HADRONIC τ DECAYS AND QCD

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Abstract

We present new results for the τ hadronic spectral functions analysis using data accumulated by the ALEPH detector at LEP during the years 1991–94. The vector and the axial-vector spectral functions are determined from their respective unfolded, *i.e.*, physical invariant mass spectra. The τ vector and axial-vector hadronic widths and certain spectral moments are exploited to measure α_s and non-perturbative contributions at the τ mass scale. The best, and experimentally and theoretically most robust, determination of $\alpha_s(M_\tau)$ is obtained from the inclusive $(V + A)$ fit that yields $\alpha_s(M_\tau) = 0.349 \pm 0.018$ giving $\alpha_s(M_Z) = 0.1212 \pm 0.0022$ after the evolution to the mass of the Z boson. The approach of the Operator Product Expansion (OPE) is tested experimentally by means of an evolution of the τ hadronic width to masses smaller than the τ mass.

1   Introduction

Spectral functions of hadronic τ decays are the key objects for various interesting studies concerning resonance structure analysis, the *Conserved Vector Current* hypothesis (CVC) and QCD tests involving strong sum rules and the measurement of α_s. The complete τ spectral function analysis is the subject of a thesis provided in Ref. [1].

In this article we deal with new data of the spectral functions from non-strange, vector and axial-vector hadronic τ decays measured by the ALEPH Collaboration. As a main topic of Quantum Chromodynamics, we determine the running strong coupling constant, α_s, at the scale of the τ mass. Such measurements were carried out for the first time time by ALEPH [2] and CLEO [3].

Tests of Quantum Chromodynamics and the precise measurement of the strong coupling constant α_s at the τ mass scale have been the subject of engaged discussions about theoretical and experimental implications, accompanied by a considerable number of interesting publications (see, *e.g.*, [4, 5, 6, 7, 8]).

2   Spectral Functions of Vector and Axial Currents

The measurement of the non-strange τ vector (axial-vector) current spectral functions requires the selection and identification of τ decay modes with a G-parity G=+1 (G=−1), *i.e.*, hadronic channels with an even (odd) number of neutral *or* charged pions. The isovector spectral function $v_{1,V-}$ ($a_{1,A-}$) of a vector (axial-vector) τ decay channel $V^- \nu_\tau$ ($A^- \nu_\tau$) is obtained by dividing the normalized invariant mass-squared distribution $(1/N_{V/A-})(dN_{V/A-}/ds)$ for a given hadronic mass $\sqrt{s}$ by the appropriate kinematic factor[1]:

$$v_{1,V-}/a_{1,A-}(s) \;=\; \frac{M_\tau^2}{6\,|V_{ud}|^2\,S_{\mathrm{EW}}}\;\frac{B(\tau^- \to V^-/A^- \nu_\tau)}{B(\tau^- \to e^- \nu_\tau \bar\nu_e)}\;\frac{dN_{V/A}}{N_{V/A}\,ds}\left[\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+\frac{2s}{M_\tau^2}\right)\right]^{-1}, \quad (1)$$

where $M_\tau = 1776.96^{+0.31}_{-0.27}$, $|V_{ud}| = 0.9752 \pm 0.0007$ denotes the CKM weak mixing matrix element [9] and $S_{\mathrm{EW}} = 1.0194 \pm 0.0040$ accounts for electroweak second order corrections [10]. The spectral functions are normalized by the ratio of the respective vector/axial-vector branching fraction $B(\tau^- \to V^-/A^- \nu_\tau)$ to the branching fraction of the electron channel $B(\tau^- \to e^- \bar\nu_e \nu_\tau) = 17.79 \pm 0.04$ [9], where the latter is additionally constrained via universality from $B(\tau^- \to \mu^- \bar\nu_\mu \nu_\tau)$ and the τ lifetime.

Assuming unitarity (which implies the *optical theorem*) and analyticity to hold, the spectral functions of hadronic τ decays are related via dispersion relations to the imaginary parts of the two-point correlation functions $\Pi^{\mu\nu}_{ij,U}(q) = i \int d^4x \, e^{iqx} \langle 0|T(U^\mu_{ij}(x)U^\nu_{ij}(0)^\dagger)|0\rangle$ of vector ($U \equiv V = \bar\psi_j \gamma^\mu \psi_i$) or axial-vector ($U \equiv A = \bar\psi_j \gamma^\mu \gamma_5 \psi_i$) colour-singlet quark currents in corresponding quantum states (see, *e.g.*, [4, 7]).

3   The Measurement Procedure

The measurement of the spectral functions defined in Eq. (1) requires the determination of the physical invariant mass-squared distribution. The details of the analysis are reported in [11, 12]. A description of the ALEPH detector and its performance is published in [13].

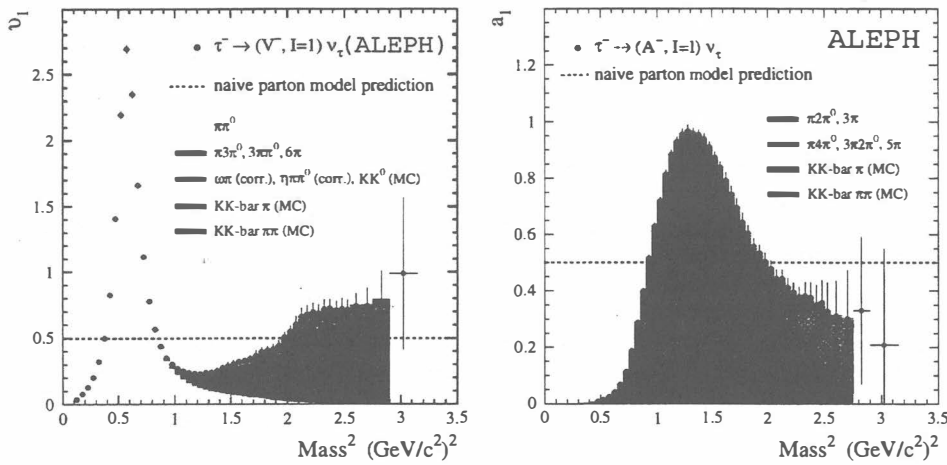[1]throughout this article, charge conjugate states are implied

Figure 1: *Total τ vector (left hand plot) and axial-vector current spectral function (right hand plot). The hatched areas indicate the respective contributions from the τ vector channels. The dashed lines represent the naive parton model predictions, while the QCD corrected predictions at M_τ^2 lie roughly 14% higher.*

The important steps of the measurement procedure are the τ pair selection, the charged particles (electrons, muons and hadrons) identification based on a maximum likelihood method, the photon and π^0 reconstruction using the electromagnetic calorimeter, the classification of the inclusive hadronic τ decay channels (performed according to [14]), the subtraction of τ and non-τ background and the unfolding of the measured mass spectra from detector effects, where the method used is based on the regularised inversion of the detector response matrix [15].

The extensive study of systematic errors affecting the measurement is subdivided into several classes according to their origin, *viz.*, the photon and π^0 reconstruction, the charged track measurement, the unfolding procedure and additional sources (such as Monte Carlo statistics, non-τ background, etc.). Since we use an unfolding procedure based upon a detector response matrix from the Monte Carlo simulation, the reliability of the simulation has to be subjected to detailed studies [11, 12, 14].

The measurement provides the physical invariant mass spectra of the measured τ decay modes including their respective bin-to-bin covariance matrices obtained after the unfolding of the spectra from the statistical errors and the study of systematic uncertainties. The inclusive vector and axial-vector τ decay channels have branching ratios of $B(\tau^- \to V^- \nu_\tau) = (31.71 \pm 0.31)\%$ and $B(\tau^- \to A^- \nu_\tau) = (30.41 \pm 0.32)\%$, respectively [11, 12]. Their spectral functions are shown in Fig 1. In the favourable case of the vector plus axial-vector $(V + A)$ spectral function we do not have to distinguish the current properties of the respective non-strange hadronic τ decay channels. Hence we measure the mixture of all contributing non-strange final states as inclusively as possible which produces a significant improvement in precision especially at higher masses.

4 The Measurement of α_s

Using the analytic property of the vector/axial-vector two-point correlation functions $\Pi_{ij,V/A}^{(J)}$, the non-strange, semileptonic τ decay rates $R_{\tau,V/A}$ can be expressed as contour integrals in the complex s-plane [4]

$$R_{\tau,V/A} = 6\pi i \oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2\left[\left(1 + \frac{2s}{M_\tau^2}\right)\Pi_{ud,V/A}^{(0+1)} - \frac{2s}{M_\tau^2}\Pi_{ud,A}^{(0)}\right]. \tag{2}$$

By means of the short distance OPE, the theoretical prediction of $R_{\tau,V/A}$ separates in the following contributions

$$R_{\tau,V/A} = \frac{3}{2}|V_{ud}|^2 S_{\mathrm{EW}}(1 + \delta_{\mathrm{P}} + \delta_{\mathrm{mass}} + \delta_{\mathrm{NP}}) . \tag{3}$$

The perturbative contribution δ_{P} of the correlators in Eq. (2) is known to next-to-next-to-leading order α_s^3. The leading quark mass correction $\delta_{\mathrm{mass}} \approx -0.05\%$ in Eq. (3) is safely neglected for u and d quarks. For the non-perturbative contribution δ_{NP} we use the SVZ-approach [16] in terms of a power series of M_τ. Long-distance QCD effects are thereby separated into unpredictable, non-perturbative contributions, absorbed in dimensional, phenomenological operators $\langle \mathcal{O}_D \rangle$ and into short distance parts C_D (Wilson Coefficients), calculable within perturbative QCD:

$$\delta_{\mathrm{NP}} \equiv \sum_{D=4,6,8} \delta^{(D)} = \sum_{D=4,6,8} C_D \frac{\langle \mathcal{O}_D \rangle}{(-M_\tau^2)^{D/2}} . \tag{4}$$

The first operator that involves non-perturbative physics appears at $D = 4$ and is linked to the gluon condensate.

Spectral Moments

To detach the measurement from theoretical constraints on the operators $\langle \mathcal{O}_D \rangle$, it is convenient to fit simultaneously $\alpha_s(M_\tau)$, $\langle \mathcal{O}_4 \rangle$, $\langle \mathcal{O}_6 \rangle$ and $\langle \mathcal{O}_8 \rangle$. In order to incorporate new experimental information, we exploit the explicit shape of the (normalized) vector/axial-vector invariant mass spectrum $(1/N_{V/A})\, dN_{V/A}/ds$ by means of the spectral moments

$$D_{V/A}^{kl} \equiv \int_0^{M_\tau^2} ds\left(1 - \frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{1}{N_{V/A}}\frac{dN_{V/A}}{ds} , \tag{5}$$

where the choice of $k = 1$ and $l = 0, 1, 2, 3$ provides four additional (correlated) degrees of freedom for each vector and axial-vector fit.

Theoretical Uncertainties

The estimate of theoretical uncertainties which is included in the fit takes into account uncertainties in the physical constants used and tiny contributions from quark mass corrections. In addition, the effect of unknown higher order perturbative contributions is estimated by varying the leading unknown coefficient K_4 between zero and $K_4 \approx 2 \times K_3(K_3/K_2) \simeq 50$ (to be compared, e.g., with the experimental estimate $K_4 = 27 \pm 5$ [17]). The uncertainties from the ambiguities of the renormalization scale μ and the renormalization scheme ($\overline{\mathrm{MS}}$) are rather small; nevertheless they are considered here. We additionally include rough estimates of higher order non-perturbative operators, which are not measured here, that lead to small contributions to the theoretical error.

Results

Computing the sum of the branching fractions of the exclusive V and A contributions yields the semileptonic widths

$$R_{\tau,V} = 1.782 \pm 0.018 \ (\pm 0.017) \ , \tag{6}$$

$$R_{\tau,A} = 1.710 \pm 0.019 \ (\pm 0.017) \ , \tag{7}$$

$$R_{\tau,V+A} = 3.492 \pm 0.015 \ (\pm 0.034) \ , \tag{8}$$

where the first errors are from experimental origin while the second ones (in parentheses) represent uncertainties in the theoretical predictions. The value of (8) is obtained from the (universality improved) leptonic branching ratios and subtracting from it the small strange contribution, $R_{\tau,S} = 0.156 \pm 0.008$ [18]. Tab. 1 lists the measured values of the (strongly correlated) spectral moments (5) for the $(V + A)$ case.

Table 1: *Spectral Moments of the $(V+A)$ τ decay modes with their experimental and theoretical uncertainties.*

ALEPH	$l = 0$	$l = 1$	$l = 2$	$l = 3$
D^{ll}_{V+A}	0.7177	0.1581	0.0585	0.0265
$\Delta^{\exp}(\Delta^{\mathrm{th}})D^{ll}_{V+A}$	0.0022(40)	0.0006(31)	0.0004(04)	0.0004(03)

The results of the various fits of $\alpha_s(M_\tau)$ and the non-perturbative terms of dimension $D = 4, 6, 8$ are given in Tab. 2. The gain from the separation of vector and axial-vector channels compared to the inclusive $V + A$ fit becomes obvious in the adjustment of the leading non-perturbative contributions of dimension $D = 6$ and $D = 8$, which cancel in the inclusive sum. The excellent agreement of the results for $\alpha_s(M_\tau)$ from the respective fits in spite of non-zero non-perturbative contributions is a remarkable confirmation of the SVZ approach used. The evolution of the "best" and most robust $\alpha_s(M_\tau)$ measurement from the inclusive $(V + A)$ observables to the mass of the Z boson, using the N^{3}LO RGE [19, 20], yields

$$\alpha_s(M_Z) = 0.1213 \pm 0.0011 \pm 0.0016 \pm 0.0010 \ .$$

The first error accounts for the experimental uncertainty, the second one gives the uncertainty of the theoretical prediction of R_τ and the spectral moments, while the last error stands for possible ambiguities in the evolution due to uncertainties of the matching scales of the quark thresholds.

Table 2: *Fit results of $\alpha_s(M_\tau)$ and the OPE non-perturbative contributions from various input parameters: the $V/A/(V+A)$ fits use $R_{\tau,V}/R_{\tau,A}/R_{\tau,V+A}$ and the corresponding moments. The $\delta^{(4)}$ contributions are tiny in all three cases.*

ALEPH	Vector (V)	Axial-Vector (A)	$V + A$
$\alpha_s(M_\tau)$	$0.349 \pm 0.015 \pm 0.015$	$0.350 \pm 0.019 \pm 0.015$	$0.349 \pm 0.010 \pm 0.015$
δ_P	0.201 ± 0.016	0.202 ± 0.018	0.201 ± 0.013
$\delta^{(6)}$	0.029 ± 0.004	-0.028 ± 0.004	0.001 ± 0.005
$\delta^{(8)}$	-0.009 ± 0.001	0.008 ± 0.001	-0.001 ± 0.001
Total δ_{NP}	0.019 ± 0.005	-0.022 ± 0.003	-0.002 ± 0.004

The Running of $\alpha_s(s)$

By means of the exclusive measurement of the vector and axial-vector spectral functions, it becomes possible to deepen the QCD analysis as the spectral functions contain the entire phenomena of QCD physics at low energy. When decreasing the energy scale, non-perturbative effects become larger, a fact characterized by the appearance of high-peaked resonances in the spectral functions, especially the $a_1(1260)$, the $\rho(770)$ and the π-pole. However, not only non-perturbative contributions arise, but also the perturbative prediction increases with decreasing energy and one might expect convergence problems of the perturbative series at $\alpha(s_0)/\pi > 0.1$ [6]. Using the spectral functions, one can simulate the physics of a hypothetical τ lepton with a mass $\sqrt{s_0}$ smaller than M_τ. The τ hadronic width $R_\tau(s_0)$ is then calculated by convoluting the spectral functions with the kinematical factor $(1 - s/s_0)^2(1 + 2s/s_0)/s_0$ corresponding to the new "τ mass" $\sqrt{s_0}$. The evolution of $R_\tau(s_0)$ provides a direct test of the running of $\alpha_s(\sqrt{s_0})$, governed by the RGE β-function.

The functional dependence of the respective $R_{\tau,V+A}$ hadronic width is plotted in Fig 2
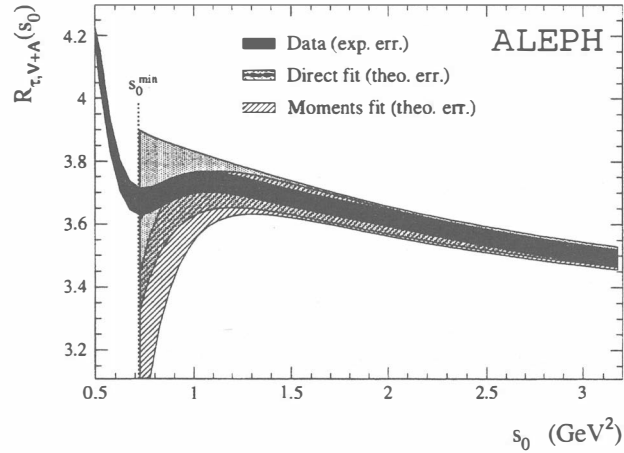

Figure 2: $(V + A)$, V and A hadronic widths versus the square "τ mass" s_0. All curves are plotted as error bands to emphasize their strong point-to-point correlations in s_0. The shaded band (with a solid line in the centre) "direct fit" ($\alpha_s(M_\tau) = 0.354 \pm 0.017$) depicts the result of the direct adjustment of the theoretical prediction to the data curve shown. The "moments fit" is the evolution of the theoretical prediction using the results for $R_{\tau,V+A}$ and the non-perturbative terms from Tab. 2.

together with the theoretical predictions. The regime of validity of the OPE is clearly above $s_0^{min} \simeq 0.7 \, \text{GeV}^2$. The experimental fact that the non-perturbative contributions (inside present accuracy) exactly cancel over the whole range $1.2 \, \text{GeV}^2 \leq s_0 \leq M_\tau^2$ appears almost magic. It is the deeper reason why the α_s determination from the inclusive $V + A$ data is so robust.

Acknowledgements

References

[1] A. Höcker, Thesis, Université de Paris-Sud, Report LAL 97-18 (1997).

[2] D. Buskulic et al. (ALEPH Collaboration), *Phys. Lett.* **B307** (1993) 209.

[3] T. Coan et al. (CLEO Collaboration), *Phys. Lett.* **B356** (1995) 580.

[4] E. Braaten, S. Narison and A. Pich, *Nucl. Phys.* **B373** (1992) 581.

[5] F. Le Diberder and A. Pich, *Phys. Lett.* **B289** (1992) 165;

[6] F. Le Diberder and A. Pich, *Phys. Lett.* **B286** (1992) 147;

[7] A. Pich, *Nucl. Phys.* **B39** (Proc. Suppl.) (1995) 326.

[8] S. Narison, *Nucl. Phys.* **B40** (Proc. Suppl.) (1995) 47; G. Altarelli, P. Nason and G. Ridolfi, *Z. Phys.* **C68** (1995) 257; M. Neubert, *Nucl. Phys.* **B463** (1996) 511; P.A. Rączka and A. Szymacha, *Z. Phys.* **C70** (1996) 125; M. Girone and M. Neubert, *Phys. Rev. Lett.* **76** (1996) 3061.

[9] R.M. Barnett et al. (Particle Data Group), *Phys. Rev.* **D54** (1996) 1.

[10] W.J. Marciano and A. Sirlin, *Phys. Rev. Lett.* **56** (1986) 22.

[11] D. Buskulic et al. (ALEPH Collaboration), CERN PPE/97-013 (1997).

[12] A. Höcker, Talk given at the TAU96 Conference, Colorado, 1996.

[13] D. Decamp et al. (ALEPH Collaboration), *Nucl. Inst. Meth.* **A294** (1990) 121; D. Buskulic et al. (ALEPH Collaboration), *Nucl. Inst. Meth.* **A360** (1995) 481.

[14] D. Buskulic et al. (ALEPH Collaboration), *Z. Phys.* **C70** (1996) 579.

[15] A. Höcker and V. Kartvelishvili, *Nucl. Inst. Meth.* **A372** (1996) 469.

[16] M.A. Shifman, A.L. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B147** (1979) 385, 448, 519.

[17] F. Le Diberder, *Nucl. Phys.* **B39** (Proc. Suppl.) (1995) 318.

[18] M .Davier, Talk given at the TAU96 Conference, Colorado, 1996.

[19] W. Bernreuther and W. Wetzel, *Nucl. Phys.* **B197** (1982) 228.

[20] S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *Nucl. Phys.* **B438** (1995) 278.

STRUCTURE FUNCTIONS