SCHOTTKY BASED INTENSITY MEASUREMENTS AND ERRORS DUE TO STATISTICAL FLUCTUATIONS

M.-E. Angoletta, C. Carli, F. Caspers, R. Jones, F. Pedersen, J. Sanchez-Quesada, L. Soby
CERN, Geneva, Switzerland

Abstract

The beam intensities at the Extra Low ENergy Antiproton ring ELENA are too low for standard beam current transformers and, thus, are measured with longitudinal Schottky diagnostics. This method is already successfully used at the Antiproton Decelerator since the commissioning of this machine. The fact that Schottky noise is a statistical phenomenon implies statistical errors of these measurements. Simple analytical formulas describing the statistical error to be expected as a function of the frequency spread of the band considered, the time resolution chosen and the background noise have been derived. On the one hand, low revolution harmonics and, in turn, frequency spread of the band analysed lead to large measurement errors as this situation corresponds to low momentum resolution of the resulting distribution describing the beam. At very large revolution harmonics and, thus, frequency spreads of the band analysed, the measurement error increases again due to additional contributions from the background noise.

INTRODUCTION

Intensities of un-bunched beams down to below 1μA during cooling plateaus of the Antiproton Decelerator (AD) [1–3] operated since 2000 and the Extra Low ENergy Antiproton ring ELENA [4–8], which is under commissioning [9] at present, cannot be measured using conventional DC beam current transformers. Thus, the intensity of the AD is determined via longitudinal Schottky diagnostics[1] with an AC beam current transformer [10] optimized for minimum background noise. Similar systems based on AC transformers [13] and electrostatic pick-ups [14, 15] are developed for ELENA.

The systems implemented for the AD and foreseen for ELENA estimate the beam intensity from the total power of one longitudinal Schottky band. Simple equations describing the statistical error due to the fact that noise is a stochastic phenomenon are derived. Both the Schottky spectrum due to the beam and background noise from the pick-up used are taken into account. This statistical error depends as well on the duration (or time resolution) of the measurement. The final result is that a broad optimum width for the analysed Schottky band exists. Small widths of the analysed Schottky band lead to poor statistics of the noise analysed. At large widths of the analysed Schottky band, background noise gives again larger statistical errors for the intensity estimate.

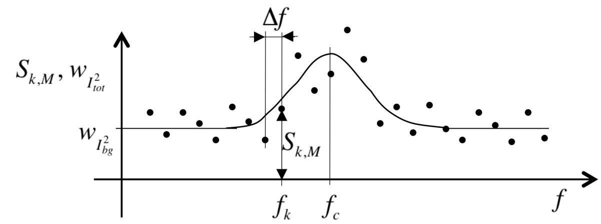

Figure 1: Spectral power density as a superposition of the Schottky signal from the beam and background noise.

DESCRIPTION OF SCHOTTKY SPECTRA

The total spectral power density $w_{I^2_{\mathrm{tot}}}$ of the signal analysed is sketched in Fig. 1 as a superposition of an approximately constant background $w_{I^2_{\mathrm{bg}}}$, where the noise generated by the pick-up used is expressed as equivalent beam current power density, and the Schottky signal generated by the beam $w_{I^2_{\mathrm{b}}}$. The central frequency of the Schottky signal $f_c = h\, f_0$ is the h-th harmonics with the (average) revolution frequency f_0. The integral of the Schottky power density $w_{I^2_{\mathrm{b}}}$ over one Schottky band gives the total Schottky power $P_{I_b} = 2N\, e^2\, f_0^2$, with N the number of particles and e the elementary charge.

Average and RMS of the Spectral Power Density Estimate for a Single Spectrum

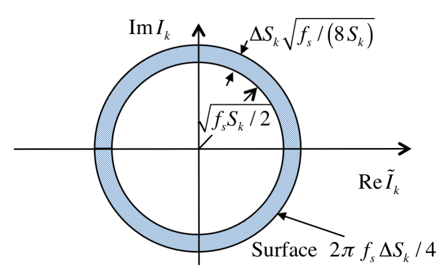

Figure 2: Computation of the average and rms of $\tilde{I}_k$

The signal to be analyzed is sampled and in general "down-mixed" by analog electronics or digitally. Then discrete Fourier transformation is applied to the time-trace of N_s samples I_n acquired with a sampling rate f_s:

$$\tilde{I}_k = \frac{1}{\sqrt{N_s}} \sum_{n=0}^{N_s-1} I_n\, e^{i\,k\,n/N_s} \quad . \qquad (1)$$

The values $\tilde{I}_k$ are complex quantities, which strongly vary with the time-trace taken due to the stochastic nature of

[1] In between, a cryogenic current comparator [11, 12] has been developed and is in use in the AD to measure very low beam intensities.

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the signal. The same probability density function describe both the real and the imaginary part of $\tilde{I}_k$. Here we assume that spectral power density of the signals analysed varies very little over the frequency resolution $\Delta f = f_s/N_s$. Then the probability density function for the real and the imaginary part of $\tilde{I}_k$ is Gaussian with an rms value of $\sigma_{\tilde{I}_k} = (1/2)\sqrt{f_s\, w_{I^2_{\rm tot}}(f_k)}$ with $f_k = f_m + f_s k/N_s$ and f_m the down-mixing frequency. The quantity $S_k = (2/f_s)|\tilde{I}_k|^2$ is introduced as estimate for the spectral density $w_{I^2_{\rm tot}}$ at the frequency f_k. The average $\langle S_k\rangle$ of S_k and its rms σ_{S_k} is determined with the help of Fig. 2. The probability that S_k is located inside the annular area is given by $(2\pi f_s \frac{\Delta S_k}{4}\, \frac{1}{2\pi}\exp(-2S_k/w_{I^2_{\rm tot}})$. The average and rms value for the quantity S_k become:

$$\langle S_k\rangle = \int_0^\infty {\rm d}S_k\, S_k\, \frac{\pi f_s}{2}\, \frac{4}{2\pi f_s\, w_{I^2_{\rm tot}}}\, e^{-\frac{S_k}{w_{I^2_{\rm tot}}}} = w_{I^2_{\rm tot}} \tag{2}$$

$$\sigma_{S_k} = \left[\int_0^\infty {\rm d}S_k \left(S_k - \langle S_k\rangle\right)^2 \frac{1}{w_{I^2_{\rm tot}}}\, e^{-\frac{S_k}{w_{I^2_{\rm tot}}}}\right]^{1/2} = w_{I^2_{\rm tot}} \tag{3}$$

Averaging over Several Spectra

Typically, noise densities are estimated by averaging over several spectra. The total number of samples over the measurement time T_m (or time resolution of the intensity measurement) is divided into M pieces[2]. This results in a frequency resolution of $\Delta f = M/T_m$. The result is sketched in Fig. 1. $S_{k,M}$ denote the spectral power density estimates obtained averaging over the M spectra obtained. The expectation value and rms value for $S_{k,M}$ are $\langle S_{k,M}\rangle = w_{I^2_{\rm tot}}$ and $\sigma_{S_{k,M}} = w_{I^2_{\rm tot}}/\sqrt{M}$.

STATISTICAL ERRORS OF SCHOTTKY BASED INTENSITY MEASUREMENTS

The errors of Schottky based intensity measurements are estimated assuming a Gaussian distribution for the beam leading to

$$w_{I^2_{\rm tot}}(f) = w_{I^2_{\rm bg}} + w_{I^2_{\rm b}} = w_{I^2_{\rm bg}} + \frac{P_{I_b}}{\sqrt{2\pi}\sigma_h}\, e^{-\frac{(f-f_c)^2}{2\sigma_h^2}} \tag{4}$$

with f_c the average frequency of the Schottky band investigated and σ_h its rms width. Other realistic assumptions for the shape of the Schottky band would yield similar results.

The total Schottky power in the band is estimated by the sum $P_{I_b\rm est} = \Delta f \sum(S_{k,M} - w_{I^2_{\rm bg}\rm est})$ with $w_{I^2_{\rm bg}\rm est} = w_{I^2_{\rm bg}} + \Delta w_{I^2_{\rm bg}}$ an estimate for the spectral power density of the background noise and $\Delta w_{I^2_{\rm bg}}$ its error. The sum extends over a band $\pm n_\sigma \sigma_h$, where σ_h sketched in Fig. 3 has to be chosen large enough to ensure that the number of particles outside this interval is negligible (e.g. for a Gaussian with $\sigma_h = 3$, about 3 ‰ of the particles are outside the region taken into account). The expectation value of the result is Δf times the sum of the expectations values of the quantities $S_{k,M} - w_{I^2_{\rm bg}\rm est}$. The sum can be approximated by an integral leading to $\langle P_{I_b\rm est}\rangle = P_{I_b}$. The two contributions to the rms of the quantity $P_{I_b\rm est}$ are:

- the uncertainty $\Delta w_{I^2_{\rm bg}}$ of the background leads to an error of $2n_\sigma\sigma_h\,\Delta w_{I^2_{\rm bg}}$
- the uncertainties of the individual values $S_{k,M}$, which have to be added quadratically to give $\Delta f\sqrt{\sum S^2_{k,M}} = \sqrt{\frac{(\Delta f)^2}{M}\sum\left(w_{I^2_{\rm bg}} + w_{I^2_{\rm b}}(f_k)\right)^2}$. Using $\Delta f/M = 1/T_m$, the last expression can be approximated by $\sqrt{\frac{1}{T_m}\int {\rm d}f\left(w^2_{I^2_{\rm bg}} + 2w_{I^2_{\rm bg}}w_{I^2_{\rm b}}(f) + w^2_{I^2_{\rm b}}(f)\right)}$. The integrals extending from $f_c - n_\sigma\sigma_h$ to $f_c - n_\sigma\sigma_h$ can be evaluated leading to $\sqrt{\left(2n_\sigma\sigma_h w^2_{I^2_{\rm bg}} + 2w_{I^2_{\rm bg}}P_{I_b} + \frac{P^2_{I_b}}{2\sqrt{\pi}\sigma_h}\right)/T_m}$.

These two independent contributions have to be added up to give the total statistical error for $P_{I_b,\rm est}$:

$$\Delta P_{I_b,\rm est} = \left[4n_\sigma^2\sigma_h^2\Delta w^2_{I^2_{\rm bg}} + \frac{2n_\sigma\sigma_h}{T_m}w^2_{I^2_{\rm bg}} + \frac{2w_{I^2_{\rm bg}}P_{I_b}}{T_m} + \frac{P^2_{I_b}}{2\sqrt{\pi}\sigma_h T_m}\right]^{(1/2)} \tag{5}$$

Background Noise Determined from the Acquisition

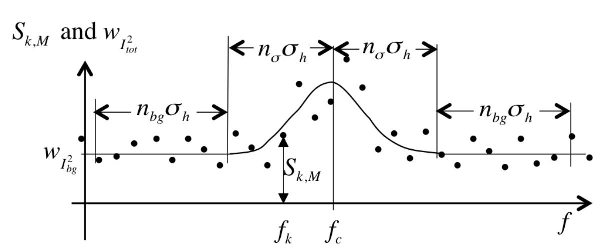

Figure 3: Estimation of background and beam intensity.

The background noise density $w_{I^2_{\rm bg}}$ may be determined from the spectrum as sketched in Fig. 3. Assuming that the dependance of the background noise from the frequency can be neglected as the frequency range considered is small compared to the average frequency f_c, the values for $S_{k,M}$ can be averaged over a suitable intervals of width $n_{\rm bg}\sigma_h$ on both sides of the spectrum outside the Schottky peak. Averaging $S_{k,M}$ over $2n_{bg}\sigma_h/\Delta f$ lines leads to $\Delta w_{I^2_{\rm bg}} = w_{I^2_{\rm bg}}/\sqrt{2n_{\rm bg}\sigma_h T_m}$. The final result for the relative error of the Schottky based intensity measurement

[2] Often the number of spectra obtained is increased by "overlapping" time-traces. However, this results in correlation of values for S_k obtained for subsequent spectra. It can be shown that the gain by overlapping is modest. 50% overlapping allows to reduce the total acquistion time by ≈25% for the same statistical fluctuation of the result.

$\Delta N/N = \Delta P_{I_b}/P_{I_b}$ become:

$$\frac{\Delta N}{N} = \frac{1}{\sqrt{F_M}}\sqrt{2n_\sigma\left(1+\frac{n_\sigma}{n_{\rm bg}}\right)\frac{F_L}{F_M}+2+\frac{1}{2\sqrt{\pi}(F_L/F_M)}}$$

with the dimensionless quantities $F_M = P_{I_b}T_m/w_{I^2_{\rm bg}}$ related to the performance of the pick-up and $F_L = \sigma_h T_m$, which can be chosen via the harmonic of the Schottky band analysed and is the maximum (for $M = 1$) number of frequencies per rms width of the band analyzed. One notes that the statistical error of the intensity measurement does not depend on the number M of time-traces but only on the total measurement time (or time resolution) T_m.

Background from a Calibration Measurement

The system now in use for the AD and proposed for ELENA uses an estimate for the background noise based on a calibration measurement. The advantage is that averaging over many spectra allows reducing the error of the calibration due to statistical fluctuations and variations over the frequency range used can be taken into account. The disadvantage is that fluctuations of the background noise or a drift since the last calibration measurement lead to systematic errors of the beam intensity estimate.

The error of the Schottky based intensity measurement for the special case assuming that the background spectral noise density is accurately known is given by the equation for $\Delta N/N$ with $n_{\rm bg} = \infty$.

NUMERICAL RESULTS

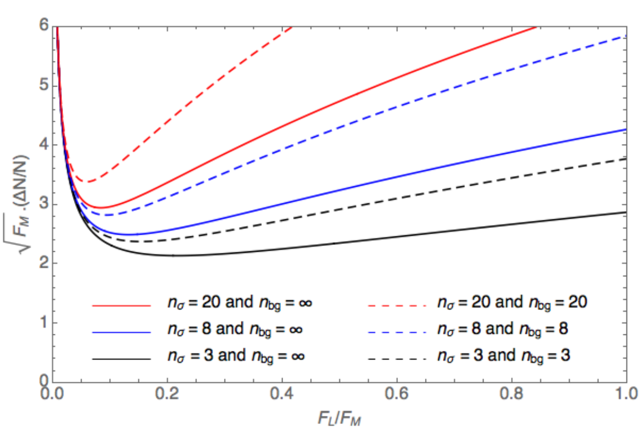

Figure 4: Statistical error of Schottky based Intensity Measurement.

Figure 4 shows a graphical representation of the final result for statistical error of Schottky based intensity measurements. In practice, the same harmonics and frequency intervals are used along a plateau for electron cooling to monitor the intensity during the cooling process. For ELENA, a reduction of the momentum spread and, in turn, the width of the Schottky band used by a factor of about three is expected for both cooling plateaus. From inspection of Fig. 4, one concludes that values of $F_L/F_M \approx 0.1$ are about optimum.

The most critical part of the ELENA cycle is the plateau with the lowest energy of 100 keV. With the revolution frequency $f_0 = 144$ kHz and for $N = 1.8 \times 10^7$ antiprotons, the total power in one Schottky band is $P_{I_b} = 19100$ pA^2. A total measurement time of $T_m = 1$ s is assumed. Furthermore the rms frequency spread of the h-th harmonic for a typical rms relative momentum spread of $\sigma_p/p = 0.5\times10^{-3}$ is $\sigma_h = h(\sigma_p/p)f_0|\eta|$ with $\eta \approx -0.75$ the momentum slip factor.

With the two different instruments available for longitudinal Schottky diagnostics, the optimum frequencies become:

- The sum signal of a single electrostatic position pick optimized for noise performance has a background noise [16] of about $w_{I^2_{\rm bg}} = 1$pA^2/Hz over a large range of frequencies, corresponding to $F_M = 19100$. This leads to an optimum harmonics $h \approx 35$, a central frequency of $f_c \approx 5$MHz an rms error of the intensity measurement due to the stochastic nature of Schottky signals of about 2%. The proposed scheme to combine the signal of all 20 position pick-ups is expected to improve the background noise by a factor 20 leads to higher optimum frequencies. The exact values are difficult to determine as at frequencies above 10 MHz, the response of the pick-ups decreases leading to an increase of the background noise.

- The signal from the magnetic pick-up has a background noise floor [17] of about $w_{I^2_{\rm bg}} \approx 3$pA^2/Hz (with some variations over the frequency range of interest, which should strictly speaking be taken into account). This leads to an optimum harmonics $h \approx 12$, a central frequency of $f_c \approx 1.7$MHz an rms error of the intensity measurement due to the stochastic nature of Schottky signals of about 4%. Optimum frequencies at the intermediate plateau with cooling are higher and the statistical error lower.

CONCLUSIONS

Statistical errors of Schottky based intensity measurements due to the stochastic nature of Schottky signals can be described by simple analytical expressions. An broad optimum of the frequency to be used has been found. Schottky based intensity measurements at too low frequencies lead, as expected, to a poor statistics over the noise generated by the beam. At high frequencies, background noise leads again to larger statistical errors.

However, systematic errors, as e.g. fluctuations or a drift of the background noise estimated from a calibration measurement, are not taken into account and are likely to change the situation. Thus, optimum frequencies for Schottky based intensity measurements obtained should be considered an upper limit. Furthermore, Schottky based intensity measurements using one single band as applied for the AD and proposed for ELENA has been proposed. Evaluation of several Schottky bands at lower frequency would as well reduce the statistical errors of the measurement.

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