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Comparing the R_ξ gauge and the unitary gauge for the standard model: An example

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Abstract

For gauge theory, the matrix element for any physical process is independent of the gauge used. However, since this is a formal statement, it does not guarantee this gauge independence in every case. An example is given here where, for a physical process in the standard model, the matrix elements calculated with two different gauge – the R_ξ gauge and the unitary gauge – are explicitly verified to be different. This is accomplished by subtracting one matrix element from the other. This non-zero difference turns out to have a subtle origin. Two simple operators are found not to commute with each other: in one gauge these two operations are carried out in one order, while in the other gauge these same two operations are carried out in the opposite order. Because of this result, a series of question are raised such that the answers to these question may lead to a deeper understanding of the Yang–Mills non-Abelian gauge theory in general and the standard model in particular.

1. Introduction

Since the standard model of Glashow, Weinberg, and Salam [1] is a Yang–Mills non-Abelian gauge theory [2], the matrix element in the standard model for any physical process is expected to be independent of the gauge chosen.

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It is the purpose of the present paper to investigate the validity of this gauge independence, by studying a specific example through explicit calculation.

For the standard model, there are two gauges that are commonly used: the R_ξ gauge and the unitary gauge [3]. Formally, these two gauge are closely related: the unitary gauge is the limit, as the parameter ξ goes to infinity, of the R_ξ gauge. Therefore, for the present investigation, these two gauges are chosen for comparison.

Which matrix element should be selected to compare the results for these two gauges? First, the physical process should be of relatively low order; otherwise it may be too difficult to compare these two gauges. Since, for tree diagrams, there is no difference between these two gauges, or any other gauge, the matrix element to be studied should be from a one-loop Feynman diagram or diagrams.

Secondly, the diagrams under consideration must contain, as internal lines, one or more W or Z propagators. Such propagators take different forms for these two gauges. Specifically, the W propagator is

$$\frac{1}{p^2 - m^2}[-g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}] \tag{1}$$

in the unitary gauge, and

$$\frac{1}{p^2 - m^2}[-g^{\mu\nu} + \frac{(1-\xi)p^\mu p^\nu}{p^2 - \xi m^2}] \tag{2}$$

in the R_ξ gauge. In both (1) and (2), m is the mass of the W boson, and an overall factor of i has been omitted.

Thirdly, it is desirable to choose a physical process that is important experimentally and theoretically. Four years ago, the ATLAS Collaboration [4] and the CMS Collaboration [5] discovered the Higgs particle [6]. In this discovery, the most important decay mode was

$$H \to \gamma\gamma. \tag{3}$$

Since the photon is massless, there is no direct coupling of the Higgs particle to the photon. This means that the decay (3) proceeds predominantly through one-loop diagrams. Since the coupling of the Higgs particle H to a fundamental particle is largest for the heaviest particles, the dominant contributions to the decay (3) are through a top loop and a W loop. The contribution from the top loop was calculated by Rizzo [7] thirty six years ago; there is no gauge dependence in this calculation. Since, as seen from (1) and (2), the W propagator depends on the choice of the gauge, the contribution from the W loop can potentially depend on the gauge used.

With these consideration, the physical process to be studied in this paper is the Higgs decay (3) through one W loop. This matrix element is to be calculated both in the R_ξ gauge and the unitary gauge, and the results are to be compared.

There is actually another reason to study this particular problem: there has been some controversy as to how this matrix element should be calculated in the standard model. The considerations in this paper is not going to solve this controversy, but may shed some light on it. In particular, a number of important question are to be raised later in Section 8, and hopefully the answer to some of these questions will teach us how this calculation, and a number of other calculations in the standard model, should be carried out.

2. Method of comparison

The comparison of the matrix element for the R_ξ gauge and that for the unitary gauge is to be carried out in the most straightforward manner, namely, by subtracting these two matrix elements.

The first step is to rewrite the W propagator (1) in the unitary gauge as follows

$$\frac{1}{p^2-m^2}[-g^{\mu\nu}+\frac{(1-\xi)p^\mu p^\nu}{p^2-\xi m^2}]+\frac{1}{p^2-\xi m^2}\frac{p^\mu p^\nu}{m^2}, \tag{4}$$

where the first term is the W propagator (2) in the R_ξ gauge. The following nomenclature is to be used: we shall call

(a) the first term of (4) the W propagator in the unitary gauge, and
(b) the second term of (4) the ϕ propagator in the unitary gauge.

Therefore, in this nomenclature for the unitary gauge, the W propagator is no longer given by (1); instead it is the same as W propagator in the R_ξ gauge. In contrast, the φ propagator is different in the R_ξ gauge and the unitary gauge.

With the original W propagator (1), there are only three Feynman diagrams, in the unitary gauge, for the decay process (3) through one W loop. That the number of diagram is so small is the major advantage of the unitary gauge. When this W propagator (1) is split into two terms via (4), then the number of diagrams in the unitary gauge increase to fourteen; this is to be compared with the twenty six diagrams in the R_ξ gauge.

In calculating the difference between the R_ξ gauge and the unitary gauge, it is essential to deal with the integrands of the Feynman diagrams instead of their integrals, because the integrals are all divergent. Let the integrands from the R_ξ gauge be denoted by I, while those from the unitary gauge by J. Thus there are twenty-six non-zero I's and fourteen non-zero J's. These forty I's and J's are to be written down explicitly in the next section.

The notation is as follows: the four-momenta of the decay products two photons are called k_1 and k_2, and their polarization indices μ and ν respectively. Therefore

$$k_1^2=k_2^2=0, \qquad (k_1+k_2)^2=m_H^2$$

and

$$k_{1\mu}=k_{2\nu}=0. \tag{5}$$

where m_H is the mass of the Higgs particle H.

Let k be the loop four-momentum to be integrated over. It is convenient to use the following notation:

$$p_1=k+\frac{k_1+k_2}{2}$$
$$p_2=k+\frac{-k_1+k_2}{2}$$
$$p_2'=k+\frac{k_1-k_2}{2}$$

and

$$p_3=k-\frac{k_1+k_2}{2}. \tag{6}$$

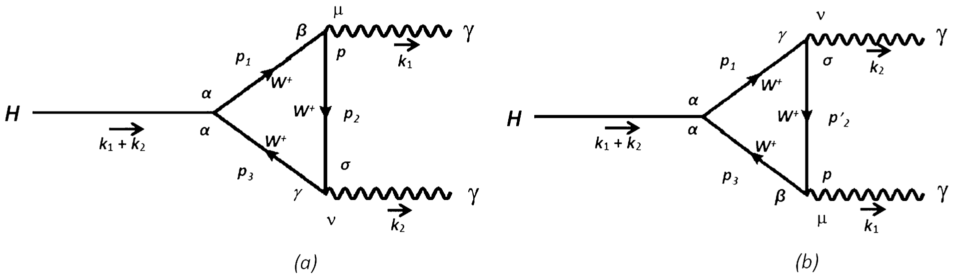

Fig. 1. Feynman diagrams whose integrands are (a) I_1 and J_1; (b) $\bar{I}_1$ and $\bar{J}_1$.

3. Feynman diagrams for the decay $H \to \gamma\gamma$

In this section, the twenty eight Feynman diagrams for $H \to \gamma\gamma$ due to one W loop are given explicitly. From these twenty eight diagrams, there are twenty six non-zero integrands in the R_ξ gauge and fourteen in the unitary gauge, as discussed in the preceding section.

The twenty eight diagrams are paired by reversing the charge of the W loop. There are thirteen such pairs together with two that are invariant under this reversal of the charge, leading to fifteen figures to be numbered from Fig. 1 to Fig. 15.

The first pair of Feynman diagram are shown in Fig. 1. The integrands for these two diagrams are

$$I_1 = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}]$$
$$\frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}] \tag{7}$$
$$m[(p_1 + k_1)_\rho g_{\beta\mu} + (p_2 - k_1)_\beta g_{\mu\rho} + (-p_1 - p_2)_\mu g_{\rho\beta}]$$
$$[(p_3 - k_2)_\sigma g_{\gamma\nu} + (p_2 + k_2)_\gamma g_{\nu\sigma} + (-p_2 - p_3)_\nu g_{\sigma\gamma}]$$

$$J_1 = I_1, \tag{8}$$

$$\bar{I}_1 = \frac{1}{p_3^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_3^\alpha p_3^\beta}{p_3^2 - \xi m^2}] \frac{1}{p_2'^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}]$$
$$\frac{1}{p_1^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{1\alpha} p_1^\gamma}{p_1^2 - \xi m^2}] \tag{9}$$
$$m[(p_3 - k_1)_\rho g_{\beta\mu} + (p_2' + k_1)_\beta g_{\mu\rho} + (-p_2' - p_3)_\mu g_{\rho\beta}]$$
$$[(p_1 + k_2)_\sigma g_{\gamma\nu} + (p_2' - k_2)_\gamma g_{\nu\sigma} + (-p_1 - p_2')_\nu g_{\sigma\gamma}]$$

and

$$\bar{J}_1 = \bar{I}_1. \tag{10}$$

In writing down these integrands (7)–(10), an overall factor of $ie^2 g$ has been omitted.

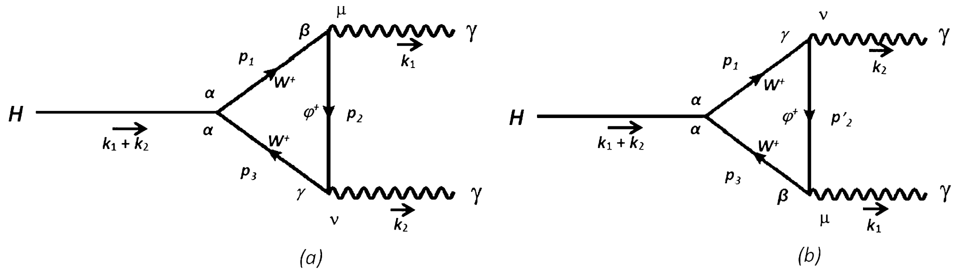

Fig. 2. Feynman diagrams whose integrands are (a) I_2 and J_2; (b) $\bar{I}_2$ and $\bar{J}_2$.

The second pair of diagrams are obtained from the first pair of Fig. 1 by replacing the vertical W line by one for the Higgs ghost φ; they are shown in Fig. 2. The four integrands for the second pair are given by

$$\begin{aligned} I_2 = & \frac{1}{p_1^2 - m^2}[- g^{\alpha\beta} + \frac{(1-\xi)p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \\ & \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}] m(m g_{\beta\mu})(m g_{\gamma\nu}), \end{aligned} \tag{11}$$

$$\begin{aligned} J_2 = & \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \frac{p_2^\rho p_2^\sigma}{m^2} \\ & \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}] \\ & m[(p_1 + k_1)_\rho g_{\beta\mu} + (p_2 - k_1)_\beta g_{\mu\rho} + (-p_1 - p_2)_\mu g_{\rho\beta}] \\ & [(p_3 - k_2)_\sigma g_{\gamma\nu} + (p_2 + k_2)_\gamma g_{\nu\sigma} + (-p_2 - p_3)_\nu g_{\sigma\gamma}], \end{aligned} \tag{12}$$

$$\begin{aligned} \bar{I}_2 = & \frac{1}{p_3^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_3^\alpha p_3^\beta}{p_3^2 - \xi m^2}] \frac{1}{p_2'^2 - \xi m^2} \\ & \frac{1}{p_1^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{1\alpha} p_1^\gamma}{p_1^2 - \xi m^2}] m(m g_{\beta\mu})(m g_{\gamma\nu}), \end{aligned} \tag{13}$$

and

$$\begin{aligned} \bar{J}_2 = & \frac{1}{p_3^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_3^\alpha p_3^\beta}{p_3^2 - \xi m^2}] \frac{1}{p_2'^2 - \xi m^2} \frac{p_2'^\rho p_2'^\sigma}{m^2} \\ & \frac{1}{p_1^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{1\alpha} p_1^\gamma}{p_1^2 - \xi m^2}] \\ & m[(p_3 - k_1)_\rho g_{\beta\mu} + (p_2' + k_1)_\beta g_{\mu\rho} + (-p_2' - p_3)_\mu g_{\rho\beta}] \\ & [(p_1 + k_2)_\sigma g_{\gamma\nu} + (p_2' - k_2)_\gamma g_{\nu\sigma} + (-p_1 - p_2')_\nu g_{\sigma\gamma}]. \end{aligned} \tag{14}$$

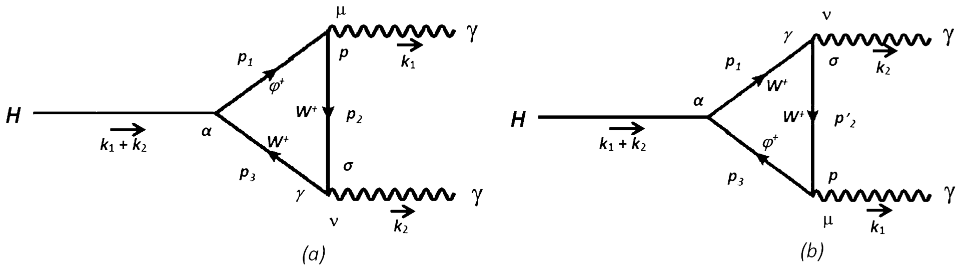

Fig. 3. Feynman diagrams whose integrands are (a) I_3 and J_3; (b) $\bar{I}_3$ and $\bar{J}_3$.

The integrands for the diagrams of Fig. 3 are given by

$$\begin{aligned} I_3 = {} & \frac{1}{p_1^2-\xi m^2}\,\frac{1}{p_2^2-m^2}[-g^{\rho\sigma}+\frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2-\xi m^2}] \\ & \frac{1}{p_3^2-m^2}[-g^{\alpha\gamma}+\frac{(1-\xi)p_3^\alpha p_3^\gamma}{p_3^2-\xi m^2}] \\ & \frac{1}{2}(p_1+k_1+k_2)_\alpha(mg_{\rho\mu}) \\ & [(p_3-k_2)_\sigma g_{\gamma\nu}+(p_2+k_2)_\gamma g_{\nu\sigma}+(-p_2-p_3)_\nu g_{\sigma\gamma}], \end{aligned} \tag{15}$$

$$\begin{aligned} J_3 = {} & \frac{1}{p_1^2-\xi m^2}\,\frac{p_1^\alpha p_1^\beta}{m^2}\,\frac{1}{p_2^2-m^2}[-g^{\rho\sigma}+\frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2-\xi m^2}] \\ & \frac{1}{p_3^2-m^2}[-g_\alpha^\gamma+\frac{(1-\xi)p_{3\alpha}p_3^\gamma}{p_3^2-\xi m^2}] \\ & m[(p_1+k_1)_\rho g_{\beta\mu}+(p_2-k_1)_\beta g_{\mu\rho}+(-p_1-p_2)_\mu g_{\rho\beta}] \\ & [(p_3-k_2)_\sigma g_{\gamma\nu}+(p_2+k_2)_\gamma g_{\nu\sigma}+(-p_2-p_3)_\nu g_{\sigma\gamma}] \end{aligned} \tag{16}$$

$$\begin{aligned} \bar{I}_3 = {} & \frac{1}{p_3^2-\xi m^2}\,\frac{1}{p_2'^2-m^2}[-g^{\rho\sigma}+\frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2-\xi m^2}] \\ & \frac{1}{p_1^2-m^2}[-g^{\alpha\gamma}+\frac{(1-\xi)p_1^\alpha p_1^\gamma}{p_1^2-\xi m^2}] \\ & \frac{1}{2}(p_3-k_1-k_2)_\alpha(mg_{\rho\mu}) \\ & [(p_1+k_2)_\sigma g_{\gamma\nu}+(p_2'-k_2)_\gamma g_{\nu\sigma}+(-p_1-p_2')_\nu g_{\sigma\gamma}], \end{aligned} \tag{17}$$

and

$$\begin{aligned} \bar{J}_3 = {} & \frac{1}{p_3^2-\xi m^2}\,\frac{p_3^\alpha p_3^\beta}{m^2}\,\frac{1}{p_2'^2-m^2}[-g^{\rho\sigma}+\frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2-\xi m^2}] \\ & \frac{1}{p_1^2-m^2}[-g_\alpha^\gamma+\frac{(1-\xi)p_{1\alpha}p_1^\gamma}{p_1^2-\xi m^2}] \\ & m[(p_3-k_1)_\rho g_{\beta\mu}+(p_2'+k_1)_\beta g_{\mu\rho}+(-p_2'-p_3)_\mu g_{\rho\beta}] \\ & [(p_1+k_2)_\sigma g_{\gamma\nu}+(p_2'-k_2)_\gamma g_{\nu\sigma}+(-p_1-p_2')_\nu g_{\sigma\gamma}]. \end{aligned} \tag{18}$$

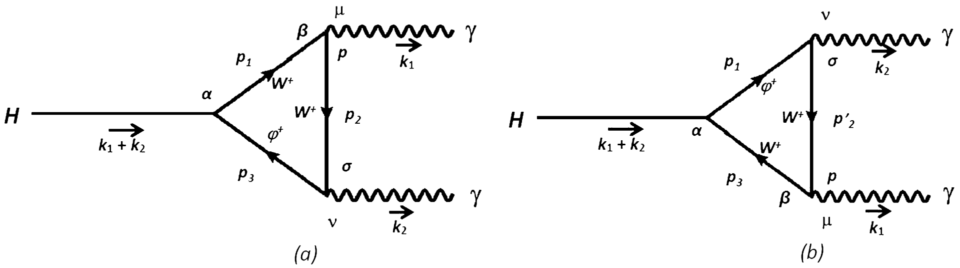

Fig. 4. Feynman diagrams whose integrands are (a) I_4 and J_4; (b) $\bar{I}_4$ and $\bar{J}_4$.

Very similarly, the integrands for the diagrams of Fig. 4 are given by

$$\begin{aligned} I_4 = &\frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \\ &\frac{1}{p_2^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}]\,\frac{1}{p_3^2 - \xi m^2}[-\frac{1}{2}(-p_3 + k_1 + k_2)_\alpha] \\ &[(p_1 + k_1)_\rho g_{\beta\mu} + (p_2 - k_1)_\beta g_{\mu\rho} + (-p_1 - p_2)_\mu g_{\rho\beta}](m g_{\sigma\nu}), \end{aligned} \tag{19}$$

$$\begin{aligned} J_4 = &\frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \\ &\frac{1}{p_2^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}]\,\frac{1}{p_3^2 - \xi m^2}\,\frac{p_{3\alpha} p_3^\gamma}{m^2} \\ &m[(p_1 + k_1)_\rho g_{\beta\mu} + (p_2 - k_1)_\beta g_{\mu\rho} + (-p_1 - p_2)_\mu g_{\rho\beta}] \\ &[(p_3 - k_2)_\sigma g_{\gamma\nu} + (p_2 + k_2)_\gamma g_{\nu\sigma} + (-p_2 - p_3)_\nu g_{\sigma\gamma}], \end{aligned} \tag{20}$$

$$\begin{aligned} \bar{I}_4 = &\frac{1}{p_3^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_3^\alpha p_3^\beta}{p_3^2 - \xi m^2}] \\ &\frac{1}{p_2'^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}]\,\frac{1}{p_1^2 - \xi m^2}[-\frac{1}{2}(-p_1 - k_1 - k_2)_\alpha] \\ &[(p_3 - k_1)_\rho g_{\beta\mu} + (p_2' + k_1)_\beta g_{\mu\rho} + (-p_2' - p_3)_\mu g_{\rho\beta}](m g_{\sigma\nu}), \end{aligned} \tag{21}$$

and

$$\begin{aligned} \bar{J}_4 = &\frac{1}{p_3^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_3^\alpha p_3^\beta}{p_3^2 - \xi m^2}] \\ &\frac{1}{p_2'^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}]\,\frac{1}{p_1^2 - m^2}\,\frac{p_{1\alpha} p_1^\gamma}{m^2} \\ &m[(p_3 - k_1)_\rho g_{\beta\mu} + (p_2' + k_1)_\beta g_{\mu\rho} + (-p_2' - p_3)_\mu g_{\rho\beta}] \\ &[(p_1 + k_2)_\sigma g_{\gamma\nu} + (p_2' - k_2)_\gamma g_{\nu\sigma} + (-p_1 - p_2')_\nu g_{\sigma\gamma}]. \end{aligned} \tag{22}$$

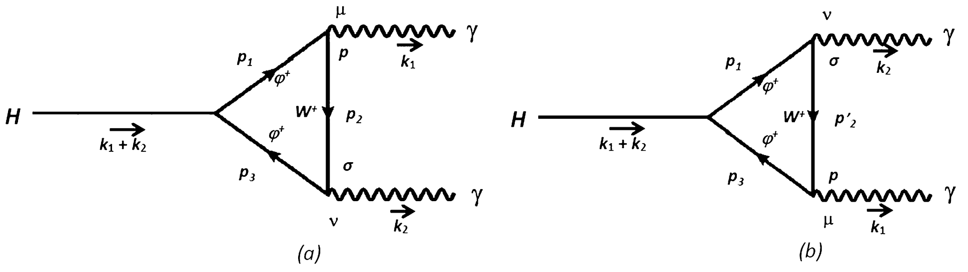

Fig. 5. Feynman diagrams whose integrands are (a) I_5 and J_5; (b) $\bar{I}_5$ and $\bar{J}_5$.

Consider next the three cases where there are one W and two φ's in the triangle. The integrands for the diagrams of Fig. 5 are give by

$$I_5 = \frac{1}{p_1^2 - \xi m^2} \frac{1}{p_2^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}] \frac{1}{p_3^2 - \xi m^2}[-\frac{1}{2}\frac{m_H^2}{m}](mg_{\rho\mu})(mg_{\sigma\nu}), \tag{23}$$

$$J_5 = \frac{1}{p_1^2 - \xi m^2} \frac{p_1^\alpha p_1^\beta}{m^2} \frac{1}{p_2^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}] \frac{1}{p_3^2 - \xi m^2} \frac{p_{3\alpha} p_3^\gamma}{m^2} m[(p_1 + k_1)_\rho g_{\beta\mu} + (p_2 - k_1)_\beta g_{\mu\rho} + (-p_1 - p_2)_\mu g_{\rho\beta}] [(p_3 - k_2)_\sigma g_{\gamma\nu} + (p_2 + k_2)_\gamma g_{\nu\sigma} + (-p_2 - p_3)_\nu g_{\sigma\gamma}], \tag{24}$$

$$\bar{I}_5 = \frac{1}{p_3^2 - \xi m^2} \frac{1}{p_2'^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}] \frac{1}{p_1^2 - \xi m^2}[-\frac{1}{2}\frac{m_H^2}{m}](mg_{\rho\mu})(mg_{\sigma\nu}), \tag{25}$$

and

$$\bar{J}_5 = \frac{1}{p_3^2 - \xi m^2} \frac{p_3^\alpha p_3^\beta}{m^2} \frac{1}{p_2' - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}] \frac{1}{p_1^2 - \xi m^2} \frac{p_{1\alpha} p_1^\gamma}{m^2} m[(p_3 - k_1)_\rho g_{\beta\mu} + (p_2' + k_1)_\beta g_{\mu\rho} + (-p_2' - p_3)_\mu g_{\rho\beta}] [(p_1 + k_2)_\sigma g_{\gamma\nu} + (p_2' - k_2)_\gamma g_{\nu\sigma} + (-p_1 - p_2')_\nu g_{\sigma\gamma}]. \tag{26}$$

Note that the mass m_H of the Higgs particle appears explicitly in I_5 and $\bar{I}_5$, but not in J_5 or $\bar{J}_5$.

Consider next the two diagrams of Fig. 6. This is the first case, as indicated in the figure caption, where there are only two non-vanishing integrand instead of four. These two integrands are

$$I_6 = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \frac{1}{p_3^2 - \xi m^2} [-\frac{1}{2}(-p_3 + k_1 + k_2)_\alpha](mg_{\beta\mu})(p_2 + p_3)_\nu \tag{27}$$

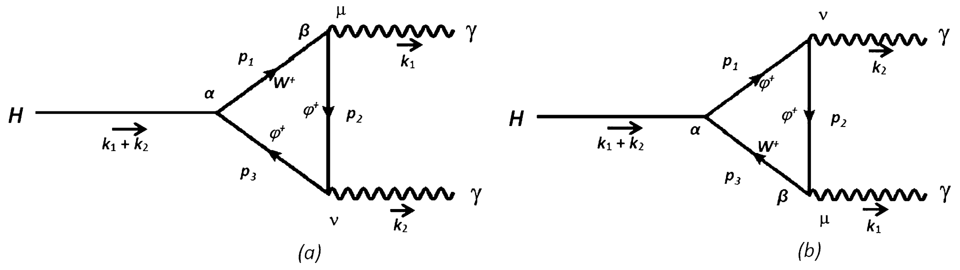

Fig. 6. Feynman diagrams whose integrands are (a) I_6 and $J_6 = 0$; (b) $\bar{I}_6$ and $\bar{J}_6 = 0$.

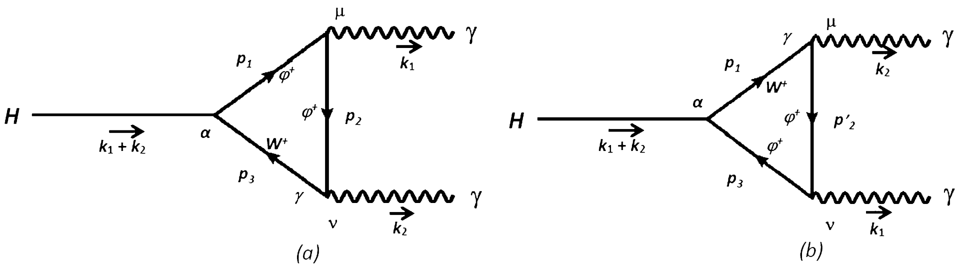

Fig. 7. Feynman diagrams whose integrands are (a) I_7 and $J_7 = 0$; (b) $\bar{I}_7$ and $\bar{J}_7 = 0$.

and

$$\begin{aligned}\bar{I}_6 = {} & \frac{1}{p_3^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_3^\alpha p_3^\beta}{p_3^2 - \xi m^2}]\,\frac{1}{p_2'^2 - \xi m^2}\,\frac{1}{p_1^2 - \xi m^2} \\ & [-\frac{1}{2}(-p_1 - k_1 - k_2)_\alpha](m g_{\beta\mu})(p_1 + p_2')_\nu .\end{aligned} \tag{28}$$

Similarly, those of Fig. 7 lead to the following two non-varnishing integrands:

$$\begin{aligned}I_7 = {} & \frac{1}{p_1^2 - \xi m^2}\,\frac{1}{p_2^2 - \xi m^2}\,\frac{1}{p_3^2 - \xi m^2}[-g^{\alpha\gamma} + \frac{(1-\xi)p_3^\alpha p_3^\gamma}{p_3^2 - \xi m^2}] \\ & [\frac{1}{2}(p_1 + k_1 + k_2)_\alpha](p_1 + p_2)_\mu (m g_{\gamma\nu})\end{aligned} \tag{29}$$

and

$$\begin{aligned}\bar{I}_7 = {} & \frac{1}{p_3^2 - \xi m^2}\,\frac{1}{p_2'^2 - m^2}\,\frac{1}{p_1^2 - m^2}[-g^{\alpha\gamma} + \frac{(1-\xi)p_1^\alpha p_1^\gamma}{p_1^2 - \xi m^2}] \\ & [\frac{1}{2}(p_3 - k_1 - k_2)_\alpha](p_2' + p_3)_\mu (m g_{\gamma\nu}).\end{aligned} \tag{30}$$

For the eighth pair of diagrams as shown in Fig. 8, all three propagators in the loop are of the Higgs ghost φ. The two non-varnishing integrands are

$$\begin{aligned}I_8 = {} & \frac{1}{p_1^2 - \xi m^2}\,\frac{1}{p_2^2 - \xi m^2}\,\frac{1}{p_3^2 - \xi m^2} \\ & [-\frac{1}{2}\,\frac{m_H^2}{m}](p_1 + p_2)_\mu (p_2 + p_3)_\nu\end{aligned} \tag{31}$$

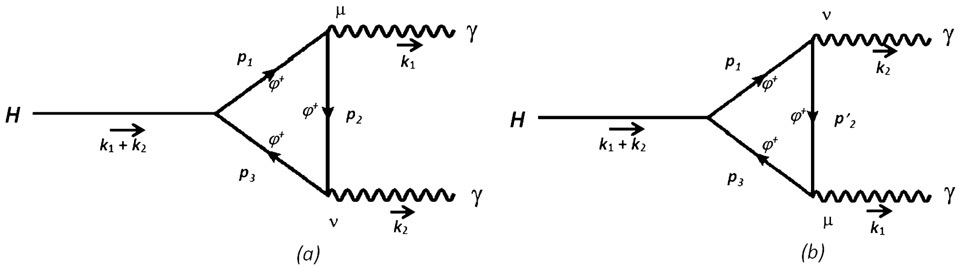

Fig. 8. Feynman diagrams whose integrands are (a) I_8 and $J_8 = 0$; (b) $\bar{I}_8$ and $\bar{J}_8 = 0$.

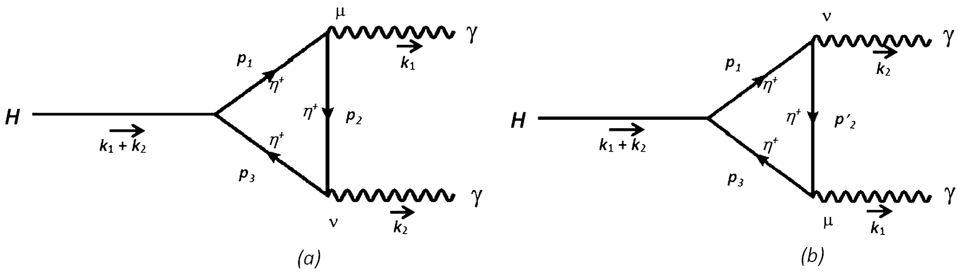

Fig. 9. Feynman diagrams whose integrands are (a) I_9 and $J_9 = 0$; (b) $\bar{I}_9$ and $\bar{J}_9 = 0$. Here η is a Faddeev–Popov ghost.

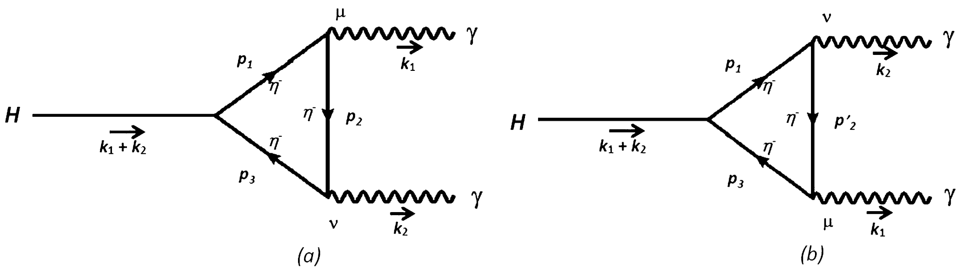

Fig. 10. Feynman diagrams whose integrands are (a) I_{10} and $J_{10} = 0$; (b) $\bar{I}_{10}$ and $\bar{J}_{10} = 0$.

and

$$\begin{aligned} \bar{I}_8 = & \frac{1}{p_3^2 - \xi m^2} \frac{1}{p_2'^2 - \xi m^2} \frac{1}{p_1^2 - \xi m^2} \\ & [-\frac{1}{2} \frac{m_H^2}{m}](p_2' + p_3)_\mu (p_1 + p_2')_\nu \end{aligned} \tag{32}$$

In each of the sixteen Feynman diagrams considered so far as shown in Figs. 1–8, the loop takes the form of a triangle, i.e., there are three propagators in the loop. Furthermore, each of these three propagators is that of a W^+ or φ^+. There are four more Feynman diagrams with a triangular loop, the remaining eight having instead only two propagators in the loop. In these four more Feynman diagrams, the triangular loop is that of a Faddeev–Popov ghost [8]. Since there is no Faddeev–Popov ghost in the unitary gauge, these four diagrams cannot lead to any non-zero J's. These four diagrams with a Faddeev–Popov loop are shown in Fig. 9 and Fig. 10.

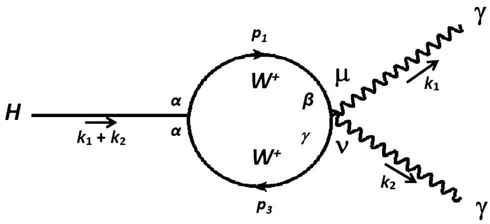

Fig. 11. Feynman diagrams whose integrand is $I_{11} = J_{11}$.

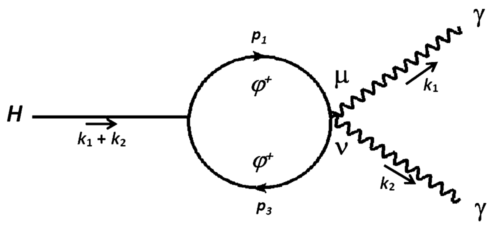

Fig. 12. Feynman diagrams whose integrands are I_{12} and J_{12}.

These four integrands are given by

$$I_9 = -\frac{1}{p_1^2 - \xi m^2} \frac{1}{p_2^2 - \xi m^2} \frac{1}{p_3^2 - \xi m^2} (-\frac{1}{2}\xi m)(-p_{2\mu})(-p_{3\nu}), \tag{33}$$

$$\bar{I}_9 = -\frac{1}{p_1^2 - \xi m^2} \frac{1}{p_2^2 - \xi m^2} \frac{1}{p_3^2 - \xi m^2} (-\frac{1}{2}\xi m)(-p'_{2\mu})(-p_{1\nu}), \tag{34}$$

$$I_{10} = I_9, \tag{35}$$

and

$$\bar{I}_{10} = \bar{I}_9. \tag{36}$$

In the twenty Feynman diagrams studied so far, there are only three vertices but no four vertex. In the remaining eight Feynman diagrams, there is a four vertex each. Because of the presence of this four vertex, there are only two propagators in each these eight diagrams.

The first of these eight diagrams to be studied is the one shown in Fig. 11. Its integrand is

$$I_{11} = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}] m[2g_{\mu\nu}g_{\beta\gamma} - g_{\mu\beta}g_{\nu\gamma} - g_{\mu\gamma}g_{\nu\beta}], \tag{37}$$

with

$$J_{11} = I_{11}. \tag{38}$$

If both of the W propagators in the diagram of Fig. 11 are replaced by φ propagators, then the resulting diagram is that of Fig. 12, whose integrands are

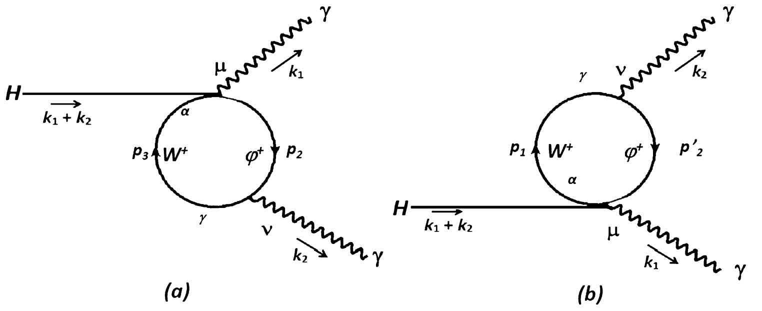

Fig. 13. Feynman diagrams whose integrands are (a) I_{13} and $J_{13} = 0$; (b) $\bar{I}_{13}$ and $\bar{J}_{13} = 0$.

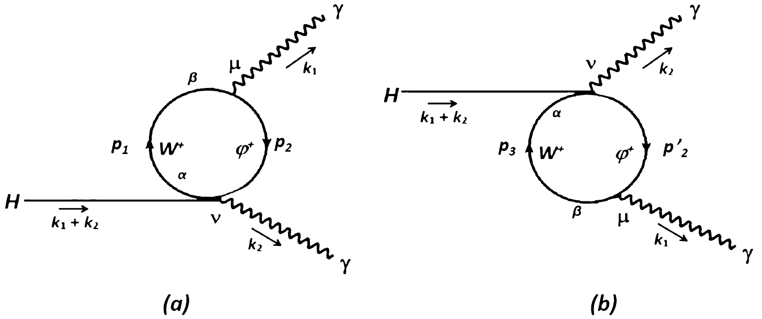

Fig. 14. Feynman diagrams whose integrands are (a) I_{14} and $J_{14} = 0$; (b) $\bar{I}_{14}$ and $\bar{J}_{14} = 0$.

$$I_{12} = \frac{1}{p_1^2 - \xi m^2} \frac{1}{p_3^2 - \xi m^2} [-\frac{1}{2} \frac{m_H^2}{m}](-2g_{\mu\nu}) \tag{39}$$

and

$$J_{12} = \frac{1}{p_1^2 - \xi m^2} \frac{p_1^\alpha p_1^\beta}{m^2} \frac{1}{p_3^2 - \xi m^2} \frac{p_{3\alpha} p_3^\gamma}{m^2} m[2g_{\mu\nu} g_{\beta\gamma} - g_{\mu\beta} g_{\nu\gamma} - g_{\mu\gamma} g_{\nu\gamma}]. \tag{40}$$

The next four diagrams are shown in Fig. 13 and Fig. 14. These four diagrams again do not lead to any non-zero J for the unitary gauge; the four I's for the R_ξ gauge are given by

$$I_{13} = -\frac{1}{p_2^2 - \xi m^2} \frac{1}{p_3^2 - m^2} [-g^{\alpha\gamma} + \frac{(1-\xi) p_3^\alpha p_3^\gamma}{p_3^2 - \xi m^2}](\frac{1}{2} g_{\alpha\mu})(m g_{\gamma\nu}) \tag{41}$$

$$\bar{I}_{13} = -\frac{1}{p_2'^2 - \xi m^2} \frac{1}{p_1^2 - m^2} [-g^{\alpha\gamma} + \frac{(1-\xi) p_1^\alpha p_1^\gamma}{p_1^2 - \xi m^2}](\frac{1}{2} g_{\alpha\mu})(m g_{\gamma\nu}) \tag{42}$$

$$I_{14} = -\frac{1}{p_1^2 - m^2} [-g^{\alpha\beta} + \frac{(1-\xi) p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} (\frac{1}{2} g_{\alpha\nu})(m g_{\beta\mu}) \tag{43}$$

and

$$\bar{I}_{14} = -\frac{1}{p_3^2 - m^2} [-g^{\alpha\beta} + \frac{(1-\xi) p_3^\alpha p_3^\beta}{p_3^2 - \xi m^2}] \frac{1}{p_2'^2 - \xi m^2} (\frac{1}{2} g_{\alpha\nu})(m g_{\beta\mu}) \tag{44}$$

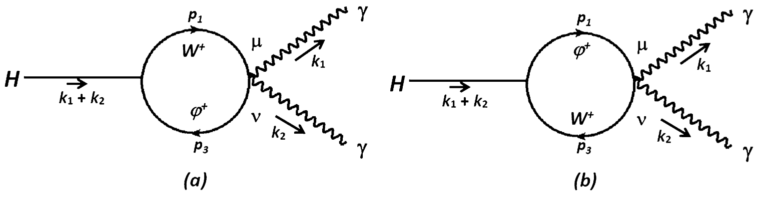

Fig. 15. Feynman diagrams whose integrands are (a) $I_{15} = 0$ and J_{15}; (b) $\bar{I}_{15} = 0$ and $\bar{J}_{15}$.

The last two of the twenty eight Feynman diagrams are shown in Fig. 15. There are only two diagrams that give non-zero J_{15} and $\bar{J}_{15}$ for the unitary gauge, but no non-zero integrands for the R_ξ gauge. The reason is that there is no $W\varphi\gamma\gamma$ four-vertex in the R_ξ gauge for the standard model [1]. The integrands J_{15} and $\bar{J}_{15}$ from the diagrams of Fig. 15 are given by

$$J_{15} = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_3^2 - \xi m^2} \frac{p_{3\alpha} p_3^\gamma}{m^2}$$
$$m[2g_{\mu\nu}g_{\beta\gamma} - g_{\mu\beta}g_{\nu\gamma} - g_{\mu\gamma}g_{\nu\beta}] \tag{45}$$

and

$$\bar{J}_{15} = \frac{1}{p_1^2 - \xi m^2} \frac{p_1^\alpha p_1^\beta}{m^2} \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}]$$
$$m[2g_{\mu\nu}g_{\beta\gamma} - g_{\mu\beta}g_{\nu\gamma} - g_{\mu\gamma}g_{\nu\beta}]. \tag{46}$$

4. Cancellation of the most divergent terms

It is interesting to consider the large-k behavior of the forty quantities give by (7) to (46) of Sec. 3. All twenty six I's behave as $(k^2)^{-2}$ for large k with k_1 and k_2 fixed. Thus each of these twenty six I's leads to a logarithmically divergent integral or a convergent integral when integrated with respect to the four-momentum k.

This behavior is also true for J_1, $\bar{J}_1$, and J_{11}, which are equal to I_1, $\bar{I}_1$, and I_{11} respectively. But the behavior is different for the other eleven J's. More precisely, for large k with k_1 and k_2 fixed,

$$J_2,\ \bar{J}_2,\ J_3,\ \bar{J}_3,\ J_4,\ \bar{J}_4,\ J_{15},\ \text{and}\ \bar{J}_{15} \tag{47}$$

behave as $(k^2)^{-1}$, while

$$J_5,\ \bar{J}_5,\ \text{and}\ J_{12} \tag{48}$$

behave as $(k^2)^0$. In other words, when integrated with respect to the four-momentum k, each of the J's listed in (47) leads to a quadratically divergent integral, while each listed in (48) a quatically divergent integral. In this sense, the three J's in (48) are conveniently referred to as the "most divergent terms".

In the sum of these three terms, the $(k^2)^0$ pieces must cancel each other. In this Sec. 4, this cancellation is to be carried out explicitly. Thus this sum behaves like $(k^2)^{-1}$, which is to be added to the eight J's listed in (47). This sum of the eleven J's of (47) and (48) is to be studied in the next section.

The cancellation of the $(k^2)^0$ terms is to be described in some detail here, because the way for this cancellation to occur will also play an important role in the next section. With (5) and (6), rewrite the J_5 and $\bar{J}_5$ of (24) and (26) as

$$J_5 = \frac{1}{p_1^2 - \xi m^2} \frac{p_1^\alpha}{m^2} \frac{1}{p_2^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi) p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}] \frac{1}{p_3^2 - \xi m^2} \frac{p_{3\alpha} p_3^\gamma}{m^2} \\ m(p_2^2 g_{\mu\rho} - p_{2\mu} p_{2\rho}) \\ [(p_3 - k_2)_\sigma g_{\gamma\nu} + (p_2 + k_2)_\gamma g_{\nu\sigma} + (-p_2 - p_3)_\nu g_{\sigma\gamma}] \tag{49}$$

and

$$\bar{J}_5 = \frac{1}{p_3^2 - \xi m^2} \frac{p_3^\alpha}{m^2} \frac{1}{p_2'^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi) p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}] \frac{1}{p_1^2 - \xi m^2} \frac{p_{1\alpha} p_1^\gamma}{m^2} \\ m(p_2'^2 g_{\mu\rho} - p'_{2\mu} p'_{2\rho}) \\ [(p_1 + k_2)_\sigma g_{\gamma\nu} + (p'_2 - k_2)_\gamma g_{\nu\sigma} + (-p_1 - p'_2)_\nu g_{\sigma\gamma}]. \tag{50}$$

Let J_5 be split into the sum of three terms as follows

$$p_2^2 g_{\mu\rho} - p_{2\mu} p_{2\rho} = [(p_2^2 - m^2) g_{\mu\rho} + \frac{1-\xi}{\xi} p_{2\mu} p_{2\rho}] + m^2 g_{\mu\rho} - \frac{1}{\xi} p_{2\mu} p_{2\rho}, \tag{51}$$

then

$$J_5 = J_5^{(1)} + J_5^{(2)} + J_5^{(3)}, \tag{52}$$

where

$$J_5^{(1)} = \frac{1}{p_1^2 - \xi m^2} \frac{p_1^\alpha}{m^2} \frac{1}{p_3^2 - \xi m^2} \frac{p_{3\alpha} p_3^\gamma}{m^2} m(-g_\mu^\sigma) \\ [(p_3 - k_2)_\sigma g_{\gamma\nu} + (p_2 + k_2)_\gamma g_{\nu\sigma} + (-p_2 - p_3)_\nu g_{\sigma\gamma}], \tag{53}$$

$$J_5^{(2)} = \frac{1}{p_1^2 - \xi m^2} \frac{p_1^\alpha}{m^2} \frac{1}{p_2^2 - m^2} [-g^{\rho\sigma} + \frac{(1-\xi) p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}] \frac{1}{p_3^2 - \xi m^2} \frac{p_{3\alpha} p_3^\gamma}{m^2} \\ m(m^2 g_{\mu\rho})[(p_3 - k_2)_\sigma g_{\gamma\nu} + (p_2 + k_2)_\gamma g_{\nu\sigma} + (-p_2 - p_3)_\nu g_{\sigma\gamma}], \tag{54}$$

and

$$J_5^{(3)} = \frac{1}{p_1^2 - \xi m^2} \frac{p_1^\alpha}{m^2} \frac{1}{p_2^2 - m^2} [-g^{\rho\sigma} + \frac{(1-\xi) p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}] \frac{1}{p_3^2 - \xi m^2} \frac{p_{3\alpha} p_3^\gamma}{m^2} \\ m(-\frac{1}{\xi} p_{2\mu} p_{2\rho})[(p_3 - k_2)_\sigma g_{\gamma\nu} + (p_2 + k_2)_\gamma g_{\nu\sigma} + (-p_2 - p_3)_\nu g_{\sigma\gamma}]. \tag{55}$$

It is immediately seen using Eqs. (5) and (6) that

$$J_5^{(3)} = 0. \tag{56}$$

Similarly,

$$\bar{J}_5 = \bar{J}_5^{(1)} + \bar{J}_5^{(2)}, \tag{57}$$

where

$$\bar{J}_5^{(1)} = \frac{1}{p_3^2 - \xi m^2} \frac{p_3^\alpha}{m^2} \frac{1}{p_1^2 - \xi m^2} \frac{p_{1\alpha} p_1^\gamma}{m^2} m(-g_\mu^\sigma) \\ [(p_1 + k_2)_\sigma g_{\gamma\nu} + (p_2' - k_2)_\gamma g_{\nu\sigma} + (-p_1 - p_2')_\nu g_{\sigma\gamma}] \tag{58}$$

and

$$\bar{J}_5^{(2)} = \frac{1}{p_3^2 - \xi m^2} \frac{p_3^\alpha}{m^2} \frac{1}{p_2'^2 - m^2} [-g^{\rho\sigma} + \frac{(1-\xi) p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}] \frac{1}{p_1^2 - \xi m^2} \frac{p_{1\alpha} p_1^\gamma}{m^2} \\ m(m^2 g_{\mu\rho})[(p_1 + k_2)_\sigma g_{\gamma\nu} + (p_2' - k_2)_\gamma g_{\nu\sigma} + (-p_1 - p_2')_\nu g_{\sigma\gamma}]. \tag{59}$$

These splittings of J_5 and $\bar{J}_5$ accomplish the following:
First, again using Eqs. (6), it is seen that

$$J_5^{(1)} + \bar{J}_5^{(1)} + J_{12} = 0, \tag{60}$$

where J_{12} is given by Eq. (40). It therefore follows that

$$J_5 + \bar{J}_5 + J_{12} = J_5^{(2)} + \bar{J}_5^{(2)}. \tag{61}$$

Secondly, as given by Eqs. (54) and (59) respectively, both $J_5^{(2)}$ and $\bar{J}_5^{(2)}$ behave as $(k^2)^{-1}$ for large k. In other words, Eq. (61) gives explicitly the cancellation of the $(k^2)^0$ terms between the J_5, $\bar{J}_5$, and J_{12} of (48).

5. Cancellation of the next most divergent terms

From (47) and (61), it is seen that, for large k with k_1, and k_2 fixed,

$$J_2,\ \bar{J}_2,\ J_3,\ \bar{J}_3,\ J_4,\ \bar{J}_4,\ J_{15},\ \bar{J}_{15},\ J_5^{(2)},\ \text{and}\ \bar{J}_5^{(2)} \tag{62}$$

behave as $(k^2)^{-1}$. In other words, when integrated with respect to the variable k, each of these ten J's leads to a quadratically divergent integral. The present section describes the way these $(k^2)^{-1}$ pieces cancel each other.

Similar to (49) and (50), these ten J's, except J_{15} and $\bar{J}_{15}$, can be rewritten as follows; they are respectively from (12), (14), (16), (18), (20), (22), (54), and (59):

$$J_2 = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi) p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} \\ \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi) p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}] \\ m(p_1^2 g_{\beta\mu} - p_{1\beta} p_{1\mu})(p_3^2 g_{\gamma\nu} - p_{3\gamma} p_{3\nu}) \tag{63}$$

$$\bar{J}_2 = \frac{1}{p_3^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi) p_3^\alpha p_3^\beta}{p_3^2 - \xi m^2}] \frac{1}{p_2'^2 - \xi m^2} \frac{1}{m^2} \\ \frac{1}{p_1^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi) p_{1\alpha} p_1^\gamma}{p_1^2 - \xi m^2}] \\ m(p_3^2 g_{\beta\mu} - p_{3\beta} p_{3\mu})(p_1^2 g_{\gamma\nu} - p_{1\gamma} p_{1\nu}) \tag{64}$$

$$J_3 = \frac{1}{p_1^2 - \xi m^2}\,\frac{p_1^\alpha}{m^2}\,\frac{1}{p_2^2 - m^2}\,[-g^{\rho\sigma} + \frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}]$$
$$\frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{3\alpha}p_3^\gamma}{p_3^2 - \xi m^2}]$$
$$m(p_2^2 g_{\mu\rho} - p_{2\mu}p_{2\rho})$$
$$[(p_3 - k_2)_\sigma g_{\gamma\nu} + (p_2 + k_2)_\gamma g_{\nu\sigma} + (-p_2 - p_3)_\nu g_{\sigma\gamma}] \tag{65}$$

$$\bar{J}_3 = \frac{1}{p_3^2 - \xi m^2}\,\frac{p_3^\alpha}{m^2}\,\frac{1}{p_2'^2 - m^2}\,[-g^{\rho\sigma} + \frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}]$$
$$\frac{1}{p_1^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi)p_{1\alpha}p_1^\gamma}{p_1^2 - \xi m^2}]$$
$$m(p_2'^2 g_{\mu\rho} - p'_{2\mu}p'_{2\rho})$$
$$[(p_1 + k_2)_\sigma g_{\gamma\nu} + (p'_2 - k_2)_\gamma g_{\nu\sigma} + (-p_1 - p'_2)_\nu g_{\sigma\gamma}] \tag{66}$$

$$J_4 = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}]$$
$$\frac{1}{p_2^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}]\,\frac{1}{p_3^2 - \xi m^2}\,\frac{p_{3\alpha}}{m^2}$$
$$m[(p_1 + k_1)_\rho g_{\beta\mu} + (p_2 - k_1)_\beta g_{\mu\rho} + (-p_1 - p_2)_\mu g_{\rho\beta}](p_2^2 g_{\nu\sigma} - p_{2\nu}p_{2\sigma}) \tag{67}$$

$$\bar{J}_4 = \frac{1}{p_3^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi)p_3^\alpha p_3^\beta}{p_3^2 - \xi m^2}]$$
$$\frac{1}{p_2'^2 - m^2}[-g^{\rho\sigma} + \frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}]\,\frac{1}{p_1^2 - \xi m^2}\,\frac{p_{1\alpha}}{m^2}$$
$$m[(p_3 - k_1)_\rho g_{\beta\mu} + (p'_2 + k_1)_\beta g_{\mu\rho} + (-p'_2 - p_3)_\mu g_{\mu\beta}](p_2'^2 g_{\nu\sigma} - p'_{2\nu}p'_{2\sigma}) \tag{68}$$

$$J_5^{(2)} = \frac{1}{p_1^2 - \xi m^2}\,\frac{p_1^\alpha}{m^2}\,\frac{1}{p_2^2 - m^2}\,[-g^{\rho\sigma} + \frac{(1-\xi)p_2^\rho p_2^\sigma}{p_2^2 - \xi m^2}]\,\frac{1}{p_3^2 - \xi m^2}\,\frac{p_{3\alpha}}{m^2}$$
$$m(m^2 g_{\mu\rho})(p_2^2 g_{\nu\sigma} - p_{2\nu}p_{2\sigma}) \tag{69}$$

and

$$\bar{J}_5^{(2)} = \frac{1}{p_3^2 - \xi m^2}\,\frac{p_3^\alpha}{m^2}\,\frac{1}{p_2'^2 - m^2}\,[-g^{\rho\sigma} + \frac{(1-\xi)p_2'^\rho p_2'^\sigma}{p_2'^2 - \xi m^2}]\,\frac{1}{p_1^2 - \xi m^2}\,\frac{p_{1\alpha}}{m^2}$$
$$m(m^2 g_{\mu\rho})(p_2'^2 g_{\nu\sigma} - p'_{2\nu}p'_{2\sigma}). \tag{70}$$

On the right-hand sides of these eight Eqs. (63)–(70), the combination $p^2 g_{\alpha\beta} - p_\alpha p_\beta$ appears with various p, α, and β, indeed twice each for J_2 and $\bar{J}_2$ while once in the other six cases. Therefore the splitting (51), which has played a central role in the preceding section, can again be used for these eight right-hand sides of Eq. (63)–(70).

Consider first the J_2 of Eq. (63); since this combination $p^2 g_{\alpha\beta} - p_\alpha p_\beta$ appears twice on the right-hand side with p_1 and p_3, Eq. (51) should be used twice leading to nine terms

$$J_2 = J_2^{(11)} + J_2^{(12)} + J_2^{(13)} + J_2^{(21)} + J_2^{(22)} + J_2^{(23)} + J_2^{(31)} + J_2^{(32)} + J_2^{(33)}, \tag{71}$$

where

$$J_2^{(11)} = \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} m(-g_\mu^\alpha)(-g_{\alpha\nu}) \tag{72}$$

$$J_2^{(12)} = \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi) p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}]\, m(-g_\mu^\alpha)\, m^2 g_{\gamma\nu} \tag{73}$$

$$J_2^{(13)} = \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi) p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}]\, m(-g_\mu^\alpha)\, (-\frac{1}{\xi} p_{3\gamma} p_{3\nu}) \tag{74}$$

$$J_2^{(21)} = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi) p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} m(m^2 g_{\beta\mu})(-g_{\alpha\nu}) \tag{75}$$

$$J_2^{(22)} = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi) p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi) p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}]\, m(m^2 g_{\beta\mu})\, m^2 g_{\gamma\nu} \tag{76}$$

$$J_2^{(23)} = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi) p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi) p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}]\, m(m^2 g_{\beta\mu})\, (-\frac{1}{\xi} p_{3\gamma} p_{3\nu}) \tag{77}$$

$$J_2^{(31)} = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi) p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} m(-\frac{1}{\xi} p_{1\beta} p_{1\mu})(-g_{\alpha\nu}) \tag{78}$$

$$J_2^{(32)} = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi) p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi) p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}]\, m(-\frac{1}{\xi} p_{1\beta} p_{1\mu})\, m^2 g_{\gamma\nu} \tag{79}$$

and

$$J_2^{(33)} = \frac{1}{p_1^2 - m^2}[-g^{\alpha\beta} + \frac{(1-\xi) p_1^\alpha p_1^\beta}{p_1^2 - \xi m^2}] \frac{1}{p_2^2 - \xi m^2} \frac{1}{m^2} \frac{1}{p_3^2 - m^2}[-g_\alpha^\gamma + \frac{(1-\xi) p_{3\alpha} p_3^\gamma}{p_3^2 - \xi m^2}]\, m(-\frac{1}{\xi} p_{1\beta} p_{1\mu})\, (-\frac{1}{\xi} p_{3\gamma} p_{3\nu}). \tag{80}$$

In this way, the ten J's of (62) are split up into $2 \times 9 + 6 \times 3 + 2 = 38$ terms. Among these 38 terms, there are cancellations similar to Eq. (60) of the preceding section.

What is found is that all these "quadratically divergent" terms cancel each other, and therefore, in the sum of the fourteen J's, the asymptotic behavior is $(k^2)^{-2}$ for large k with k_1 and k_2 fixed. This asymptotic behavior is the same as the sum of the twenty six I's in the R_ξ gauge.

6. Difference between the R_ξ gauge and the unitary gauge

In terms of the integrands of the Feynman diagrams, the difference between the R_ξ gauge and the unitary gauge is given by

$$\begin{aligned}\delta_{\mu\nu} = &\text{sum of the fourteen non-zero } J\text{'s} \\ &- \text{sum of the twenty six non-zero } I\text{'s},\end{aligned} \tag{81}$$

where the forty I's and J's are given explicitly by the forty equations of Sec. 3. Because of the cancellations shown in the preceding two sections, this $\delta_{\mu\nu}$ behaves as $(k^2)^{-2}$ for large k with k_1 and k_2 fixed.

Although somewhat lengthy, it is completely straightforward to simplify this expression (81) for the difference $\delta_{\mu\nu}$, and the resulting formula is:

$$\begin{aligned}\delta_{\mu\nu} = -\frac{1}{m}[(&\frac{1}{p_1^2-\xi m^2}+\frac{1}{p_3^2-\xi m^2}-\frac{1}{p_2^2-\xi m^2}-\frac{1}{p_2'^2-\xi m^2})g_{\mu\nu} \\ &+\frac{1}{p_1^2-\xi m^2}\,\frac{1}{p_2^2-\xi m^2}p_{2\mu}k_{1\nu} \\ &-\frac{1}{p_2^2-\xi m^2}\,\frac{1}{p_3^2-\xi m^2}k_{2\mu}p_{2\nu} \\ &-\frac{1}{p_2'^2-\xi m^2}\,\frac{1}{p_3^2-\xi m^2}p'_{2\mu}k_{1\nu} \\ &+\frac{1}{p_1^2-\xi m^2}\,\frac{1}{p_2'^2-\xi m^2}k_{2\mu}p'_{2\nu}].\end{aligned} \tag{82}$$

As expected, the right-hand side of this Eq. (82) has a great deal of symmetry. Specifically, it has the following properties:

(a) Consider the first term; by Eqs. (6), there are the following relations:

$$\frac{1}{p_2'^2-\xi m^2}=\left.\frac{1}{p_1^2-\xi m^2}\right|_{k\to k-k_2},$$

$$\frac{1}{p_3^2-\xi m^2}=\left.\frac{1}{p_2^2-\xi m^2}\right|_{k\to k-k_2},$$

$$\frac{1}{p_2'^2-\xi m^2}=\left.\frac{1}{p_3^2-\xi m^2}\right|_{k\to k+k_1},$$

and

$$\frac{1}{p_1^2-\xi m^2}=\left.\frac{1}{p_2^2-\xi m^2}\right|_{k\to k+k_1}. \tag{83}$$

From these four Eqs. (83), it follows immediately that

$$\frac{1}{p_2'^2-\xi m^2}-\frac{1}{p_3^2-\xi m^2}=\left[\frac{1}{p_1^2-\xi m^2}-\frac{1}{p_2^2-\xi m^2}\right]_{k\to k-k_2} \tag{84}$$

and

$$\frac{1}{p_2'^2-\xi m^2}-\frac{1}{p_1^2-\xi m^2}=[\frac{1}{p_3^2-\xi m^2}-\frac{1}{p_2^2-\xi m^2}]_{k\to k+k_1} \tag{85}$$

The advantage of rewriting Eqs. (83) as Eq. (84) and Eq. (85) is that, when integrated over the four-momentum k, both expressions are *linearly* divergent instead of being quadratically divergent. Since shifting the variable of integration for a linearly divergent integral leads to an additional term that is finite, it follows Eq. (84) that

$$\int d^4k[(\frac{1}{p_1^2-\xi m^2}-\frac{1}{p_2^2-\xi m^2})-(\frac{1}{p_2'^2-\xi m^2}-\frac{1}{p_3^2-\xi m^2})]=A_1^\alpha k_{2\alpha}, \tag{86}$$

where A_1^α is a non-zero, finite four-vector. Similarly, it follows from Eq. (85) that

$$\int d^4k[(\frac{1}{p_3^2-\xi m^2}-\frac{1}{p_2^2-\xi m^2})-(\frac{1}{p_2'^2-\xi m^2}-\frac{1}{p_1^2-\xi m^2})]=A_2^\alpha k_{1\alpha}, \tag{87}$$

where A_2^α is another non-zero, finite four-vector.

But the left-hand side of Eq. (87) is the same as the left-hand side of Eq. (86). It therefore follows from Eqs. (86) and (87) that

$$\int d^4k[\frac{1}{p_1^2-\xi m^2}+\frac{1}{p_3^2-\xi m^2}-\frac{1}{p_2^2-\xi m^2}-\frac{1}{p_2'^2-\xi m^2}]=A_0(k_1.k_2), \tag{88}$$

where A_0 is merely a non-zero, finite number. Note that this number A_0 must be independent of ξ.

(b) The other four terms on the right-hand side of Eq. (82) can be treated in an entirely similar matter. In fact, due to the appearance of either $k_{1\nu}$ or $k_{2\mu}$ in each of these four terms, the necessary considerations are simpler.

Similar to Eqs. (83), it again follows from Eqs. (6) that

$$\frac{1}{p_2'^2-\xi m^2}\frac{1}{p_3^2-\xi m^2}p'_{2\mu}k_{1\nu}=\frac{1}{p_1^2-\xi m^2}\frac{1}{p_2^2-\xi m^2}p_{2\mu}k_{1\nu}\Big|_{k\to k-k_2} \tag{89}$$

and

$$\frac{1}{p_1^2-\xi m^2}\frac{1}{p_2'^2-\xi m^2}k_{2\mu}p'_{2\nu}=\frac{1}{p_2^2-\xi m^2}\frac{1}{p_3^2-\xi m^2}k_{2\mu}p_{2\nu}\Big|_{k\to k+k_1}. \tag{90}$$

This time, when integrated over the four-momentum k, both expressions in Eq. (89) and Eq. (90) are already linearly divergent. Therefore, similar to Eq. (86) and Eq. (87), these Eq. (89) and Eq. (90) lead to, respectively

$$\int d^4k[\frac{1}{p_1^2-\xi m^2}\frac{1}{p_2^2-\xi m^2}p_{2\mu}k_{1\nu}-\frac{1}{p_2'^2-\xi m^2}\frac{1}{p_3^2-\xi m^2}p'_{2\mu}k_{1\nu}]=A_3k_{2\mu}k_{1\nu} \tag{91}$$

and

$$\int d^4k[\frac{1}{p_2^2-\xi m^2}\frac{1}{p_3^2-\xi m^2}k_{2\mu}p_{2\nu}-\frac{1}{p_1^2-\xi m^2}\frac{1}{p_2'^2-\xi m^2}k_{2\mu}p'_{2\nu}]=A_4k_{2\mu}k_{1\nu}, \tag{92}$$

where A_3 and A_4 are again merely non-zero, finite numbers, which are independent of ξ.

(c) Because of the above results (88), (91), and (92), we are now in a position to study the difference between the matrix elements, calculated in the R_ξ gauge and the unitary gauge, for the Higgs decay process $H \to \gamma\gamma$ through one W loop. Leaving out an overall factor of ie^2g, this difference between these two matrix elements is given by

$$\Delta_{\mu\nu} = \frac{1}{(2\pi)^4} \int d^4k \delta_{\mu\nu}, \tag{93}$$

where $\delta_{\mu\nu}$ is given explicitly by Eq. (82).

Since this $\Delta_{\mu\nu}$ satisfies

$$\Delta_{\mu\nu} k_1^\mu = \Delta_{\mu\nu} k_2^\nu = 0, \tag{94}$$

it must take the form

$$\Delta_{\mu\nu} = C[(k_1.k_2)g_{\mu\nu} - k_{1\nu}k_{2\mu}]. \tag{95}$$

Furthermore, it follows from Eqs. (82) and (88) that the C here is a non-zero, finite constant independent of ξ.

This Eq. (95) is the main result of the present paper.

In the remainder of this Section 6, some aspects of this main result are to be discussed in more detail.

(d) Since this main result Eq. (95) gives a finite difference between the matrix element calculated for the R_ξ gauge and that with the unitary gauge, there is neither necessity nor justification to alter the physical space–time dimension of $3+1$. This is fortunate because it is always difficult to show that any particular matrix element is a continuous function of the dimension.

(e) The present result as given by Eq. (82) has been obtained by a fairly lengthy although elementary calculation. What does this calculation really accomplish?

Because of the Feynman rules for the R_ξ gauge and the unitary gauge together with Eq. (94), the right-hand side of Eq. (81) must necessarily take this form. For example, arguments can be given that the first term there, the one proportional to $g_{\mu\nu}$, must be this particular combination of four terms, each with just one denominator. Indeed, this particular combination of four terms has been seen much before the present considerations have been carried out. Once this first term is obtained, the others follow from Eq. (94).

What the present calculation has really accomplished is to show that this right-hand side of Eq. (82) is not zero, a result that does not follow from any simple argument.

(f) Historically, the calculation of the Higgs decay $H \to \gamma\gamma$ was first carried out by Ellis, Gaillard, and Nanopoulis forty years ago using the R_ξ gauge [9]. Their result was reproduced and generalized shortly thereafter.

The corresponding calculation in the unitary gauge was carried out much later by Gastmans, Wu, and Wu [10]. This result with the unitary gauge was later verified by Christova and Todorov using an unsubtracted dispassion relation [11]. It has been puzzling why these two results, one with the R_ξ gauge and the other with the unitary gauge, do not agree with other; the main result Eq. (95) of the present paper shows, through a detailed analysis, why they should not agree.

(g) It is interesting, and important, to delve into the underlying reason in more detail why, as found here and contrary to expectation, the R_ξ gauge and the unitary gauge do not give the same answer for this Higgs decay $H \to \gamma\gamma$ through one W loop.

As already pointed out in the Introduction, the unitary gauge is the formal limit, as $\xi \to \infty$, of the R_ξ gauge. How does this formal property manifest itself in the quantities $\delta_{\mu\nu}$ and $\Delta_{\mu\nu}$ of Eqs. (81) and (93)?

It follows immediately from Eq. (82) that

$$\lim_{\xi\to\infty} \delta_{\mu\nu} = 0 \tag{96}$$

for every values of k, k_1 and k_2. The important point here is that this limiting formula (96) is *not uniform* in k. This non-uniformity has the following consequence.

While it follows from (96) that

$$\frac{1}{(2\pi)^4}\int d^4k \lim_{\xi\to\infty} \delta_{\mu\nu} = 0, \tag{97}$$

reversing the integration over k and the limiting process $\xi \to \infty$ gives instead, from Eq. (95),

$$\lim_{\xi\to\infty} \frac{1}{(2\pi)^4}\int d^4k\delta_{\mu\nu} = C[(k_1.k_2)g_{\mu\nu} - k_{1\nu}k_{2\mu}] \tag{98}$$

with $C \neq 0$. This shows explicitly that

$$\frac{1}{(2\pi)^4}\int d^4k \lim_{\xi\to\infty} \delta_{\mu\nu} \neq \lim_{\xi\to\infty} \frac{1}{(2\pi)^4}\int d^4k\delta_{\mu\nu}. \tag{99}$$

This means that the two operations

$$\lim_{\xi\to\infty} \tag{100}$$

and

$$\frac{1}{(2\pi)^4}\int d^4k \tag{101}$$

do *not* commute.

What we have shown here is that the underlying reason for the disagreement between the R_ξ gauge and the unitary gauge is very subtle for the present case. It is due to the failure of the two operations (100) and (101), one a limiting process to go from the R_ξ gauge to the unitary gauge and the other the integration over the loop momentum, to commute.

(h) The matrix element for the decay $H \to \gamma\gamma$ through one W loop is given by

$$-\frac{e^2 g}{8\pi^2 m}[k_{2\mu}k_{1\nu} - g_{\mu\nu}(k_1.k_2)][3\tau^{-1} + 3(2\tau^{-1} - \tau^{-2})f(\tau)] \tag{102}$$

when the unitary gauge is used, and by

$$-\frac{e^2 g}{8\pi^2 m}[k_{2\mu}k_{1\nu} - g_{\mu\nu}(k_1.k_2)][2 + 3\tau^{-1} + 3(2\tau^{-1} - \tau^{-2})f(\tau)] \tag{103}$$

when the R_ξ gauge is used [10]. In (102) and (103), the quantity τ is

$$\tau = \frac{m_H^2}{4m^2} \tag{104}$$

and

$$f(\tau) = \begin{cases} [\sin^{-1}\sqrt{\tau}]^2 & \text{for } \tau \leq 1, \\ -\frac{1}{4}[\ln\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi]^2 & \text{for } \tau > 1. \end{cases} \tag{105}$$

The difference between (102) and (103) is just due to the $\Delta_{\mu\nu}$ as given by Eq. (95).

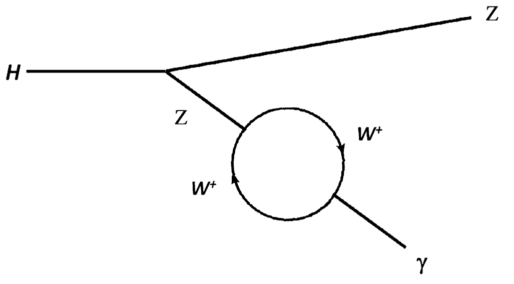

Fig. 16. A possible additional Feynman diagram for the decay (106) of $H \to Z\gamma$. In the standard model, the matrix element for this diagram is zero.

7. Higgs decay $H \to Z\gamma$

For the Higgs particle, there is another decay process that is similar to that of (3), namely,

$$H \to Z\gamma \tag{106}$$

through one W loop.

The Feynman diagrams for this decay (106) are identical to those shown in Fig. 1 to Fig. 15 provided that one of the outgoing photons, say the one with the four-momentum k_1, is replaced by the Z. With this replacement, the number of diagrams for the R_ξ gauge remains twenty six, but the corresponding number of diagrams in the unitary gauge increases by two, from fourteen to sixteen, because J_7 and $\bar{J}_7$ of Fig. 7 are no longer zero.

The matrix element for this decay (106) through one W loop depends on the mass m_Z of the Z boson. In the standard model [1], the value of this m_Z is an independent parameter. For both the R_ξ gauge and the unitary gauge, the Feynman rule for any vertex involving this Z boson, in the limit of $m_Z \to 0$, agrees with that for the same vertex with the Z replaced by a photon. Therefore, except for an overall constant,

$$\begin{aligned} &\text{matrix element for the decay (106)} \,|_{m_Z \to 0} \\ &= \text{corresponding matrix element for the decay (3).} \end{aligned} \tag{107}$$

In writing down this Eq. (107), it should be noted that the matrix element for the diagram of Fig. 16 is zero.

As seen from the explicit calculation carried out in the present paper, the matrix elements for the decay , as obtained for the R_ξ gauge and the unitary gauge, do not agree with each other, and in fact differ by a non-zero constant as given by Eq. (95). It therefore follows rigorously from Eq. (107) that

$$\begin{aligned} &\text{matrix element for the decay (106) in the } R_\xi \text{ gauge} \\ &\neq \text{matrix element for the decay (106) in the unitary gauge.} \end{aligned} \tag{108}$$

It is in principle possible that there is a specific value of m_Z for which these matrix elements from the R_ξ gauge and the unitary gauge agree. This is however not the case for the one W loop diagrams.

There are therefore two known cases, the Higgs decays (3) and (106), where the R_ξ gauge and the unitary gauge lead to different values for the matrix element.

8. Discussions and conclusion

It has been shown that in the standard model, for the decay of the Higgs particle into two photons through one W loop, the resulting matrix element is different depending on whether the R_ξ gauge or the unitary gauge is used. The difference in these two matrix element is calculated directly, and this calculation is lengthy but completely elementary.

Since this rate of the Higgs particle decay into two photons is well defined experimentally, there cannot be two different correct predictions from the same standard model [1]. Thus there are the following two distinct possibilities:

the prediction using the unitary gauge is correct but the one using the R_ξ gauge is wrong; or

the prediction using the R_ξ is correct but the one using the unitary gauge is wrong.

[It is in principle possible that both of these predictions are wrong for the standard model, but this seems unlikely.]

In order to ascertain which one of these two possibilities is the right one, it is necessary to gain a much deeper understanding of the standard model. Such a deeper understanding does not exist yet; instead, we raise a number of questions, listed below, to be answered before such a deeper understanding can be achieved.

(a) In view of the explicit verification that the operations of taking a limit (100) and integration over the internal momentum (101) do not commute, the most important question to be answered is the following.

Question 1: How can the development of quantum field theory be carried out without any implicit assumption that various operations commute?

In the context of the standard model, this question is to be raised for the derivation of the Feynman rules for the R_ξ gauge as well as those for the unitary gauge.

(b) Numerous aspects of quantum field theories need to be re-evaluated; here is one of many examples. In the usual derivation of the Feynman rules for the R_ξ gauge, the first step is to add a gauge–fixing term to the Lagrangian density.

Question 2: Is the addition of such a gauge-fixing term justified?

It was over eighty years ago when Fermi [12] used the addition of such a gauge-fixing term in the case of quantum electrodynamics. What Fermi did was completely justified, but, to the best knowledge of the authors, such a justification has never been successfully extended to the case of the Yang–Mills non-Abelian gauge theory [2]. It would be a most important step forward to find out whether the addition of such a gauge-fixing term is correct or not in the case of the standard model.

(c) For a number of years, the Higgs decay $H \to \gamma\gamma$ was the only known process where the matrix element is different when calculated in the R_ξ gauge and the unitary gauge. There are now two known cases, as give by (3) and (106), where there is a difference in the matrix elements as calculated these two ways, namely,

i) $H \to \gamma\gamma$ through one W loop, and

ii) $H \to Z\gamma$ through one W loop.

In spite of the similarity between these two processes, there are major differences in the calculations.

Question 3: How can the considerations in the present paper be generalized from the process of i) to that of ii)?

Let the reader be assured that this generalization is far from being straightforward.

The process of ii) serves another useful purpose: whenever an argument is proposed in favor of either the unitary gauge or the R_ξ gauge for the process $H \to \gamma\gamma$ of i), it is essential to check whether this argument makes equally good sense for the process $H \to Z\gamma$ of ii).

(d) As already emphasized in the Introduction, the important feature of the diagrams that can lead to different results for the R_ξ gauge and the unitary gauge is the presence of internal W and/or Z lines, the reason being the qualitative difference between these propagators in the two gauges as given by (1) and (2). The presence or absence of any external line for the Higgs particle does not seem to play any important role; nevertheless, the two known cases (3) and (106) are both for the decay of the Higgs particle.

Question 4: What are the other processes beyond Higgs decay where their matrix elements are different when calculated in the unitary gauge and the R_ξ gauge?

Question 5: How can such processes be characterized where their matrix elements are different when calculated in the unitary gauge and the R_ξ gauge?

(e) All the considerations have been limited to one-loop diagrams so far. This limitation should of course be removed.

Question 6: How can the above considerations, including the five questions listed above, be generalized first to the case of two loops and eventually more loops?

This question is of course very difficult to answer even partially, and is therefore not for the immediate future.

Yang–Mills non-Abelian gauge theory in general and the standard model in particular are much more subtle than what has been generally realized.

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