model dependent) inclusion of vector mesons in the chiral perturbation theory framework [53], that parameter can be fixed and the lower bound comes to the prediction for the branching ratio of 5.8×10^{-7} instead of the value in Table IV. Also, the form of the $\gamma\gamma$ invariant mass spectrum is characteristic, as it closely reflects the dynamics underlying such a calculation, so it should be interesting to try to measure it. In this regard, the experimental (preliminary) limit reported in Table IV has been obtained by the AGS experiment 787 [63] as an improvement of the previous limit found by [64]. It assumes a phase space distribution for the $\gamma\gamma$ mass spectrum. This assumption is model dependent, and, if the characteristic $\gamma\gamma$ spectrum coming from the theoretical approach is used, then a much less stringent limit is obtained.

A CP–violating asymmetry can be defined also for $K^\pm \to \pi^\pm \gamma\gamma$ decays, and one can obtain qualitatively [55]:

$$\frac{\Gamma(K^+ \to \pi^+\gamma\gamma) - \Gamma(K^- \to \pi^-\gamma\gamma)}{\Gamma(K^+ \to \pi^+\gamma\gamma) + \Gamma(K^- \to \pi^-\gamma\gamma)} \leq 10^{-3} \quad , \qquad (13)$$

corresponding to the lower bound on the rate mentioned above. While it should be possible to detect this decay in the existing or future facilities, the measurement of the CP-violating asymmetry seems out of reach.

The decay $K_L \to \pi^0\gamma\gamma$ is a nice test of the chiral lagrangian realization of the standard model, in the same sense as (or even more strictly than) $K^\pm \to \pi^\pm\gamma\gamma$. It can be predicted with no free parameters in chiral pertubation theory [55] (for alternative calculations in this framework see ref. [65], [66], [67]). Also, analogously to $K^\pm \to \pi^\pm\gamma\gamma$, there is a characteristic two-photon mass distribution. Particularly important in this process, from the theoretical point of view, is the rôle played by the inclusion of vector meson dominance [53], [68].

Quite recently, this decay mode has been observed, with the branching ratio $BR(K_L \to \pi^0\gamma\gamma) = (2.1 \pm 0.6) \times 10^{-6}$ for decays with invariant two–photon mass above 280 MeV [69]. It is superfluous to underline the relevance of the above experimental measurement within the class of GIM suppressed radiative K decays. Although somewhat higher, the above result for the branching ratio is not in disagreement with estimates based on chiral perturbation theory. Actually, it is encouraging that the experimental two–photon spectrum strongly favors the behaviour indicated by this kind of theoretical approach.

The other important point is that $K_L \to \pi^0\gamma\gamma$ is needed to estimate the CP-conserving, two-photon exchange contribution to the much more rare decay (see later) $K_L \to \pi^0 e^+ e^-$, and thus to decide whether the CP-violating one-photon amplitude can dominate the latter process. It follows that the measurement of $K_L \to \pi^0\gamma\gamma$ has important bearing on CP-violation studies. Concerning the decays $K_S \to \pi^0\gamma\gamma$, this is expected to be dominated by the long-distance $K_S \to \pi^0\pi^0 \to \pi^0\gamma\gamma$ over the entire Dalitz plot, as there is no cancellation between π^0 and η (actually the latter contribution seems to be quite suppressed). Notice that at present there is no experimental limit for this decay, which seems a good candidate to be studied at a ϕ factory.

It might be interesting to mention, at this point, that amplitudes of non-leptonic radiative K decays with at most one final pion are necessarily of higher order in chiral perturbation theory (the lowest order vanishes). This implies in general a characteristic suppression of order $m_K^2/(16\pi^2 f_\pi^2) \ll 1$, which can overcome the $\Delta I = \frac{1}{2}$ enhancement of the basic non-leptonic $\Delta S = 1$ weak hamiltonian. Indeed, such a suppression effect is observed in the $K^\pm \to \pi^\pm(e^+e^-)$ decay discussed later on.

We turn now to the decays $K \to \pi\pi\gamma$. They could give in principle additional information on direct CP violation, via charge asymmetries in Dalitz plots or through interference between $K_S \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-\gamma$, similarly to $K_{L,S} \to 3\pi$. They bear on the question whether the $\Delta I = \frac{1}{2}$ enhancement in $K^\pm \to \pi^\pm\pi^0$ persists in $K^\pm \to \pi^\pm\pi^0\gamma$. Moreover, they have a rôle in the determination of ϵ' from $K_L \to 2\pi$ as they are a radiative correction there [70]. Also, they should allow, in principle, to clarify the relevance of vector mesons in chiral lagrangians. The $K \to \pi\pi\gamma$ amplitudes have two components which can be measured separately: the inner bremsstrahlung (IB), reliably computed in terms of $K \to 2\pi$, and the direct emission (DE) (in turn split into short–distance plus long–distance polar amplitudes), which suffers from theoretical uncertainties coming from the incomplete knowledge of the confining physics. Recent attempts to estimate the DE amplitudes have appeared [67], [71], [72]. In Table V we have reported the results obtained in these refs., as well as the available experimental data [73], [74], [75], [76], [77]. The comparison looks acceptable, keeping in mind the complications due to one more pion in the final state, compared to the cases above. However, some more theoretical effort may be needed, to achieve more model independent predictions.

TABLE V. Branching ratios of $K \to \pi\pi\gamma$ decays. The theoretical predictions of the two components of inner bremsstrahlung (IB) and direct emission (DE) come from ref. [67, 71]. The experimental references are also given.

mode	theory	experiment	reference
$K^+ \to \pi^+\pi^0\gamma\vert_{IB}$	2.88×10^{-4}	$(2.55 \pm 0.16) \times 10^{-4}$	[73, 74]
$K^+ \to \pi^+\pi^0\gamma\vert_{DE}$	$(1.05\text{-}1.94) \times 10^{-5}$	$(1.56 \pm 0.35) \times 10^{-5}$	[73]
		$(2.05 \pm 0.46) \times 10^{-5}$	[75]
		$(2.3 \pm 3.2) \times 10^{-5}$	[74]
$K_L \to \pi^+\pi^-\gamma\vert_{IB}$	1.45×10^{-5}	$(1.52 \pm 0.16) \times 10^{-5}$	[76]
$K_L \to \pi^+\pi^-\gamma\vert_{DE}$	$(7.0\text{-}1.19) \times 10^{-5}$	$(2.89 \pm 0.28) \times 10^{-5}$	[76]
$K_S \to \pi^+\pi^-\gamma\vert_{IB}$	2.41×10^{-3}	$(1.85 \pm 0.10) \times 10^{-3}$	[77]
$K_S \to \pi^+\pi^-\gamma\vert_{DE}$	2.02×10^{-7}	$< 6 \times 10^{-5}$	[77]

Finally, the possibility of observing CP violation in the $K \to \pi\pi\gamma$ decays has been examined in [78]. The perspectives do not seem very favorable, even with high statistics.

V. Rare decays: $K^\pm \to \pi^\pm(\ell^+\ell^-)$, $K^\pm \to \pi^\pm(\nu\bar{\nu})$, $K \to \mu^+\mu^-$, $K \to e^+e^-$

All of these decays are extremely important, and, at the same time, accessible through the present experimental techniques: most of them belong to the category that can be studied with precision only at the existing high intensity K beams and at the future kaon factories, as their branching ratios fall below 10^{-8}.

They are all allowed within the standard model, but only through one–loop graphs, in this way testing more intimately the standard model itself. At the same time, they may become sensitive to specific extensions of the standard model, so representing probes of "new physics". Let us consider their main properties.

$K^\pm \to \pi^\pm(\ell^+\ell^-)$

This decay is a typical process in which effects due to (virtual) heavy quarks may play an important rôle. It is expected to be dominated by the so-called "electromagnetic penguin diagram", which involves the replacement of a gluon by a virtual photon in the usual penguin: the short–distance contribution is then represented by the quark diagrams depicted in Fig. 10. The largest contribution in Fig. 10 comes from the charge radius-like term in the effective $sd\gamma$ vertex of the first graph. The loop is only logarithmically GIM suppressed, so that the charm quark contribution dominates even for large top quark mass. The original estimate of ref. [51] within the four-quark model led to a prediction for the branching ratio of the right order of magnitude to be in agreement with the measured value. The latter is [79]

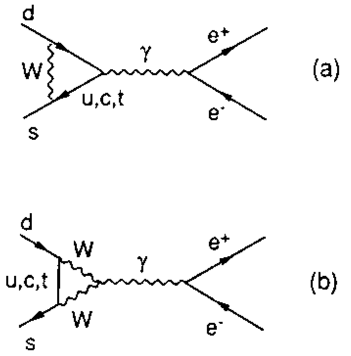

Fig. 10 - The short distance contribution to the $K^\pm \to \pi^\pm e^+ e^-$.

$$\mathrm{BR}[K^\pm \to \pi^\pm(e^+e^-)]_{\mathrm{exp}} = (2.6 \pm 0.5) \times 10^{-7} \quad , \tag{14}$$

while the present upper limit for the muon mode is [80]

$$\mathrm{BR}[K^\pm \to \pi^\pm(\mu^+\mu^-)]_{\mathrm{exp}} < 2.3 \times 10^{-7} \quad . \tag{15}$$

Actually, later calculations showed that QCD effects can change the free (from strong interactions) quark model result of [51] both in magnitude and in sign. Furthermore, other diagrams have been found to give contributions to the amplitude of $K^\pm \to \pi^\pm(e^+e^-)$, which are comparable to that of Fig. 10, and which unfortunately enter with different signs [81]. Therefore, the dynamics responsible for this process is much more complex than originally envisaged in Fig. 10. These extra effects are induced by the action of the effective hamiltonian for $\Delta S = 1$ non leptonic weak interactions, and include both short-distance and long-distance mechanisms, as pictorially represented in Fig. 11. This figure is taken from ref. [82], where this situation has been systematically analysed in the framework of the six-quark model.

It follows that the decay $K^\pm \to \pi^\pm(\ell^+\ell^-)$ is regarded with considerable interest within the approach of the chiral perturbation theory, and indeed it is used to fix one free parameter in the chiral lagrangian. Once having done that, the $(\ell^+\ell^-)$ invariant mass spectra and the ratio of the $\mu^+\mu^-$ to the e^+e^- modes become well-defined predictions which should be possible to test at a high statistics facility. In this regard, a factor 50 improvement for the $K^+ \to \pi^+e^+e^-$ and a factor 10 for $K^+ \to \pi^+\mu^+\mu^-$ are foreseen at existing experiments.

It is worth mentioning that the transition $K^\pm \to \pi^\pm(\ell^+\ell^-)$ has been one of the most promising ones in the search of a light Higgs ($K^\pm \to \pi^\pm H$) with H decaying into $(\ell^+\ell^-)$ or of other light scalar or pseudoscalar particles, as familons, axions, etc., a subject which has been intensively pursued until recently (a very recent approach to the decays $K \to \pi H$ and $K \to \eta H$ is given in [83]). One of the most interesting results of the first phase of LEP is, however, the very strong limit posed on the H mass: 32 MeV $< m_H <$ 24 GeV at 95% C.L. [7], which discourages further light Higgs

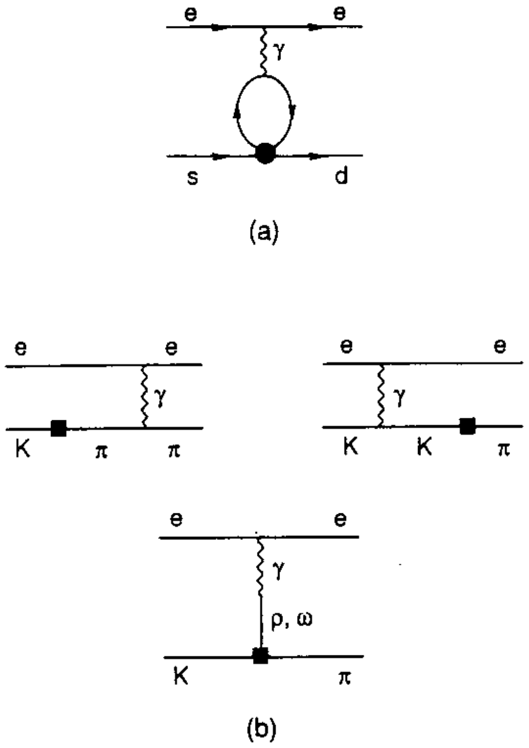

Fig. 11 - (a) Strong interaction corrections to the electromagnetic penguin diagram of the decay $K^\pm \to \pi^\pm e^+ e^-$. The black circle represents W exchange plus all strong-interaction corrections. (b) Long distance contributions to $K \to \pi e^+ e^-$. The black box represents the action of the effective Hamiltonian for $\Delta S = 1$ non leptonic weak decays.

searches.

$K^\pm \to \pi^\pm(\nu\bar{\nu})$

The experimental limit on the branching ratio of this decay has been recently improved. At present we have [84]

$$\mathrm{BR}[K^\pm \to \pi^\pm(\nu\bar{\nu})]_{\mathrm{exp}} < 3.4 \times 10^{-8} \quad , \tag{16}$$

to be compared with the previous limit 1.4×10^{-7} [85]. This is one of the decays best studied within the standard model, and it proceeds through the second order weak diagrams of Fig. 12. Within the standard model with three generations (in

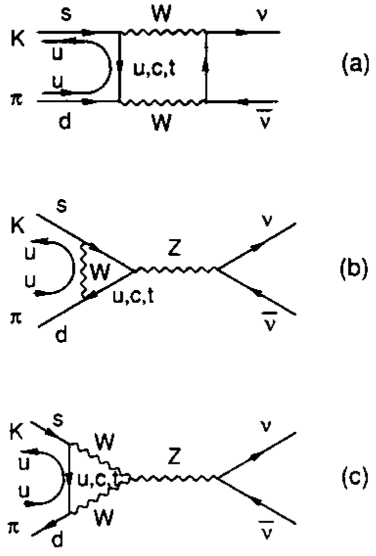

Fig. 12 - Second-order weak diagrams for $K^\pm \to \pi^\pm \nu\bar{\nu}$.

particular with three light neutrinos) one can distinguish [86] two main contributions to the total amplitude, coming from the charm and the top quark loops (including the interference, which is generally constructive). The recent prediction about the t–quark mass [87] leads to expect that the top contribution is likely to dominate: taking $|V_{ts}^* V_{td}/V_{cs}^* V_{cd}| \simeq 4 \times 10^{-3}$ and m_t in the interval 100 – 170 GeV [87], one derives

$$\mathrm{BR}[K^{\pm} \to \pi^{\pm}(\nu\bar{\nu})]_{\mathrm{th}} \simeq (1-7) \times 10^{-10} \quad . \tag{17}$$

A detailed account of the dependence of the above branching ratio on the t-quark mass and on other parameters of the standard model can be found in [86], from which Fig. 13 has been taken.

The important feature of this process is that the short–distance contributions to the branching ratio (17), estimated from the diagrams in Fig. 12, are numerically much larger than the long–distance corrections, which have been recently estimated in [88], while QCD effects are also found to be small and reasonably unambiguous [89]. Consequently, these long–distance effects should not obscure the short–distance H_{eff} one is interested in, so that the decay $K^{\pm} \to \pi^{\pm}(\nu\bar{\nu})$ offers a unique possibility to directly test the electroweak short–distance dynamics at quark level, and thus to significantly constrain the parameters of the heavy quarks which mediate the transition.

A branching ratio larger than that reported in eq. (17) can be an indication of some effect difficult to be controlled within the standard model: one would expect either a very large top mass, but this would be in contradiction with the prediction coming from the compatibility of the radiative correction effects in the neutral current sector when compared with the present estimates of the vector boson masses [87], or a large value of $|V_{ts}^* V_{td}|$, which, however, tends to violate the three generation unitarity constraint analogous to eq. (11). Moreover, one has to take into account the bound on both $|V_{ts}^* V_{td}|$ and the t-quark mass coming from the estimate of the short–distance amplitudes of the $K_L \to \mu^+\mu^-$ decay, which is discussed later on.

It follows that a branching ratio larger than expected would be a signal of "new physics", such as the existence of a fourth generation heavy quark dominating the quark

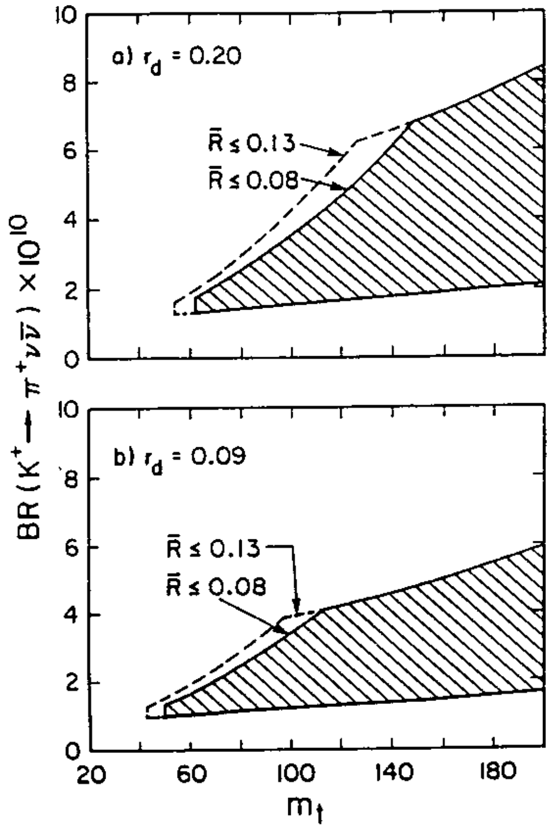

Fig. 13 - Allowed branching ratio for $K^+ \to \pi^+\nu\bar{\nu}$ under the $B - \bar{B}$ mixing versus the top-quark mass m_t (GeV) from Ref. [86]. In this figure $\bar{R} = \Gamma(b \to u)/\Gamma(b \to c)$ whereas $r_d = \Gamma(B_d^0 \to \ell^+ X)/\Gamma(B_d^0 \to \ell^- X)$.

loop (but with a heavy neutrino and requiring a rather large mixing between third and fourth generation), or of some exotic quark mixing [90], [86]. Indeed, a fourth generation with standard couplings and light neutrinos is practically excluded by LEP experiments [9]. There remains, however, the possibility of a fourth fermion family characterized by a heavy neutral lepton, which cannot be excluded experimentally and still has some room from the theoretical point of view.

Other candidates of "new physics" can also be signalled by an enhancement of the transition $K^+ \to \pi^+$, via its connection to the phenomenology of $K^+ \to \pi^+ x x'$, which simulates the $K^\pm \to \pi^\pm(\nu\bar{\nu})$ decay if x, x' are light neutral scalar or pseudoscalar particles which only weakly interact with the apparatus and thus escape detection. One example could be the lepton flavour violating process $K^\pm \to \pi^\pm(\nu\bar{\nu}')$, which can be mediated, in standard model estensions, by hypothetical horizontal gauge bosons, by leptoquarks, and also by specific Higgs bosons. Although severely constrained by the measured $K_S - K_L$ mass difference [91], nevertheless the process $K^\pm \to \pi^\pm(\nu\bar{\nu}')$ could be enhanced by specific choices of the masses and mixings [92]. Alternatively, x and x' could be the lightest supersymmetric particles, e.g. photinos [93].

An unambiguous signature of new physics would be indeed the observation of the two-body decay $K^+ \to \pi^+ x$, signalled by a monochromatic π^+, and where x could be in principle any light or massless neutral particle, such as the axion [94], or the hyperphoton [95], or the familon [96], i. e. a light or massless particle arising either from a spontaneously broken global symmetry or mediating new fundamental interactions. The recent experimental limit for this kind of decay [84] is:

$$\mathrm{BR}(K^+ \to \pi^+ x)_{\mathrm{exp}} < 6.4 \times 10^{-9} \quad . \tag{18}$$

This limit excludes the conventional axion, which should have been already observed at such a level, and also the light hyperphoton mediating the fifth force. Other possibilities remain to be tested, however, as they would require better sensitivities than eq. (18). From the considerations above it emerges that $K^+ \to \pi^+ \nu\bar{\nu}$ is of paramount importance, since it is in practice one of the cleanest tests of the standard model, free of hadronic

uncertainties. The foreseen factor 10 of increase in statistics at present experiments will hopefully approach the experimental sensitivity close to the range indicated by (17).

$K_L \to \mu^+\mu^-$, e^+e^-

It is well known that the decay $K_L \to \mu^+\mu^-$ played an important rôle in the study of the electroweak theory. The branching ratio is of the order 10^{-9}, the two recent experimental determinations coming from KEK [97] and BNL AGS E791 [98] experiments give in fact

$$\mathrm{BR}(K_L \to \mu^+\mu^-) = \begin{cases} (8.4 \pm 1.1) \times 10^{-9} & [97] \\ (5.8 \pm 0.6\,\mathrm{stat} \ \pm 0.4\,\mathrm{syst}) \times 10^{-9} & [98] \end{cases} \quad . \tag{19}$$

These improve the old result $(9.1 \pm 1.9) \times 10^{-9}$[85]. This small branching ratio indicates

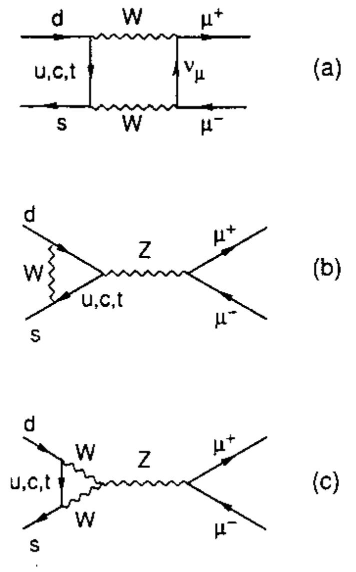

Fig. 14 - Short distance diagrams for $K_L^0 \to \mu^+\mu^-$.

the suppression of weak processes induced by strangeness changing neutral currents, and actually it motivated the introduction of the c-quark and of the GIM mechanism [6]. In addition there is a suppression from helicity selection rules. The second order electroweak short–distance mechanism for this transition is represented by the $s\bar{d} \to \mu^+\mu^-$ annihilation of Fig. 14. This short–distance amplitude has been estimated by several authors [99], and is clearly dominated by the t-quark contribution for large enough m_t and for $|V_{ts}^* V_{td}|$ not excessively small. However, a recent analysis in the light of the measured $B-\bar{B}$ mixing [86], including QCD effects to the graphs of Fig. 14, leads to the conclusion that for m_t large but constrained by the flavour-conserving neutral current data and including the QCD effects the short distance contribution is not able to account for the value of the branching ratio in (19). In fact the latter turns out to be almost completely saturated by the long-distance non-perturbative diagram where the transition proceeds via a two-photon intermediate exchange, i.e. $K_L \to \gamma\gamma \to \mu^+\mu^-$ (Fig. 15). The imaginary (absorptive) part of the amplitude of the process $K_L \to \gamma\gamma \to \mu^+\mu^-$ can be reliably estimated, since the two photons are on the mass-shell there, in terms of the measured $K_L \to \gamma\gamma$ amplitude and of the pure QED amplitude for the process $\gamma\gamma \to \mu^+\mu^-$. In terms of the rate of the process $K_L \to \gamma\gamma$ it is possible to derive [100]

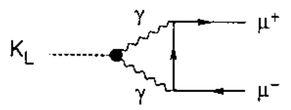

Fig. 15 - Long-distance contribution to the $K_L \to \mu^+\mu^-$ decay with two photon intermediate state.

$$\Gamma(K_L \to \mu^+\mu^-)_{\rm abs} = 1.2 \times 10^{-5} \cdot \Gamma(K_L \to \gamma\gamma) \quad , \tag{20}$$

so that, using the recent estimate of the experimental rate of $K_L \to \gamma\gamma$ [16], one finds

$$\frac{\Gamma(K_L \to \mu^+\mu^-)_{\rm abs}}{\Gamma(K_L \to {\rm all})} = (7.6 \pm 0.1) \times 10^{-9} \quad , \tag{21}$$

quite close to the experimental values in (19).

This leaves very little room to the dispersive contribution, made of the short–distance amplitude, which is reliably estimated as discussed above, and of the long–distance part. The latter involves the integration over the photon momenta in Fig. 15 and thus is sensitive to the confinement non-perturbative physics. It follows that the final result could be somewhat model–dependent [56], [101], and that the experimental measurement of $\mathrm{BR}(K_L \to \mu^+\mu^-)$ can be utilized only to constrain the value of the short–distance amplitude (assuming that it does not interfere with the long–distance one). This can be turned into a constraint (for large m_t) on the quantity $|\mathrm{Re}(V_{ts}^* V_{td})|$ as a function of m_t, which could be of some use, and indeed it has been mentioned in the previous discussion on $K^\pm \to \pi^\pm(\nu\bar{\nu})$.

All of the above properties can be extended to the process $K_L \to e^+e^-$. This decay has not been seen yet: preliminary limits has been recently reported by KEK [97] and BNL AGS E791 [102] experiments:

$$\mathrm{BR}(K_L \to e^+e^-) < \begin{cases} 5.4 \times 10^{-10} & [97] \\ 3.2 \times 10^{-10} & [102] \end{cases} \quad . \tag{22}$$

From the theoretical point of view, the helicity suppression is much larger than for $K_L \to \mu^+\mu^-$ due to $m_e/m_\mu \ll 1$, and the long–distance amplitude should similarly dominate the transition. To have an idea of the order of magnitude expected in the standard model, we can take the unitarity limit, namely just the absorptive part of the long–distance amplitude [100]. Using again the rate of the process $K_L \to \gamma\gamma$ [16] one finds:

$$\mathrm{BR}(K_L \to e^+e^-) \geq 2.5 \times 10^{-11} \quad , \tag{23}$$

about one order of magnitude far from the experimental limit (22). The data already on tape should allow soon an improvement by about a factor of three, and the sensitivity of the existing experiments, in particular KEK 137, seems adequate to match the theoretical limit (23).

As far as the $K_S \to \mu^+\mu^-$ is concerned, the partial width from $K_S \to \gamma\gamma \to \mu^+\mu^-$ is numerically comparable to the K_L case, but there is a suppression of two orders of magnitude in the branching ratio due to $\Gamma_L \ll \Gamma_S$.

The decay $K_L \to \mu^+\mu^-$ can provide information on the CP violation through a search for the muon longitudinal polarization [103], defined as

$$P_L = \frac{N_R - N_L}{N_R + N_L} \quad , \tag{24}$$

where N_R (N_L) is the number of μ^- with positive (negative) helicity. It is possible to see that, in the standard model, the longitudinal polarization for $K_L \to \mu^+\mu^-$ essentially comes from the CP impurity in the K_L state, and can be related to the absorptive part of the (dominant) process $K_L \to \gamma\gamma \to \mu^+\mu^-$: this contribution is rather small and has been estimated as

$$|P_L^{(K)}| \simeq 7.1 \times 10^{-4} \quad . \tag{25}$$

A potentially important source of enhancement could arise from the Higgs exchange diagram reported in Fig. 16 and estimated in [104]. However, the recent results of LEP concerning the limits on the Higgs mass [7] makes such a contribution too small to be observed. Thus a muon polarization larger than that of eq. (25) would suggest the existence of a non-standard CP-violating quark-lepton interaction. Some possibilities, related to flavour-violating Higgs boson exchanges or to leptoquark exchanges, are discussed in [105].

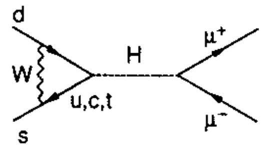

Fig. 16 - The Higgs exchange diagram which contributes to the longitudinal polarization of $K_L \to \mu^+\mu^-$ decay.

Although some hundreds of events have already been observed, and many more are expected in the coming years, the μ polarization is not being measured in the existing experiments (even if it was foreseen in the proposal of the Brookhaven E791 experiment). If this study is continued at higher statistics, μ polarimetry would be an important tool in distinguishing between the various sources of CP violation.

VI. "Very rare" decays: $K^0 \to \pi^0(\ell^+\ell^-)$, $K^0 \to \pi^0(\nu\overline{\nu})$

Although still in the class of the GIM suppressed decays considered so far, these transitions are expected to be extremely rare in the standard model, at the branching ratio level of 10^{-11} or less. On the other hand, they are particularly interesting as they can offer an alternative opportunity (to the $K^0 - \overline{K}^0$ system) to study CP violation [106], and for this reason they have received much attention recently.

The point which makes the decay $K_L \to \pi^0(\ell^+\ell^-)$ extremely rare is that, differently from the corresponding charged one (and from the decay $K_S \to \pi^0(\ell^+\ell^-)$), it cannot proceed through a one-photon exchange diagram without violation of CP [107]. The CP-conserving part of the decay $K_L \to \pi^0(\ell^+\ell^-)$ comes from the diagram involving two photons ($K_L \to \pi^0\gamma^*\gamma^* \to \pi^0(\ell^+\ell^-)$), suppressed by a factor (α/π) (see Fig. 17,

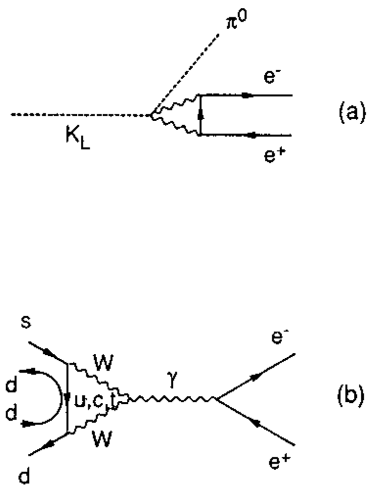

Fig. 17 - Diagrams for $K_L^0 \to \pi^0 e^+ e^-$: (a) CP conserving two photon process; (b) CP violating penguin diagram.

where both diagrams are reported). Calculations in the framework of chiral perturbation theory [55], [108] indicate indeed a clear dominance of the one-photon exchange amplitude, and thus confirm the nice perspective to directly measure CP violation from this process. Instead, the opposite situation, namely a one-photon amplitude smaller or (at most) equal to the two-photon one, has been found in [109] using a vector dominance model. Results midway between [108] and [109] have been obtained in [110]. Thus more theoretical work is needed, and probably a clarification will be possible by the inclusion of vector mesons in the chiral effective lagrangians [53], [54].

The indication of the recent experimental measurement of $K_L - \pi^0\gamma\gamma$ [69] is, within the uncertainties, that the absorptive part of the two-photon diagram in Fig. 17a should be reasonably small, so that the possibilitiy of testing CP violation should be

considered with optimism. To give an idea of the orders of magnitude involved, we can say that the various references agree in predicting $\mathrm{BR}(K_L \to \pi^0 e^+ e^-) \simeq 10^{-11} - 10^{-12}$, so that this is the sensitivity required to perform this interesting measurement of CP. From the experimental point of view, the current limits

$$\mathrm{BR}(K_L \to \pi^0 e^+ e^-) < \begin{cases} 3.2 \times 10^{-7} & [111] \\ 4.0 \times 10^{-8} & [112] \\ 7.5 \times 10^{-9} & [113] \\ 5.5 \times 10^{-9} & [114] \end{cases} \tag{26}$$

lie more than two orders of magnitude above the standard model prediction, affording a large window for new physics. The sensitivity of the existing experiment should allow to reach the 10^{-11} region, during the next few years.

We may also mention that other interesting CP-odd observables have been proposed, such as the asymmetry between the energy spectra of the electron and of the positron [109], or the muon polarization in the $K_L \to \pi^0 \mu^+ \mu^-$ mode [55]. This asymmetry depends on the interference between the one-photon and the two-photon amplitudes.

The decay $K_L \to \pi^0 \nu\bar{\nu}$ should represent an even cleaner test of CP violation effects, because there is no photon contributions to the amplitude. In addition, and very interestingly, the largest contribution should come from direct CP violation [115]. Unfortunately, the expected branching ratio is tiny, of the order of 10^{-12}, while a possible experimental upper limit is $\mathrm{BR}(K_L \to \pi^0 \nu\bar{\nu}) < 1 - 4 \times 10^{-3}$. We note here that the study of this decay looks extremely difficult, if not impossible, at high intensity K beams, because of background and lack of experimental constraints. A significant improvement on the branchin ratio limit both for K_S and K_L would be expected at a ϕ factory (and at LEAR) thanks to low background and $K_S(K_L)$ tagging capability. However, the expected statistics at these facilities will not allow to reach the theoretical value.

VII. "Not expected or forbidden" decays: $K \to \mu e$, $K \to \pi\mu e$

The decays $K \to \mu e$ and $K \to \pi\mu e$ are strictly forbidden in the standard model (with massless neutrinos) by the separate conservation of muon and of electron leptonic numbers. Actually, in an extension of the standard model implying massive neutrinos these transitions could proceed via the neutrino mixing. The rates would be, however, unmeasurably small, as being proportional to the small neutrino mass. Therefore, the observation of these decays would represent the unambiguous experimental evidence for new physics outside the standard model, and indeed in this regard they probably are the most interesting ones. The most stringent experimental upper limits available at present are:

$$\begin{aligned} &\mathrm{BR}(K_L \to \mu e) < 2.2 \times 10^{-10} \quad [102] \;, \\ &\mathrm{BR}(K \to \pi\mu e) < 2.1 \times 10^{-10} \quad [116] \;. \end{aligned} \tag{27}$$

These results represent the status of the art in high statistics K experiments.

Various theoretical schemes for extending the standard model have been proposed, in the attempt to find an answer to some challenging conceptual difficulties of the model, such as the excessive number of parameters, the existence of fermion generations with their observed masses and mixing angles, and some disappointing features of the Higgs particle. Clearly, such questions can find a solution only in an extended framework, containing new physics beyond the standard model. Examples of these schemes are grand unification [117], left-right symmetry [118], supersymmetry [119], technicolor [120], "horizontal" gauge symmetry [121], compositeness of quarks and leptons and/or of gauge bosons W, Z [122], and combinations thereof. More recently, a well-defined class of grand-unified supersymmetric models have been inspired by superstring theory [123], [124].

The important common feature of these schemes is that in general they predict the existence of new fundamental fields, some of which mediate new, flavour violating interactions among quarks and leptons. A list of model extensions and of the corresponding quantum exchanges leading to flavour violation is presented in Table VI. In Fig. 18 we schematically show the various mechanisms for $\bar{s} + d \to \mu e$, relevant to $K \to \mu e$ (and to $K \to \pi\mu e$).

TABLE VI. Some theoretical extensions to the standard model.

Model	Flavour violator
Additional generations	Heavy neutral leptons
L-R Symmetric models	Majorana Particles
Extended Higgs sector	Horizontal gauge models
Extended technicolor	Gauge bosons, leptoquarks
Supersymmetry	Scalar partners of fermions
Substructure models	Leptoquarks, gauge bosons
Family symmetry	Familon
Superstrings	Leptoquarks

Another remarkable feature is that these new interactions are supposed to be effective at some extremely large mass scales $\Lambda \gg M_W$, which can be identified with the mass of the exchanged heavy objects in Fig. 18, or with the compositeness scale Λ_c. These masses determine in fact the big suppression of these (otherwise totally forbidden) flavour violating transitions, and thus are new constants of nature. Consequently, these "forbidden" K-decays have the rôle of being a testing ground for the standard model extensions and of providing, in addition, glimpses on the above-mentioned large mass scales, which may well be outside the reach of the biggest accelerators. We may also notice that $K \to \pi\mu e$ tests vectorial couplings, while $K \to \mu e$ only proceeds via axial-like couplings, so that these two processes are logically independent.

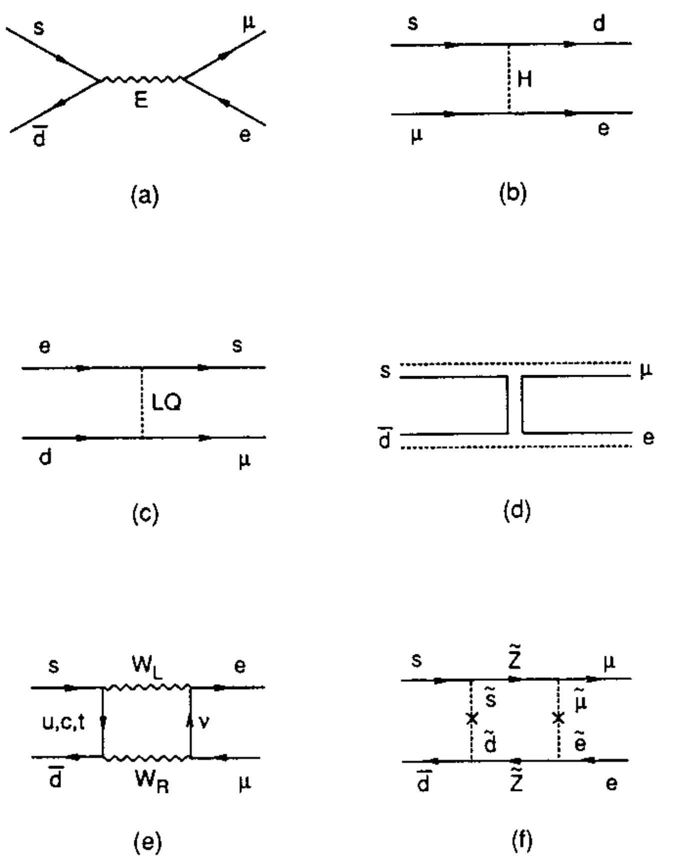

Fig. 18 - Contribution to $K_L \to \mu e$ from (a) horizontal boson exchange, (b) Higgs exchange, (c) leptoquark exchange, (d) composite model mechanisms, (e) left-right models, and (f) supersymmetric particles.

To give an idea of the experimental sensitivities which are expected to be significant, we should notice that the "horizontal" exchange of Fig. 18a can also directly mediate the $\Delta S = 2\ K^\circ\overline{K^\circ}$ mixing, and thus this mechanism is requested by the measured $\Delta m_K = m_{K_L} - m_{K_S} = 3.5 \times 10^{-15}$ GeV. As a rule, the corresponding branching ratios should be not larger than $10^{-12} - 10^{-13}$ [91], [125]. The same should be true for the Higgs exchange mechanism of Fig. 18b, although examples of models have been proposed where the Δm_K constraint can be evaded [126].

The leptoquark exchange of Fig 18c is not constrained by Δm_K, because it cannot induce direct $K^\circ\overline{K^\circ}$ transitions, and therefore leptoquark masses are directly probed by the decays discussed here. From the limits (27), and by making the (model) assumption of gauge couplings in Fig. 18a equal to the standard W couplings, one can derive [92]for the typical mass scale of these new interactions, represented by the mass of the exchanged heavy object:

$$M > 57\ \mathrm{TeV}/c^2 \quad . \tag{28}$$

It is a little unfortunate that the bound on M only grows like the 1/4 power of the experimental sensitivity on the branching ratio. Nevertheless, M in eq. (28) is quite a large mass, which can be already considered as significant in order to test models. For example, in extended technicolor models the typical heavy boson masses are requested by the values of the dynamically generated lepton and quark masses to be in the range 10-100 TeV.

The composite model mechanism of Fig. 18d represents the exchange of a heavy bound state made of quark and lepton "subconstituents", with a mass of the order of the compositeness scale Λ_c. Then the bound in eq. (28) applies to Λ_c. The constraint from Δm_K also works for composite models, and limits the predicted branching ratios for $K \to \mu e$ and $K \to \pi\mu e$. The transition amplitude estimates are very model dependent, but roughly speaking branching ratios can be expected in the range $10^{-12} - 10^{-13}$ or less. Also in this case there are possibilities for larger values of the branching ratios, such as e.g. the model in ref. [127], which interestingly predicts $\sim 10^{-10}$, and thus is at the point of being tested by experiment.

Finally, we mention that left-right symmetric models with heavy neutrino masses can induce contributions like that of Fig. 18e, with branching ratios of the same order of magnitude [128]. Also, useful constraints on supersymmetric parameters can be obtained from the decays considered here, because there are no lepton number violations at the tree level as the effect of direct exchange of new superparticles [129]. We show in Fig. 18f, taken from [130], an example of a supersymmetry induced lepton number violating process contributing to $K \to \mu e$.

On the basis of the previous considerations, an experimental sensitivity to branching ratios of the order of at least 10^{-12}–10^{-13} is expected to be necessary, to test models of lepton flavour violation. In order to significantly improve the sensitivity of the existing experiments, very substantial upgrades of beams and experimental capabilities are needed. The ongoing or proposed programs (at FNAL, KEK, BNL and later at kaon factories) are very challenging and they face experimental problems (like high precision tracking, calorimetry and particle identification in the presence of very high fluxes, powerful trigger rejection, radiation damage and so on) similar to those which will be encountered in SSC/LHC environment. The final goal is to reach the level of 10^{-13} in sensitivity during next decade.

VIII. Concluding remarks

The presentation of K decay physics given in the preceding sections is intended to give a general view on this subject, trying to emphasize as much as possible the main present and future points of interest. Thus unavoidably some specific items could not be discussed in as much details as they would have deserved. Nevertheless, in our hope, the considerations above clearly demonstrate that K decays have maintained unaltered their prime rôle of providing an unique testing ground for the theory of electroweak interactions and its model extensions.

Such a program demands for high precison measurements, suitable to scrutinize the standard model predictions and to constrain the values of parameters which are essential to the theoretical description, such as quark masses and mixing angles, and the hadronic matrix elements of the electroweak hamiltonian. In this respect kaons are rather unique, in that they are heavy enough to allow for an interesting range of decay modes, and at the same time are light enough to be produced so copiously that high precision, and sensitivities to extremely small branching ratios, could be reached.

From the experimental point of view there has been an impressive progress made over the past few years: suffice to mention, as examples, the CP violation studies in $K \to 2\pi$ decays, and the improvement by four orders of magnitude in the sensitivities to some rare (or forbidden) decays. These are now in the range 10^{-8}–10^{-10} at existing facilities, thanks to dedicated kaon beams and to specially designed experimental devices.

Equally substantial is the progress that has been achieved on the theoretical side. One outstanding example is the development of the chiral lagrangian approach, which allows to evaluate the non-perturbative hadronic matrix elements of the $\Delta S = 1$ weak effective hamiltonian $H_{\rm eff}$, accounting for the basic principles that underlie the theory of strong interactions. This subject is still developing, as there are a number of important open questions, such as the rôle of vector meson dominance and of the higher order terms in the pseudoscalar meson momenta, and, above all, the comprehension of the fundamental $\Delta I = 1/2$ rule, which has not been achieved yet. Great progress has been obtained also by the lattice QCD, which in the future should accomplish the estimate of relevant coupling constants and transition matrix elements, starting from the QCD first principles. Furthermore, quite remarkable has been the reanalysis of perturbative, short-distance calculations of $H_{\rm eff}$ for large values of m_t, leading to the conclusion that top quark physics should be effective in numerous K decays. Such a conclusion would have been totally unorthodox until few years ago.

Thus, briefly summarizing the content of the previous sections, we may underline once again in what follows the interests of K decay physics, so as to evidence the richness of this field as well as the facilities required in perspective.

First of all, it is very true that the generation changing decays $K \to \mu e$ and $K \to \pi \mu e$ have been the big (if not the only) motivation for the renewed enthusiasm for the K decay experiments, as these transitions promise competitive searches of new physics and of the corresponding new large mass scales. The sensitivities presently attained, although possibly close to being significant in some respects, should be pushed to at least 10^{-12}–10^{-13} to probe models of flavour violation in a decisive way. This will require the intense kaon beams available at the planned kaon factories.

However, keeping in mind the recent progress mentioned above (both theoretical and experimental), and the availability of upgraded facilities, the program of a systematic analysis of the K decays allowed in the standard model appears now as strongly motivated, and promising of profound results, as the lepton flavour changing decays.

Indeed, we could start the list of interests by the CP violation measurements in $K \to 2\pi$, hopefully bringing the experimental results on ϵ'/ϵ to an agreement. Alternative sources of CP violation, such as $K_L \to \pi^0 e^+ e^-$ and the asymmetries in $K \to 3\pi$ and in $K \to \gamma\gamma$ should become reachable, and could give supplementary information on ϵ'/ϵ.

Essential progress regarding the precision test of the standard model, in particular of the fundamental GIM mechanism, would follow from the observation of $K^+ \to \pi^+ \nu\bar{\nu}$, with the predicted branching ratio in the range of 10^{-10}. In searching for this process down to the standard model level, a considerable window on non-standard physics would be explored.

The non-leptonic, $\Delta S = 1$ radiative rare K decays, with branching ratios in the range 10^{-4}–10^{-9}, are characterized by a significant interplay between standard model electroweak physics and the non-perturbative confinement dynamics, governing hadronic matrix elements. Thus they offer crucial tests of this correlatioon, in particular of the chiral lagrangian realization of the electroweak hamiltonian. The advent of ϕ factories will allow a systematic study of the K_S sector. For what K_L and K^+ are concerned, they are already in the range of present available sensibilities, and indeed the very recent measurement of $K_L \to \pi^0 \gamma\gamma$ represents the initial step in this important and interesting program.

Last (but not least), come the "traditional" leptonic and semileptonic K decays. These could be measured with really great accuracy, and would give, in first instance, very useful information on the meson structure as embodied in the form factors. This would test the chiral lagrangian approach in a quite stringent way. In addition, however, since the precision will be high, these decays should also be capable to give determinations, or at least significant upper limits, on a variety of exotic quark-lepton couplings. In conclusion, K decay physics is not only still quite alive, but is a flourishing field of particle physics, full of interesting and challenging perspectives.

References

[1] T. D. Lee and C. N. Yang, Phys. Rev. **104** (1956) 254;
C. S. Wu et al., Phys. Rev. **105** (1957) 1413.

[2] R. H. Dalitz, Repts. Progr. in Phys. **20** (1957) 163.

[3] For an update review, see *"CP Violation"*, C. Jarlskog ed. (World Scientific, 1989).

[4] J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, Phys. Rev. Lett. **13** (1964) 138.

[5] M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[6] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. **D2** (1970) 1285.

[7] OPAL Collaboration, M. Z. Akrawy et al., Phys. Lett. **236B** (1990) 224;
ALEPH Collaboration, D. Decamp et al., Phys. Lett. **236B** (1990) 233; Phys. Lett. **241B** (1990) 141.

[8] S. Weinberg, Phys. Rev. Lett. **37** (1976) 657;
T. D. Lee, Phys. Rep. **9C** (1974) 144;
S. Weinberg, Phys. Rev. Lett. **63** (1989) 2333.

[9] L3 Collaboration, M. Aguilar-Benitez et al., Phys. Lett. **231B** (1989) 509; ALEPH Collaboration, J.-P. Lees et al., Phys. Lett. **231B** (1989) 519; D. Decamp et al., Phys. Lett. **235B** (1990) 399; OPAL Collaboration, J. Allison et al., Phys. Lett. **231B** (1989) 530; DELPHI Collaboration, W. Adam et al., Phys. Lett. **231B** (1989) 539.

[10] A. J. Malensek, Fermilab Report FN-341 (1981), unpublished.

[11] See for instance: M. K. Craddock, in *Proceedings of the International Conference on High-Energy Accelerators* (Tsukuba, 1989), to appear.

[12] B. Winstein, talk at the *Workshop on Physics at the Main Injector* (Fermilab, 1989), University of Chicago preprint EFI 89-59 (1989).

[13] D. Cocolicchio, G. L. Fogli, M. Lusignoli and A. Pugliese, Phys. Lett. **238B** (1990) 417.

[14] G. Barbiellini and C. Santoni, preprint CERN–EP/89-88 (1989).

[15] R. Landua, talk at *Fermilab Low-Energy Antiproton Facility Workshop* (Batavia, 1986), preprint CERN-EP/86-136 (1986), unpublished.

[16] NA31 Collaboration, H. Burkhardt et al., Phys. Lett. **206B** (1988) 169.

[17] E731 Collaboration, J. P. Patterson et al., Phys. Rev. Lett. **64** (1990) 1491.

[18] D. A. Bryman, Int. Jour. Mod. Phys. **A4** (1989) 79.

[19] I.-H. Chiang et al., "*A Study of the Decay* $K^+ \to \pi^+\nu\bar{\nu}$", AGS Proposal, Experiment BNL E787 (1983).

[20] AGS Experiment 777, C. Campagnari et al., Phys. Rev. Lett. **61** (1988) 2062.

[21] A. Drescher et al., Nucl. Instr. and Methods **205** (1983) 125.

[22] S. Smith, talk given at the SLAC Summer Institute, 1988.

[23] K. Kleinknecht, Ann. Rev. Nucl. Sci. **16** (1976) 1;
J. W. Cronin, Rev. Mod. Phys. **53** (1981) 373.

[24] J.Donoghue, B. R. Holstein and G. Valencia, Int. J. Mod. Phys. **2** (1987) 319;
E. Paschos and U. Turke, Phys. Rep. **178** (1989) 145.

[25] J. M. Flynn and L. Randall, Phys. Lett. **224B** (1989) 1221;
G. A. A. Buchalla, A. J. Buras, M. K. Harlander and P. Krawczyk, Max-Planck Institut preprint MPI-PAE/Pth 63/89 (1989).

[26] I. Dunietz, J. Hauser and J. L. Rosner, Phys. Rev. **D35** (1987) 2166;
J. Bernabeu, F. J. Botella and J. Roldan, Phys. Lett. **211B** (1988) 226.

[27] Y. Fukushima et al., KEK preprint 89-159 (1989).

[28] S. Nussinov and T. N. Truong, Phys. Rev. Lett. **63** (1989) 2003.

[29] V. V. Barmin et al., Phys. Lett. **128B** (1985) 129.

[30] T. J. Devlin and J. O. Dickey, Rev. of Mod. Phys. **51** (1979) 237.

[31] For a review of the subject see L. Wolfenstein, Ann. Rev. Nucl. Part. Sci. **36** (1986) 137.

[32] L.-F. Li and L. Wolfenstein, Phys. Rev. **D21** (1980) 178.

[33] J. F. Donoghue, B. R. Holstein and G. Valencia, Phys. Rev. **D36** (1987) 798.

[34] S. Fajfer and J. M. Gérard, Zeit. Phys. – Particles and Fields **C42** (1989) 425.

[35] H.-Y. Cheng, C. Y. Cheung and W. B. Yeung, Mod. Phys. Lett. **A4** (1989) 869.

[36] R. S. Chivukula and A. V. Manohar, Harvard preprint HUTP–86/A050 (1986), unpublished;
H.-Y. Cheng, Brookhaven Laboratory preprint BNL-43594 (1989).

[37] A. A. Bel'kov, G. Bohm, D. Ebert and A. V. Lanjov, Phys. Lett. **232B** (1989) 118;
A. A. Bel'kov, D. Ebert and A. V. Lanjov, Serpukhov preprint 89-11 (1989).

[38] R. Shrock, Phys. Lett. **96B** (1970) 159.

[39] H. Leutwyler and M. Roos, Zeit. Phys. – Particles and Fields **C25** (1984) 91.

[40] W. J. Marciano and A. Sirlin, talk presented at the *13th Annual Meeting of Div. of Particles and Fields of the APS*, Salt Lake City, Utah, Jan. 14-17, 1987, Brookhaven Laboratory preprint BNL-39658 (1987).

[41] J. F. Donoghue and B. R. Holstein, Phys. Rev. **D40** (1989) 3700.

[42] C. G. Callan and S. B. Treiman, Phys. Rev. Lett. **16** (1966) 153.

[43] G. Kane et al., *Proceedings of the Workshop on CP Violation at a Kaon Factory*, J. N. Ng ed. (1989) p. 1 ;
W. M. Morse et al., ibidem, p. 58.

[44] W. M. Morse et al., Phys. Rev. **21** (1980) 1750;

S. R. Blatt et al., Phys. Rev. **D27** (1983) 1056.

[45] M. Leurer, Phys. Rev. Lett. **62** (1989) 1697;
P. Castoldi, J. M. Frère and G. L. Kane, Phys. Rev. **D39** (1989) 2633.

[46] L. Rosselet et al., Phys. Rev. **D15** (1977) 574.

[47] See e.g. E.P. Shabalin, Sov. J. Nucl. Phys. **49** (1989) 365, and reference therein.

[48] K. S. Heard et al., Phys. Lett. **55B** (1975) 324;
J. Heintze et al., Nucl. Phys. **B149** (1979) 365.

[49] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158** (1984) 142; Nucl. Phys. **B250** (1985) 465, 517.

[50] B. R. Holstein, Univ. of Amherst preprint UMHEP-337 (1990).

[51] M. K. Gaillard and B. W. Lee, Phys. Rev. **D10** (1974) 897.

[52] I. Kambor, J. Missimer and D. Wyler, Zurich Univ. preprint ETH-TH/89-42 (1989).

[53] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. **B237** (1990) 481.

[54] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **321B** (1989) 311.

[55] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B291** (1987) 692; **B303** (1988) 665; Phys. Lett. **189B** (1987) 363.

[56] L. L. Chau and H. Y. Cheng, Phys. Rev. Lett. **54** (1985) 1768;
G. D'Ambrosio and D. Espriu, Phys. Lett. **175B** (1986) 237;
J. L. Goity, Zeit. Phys. – Particles and Fields **C34** (1987) 341.

[57] A. A. Bel'kov, Yu. L. Kalinowski, V. N. Pervushin and N. A. Sarikov, Sov. J. of Nucl. Phys. **44** (1986) 448.

[58] J. F. Donoghue, B. R. Holstein and Y.-C. Lin, Nucl. Phys. **B277** (1986) 651.

[59] A. Pich, talk at the *Workshop on Rare K–Decays and CP Violation* (Upton, NY, 1988).

[60] NA31 Collaboration, H. Burkhardt et al., Phys. Lett. **199B** (1987) 139.

[61] NA31 Collaboration, G. D. Barr, et al., Phys. Lett. **B240** (1990) 283.

[62] R. Decker, P. Pavlopoulos and G. Zoupanos, Zeit. Phys. – Particles and Fields **C28** (1985) 117;
L. L. Chau and H. Y. Cheng, Phys. Lett. **195B** (1987) 275;
F. Buccella, G. D'Ambrosio and M. Miragliuolo, Univ. of Napoli preprint "*CP violation in the decays of neutral kaons into two photons*" (1989);
J. O. Eeg, B. Nizic and I. Picek, Oslo Univ. preprint OSLO-90-06.

[63] L. S. Littenberg, Brookhaven Laboratory preprint, BNL-43660 (1989).

[64] Y. Asano et al., Phys. Lett. **113B** (1982) 195.

[65] P. Ko and J. L. Rosner, Phys. Rev. **D40** (1989) 3775;
P. Ko, Phys. Rev. **D41** (1990) 1531;
T. N. Truong, Univ. of Chicago preprint EFI 89-57 (1989).

[66] T. Morozumi and H. Iwasaki, Progr. Theor. Phys. **82** (1989) 371.

[67] H.-Y. Cheng, Brookhaven Laboratory preprint BNL 43466 (1989).

[68] L. M. Sehgal, Phys. Rev. **D41** (1990) 161.

[69] NA31 Collaboration, G. D. Barr et al., preprint CERN-EP/90-69 (1990).

[70] P. K. Kabir and A. Sirlin, presented by P. K. Kabir, *Proceedings of the XXIV Int. Conference on High Energy Physics* R. Kotthaus and J. Kühn eds. (Münich, 1988) p. 541.

[71] S. Fajfer, Zeit. Phys. – Particles and Fields **C45** (1989) 293.

[72] H. Y. Cheng, S. C. Lee, H. -L. Yu, Zeit. Phys. – Particles and Fields **C41** (1988) 223.

[73] R. J. Abrams et al., Phys. Rev. Lett. **29** (1972) 1118.

[74] K. M. Smith et al., Nucl. Phys. **B109** (1976) 173.

[75] V. N. Bolotov et al., in *Proceedings of the XXIII Int. Conference on High Energy Physics*, S. C. Loken ed. (Berkeley, 1986) p. 887.

[76] A. S. Carroll et al., Phys. Rev. Lett. **44** (1980) 529.

[77] H. Taureg et al., Phys. Lett. **65B** (1976) 92.

[78] M. McGuigan and A. I. Sanda, Phys. Rev. **D36** (1987) 1415;
Y. C. Lin and G. Valencia, Phys. Rev. **D37** (1988) 143.

[79] P. Bloch et al., Phys. Lett. **56B** (1975) 201.

[80] BNL-E787 Collaboration, M.S. Atiya et al., Phys. Rev. Lett. **63** (1989) 2177.

[81] A. I. Vainshtein. V. I. Zakharov, L. B. Okun and M. A. Shifman, Sov. J. of Nucl. Phys. 24 (1976) 427.

[82] F. J. Gilman and M. B. Wise, Phys. Rev. **D21** (1980) 3150.

[83] H. Leutwyler and M. A. Shifman, University of Bern preprint BUTP-89/29 (1989).

[84] BNL-E787 Collaboration, M. S. Atiya et al., Phys. Rev. Lett. **64** (1990) 21.

[85] Particle Data Book, Review of Particle Properties, Phys. Lett. **204B** (1988) 1.

[86] J. Ellis, J. S. Hagelin and S. Rudaz, Phys. Lett. **192B** (1987) 201;
J. Ellis, J. S. Hagelin, S. Rudaz and D.D. Wu, Nucl. Phys. **B304** (1988) 205.

[87] J. Ellis and G. L. Fogli, Phys. Lett. **232B** (1989) 139.

[88] D. Rein and L. M. Sehgal, Phys. Rev. **D39** (1989) 3325.

[89] C. Dib, I. Dunietz and F. J. Gilman, Stanford preprint SLAC PUB 4840 (1989).

[90] U. Turke, Phys. Lett. **168B** (1986) 296;
W. Marciano and Z. Parsa, Ann. Rev. Nucl. Part. Sci. **36** (1986) 171.

[91] G. L. Kane and R. Thun, Phys. Lett. **94B** (1980) 513.

[92] R. N. Cahn and H. Harari, Nucl. Phys. **176** (1980) 135.

[93] S. Bertolini and A. Masiero, Phys. Lett. **174B** (1986) 343.

[94] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 40 (1978) 233;
F. D. Wilczek, Phys. Rev. Lett. **40** (1978) 279.

[95] S. Aronson, E. Fishbach, D. Sudarsky and C. Talmadge, BNL report BNL–41475 (1988), unpublished.

[96] F. Wilczek, Phys. Rev. Lett. **49** (1982) 1549.

[97] E137 Collaboration, T. Inagaki et al., Phys. Rev. **D40** (1989) 1712.

[98] C. Mathiazhagan et al., Phys. Rev. Lett. **63** (1989) 2185.

[99] M. K. Gaillard, B.W. Lee and R. E. Shrock, Phys. Rev. **D13** (1976) 2674;
R. E. Schrock and M. B. Voloshin, Phys. Lett. **87B** (1979) 375;
T. Inami and C. S. Lim, Prog. Theor. Phys. **65** (1981) 297, 1772(E).

[100] L. M. Sehgal, Phys. Rev. **183** (1969) 1511;
B. R. Martin, E. de Rafael and J. Smith, Phys. Rev. **D2** (1970) 179.

[101] V. Barger, W. F. Long, E. Ma and A. Pramudita, Phys. Rev. **D25** (1982) 1860.

[102] C. Mathiazhagan et al., Phys. Rev. Lett. **63** (1989) 2181.

[103] A. Pais and S. B. Treiman, Phys. Rev. **176** (1968) 1974;
L. M. Sehgal, Phys. Rev. **181** (1969) 2151.

[104] F. J. Botella and C. S. Lim, Phys. Rev. Lett. **56** (1986) 1651.

[105] P. Herczeg, Phys. Rev. **D27** (1983) 1512.

[106] L. M. Sehgal, Phys. Rev. **D6** (1972) 367.

[107] M. Baker and S. L. Glashow, Nuovo Cimento **25** (1962) 857;
A. Pais and S. B. Treiman, Phys. Rev. **176** (1968) 1974.

[108] J. F. Donoghue, B. R. Holstein and G. Valencia, Phys. Rev. **D35** (1987) 2769.

[109] L. M. Sehgal, Phys. Rev. **D38** (1989) 808.

[110] J. M. Flynn and L. Randall, Phys. Lett. **216B** (1989) 221; Nucl. Phys. **B326** (1989) 31;
C. Dib, I. Dunietz and F. J. Gilman, Phys. Lett. **218B** (1989) 487; Phys. Rev. **D39** (1989) 2639.

[111] E780 Collaboration, E. Jastrzembski et al., Phys. Rev. Lett. **61** (1988) 2300.

[112] NA31 Collaboration, G. D. Barr et al., Phys. Lett. **214B** (1988) 303; Phys. Lett. **235B** (1990) 356.

[113] E731 Collaboration, A. Barker et al., Univ. of Chicago preprint EFI 90-15 (1990).

[114] AGS Experiment 845, K. E. Ohl et al., Yale preprint (1990).

[115] L. S. Littenberg, Phys. Rev. **D39** (1989) 3322.

[116] A. M. Lee et al., Phys. Rev. Lett. **64** (1990) 165.

[117] For a review see e. g. D. V. Nanopoulos, *Proceedings of the XXII International Conference on High Energy Physics*, eds. A. Meyer and E. Wieczorek (Leipzig, 1984) p. 36.

[118] For a general survey of the left-right symmetric models, see R. Mohapatra, *Unification and Supersymmetry* (Springer Verlag, 1986).

[119] For a review see H. Haber and G. L. Kane, Phys. Rep. **117** (1985) 75.

[120] E. Farhi and L. Susskind, Phys. Rep. **74** (1981) 277.

[121] See e.g. A. Davidson and K. C. Wali, Phys. Rev. Lett. **26** (1981) 691.

[122] For a review see R. Peccei, *Proceedings of the XXIII International Conference on HEP*, S. Loken ed. (World Scientific, Singapore, 1986) p. 3.

[123] For a review on phenomenological implications see B. A. Campbell, J. Ellis, K. Enqvist, M. K. Gaillard and D. V. Nanopoulos, Int. J. Mod. Phys. **A2** (1987) 831.

[124] G. G. Ross, *Proceedings of the 1987 International Symposium on Lepton and Pho-*

ton Interactions at High Energies, W. Bartel and P. Rückl eds. (Hamburg, 1987) p. 743; this paper also contains a comprehensive survey on recent developments in standard model extensions.

[125] R. Cahn and H. Harari, Nucl. Phys. **B185** (1981) 382;
E. J. Eichten and D. Lane, Phys. Lett. **90B** (1980) 125.

[126] O. Shanker, Nucl. Phys. **B206** (1982) 253;
D. Cocolicchio and G. L. Fogli, Phys. Rev. **D30** (1984) 2391.

[127] J. C. Pati and H. Stremnitzer, Phys. Lett. **172B** (1986) 441.

[128] A. Barroso, G. C. Branco and M. C. Bento, Phys. Lett. **134B** (1984) 123.

[129] B. A. Campbell, Phys. Rev. **D28** (1983) 209.

[130] J. S. Hagelin and S. Kelley, Maharishi Univ. preprint MIU-THP-88/34 (1988), unpublished.

Imprimé au CERN. Prix : 1.00 Fr.