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Does experiment show that beamstrahlung theory - strong field QED - can be trusted?

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Abstract. We present a discussion of the experimental foundation for the strong field effects that are applied in beamstrahlung calculations for the planned next generation of linear colliders. In particular, quantum suppression of synchrotron radiation, coherent pairs and tridents are treated. It is shown, that many of these phenomena can be - and some have been - tested using present day technology.

1. Introduction

The admirable successes of precision tests of Quantum ElectroDynamic (QED) processes, where experiment and theory have been shown to agree with an accuracy down to 10^{-14} - for instance in the hydrogen $1S-2S$ transition or the $g-2$ experiments [1] - has naturally led to a high degree of confidence in extrapolations of QED to hitherto untested regimes. New experiments, demanding extreme precision, as e.g. the $1S-2S$ transition comparison between hydrogen and antihydrogen which aim at an ultimate accuracy approaching 10^{-18}, see e.g. [2], are for at least the first 12 orders of magnitude - corresponding to the foreseeable future - purely QED experiments. Therefore, and since physics is basically an experimental science, it still makes sense to ask the question: Is QED really correct in new regimes, and to what extent can we - based on experimental evidence - have confidence in QED, e.g. in the strong field regime?

In the following, we present a number of measurements that have been performed using strong crystalline fields in order to access the QED regime in which one of the three relativistic invariants that can be formed from the electromagnetic field tensor and the four-momentum of the particle (electron or photon) is comparable to, and in some cases even substantially exceeds, one. The strong-field regime is where the field in the rest frame of the electron becomes comparable to the so-called critical field - in QED the only combination of the electron mass and charge, Planck's constant and the speed of light that yields an electric field, signifying that relativistic quantum field theory must be applied. This regime is interesting in its own right, showing clear quantum corrections to a theory that is to a large extent describable by purely classical concepts [3]. But it is also of crucial importance to the construction of the next generation of linear colliders. In these machines, the electromagnetic field of one bunch acts upon a particle in the opposing bunch in exactly this manner: a critical field is experienced by the electron. This leads to a new behaviour of the synchrotron radiation equivalent of particle deflection in the field of the bunch, instead of in a magnetic dipole: Beamstrahlung. As the emission of beamstrahlung has a direct and significant impact on the energy of the colliding particles, it is a decisive factor for e.g. the energy-weighted luminosity. It is therefore important to know if beamstrahlung theory is correct for the conceptual and technical design of the collision region - the center about which the rest of the machine is based.

Three dimensionless invariants can be constructed from the the electromagnetic field strength tensor, $F_{\mu\nu}$, and the momentum four-vector p^ν (or, in the case of a photon, $\hbar k^\nu$). Two of these, $\Xi = F_{\mu\nu}^2/\mathcal{E}_0^2 = 2(\vec{B}^2 - \vec{\mathcal{E}}^2)/\mathcal{E}_0^2$ and $\Gamma = e_{\lambda\mu\nu\rho}F^{\lambda\mu}F^{\nu\rho}/\mathcal{E}_0^2 = 8\vec{\mathcal{E}}\cdot\vec{B}/\mathcal{E}_0^2$ express the 'inherent' field strength and 'nature' (pure electric or magnetic in some frame) of the field, respectively, taken in units of the critical field $\mathcal{E}_0 = m^2c^3/\hbar e$. The third is given by

$$\chi^2 = \frac{(F_{\mu\nu}p^\nu)^2}{m^2c^4\mathcal{E}_0^2} \tag{1}$$

expressing the field experienced by the particle. For an ultra-relativistic particle moving across fields $\mathcal{E} \ll \mathcal{E}_0, B \ll B_0$ with an angle to the field direction $\theta \gg 1/\gamma$ the invariants fulfill $\chi \gg \Xi, \Gamma$ and $\Xi, \Gamma \ll 1$. The relation of χ to the fields $\vec{\mathcal{E}}$ and $\vec{B}$ is given by [4]

$$\chi^2 = \frac{1}{\mathcal{E}_0^2 m^2 c^4}((c\vec{p}\times\vec{B} + E\cdot\vec{\mathcal{E}})^2 - (c\vec{p}\cdot\vec{\mathcal{E}})^2) \tag{2}$$

For an ultrarelativistic particle moving perpendicularly to a pure electric or pure magnetic field this reduces to

$$\chi = \frac{\gamma\mathcal{E}}{\mathcal{E}_0} \quad \text{or} \quad \chi = \frac{\gamma B}{B_0} \tag{3}$$

Thus, a Lorentz transformation reveals that electromagnetic fields that in the restframe are transversal to the boost remain transversal and are larger by a factor of γ. This is a consequence of the relativistic invariance of the parameter χ in equation (3). The critical field $\mathcal{E}_0$ is frequently referred to as the Schwinger field [5], although it was treated as early as 1931 by Sauter [6, 7], following a supposition by Bohr. The χ parameter - usually called Υ in the accelerator physics community - is decisive for the behaviour of the emission of beamstrahlung in 0.5 TeV linear colliders. The question is: Can processes involving $\chi \simeq 1$ be addressed experimentally with present day technology, such that estimates of the reliability of QED in beamstrahlung for future machines can be given now? In the following, we answer this question in the affirmative.

2. Quantum synchrotron

As the field in a synchrotron radiation experiment grows larger, the recoil imposed on the electron by emitting increasingly energetic photons becomes more and more important. If this correction is not taken into account, the calculated synchrotron radiation will carry away more energy than the energy of the charged particle - clearly an impossible situation. This enforces an inclusion of the quantum nature of the radiation process in the description of synchrotron radiation in strong fields. The replacement in the classical formula of synchrotron radiation of the frequency ω with $\omega^\star = \omega E/(E - \hbar\omega)$ provides an excellent description of the spinless quantum regime, i.e. radiation from a charged scalar particle [8]. However, for $\chi \gg 1$ the hard end of the photon spectrum becomes dominated by a spin flip transition, making it develop a pronounced peak near the incident electron energy. This feature allows the transfer of helicity in strong fields at large photon energies [9]. Meanwhile, electron motion in a transversal electric field generates linearly polarized light in non spin-flip transitions.

2.1. Quantum treatment of beamstrahlung

The discussion of quantum effects - and in particular a suppression of the intensity - in radiation emission from energetic particles in collision with a counterpropagating bunch, was started in the mid-80's [10, 11, 12] and later continued with treatments of pair creation [13, 14]. It turned

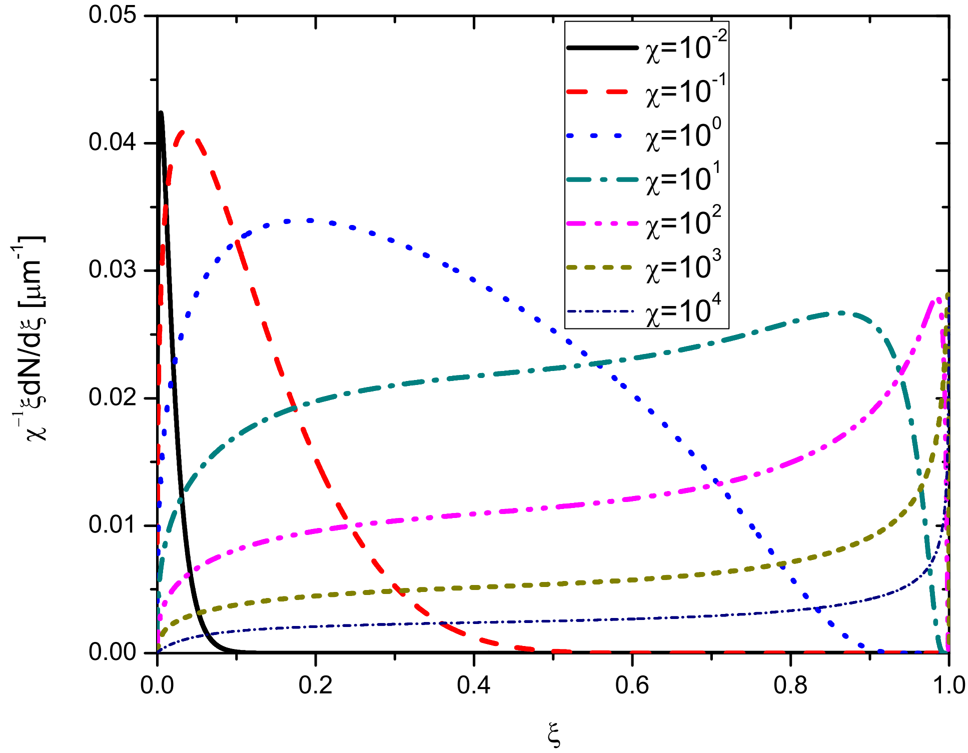

Figure 1. *Synchrotron spectrum dependence on the magnitude of χ. For small χ values the spectrum tends to a classical synchrotron spectrum. For $\chi \ll 1$ the spectrum, when scaled to the Lienard energy loss and the critical frequency, is invariant [3]. On the other hand, for χ0.1, the spectrum shape becomes strongly dependent on the exact value of χ and for $\chi \gg 1$ peaked towards the energy of the incoming particle, i.e. typically $\xi = \hbar\omega/E_0 \to 1$.*

out, that the emission of synchrotron-like radiation from the beams enters the regime where χ becomes of the order one, and the spectrum changes character completely, see figure 1. Of particular relevance to the connection of beamstrahlung with emission from electrons penetrating crystals is the contribution by Baier and Katkov - two of the theory pioneers in strong field effects in crystals - to the early development of the theory of beamstrahlung [15].

Shortly after the first publications on the relevance of quantum theory to beamstrahlung, Blankenbecler and Drell contributed a full quantum treatment of the problem, based on the eikonal approximation [16]. The scaling parameter in this case is given by

$$C = \frac{m^2c^3RL}{4Ne^2\gamma^2\hbar} \tag{4}$$

representing the electric field from a homogeneously charged cylinder of length L and radius R holding N charges, in units of $\mathcal{E}_0 = m^2c^3/\hbar e$. The applicability of this scaling parameter was later elaborated upon by Solov'yov and Schäfer [17, 18]. From this model of a beam, the form factor $F = \delta/\delta_{\text{classical}}$, describing the quantal energy loss in units of the classical, was derived and approximated by:

$$F(C) = [1 + \frac{1}{b_1}[C^{-4/3} + 2C^{-2/3}(1 + 0.20C)^{-1/3}]]^{-1} \tag{5}$$

with $b_1 = 0.83$, see also [17]. Clearly, as stated by Blankenbecler and Drell, in the classical

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regime $\hbar \to 0$ in equation (4) such that C tends to infinity, and therefore the form factor tends to 1 according to equation (5), as must be required.

Even earlier, Himel and Siegrist [10], developed a fairly compact expression for the fraction of the energy radiated by the electron

$$\delta_{QM} = \frac{8mc^2}{\sqrt{3}}(\frac{r_e}{2\sqrt{3}\hbar c})^{4/3}(\frac{DP}{\gamma f})^{1/3} \tag{6}$$

where $r_e = \alpha\lambda$ is the classical electron radius, $D = N r_e \sigma_z / 2\gamma\sigma_r^2$ is the disruption parameter for which σ expresses the beam radius, N the number of particles per bunch, $P = fN\gamma mc^2$ the beam power and f the repetition rate.

Blankenbecler and Drell in their paper state "'Their [Himel and Siegrist's [10]] final formula [eq. (6)] is remarkably accurate in the full quantum regime: $C \ll 1$."' It is remarkable, but not surprising, as several thorough investigations have since shown, e.g. [19, 15] and as we will now show with relatively simple means. The approach of Himel and Siegrist rests on an estimate based on the "'... full quantum treatment of synchrotron radiation [which] was done in 1952 by Sokolov, Ternov and Klepikov [20]"', and also by Julian Schwinger [21].

As a result of the quantum correction, the total radiated intensity for the classical emission is according to Schwinger reduced by a factor $1 - 55\sqrt{3}\lambda_c\omega_0\gamma^2/16c$ due to first order quantum corrections when $\chi \ll 1$ [20, 21, 22]. Including the second order term the reductions for small and large values of χ are [4, 23]

$$I/I_{\text{cl}} = 1 - 55\sqrt{3}\chi/16 + 48\chi^2 \qquad \chi \ll 1 \tag{7}$$

$$I/I_{\text{cl}} \simeq 0.56\chi^{-4/3} \qquad \chi \gg 1 \tag{8}$$

Furthermore, an approximate expression ("accuracy better than 2% for arbitrary χ" [23, eq. (4.57)])

$$I/I_{\text{cl}} \simeq (1 + 4.8(1+\chi)\ln(1+1.7\chi) + 2.44\chi^2)^{-2/3} \tag{9}$$

gives a compact analytical expression applicable e.g. in computer codes.

In figure 2 is shown graphs based on equation (5) and equation (9), where it has been assumed that $C = 1/\chi$. Clearly the curves are similar, and the likelihood of the formulas originating from the same phenomenon becomes even more evident by adjusting to $C = 1.3/\chi$ which results in the curves being indistinguishable on the plot. This is remarkable, given a range of more than five orders of magnitude in χ. In Blankenbecler and Drell's theory, the bunch is treated as a homogeneously charged cylinder of length L and radius R holding N charges. At the distance r from the center of this cylinder the electric field is $2\gamma Ne/Lr$, which leads to an average (over r) field

$$\mathcal{E} = \frac{\gamma Ne}{LR} \tag{10}$$

which can be combined with equation (4) to give

$$C = \frac{m^2c^3}{\gamma\mathcal{E}e\hbar} = \frac{\mathcal{E}_0}{\gamma\mathcal{E}} = \frac{1}{\chi} \tag{11}$$

so it is legitimate to interchange C and $1/\chi$. The reason for the additional factor 1.3 that brings the curves into almost exact agreement, is due to the radiation intensity being non-linear in C, i.e. averaging over the field encountered and then calculating the intensity from this field is not the same as calculating the intensity from the fields encountered and then averaging.

In addition, early studies by Chen and Yokoya [24] showed that field gradient effects are small for a collider operating near $\Upsilon = \chi = 1/C = 1$, i.e. also for the planned Compact LInear Collider (CLIC) at CERN where the expected value is $\Upsilon \simeq 4$. It is therefore to a high degree of accuracy sufficient to use equation (5) derived for the homogeneously charged cylinder.

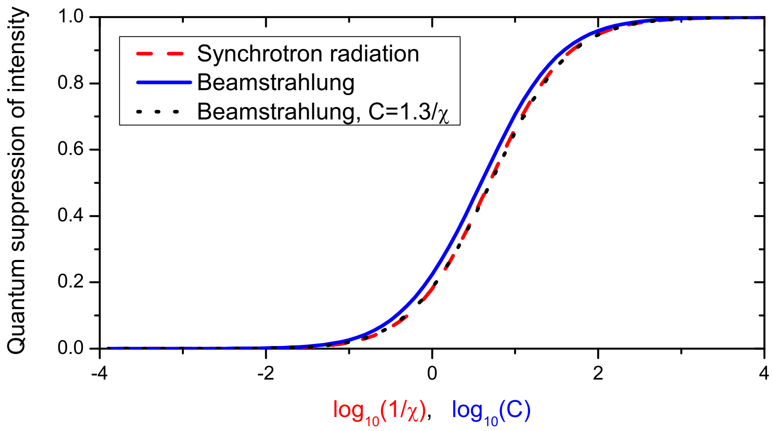

Figure 2. *The quantum suppression of radiation emission intensity, according to eq. (9) and eq. (5).*

2.2. Experiments that support the theory

With the acceleration techniques at hand today, the energy of electrons available for laboratory experiments is limited to $\gamma \leq 10^6$. This means that *macroscopic* magnetic fields of the order a few thousand Tesla or electric fields of the order $\mathcal{E} = 10^{11}$ V/cm are needed to experimentally test the theory for $\chi = \gamma\mathcal{E}$ [V/cm]$/1.32 \cdot 10^{16} \simeq 1$. At present this is certainly beyond reach by conventional means. However, an analogous case is seen for the pair production and radiation emission in single crystals and these phenomena enable a verification of the essential expressions in beamstrahlung theory.

In figure 3 is shown results for radiation emission from electrons impinging on a tungsten crystal close to the $\langle 111 \rangle$ axis. As a consequence of the strong deflection upon the passage of the string of nuclei composing the axis, the electron is forced to emit radiation as in a constant field. This happens much like in normal synchrotron radiation emission, only in a much more intense field, 10^{11} V/cm, corresponding to 30.000 T. Put in one sentence only, such fields are obtained from the coherent addition of the nuclear electric fields in the crystal - fields that for unscreened nuclei of high Z are themselves only slightly smaller than $\mathcal{E}_0$ at a distance of a 1s electron, $\mathcal{E}_{1s}/\mathcal{E}_0 = \alpha^3 Z^3$. As a result of the high peak value of the χ parameter ($\chi_{W,\langle 111\rangle} \simeq 0.03 \cdot E$[GeV]), the radiation emission is subjected to strong quantum suppression. For a brief introduction to these effects in crystals, see Appendix A. In the limit $\chi \ll 1$ the enhancement - defined as the intensity from the aligned crystal divided by the intensity for a non-aligned (so-called 'random' i.e. effectively amorphous) crystal - would be linear with increasing energy, as shown by the dash-dotted line. This is the case because synchrotron radiation emission is quadratic in energy and radiation from an amorphous foil is linear in energy, but due to the strong quantum suppression, the enhancement is reduced to the level shown by the dashed line, as also expected from equation (9). The good agreement between experimental values and theory shown in figure 3, combined with the equality of beamstrahlung and strong field theory shown in figure 2 provide a strong experimental indication that QED theory as applied to beamstrahlung in the regime $1\Upsilon10$ is correct.

The accuracy of the experimental values is $5 - 10\%$, enough to ascertain the validity of

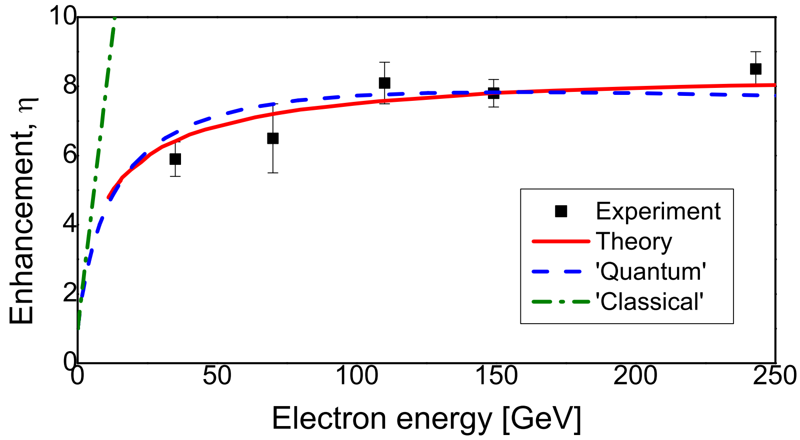

Figure 3. *Experiment [25] and theory [26] for radiation emission from electrons penetrating a tungsten crystal near the* $\langle 111 \rangle$ *axis. For comparison, a curve based on equation (9) with a slightly arbitrary, but realistic* $\overline{\chi} = 0.02 \cdot E[GeV]$ *(and vertical scale obtained as the best fit) is shown as the dashed line ('quantum'), and the corresponding classical expression as the dash-dotted line ('classical'). The enormous difference between the 'classical' and 'quantum' curves directly show the strong quantum suppression in the experimentally accessible regime.*

the theoretical approach. We can therefore confidently claim that the 'quantum synchrotron' behaviour of radiation emission in a strong field, is experimentally well confirmed.

3. 'Coherent pairs'

Having now established the equality between the variables $1/C$ and Υ used in the beamstrahlung community and the variable χ used to describe synchrotron radiation in a strong field, we proceed with a discussion of pair creation in strong fields, described, as expected from crossing symmetry of the processes, by similar variables.

3.1. Pair production in a strong electromagnetic field

In the following, pair production and radiation emission processes are considered as taking place in magnetic or electric fields that are homogenous (constant in space over the formation length of the corresponding process). The pair production probability differential in the energy of one of the final state particles, ϵ, is given as [23]

$$\frac{\mathrm{d}W}{\mathrm{d}\epsilon} = \frac{\alpha m^2 c^4}{\sqrt{3}\pi\hbar^3\omega^2}\left[\frac{\epsilon^2+\epsilon_f^2}{\epsilon\epsilon_f}K_{2/3}(\xi) + \int_\xi^\infty K_{1/3}(y)\mathrm{d}y\right] \tag{12}$$

where $\epsilon_f = \hbar\omega - \epsilon$, $\xi = 8u/3\chi_p$, $u = \gamma^2$, and $\mathrm{d}W = \mathrm{d}w/\mathrm{d}t$ is the probability per unit time. Here ϵ denotes the energy of one of the produced particles, ϵ_f that of the other and $\hbar\omega$ the energy of the photon. The functions $K_{1/3}$ and $K_{2/3}$ are the modified Bessel functions of order 1/3 and 2/3, respectively. The Lorentz invariant pair production strong field parameter is given as $\chi_p = \gamma_p B/B_0$ where the critical field once again appears. γ_p is understood here as $\hbar k_\perp/mc$, i.e. the photon momentum transverse to the magnetic field, in units of mc. The 'strong field regime' is reached when χ_p becomes comparable to or exceeds 1 - but already at $\chi_p \simeq 0.1$ strong field effects become significant.

The differential probability develops a pronounced minimum around $x = \epsilon/\hbar\omega = 1/2$ and peaks at $x \simeq 1.6/\chi_p$ for large χ_p.

The total pair production probability per unit time is given as [23]

$$W = \frac{\alpha m^2 c^4}{6\sqrt{3}\pi\hbar^2\omega} \int_1^\infty \frac{8u+1}{u^{3/2}\sqrt{u-1}} K_{2/3}(\xi)\mathrm{d}u \tag{13}$$

In the limit $\chi_p \ll 1$ the probability is exponentially small, $W \simeq 3\sqrt{3}\alpha m^2 c^4 \chi_p / 16\sqrt{2}\hbar^2\omega \exp(8/3\chi_p)$, and in the limit $\chi_p \gg 1$ the result is $W \simeq 0.38\alpha mc^2 B/B_0\hbar\chi_p^{1/3}$, i.e. the probability actually diminishes once the energy surpasses the domain where $\chi_p \simeq 10$ [23].

The equations (12) and (13) are precise expressions (for $B \ll B_0$), but eq. (13) has an 'analytic approximation' that does not involve integration [27] (where $\chi_e = \hbar\omega B/2mc^2 B_0$)

$$n_{\text{pairs}} \simeq n_{\text{photons}} d \frac{\alpha B}{2\lambda B_0} \cdot 0.16 \frac{1}{\chi_e} K_{1/3}^2(2/3\chi_e) \tag{14}$$

and which generally forms the basis for many calculations.

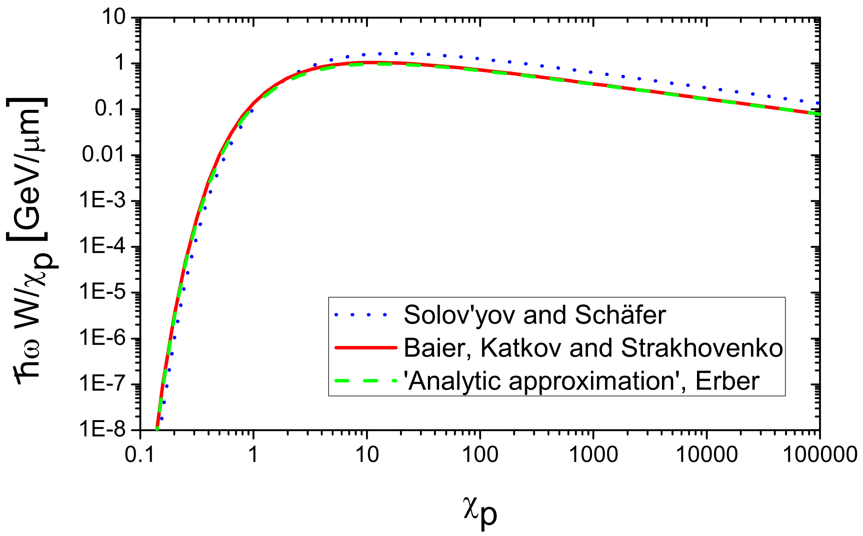

Figure 4. *Comparison between the total pair production probabilities in strong fields of colliding beams according to Solov'yov and Schäfer [18] (dotted line), in a general strong field according to Baier, Katkov and Strakhovenko [23] (full line) and the 'analytic approximation' to the latter, eq. (14), from Erber [27] (dashed line).*

In an article by Solov'yov and Schäfer [18] pair creation by photons colliding with a relativistic beam is treated. Once again, their central scaling parameter

$$C_p = \frac{m^3 c^5 r}{2e\mathcal{E}\gamma\omega R} \tag{15}$$

is closely related to χ_p by $C_p = r/\chi_p R$ where R is the radius of the beam colliding with the photons and r is the position of the photon with respect to the beam. In their article an averaging over r is done, but naively assuming $r = R$ one can compare directly with the results of Baier and Katkov, cf. equation ((13)). This simple comparison, shown in figure 4, shows that the functional dependence on the strong field parameter of pair creation is the same.

3.2. Experiments that support the theory

In crystals, the process of axial pair creation is well described by the strong field equations mentioned above. Multiple scattering events give rise to an LPM (Landau-Pomeranchuk-Migdal) effect [28] which is a reduction of the order of a few percent to the total pair production rate and especially dominant for soft photons. Furthermore there is an incoherent contribution to the pair production rate which dominates the spectrum at low photon energies, while this contribution is nearly negligible compared to the total rate when the photon energy is large. Pair production probability rises above the usual Bethe-Heitler one with increasing photon energy. This is a consequence of the strong field and has been measured in several crystalline media [29].

The experimental results are well understood theoretically, and though the constant field approximation does well in estimating the yield, a more careful theoretical treatment is needed accounting for the incoherent contribution as well as the LPM effect and atomic scattering nature of the process. Again QED theory proves sufficient for description of the coherent pairs in crystalline matter, see figure 5.

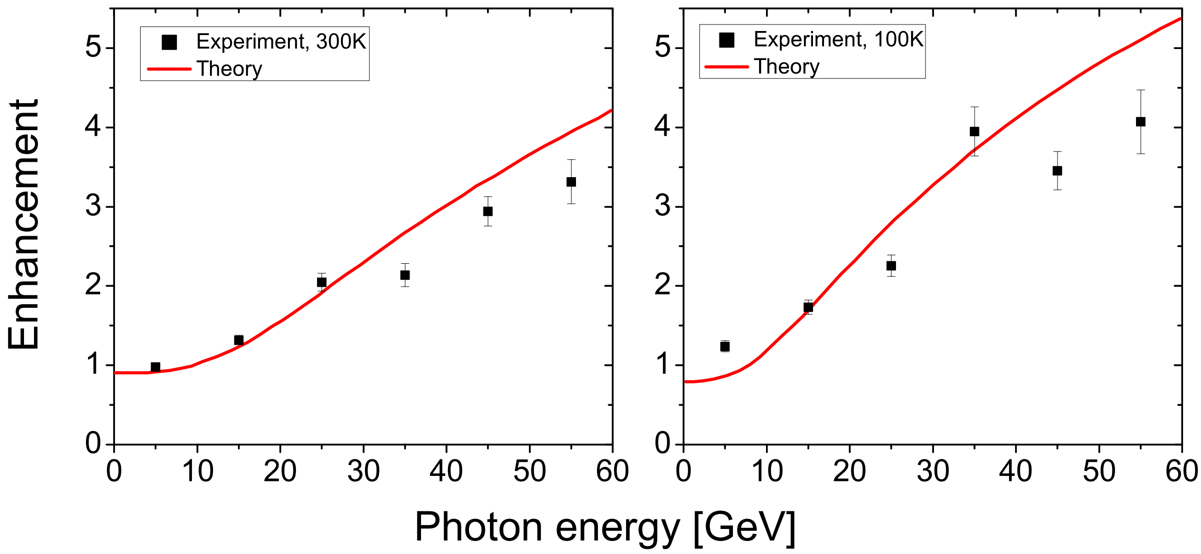

Figure 5. *Total pair production enhancement in W ⟨111⟩ as function of photon energy. Theoretical curves from [28] and experiment from [29]. The coherence rises due to reduced thermal vibrations, making cooling of the crystals a way of pronouncing the strong field effect.*

4. 'Landau-Lifshitz' pairs and tridents

In a strong field, electron/positron pairs may be produced via a virtual photon. The direct production of a pair via virtual photons of a constant field can be calculated by the Weizsäcker-Williams method of virtual quanta. In this approach, the spectral density of virtual photons carried by an electron of energy E_0 may be estimated by

$$\frac{dN_{ww}}{\hbar\omega} = \frac{2}{\pi}\frac{\alpha}{\hbar\omega}W\left(\frac{\hbar\omega}{E_0}\right), \tag{16}$$

where

$$W(x) = xK_0(x)K_1(x) - \frac{x^2}{2}(K_1^2(x) - K_0^2(x)) \tag{17}$$

The amount of these photons converting in a strong field corresponds to the number of Landau-Lifschitz pairs. The subject of equivalent-quanta trident production in a homogenous field has been studied by several authors [30, 27] or in crystals using the Weizsäcker-Williams method [31]. Since no electromagnetic quanta need to be created, the dependence on the thickness of the crystal is linear. The total probability of this process in a homogenous field is [30]

$$W_v = \frac{13\alpha^2}{18\sqrt{3}\pi\lambda_c}\frac{\chi mc^2 t}{E_0}(\ln(\chi) - 0.577 - \ln(4\sqrt{3}) - 77/39) \tag{18}$$

where t is the extent of the magnetic field. The characteristic length of formation for this process is $l_v = \lambda B_0/B$.

A competing process of a bremsstrahlung photon being emitted and the subsequent conversion of the photon into a pair occurs, but it has a quadratic dependence on the thickness. The combined virtual (direct) and real (cascade) photon quanta processes have become known as the 'trident' process [27] due to the shape of the tracks in emulsions for this type of event. The total probability of cascade tridents is within 4% accuracy for arbitrary χ [30] found by the expression

$$W_r = \frac{3\chi}{16}\left(\frac{\alpha t}{\gamma\lambda_c}\right)^2 \ln(1+\frac{\chi}{12})\exp(-\frac{16}{3\chi})(1+0.56\chi+0.013\chi^2)^{1/6}, \tag{19}$$

where λ_c is the reduced Compton wavelength, $\lambda_c/2\pi$. The formation length measured in the laboratory is given for soft photons as

$$l_f = \frac{2\gamma^2 c}{\omega} \tag{20}$$

this is the characteristic length for radiation emission in a classical synchrotron experiment. It may be seen as the distance an electron needs to travel before the created photon is separated by one reduced wavelength. By choosing an extent of the magnetic field comparable to the formation length, the cascade contribution to the trident yield will be suppressed since the formation length of the cascade process is larger than the formation length for a direct trident.

In beam-beam collisions, it is crucial to know the yield of the direct process since the dependence on χ is very strong for relevant values of χ, and the lack of a photon track may make it a difficult background to account for.

A recent experiment [32] addressed trident production using aligned Ge $\langle 110\rangle$ crystals. Results are still preliminary, but a strong field effect on the trident process was seen. The trident crystal-to-amorphous enhancement shows that crystals can be used to observe higher order QED processes in strong fields. Theoretical work on the issue [33] has recently been published suggesting that the dominant process is the cascade process for the configuration investigated, meaning that the enhancement of the trident yield is closely connected to the enhancement of bremsstrahlung photons.

5. Spin-flip in a strong field

In the early 90's, Lindhard[8] showed that the contribution from the spin can be derived from a Weizsäcker-Williams type calculation, replacing the spin-less Thomson cross section by the

Klein-Nishina cross section. He and Sørensen [34] convincingly demonstrated that the contribution from the spin dominates the hard end of the photon spectrum as soon as $\chi \gg 1$. This means that - apart from the reverse action of the photon on the electron, the recoil - an additional quantum effect of the spin of the electron influences the spectrum.

Why the spin makes its influence can be seen by a rough, but simple argument [35]: The energy of a magnetic moment $\vec{\mu}$ at rest in a magnetic field $\vec{B}$ is given by

$$E_\mu = -\vec{\mu} \cdot \vec{B} \tag{21}$$

Therefore spin-flip transitions of electrons with $\mu = e\hbar/2mc$ have an energy $\Delta E_\mu = e\hbar B/mc$ in the rest system. In this system, the purely electric field in the crystal is by the Lorentz transformation converted into a magnetic field $B = \gamma\beta\mathcal{E}_{\rm lab}$ plus an electric field which does not influence $\vec{\mu}$. Transformation back to the laboratory yields a factor γ (as in the case of channeling radiation arising from transitions in the transverse potential) such that the result is

$$\Delta E_\mu = \gamma^2\beta\frac{\mathcal{E}}{\mathcal{E}_0}mc^2 = \chi\beta\gamma mc^2 \tag{22}$$

which is equal to the initial energy of the electron, E, when $\chi\beta = 1$. This simple estimate shows why the radiation from spin-flip concentrates near the endpoint of the spectrum. Asymptotically, the spin contribution becomes $\xi dN/d\xi \propto (\xi^7/(1-\xi))^{1/3} \cdot \chi^{2/3}$ for $\chi \to \infty$ such that it is strongly peaked at the upper end of the spectrum, see figure 1.

Considering the process of radiative polarization of the electron, the typical time of spin-flip transitions, τ, it is given by [22]

$$\tau = \frac{8\hbar}{5\sqrt{3}\alpha m}(\frac{B_0}{B})^3\frac{1}{\gamma^2} = \frac{8\hbar}{5\sqrt{3}\alpha m}\frac{\gamma}{\chi^3} \tag{23}$$

such that $c\tau$ becomes 15 μm for a 150 GeV electron in a $\chi = 1$ field. Therefore a substantial fraction of the radiation events originate from spin-flip transitions as one gets to and beyond $\chi \simeq 1$, even in a target as thin as 0.1 mm.

5.1. Experiments that support the theory

In figure 6 is shown the radiated power as a function of the fractional photon energy $\xi = \hbar\omega/E$ for 2 energies of the impinging electron, 70 and 243 GeV (more examples can be found in [35]). The experimental points are compared to two theoretical curves: one where the effect of spin has been neglected and one where the electron is treated properly as a spin $^1/_2$ particle. Although the experiment suffers a bit from lack of sufficient statistics (given indirectly by the scatter of points), the trend is clear: The spin of the electron must be included to get the best description of the experimental values. This is not surprising, but it shows that the above description of spin-flip processes is likely to be correct, even in a strong field. It must be emphasized, though, that neither the electron nor the photon beam becomes polarized in the experiment - the electron is exposed to strong fields, but in the radial direction from the crystal axis. Since this does not single out any preferred direction, the beams cannot become polarized, yet the emission spectrum may bear evidence for the spin-flip process.

6. Related areas where similar theories are applied

Strong field effects can be investigated by other means. One example is in heavy ion collisions where the field becomes comparable to the critical field, but the collision is of extremely short duration. Another - technically demanding - example is in multi-GeV electron collisions with

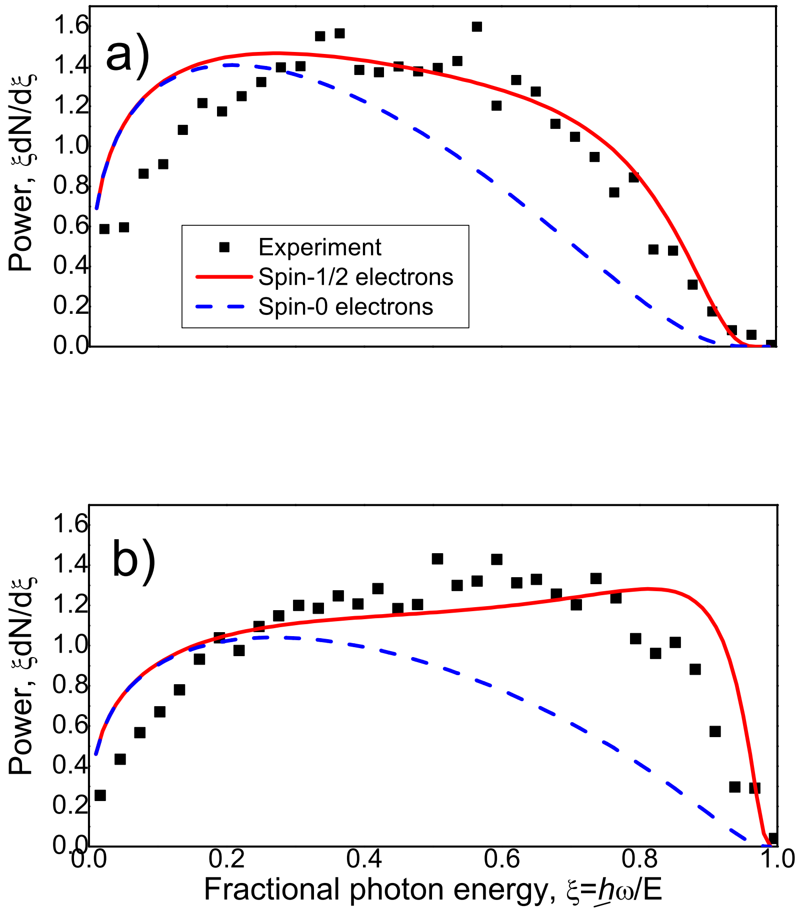

Figure 6. *Radiation emission as a function of photon energy for a) 70 GeV electrons and b) 243 GeV electrons. The full-drawn line shows the calculated radiation spectrum for spin one-half electrons at $\chi = 2.1$ and $\chi = 7.3$ (i.e. not averaged over the fields encountered, see also [36]), respectively, while the dashed line shows the corresponding quantity for scalar charged particles. The full-drawn lines can be compared to the dotted and dash-dotted lines in figure 1.*

terawatt laser pulses where non-linear Compton scattering and so-called Breit-Wheeler pair production are observed [37]. In none of these cases can the field be considered macroscopic.

From the foregoing discussion, it is clear that the two groups of Blankenbecler and Drell and Baier and Katkov have contributed substantially to the modern understanding of beamstrahlung. For the latter group, they - together with V. Strakhovenko - have been, and are still, world leaders in the theory of strong field effects in crystals. Both groups have furthermore contributed to other, related areas of recent experimental interest, as we discuss in short in the following.

6.1. Landau-Pomeranchuk-Migdal effect

The Landau-Pomeranchuk-Migdal (LPM) effect arises because of multiple Coulomb scattering that leads to a reduced radiation emission probability. The criterion for the onset of the effect is that the multiple scattering angle within the formation length must be larger than the Lorentz transformed $1/\gamma$ opening cone of radiation emission, such that emission amplitudes add destructively. The formation length is loosely speaking the distance of travel for the electron to separate by one reduced wavelength from the emitted photon ('the photon is set free'). For details on the formation length - also an extremely useful concept in strong field effects - the reader may consult [38, 39].

Blankenbecler and Drell developed their eikonal approximation methods to include the effect of a finite target for the LPM effect. This effect is known as the Ternovskii-Shul'ga-Fomin effect in radiation emission: that if the target becomes sufficiently thin, the radiation emission has the same dependence on photon energy as the normal amorphous (Bethe-Heitler) spectrum, only it is suppressed by up to an order of magnitude. Likewise, Baier and Katkov [40] increased the accuracy of calculations compared to the original approach by Migdal. In both cases, theory is in good agreement with measurements [41, 42]. It is clear that similar phenomena may appear in beamstrahlung, for sufficiently small bunch lengths, although this regime has not been touched upon in collider concepts so far.

6.2. Structured target resonances

As a continuation of the finite target approach developed with Drell, Blankenbecler enlarged the region of applicability of the theory by also considering resonances appearing from a stack of thin foils with constant separation, a so-called structured target (or sandwich target). It turned out that for such a target, resonance-like structures in the radiation emission spectrum appear for photon energies where the formation length is approximately equal to the foil separation. This calculation was confirmed shortly after by Baier and Katkov. Given a micro-structure in the colliding beams, such effects may also be present for beamstrahlung. Seen from an experimental viewpoint, however, the structured target resonances have proven very difficult to establish, since they amount to a $\simeq 10\%$ correction to the effect of one finite target.

7. Conclusion

We have shown the close connection between strong fields in crystals and in beamstrahlung. Crystals provide experimental data supporting calculations of QED processes in strong electromagnetic fields. This confirmation is important for the development of future electron/positron collider experiments, since simulations of the interaction region must rely on theory in order to calculate e.g. QED backgounds. Experimental studies of quantum synchrotron radiation, pair production and the trident process have been performed. All of them address important issues of strong field QED and all of them have confirmed the validity of the theory in the tested strong field regime relevant to for example the interaction region of CLIC.

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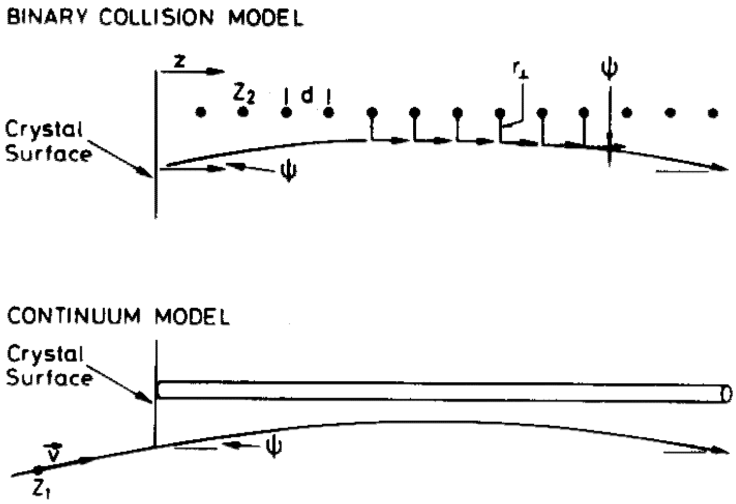

Figure A1. *A schematical drawing of the discrete nature of the scattering centers in a crystal and the resulting continuum approximation.*

Appendix A. Strong fields in crystals

In the following, a brief introduction to strong field effects in crystals is given with particular emphasis on pair production. However, essentially all conclusions and expressions can be applied also to the case of radiation emission and thus apply for the development of electromagnetic showers as well. For a more complete introduction to strong field effects in crystals, see [23], [34] and [43].

The large fields present near the nuclei in solids may in the case of single crystals add coherently such that a penetrating charged particle experiences a continuous field along its direction of motion, see figure A1. As a special case, if the particle is incident with a sufficiently small angle to a particular crystallographic direction, inside the so-called Lindhard angle, the negatively/positively charged particle is constrained to move near to/far from the nuclei and the electron clouds surrounding these. This is the channeling phenomenon [44] which has found widespread applications in physics. Another well-known example of coherent action is the emission of so-called coherent bremsstrahlung. This appears when crystal planes are crossed by the particle at regular intervals *and* at small angles to crystallographic directions during penetration [45]. The small angle requirement is essential for the coherence of the scattering that results in amplification of e.g. the radiation spectra.

On the other hand, at high energies (generally above a few GeV for electrons) a seemingly different coherent effect starts to appear. This effect arises due to the continuous field seen by the penetrating particle which acts on an electron with sufficient strength to impose a strongly curved trajectory (as compared to the energy - and thus typical emission angles - of the particle), see figure A1. The curvature - independent of the origin being magnetic or electric fields - gives rise to radiation emission of synchrotron character. Furthermore, this effect persists out to the so-called Baier angle, Θ_0, which is of the order of a few mrad and independent of energy

$$\Theta_0 \simeq Ze^2/dmc^2 \tag{A.1}$$

where d is the lattice distance along the axis. For particle energies above a few GeV (depending on crystal type), the Baier angle exceeds the Lindhard angle, and the particle is subjected to a continuously strong field even though it is not channeled, as long as it is incident within Θ_0.

The electric fields, $\mathcal{E}$, can be locally up to a few 10^{11} V/cm, depending on the crystal type and orientation. The incident particle moves in these immensely strong fields over distances up to that of the crystal thickness, i.e. up to several cm. For a sufficiently energetic particle with γ approaching 10^6, the critical parameter $\chi = \gamma\mathcal{E}/\mathcal{E}_0$ can thus reach values near and even beyond unity since $\mathcal{E}_0 = 1.32 \cdot 10^{16}$ V/cm. Thereby the behaviour of charged particles in macroscopic strong fields as $\mathcal{E}_0$ or B_0 can be investigated in accelerator based experiments.

It is essentially the same phenomenon which is responsible for the mrad deflection of multi-GeV protons [46] and ions [47] during the passage of a few cm of a bent crystal. In this case, the equivalent magnetic field is as high as several times 1000 T [48].

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