From R^2 gravity to no-scale supergravity

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We show that R^2 gravity coupled conformally to scalar fields is equivalent to the real bosonic sector of $\mathrm{SU(N,1)/SU(N)\times U(1)}$ no-scale supergravity, where the conformal factor can be identified with the Kähler potential, and we review the construction of Starobinsky-like models of inflation within this framework.

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I. INTRODUCTION

The singularity-free cosmological model which incorporates inflation [1], and that in which quantum perturbations were first calculated [2], was that based on $R + R^2$ gravity. Remarkably, almost four decades later, the perturbation spectrum calculated in this pioneering model of inflation remains in excellent agreement with the growing wealth of measurements of the cosmic microwave background (CMB) radiation and data on a large-scale structure [3], whereas many more junior models have fallen by the wayside.

Since we are unabashed fans of supersymmetry at the TeV scale and above, we have long advocated supersymmetric models of inflation [4]. Since any cosmological model must incorporate gravity, we have also long advocated models of inflation based on the framework of local supersymmetry, i.e., supergravity [5]. In particular, for over 30 years we have been advocating models of inflation [6–9] formulated within no-scale supergravity [10–12], which offers a positive semidefinite scalar potential and mitigates the η-problem [13] that is the bane of generic supergravity models of inflation [14]. This conclusion is unaffected by radiative corrections [13,15].

The advent of a new generation of CMB data in the past few years encouraged us to return to the construction of no-scale supergravity models of inflation. Imagine our surprise when we discovered that a simple model of an inflaton field coupled to $\mathrm{SU}(1,1)/\mathrm{U}(1)$ no-scale supergravity [16] could yield an effective scalar potential that is identical to that obtained in the original $R + R^2$ model after a conformal transformation [17], a realization that had also been reached in 1987 [18], though without making the connection to cosmological inflation. This convergence between $R + R^2$ gravity and no-scale supergravity was very intriguing [19], but the nature of any deeper connection remained obscure.

In this paper we make a simple point that, to our knowledge, has not been made previously in the way described here. We consider pure R^2 gravity supplemented by a set of complex scalar fields ϕ^i with conformal couplings to R of the form $(\sum_{i=1}^{N-1}|\phi^i|^2)R/3$. Within this extended R^2 theory, we consider a generalization of the conformal transformation [20] that rewrites $R + R^2$ gravity as minimal R gravity coupled to a scalar field with a potential of the form

$$V = \frac{3M^2}{4\kappa^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2, \tag{1}$$

which yields successful inflation. The multifield generalization yields a scalar Lagrangian that is identical to that obtained in a $\mathrm{SU(N,1)/SU(N)\times U(1)}$ no-scale supergravity model [11] with Kähler potential

$$K = -3\ln\left(T + T^\dagger - \left(\sum_{i=1}^{N-1}|\phi^i|^2\right)/3\right), \tag{2}$$

when we discard the imaginary parts of the scalar fields. Within this framework, the inflaton field ϕ in (1) can be identified (up to a suitable normalization) with either the combination $T + T^\dagger$ or the real part of one of the conformal fields ϕ^i. Moreover, the conformal factor Ω that transforms the extended R^2 theory to the Einstein frame is identical to the Kähler potential (2), up to a numerical factor: $\Omega = -K/6$. This identification reinforces the connection between R^2 gravity and no-scale supergravity that emerged in [17] (see also [21]), and was developed further in [22–30].

II. R^2 GRAVITY AND A DE SITTER UNIVERSE

As a preliminary to developing this connection, we first review the case of pure R^2 gravity, which is described by the action

$$\mathcal{A} = \frac{1}{2}\int d^4x\sqrt{-g}\alpha R^2, \tag{3}$$

where α is an arbitrary dimensionless constant. The pure R^2 theory (3) is scale invariant. It may be rewritten in the following form, using a Lagrange multiplier field Φ:

$$\mathcal{A} = \frac{1}{2}\int d^4x\sqrt{-g}(2\alpha\Phi R - \alpha\Phi^2). \tag{4}$$

Note that the field Φ has a noncanonical mass dimension, $[\Phi] = 2$, not the canonical mass dimension $[\Phi] = 1$.

In order to rewrite the action (4) in the Einstein-Hilbert form, one rescales the metric by introducing a conformal factor Ω as follows:

$$\tilde{g}_{\mu\nu} = e^{2\Omega}g_{\mu\nu} = \frac{2\alpha\Phi}{\mu^2}g_{\mu\nu}, \tag{5}$$

where μ is a mass scale to be determined. The pure R^2 action (3) then becomes

$$\mathcal{A} = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}e^{-4\Omega}\alpha R^2, \tag{6}$$

where

$$R = e^{2\Omega}\tilde{R} + 6\Box\Omega + 6\partial^\mu\Omega\partial_\mu\Omega, \tag{7}$$

which implies that

$$e^{-4\Omega}\alpha R^2 = e^{-2\Omega}2\alpha\Phi\tilde{R} + 12e^{-4\Omega}\alpha\Phi(\Box\Omega + \partial^\mu\Omega\partial_\mu\Omega) - e^{-4\Omega}\alpha\Phi^2. \tag{8}$$

After rewriting contractions and covariant derivatives in terms of the new metric $\tilde{g}$, we have

$$e^{-4\Omega}\alpha R^2 = \mu^2\tilde{R} + 6\mu^2(\Box\Omega - \partial^\mu\Omega\partial_\mu\Omega) - \frac{\mu^4}{4\alpha}. \tag{9}$$

After further eliminating a total divergence, we see that the action appears now in the Einstein frame

$$\mathcal{A} = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\left(\mu^2\tilde{R} - 6\mu^2\partial^\mu\Omega\partial_\mu\Omega - \frac{\mu^4}{4\alpha}\right) \tag{10}$$

or equivalently

$$\mathcal{A} = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\left(\mu^2\tilde{R} - \frac{3\mu^2}{2}\frac{\partial^\mu\Phi\partial_\mu\Phi}{\Phi^2} - \frac{\mu^4}{4\alpha}\right). \tag{11}$$

Finally, in order to write the scalar kinetic term in canonical form, we introduce $\phi \equiv \sqrt{6}\mu\Omega$, leading to

$$\mathcal{A} = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\left(\mu^2\tilde{R} - \partial^\mu\phi\partial_\mu\phi - \frac{\mu^4}{4\alpha}\right). \tag{12}$$

We see now that the mass scale μ can be identified with the Planck scale: Newton's constant $8\pi G_N^2 = \kappa^2 = 1/\mu^2$.

Thus we have recovered the well-known result [21,31] that pure R^2 gravity is equivalent to the conventional Einstein-Hilbert theory with a massless scalar field ϕ and a cosmological constant $\Lambda = \mu^4/8\alpha$. The dimensionless parameter α in (3) specifies the magnitude of Λ in Planck units, and we see that $\alpha \gg 1$ is required.

III. THE STAROBINSKY MODEL OF INFLATION

Next we recall the Starobinsky model of inflation [1], which is derived by adding to the pure R^2 action (3) the conventional linear Einstein-Hilbert term

$$\mathcal{A} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}(R + \tilde{\alpha}R^2), \tag{13}$$

where we have introduced the dimensionful constant $\tilde{\alpha} = \kappa^2\alpha$. As is well known, after rewriting $\alpha R^2 \to 2\alpha\Phi R - \alpha\Phi^2$ and making a conformal transformation analogous to that in the pure R^2 case

$$\tilde{g}_{\mu\nu} = e^{2\Omega}g_{\mu\nu} = (1 + 2\tilde{\alpha}\Phi)g_{\mu\nu}, \tag{14}$$

one finds

$$\mathcal{A} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\tilde{g}}\left[\tilde{R} - \frac{6\tilde{\alpha}^2}{(1+2\tilde{\alpha}\Phi)^2}\left(\partial^\mu\Phi\partial_\mu\Phi + \frac{\Phi^2}{6\tilde{\alpha}}\right)\right]. \tag{15}$$

Setting $\kappa\phi \equiv \sqrt{3/2}\ln(1 + 2\tilde{\alpha}\Phi)$, (13) may be written as follows in the Einstein frame:

$$\mathcal{A}=\frac{1}{2\kappa^2}\int d^4x\sqrt{-\tilde{g}}\left[\tilde{R}-\kappa^2\partial^\mu\phi\partial_\mu\phi-\frac{1}{4\tilde{\alpha}}\left(1-e^{-\sqrt{\frac{2}{3}}\kappa\phi}\right)^2\right]. \tag{16}$$

Thus one recovers the successful inflationary potential (1) with $\tilde{\alpha}=1/6M^2$ [1]. The scale invariance of the pure R^2 theory (3) is broken explicitly by the Einstein-Hilbert term in (13) and leads to an effective potential (1) with a constant, scale-invariant asymptotic limit that is approached exponentially at a rate controlled by the Planck scale κ.

IV. GENERALIZATION WITH ADDITIONAL CONFORMALLY COUPLED FIELDS

With a view to the later comparison with generalized no-scale supergravity models [11], we now consider adding to the R^2 action (13) $N-1$ additional complex fields ϕ^i with conformal couplings to R,

$$\mathcal{A}=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Bigg[\delta R+\tilde{\alpha}R^2 - 2\kappa^2\sum_{i=1}^{N-1}\left(\partial^\mu\phi^i\partial_\mu\phi_i^\dagger+\frac{1}{3}|\phi^i|^2R\right)\Bigg], \tag{17}$$

where $\delta=0$ corresponds to the R^2 theory and we allow $\delta=1$ to describe the $R+\alpha R^2$ Starobinsky model. As previously, we introduce a Lagrange multiplier field Φ and replace αR^2 in (17) by $2\alpha\Phi R-\alpha\Phi^2$, as in (4). In order to transform to the Einstein frame, we must now rescale the metric by the modified conformal factor Ω,

$$\tilde{g}_{\mu\nu}=e^{2\Omega}g_{\mu\nu}=\left(\delta+2\tilde{\alpha}\Phi-\frac{\kappa^2}{3}\sum_{i=1}^{N-1}|\phi^i|^2\right)g_{\mu\nu}. \tag{18}$$

Thus we arrive at the following generalization of (10):

$$\mathcal{A}=\frac{1}{2\kappa^2}\int d^4x\sqrt{-\tilde{g}}\left[\tilde{R}-6\partial^\mu\Omega\partial_\mu\Omega-\sum_{i=1}^{N-1}\frac{2\kappa^2\partial^\mu\phi^i\partial_\mu\phi_i^\dagger}{(\delta+2\tilde{\alpha}\Phi-\frac{\kappa^2}{3}\sum_{i=1}^{N-1}|\phi^i|^2)}-\frac{\tilde{\alpha}\Phi^2}{(\delta+2\tilde{\alpha}\Phi-\frac{\kappa^2}{3}\sum_{i=1}^{N-1}|\phi^i|^2)^2}\right] \tag{19}$$

that we compare below with the effective action of $\mathrm{SU(N,1)/SU(N)\times U(1)}$ no-scale supergravity [17].

V. FROM R^2 GRAVITY TO SU(1,1)/U(1) NO-SCALE SUPERGRAVITY

The pure R^2 supergravity model was constructed in [26]. Here we compare the action in (11) and (12) with that of the simplest $\mathrm{SU}(1,1)/\mathrm{U}(1)$ no-scale supergravity model [16] (see also [21]).

We recall that, in addition to the supergravity multiplet, which contributes the bosonic term $\mathcal{A}=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}R$ to the effective action, the structure of the matter sector of a supergravity theory is characterized by the Kähler function

$$G=K+\ln|W|^2, \tag{20}$$

where the Kähler potential K is a Hermitian function of the complex scalar fields, and W is the superpotential, which is a holomorphic function of these fields. The simplest $\mathrm{SU}(1,1)/\mathrm{U}(1)$ no-scale supergravity model can be written in terms of a single complex field T with Kähler potential

$$K=-3\ln\kappa(T+T^\dagger), \tag{21}$$

whose Lagrangian takes the very simple form

$$\mathcal{L}=-\frac{3}{(T+T^\dagger)^2\kappa^2}\partial^\mu T\partial_\mu T^\dagger=-\frac{1}{12\kappa^2}(\partial_\mu K)^2 -\frac{3}{4}e^{2K/3}|\partial_\mu(T-T^\dagger)|^2, \tag{22}$$

with vanishing potential.

In order to establish the correspondence with R^2 gravity, we consider a superpotential of the form [16,28]

$$W=T^3-\frac{\mu^3}{12\alpha}, \tag{23}$$

which generates a scalar potential of the form

$$V(T,T^\dagger)=\frac{\mu^4}{4\alpha}\frac{T^2+T^{\dagger 2}}{(T+T^\dagger)^2}. \tag{24}$$

We restrict our attention initially to the real direction in field space.[1] With this restriction, the last term in (22) can be discarded, and the effective potential (24) is constant along the real $T+T^\dagger$ direction, *à la* de Sitter. Comparing with the action (12), which is equivalent to the pure R^2 theory (3), we see that there is a direct correspondence and that we can identify

$$K=-6\Omega. \tag{25}$$

Thus we have made the association $2\tilde{\alpha}\Phi=\kappa(T+T^\dagger)$ and $\Omega=\frac{1}{2}\ln\kappa(T+T^\dagger)$. It is striking that this correspondence is realized with a superpotential (23) that is a simple combination of trilinear and constant terms.

[1] As noted in [28], this theory is unstable in the imaginary $T-T^\dagger$ direction, but could be stabilized by some suitable mechanism such as quartic terms in the Kähler potential, as considered in [11,22].

We note that the relation (25) holds in general when one matches to supergravity any theory whose gravitational kinetic term can be written in the form ΦR, including R^2 gravity. The supergravity Lagrangian can be written as [32]

$$\mathcal{L}_{SG} = -\frac{1}{6}\Phi R - \frac{\partial^2\Phi}{\partial\phi^i\partial\phi_j^*}(\partial_\mu\phi^i\partial^\mu\phi_j^*) - \frac{1}{4\Phi}\left(\frac{\partial\Phi}{\partial\phi^i}\partial_\mu\phi^i - \frac{\partial\Phi}{\partial\phi_j^*}\partial_\mu\phi_j^*\right)^2 + \cdots, \quad (26)$$

where the $\cdots$ represent terms containing gauge fields and fermions, as well as potential terms. Here, Φ is a real function of the scalar components of chiral superfields. Upon transformation to the Einstein frame via a conformal transformation with $e^{2\Omega} = -\kappa^2\Phi/3$, we recover the standard kinetic terms for supergravity with

$$\mathcal{L} = \frac{1}{2\kappa^2}\tilde{R} - \frac{1}{\kappa^2}K_i^j(\partial_\mu\phi^i)(\partial^\mu\phi_j^*), \quad (27)$$

where $K_i \equiv \partial K/\partial\phi^i$ and $K^i \equiv \partial K/\partial\phi_i^*$ and we have the same relation (25) between K and Ω. We further note that, since the pure R^2 gravitational theory contains no kinetic term for scalar fields, i.e., the middle term in Eq. (26) vanishes, the kinetic term of the scalar degree of freedom in the Einstein frame arises solely from the conformal transformation [16], and we can write $-\kappa\Phi/3 = T + T^\dagger$, i.e., $K = -3\ln\kappa(T + T^\dagger)$, without loss of generality, thus pointing to the root of the R^2 conformal equivalence to no-scale supergravity.

The correspondence between the kinetic terms for the conformal scalar field in the Starobinsky model (16) and the no-scale field in (22), namely $\delta + 2\tilde{\alpha}\Phi = \kappa(T + T^\dagger)$, was already noted in [22]. This identification reflects the partial invariance of both theories under the noncompact U(1) scale transformations $t \to \alpha t$, which are included in the SU(1,1) group of isometric transformations as dilations, though neither K nor W are themselves invariant, and the analogous transformation for the scalar kinetic term in the Starobinsky model (16). The potential of the Starobinsky model or the corresponding no-scale $\mathrm{SU}(1,1)/\mathrm{U}(1)$ model is, however, not invariant under this rescaling of the corresponding scalar field, as this scale invariance is explicitly broken by the Einstein-Hilbert term, which is linear in the curvature R, or by the superpotential, causing a deviation from pure de Sitter.

The kinetic term for the imaginary part of T can also be accounted for if we extend the gravitational action to include an auxiliary field, b_μ, coupled as follows in the Einstein frame

$$\Delta\mathcal{A} = -\frac{1}{\kappa^2}\int d^4x\sqrt{-\tilde{g}}\left(\frac{1}{3}b_\mu b^\mu - b_\mu J^\mu\right) \quad (28)$$

to a current J_μ:

$$J_\mu = -2(\Omega_T\partial_\mu T - \Omega_{T^*}\partial_\mu T^*) = \frac{1}{3}(K_T\partial_\mu T - K_{T^*}\partial_\mu T^*). \quad (29)$$

The field b_μ satisfies the equation of motion $b_\mu = \frac{3}{2}J_\mu$, so that the action becomes

$$\Delta\mathcal{A} = \frac{3}{4\kappa^2}\int d^4x\sqrt{-\tilde{g}}(J_\mu J^\mu) = -\frac{3}{4\kappa^2}\int d^4x\sqrt{-\tilde{g}}|J_\mu|^2, \quad (30)$$

which corresponds to the final term in (22); see also [16].

Before generalizing the $\mathrm{SU}(1,1)/\mathrm{U}(1)$ theory, we return to the question of flat potentials and the SU(1,1) invariance. It was argued in [16] that, in order to solve the hierarchy problem, the theory should have constant Kähler curvature, $\mathcal{R} = 2/3$ which is guaranteed by the choice of Kähler potential given in (21). As was also shown in [16], the SU (1,1) invariance also allows, more generally, any space with Kähler curvature given by $\mathcal{R} = 2/3a$ which is obtained when (21) is generalized to

$$K = -3a\ln\kappa(T + T^\dagger). \quad (31)$$

This theory will also produce a flat potential [12,16,28] when (i) $W(T) = 1$ leading to zero cosmological constant, or (ii) when $W(T) = T^{3a/2}$ leading to an anti–de Sitter solution [22], or (iii) when $W(T) = T^{3a/2}(T^{-3\sqrt{a}/2} - T^{3\sqrt{a}/2})$. The latter corresponds to the choice (23) for $a = 1$ and is stable for $a > 1$ [28]. It can be used for single-field inflation as in the so-called α-attractor models [33]. We note that this class of maximally symmetric models can also be matched to the R^2 theory with $K = -6a\Omega$, with the special case of $a = 1$ corresponding to the no-scale models we discuss here.

VI. GENERALIZATION TO SU(N,1)/SU(N) × U(1) NO-SCALE SUPERGRAVITY

We now show that the generalization (17) of the R^2 theory with additional conformally coupled fields corresponds in a similar way to the $\mathrm{SU(N,1)/SU(N)} \times \mathrm{U}(1)$ no-scale supergravity model [11] with Kähler potential (2).

In this model, the relevant scalar-bosonic kinetic terms can (after some simple algebraic manipulations) be written as

$$-\frac{1}{12\kappa^2}(\partial_\mu K)^2 - e^{K/3}|\partial_\mu\phi^i|^2 - \frac{3}{4}e^{2K/3}\left|\partial_\mu(T - T^\dagger) + \sum_1^{N-1}\frac{1}{3}\kappa(\phi_i^*\partial_\mu\phi^i - \phi^i\partial_\mu\phi_i^*)\right|^2. \quad (32)$$

Comparing (32) with (19), we can make the same identification as in (25), after identifying $\delta + 2\tilde{\alpha}\Phi = \kappa(T + T^*)$. As in the $SU(1,1)/U(1)$ case, we assume that the imaginary component of T is stabilized as well as the imaginary parts of ϕ^i, in which case the last term in (32) can be discarded, and the correspondence to the kinetic terms in (19) is direct.

The remaining kinetic terms associated with the imaginary parts of the scalar fields can be mirrored by making the same addition to the action as in (30), with an extension of the current to include the remaining $N - 1$ fields,

$$J_\mu = \frac{1}{3}\sum_a (K_a \partial_\mu \Phi^a - K^a \partial_\mu \Phi_a^*), \qquad (33)$$

where the index a runs over the fields T and the ϕ^i. The current-current interaction in (30) corresponds to the final term in (32) when the current is defined as in (33).

In order to complete the correspondence with the generalized R^2 gravity theory (17), we must introduce an effective scalar potential term corresponding to the last term in (19). This is easily done by including the same superpotential term as in (23), which yields the scalar potential

$$V(T,\phi) = \frac{\mu^2}{4\alpha}\frac{T^2 + T^{\dagger 2}}{(\kappa(T + T^\dagger) - \frac{\kappa^2}{3}\sum_{i=1}^{N-1}|\phi^i|^2)^2}. \qquad (34)$$

This reproduces the last term in Eq. (19) for $\delta = 0$ when we restrict our attention to the real direction in T, as per our previous assumption that the imaginary direction in T is stabilized.

To summarize this part of our paper, the conformal factor that transforms the generalized scale-invariant R^2 theory with multiple conformally coupled scalar fields (17) to the Einstein frame is identical with the Kähler potential of $SU(N,1)/SU(N) \times U(1)$ no-scale supergravity. The kinetic term for Ω in (19) matches exactly the first term in the no-scale scalar kinetic term (32), and the second term in (32) also exactly matches the kinetic term for the ϕ^i in (19). When we restrict our attention to the real parts of the complex fields T and ϕ^i, the last term in (32) vanishes, and the identification is complete. This final term can also be mirrored by introducing into the gravity theory a suitable interaction of the current (33).

VII. INTRODUCING A STAROBINSKY-LIKE INFLATIONARY POTENTIAL

We now discuss how a potential $\hat{V}$ for the fields ϕ^i may be introduced into the multifield R^2 action (17), i.e.,

$$\Delta\mathcal{A} = -\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,2\kappa^2\hat{V}(\Phi,\phi^i), \qquad (35)$$

which corresponds to a term of the form

$$\Delta\mathcal{A} = -\frac{1}{2\kappa^2}\int d^4x\sqrt{-\tilde{g}}\frac{2\kappa^2\hat{V}(\Phi,\phi^i)}{(\delta + 2\tilde{\alpha}\Phi - \frac{\kappa^2}{3}\sum_{i=1}^{N-1}|\phi^i|^2)^2} \qquad (36)$$

in the Einstein frame.

We recall that the effective potential in no-scale supergravity takes the following form for general W:

$$V = \frac{\hat{V}}{(\kappa(T + T^\dagger) - \frac{\kappa^2}{3}\sum_{i=1}^{N-1}|\phi^i|^2)^2}, \qquad (37)$$

where

$$\hat{V} \equiv \sum_1^{N-1}\left|\frac{\partial W}{\partial \phi^i}\right|^2 + \frac{\kappa}{3}(T + T^*)|W_T|^2 + \frac{\kappa^2}{3}\left(W_T\left(\sum_1^{N-1}\phi_i^* W_{\phi_i}^* - 3W^*\right) + \text{H.c.}\right). \qquad (38)$$

The following specific, separable form for W,

$$W = T^3 - \frac{\mu^3}{12\alpha} + f(\phi^i), \qquad (39)$$

yields the following form for the scalar potential $\hat{V}$:

$$\hat{V}(T,\phi^i) = (\kappa^2(\phi_i f_{\phi_i} - 3f)T^{\dagger 2} + \text{H.c.}) + |f_{\phi^i}|^2 + \kappa\frac{\mu^3}{4\alpha}(T^2 + T^{\dagger 2}). \qquad (40)$$

We note that the last term in (40) is precisely that in (34) and does not contribute to the construction of $\hat{V}(\phi,\phi^i)$ in (36).

In order to realize inflation in this framework, we restrict our attention, for simplicity, to a single matter field ϕ^1 in addition to the modulus T in the no-scale picture. This corresponds to a noncompact $SU(2,1)/SU(2) \times U(1)$ coset structure, which we have argued previously is the minimal structure required to construct a suitable inflationary model [17]. We consider two forms of the superpotential which can accommodate the Starobinsky model of inflation.

The first is a Wess-Zumino model in which the inflaton is identified with ϕ^1. It is described by the ϕ^1-dependent superpotential [17]

$$W^{WZ} = M\left[\frac{\phi^{1^2}}{2} - \frac{\kappa\phi^{1^3}}{3\sqrt{3}}\right]. \qquad (41)$$

As discussed in [17], if we assume that T is constrained by Planck-scale dynamics to have the specific value

$\kappa T = 1/2^2$, the resulting no-scale model yields the Starobinsky potential (1), as we now show.[2]

Restricting to real fields as discussed previously, we can match this theory with the potential

$$\hat{V} = M^2 \phi^{1^2} (1 - \kappa\phi^1/\sqrt{3})^2 - \frac{1}{2\kappa^2}\tilde{\alpha}\Phi^2, \tag{42}$$

where the last term in (42) is needed to cancel the last term in (19). Then, assuming that the value of Φ is fixed, $\delta + 2\tilde{\alpha}\Phi = 1$, and combining with (19), we find the following scalar potential for ϕ^1:

$$V(\phi^1) = \frac{\hat{V} + \frac{(1-\delta)^2}{8\tilde{\alpha}\kappa^2}}{(1 - \frac{\kappa^2}{3}|\phi^1|^2)^2} = \frac{M^2 \phi^{1^2}(1 - \kappa\phi^1/\sqrt{3})^2}{(1 - \frac{\kappa^2}{3}|\phi^1|^2)^2}, \tag{43}$$

when one recalls that $\tilde{\alpha} = 1/6M^2$, and remembers that the last term in (42) cancels. Finally, making the transformation

$$\phi^1 = \sqrt{3}\tanh(\phi/\sqrt{6}), \tag{44}$$

one recovers the standard form of the Starobinsky potential (1) as a function of this ϕ field.

The second route to a Starobinsky-like model of inflation is the Cecotti model [18],

$$W^C = \sqrt{3} M \phi^1 (T - 1/2), \tag{45}$$

where the inflaton is identified with T. This offers a simpler realization of inflation, since it does not require any additional superpotential term as in Eq. (23). In terms of Φ and ϕ^1, the theory is equivalent (when fields are again taken to be real) to

$$2\kappa^2 \tilde{\alpha}\,\hat{V} = \frac{1}{4} - \frac{\delta + 2\tilde{\alpha}\Phi}{2} + \frac{\delta^2 + 4\delta\tilde{\alpha}\Phi}{4} + \frac{2}{3}\kappa^2 \phi^{1^2}\left(1 - \frac{\delta + 2\tilde{\alpha}\Phi}{2}\right). \tag{46}$$

When ϕ^1 is constrained to have vanishing expectation value and when combined with (19), this yields the following scalar potential for Φ:

$$V(\phi) = \frac{\hat{V} + (\tilde{\alpha}\Phi^2/2\kappa^2)}{(\delta + 2\tilde{\alpha}\Phi)^2} = \frac{3M^2}{16\kappa^2 t^2} - \frac{3M^2}{4\kappa^2 t} + \frac{3M^2}{4\kappa^2}, \tag{47}$$

where we have defined $2t = \delta + 2\tilde{\alpha}\Phi$. Making the transformation $\kappa\phi \equiv \sqrt{6}\Omega = (\sqrt{6}/2)\ln 2t$ we recover once again the Starobinsky potential as a function of this ϕ field.

[2] The choice of this value is for illustration: other choices yield similar results when combined with the corresponding change in (41).

VIII. SUMMARY

We have explored in this paper the connection between R^2 gravity and minimal $\mathrm{SU}(1,1)/\mathrm{U}(1)$ no-scale supergravity [17]. The supersymmetric completion of pure R^2 was considered in [26], and we have shown that by an extension of the theory to include many conformally coupled scalar fields can be matched to more general $\mathrm{SU(N,1)/SU(N)\times U(1)}$ no-scale supergravity theories (see also [21]). In the R^2 frame, we are able to perform a conformal transformation e^Ω to the Einstein frame which introduces a dynamical real scalar degree of freedom, Ω. In the pure R^2 theory, the field is massless and there is a nonzero cosmological constant characterized by the coupling of the R^2 term in the action. In either the $R + R^2$ theory, or one with conformally coupled scalar fields, the scalar potential is nontrivial and possesses a second-order pole, whereas the kinetic terms of the conformally coupled scalars contain a first-order pole. The presence of this pole leads to the asymptotically flat feature at large field values that is characteristic of the Starobinsky model [1]. In the no-scale supergravity theory, due to the logarithmic structure of the Kähler potential and the definition of the kinetic and potential terms in terms of derivatives of K, these terms (in the real directions) possess exactly the same pole structure, leading to the asymptotical flatness so useful for inflation.

We have shown that the parts of the action involving the real components of the scalar fields are identical in R^2 gravity and no-scale supergravity, and we have shown how this correspondence can be extended to the imaginary components by adding a suitable current-current interaction to the R^2 gravity theory. Our analysis deepens understanding of the connection between no-scale supergravity and scale-invariant extensions of Einstein's theory of gravity. Our interest in this connection was triggered by the observational success [3] of the Starobinsky model of inflation [1], and we have reviewed briefly above two examples for how a Starobinsky-like inflationary potential can emerge in simple ways from the $\mathrm{SU}(2,1)/\mathrm{SU}(2)\times \mathrm{U}(1)$ no-scale supergravity theory. We think that this is the most promising avenue for eventually constructing a complete theory of everything below the Planck scale, connecting inflationary model building to accessible physics beyond the Standard Model [23,29,30,34,35].

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