Isothermal compressibility of hadronic matter formed in relativistic nuclear collisions

Maitreyee Mukherjee^{a,*}, Sumit Basu^{b,**}, Arghya Chatterjee^c, Sandeep Chatterjee^d, Souvik Priyam Adhya^c, Sanchari Thakur^c, Tapan K. Nayak^{c,e}

^aBose Institute, Department of Physics and CAPSS, Kolkata-700091, India
^bDepartment of Physics and Astronomy, Wayne State University, Detroit, MI 48201, USA
^cVariable Energy Cyclotron Centre, HBNI, Kolkata-700064, India
^dAGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland
^eCERN, Geneva 23, Switzerland

Abstract

We present the first estimates of isothermal compressibility $(k_{\rm T})$ of hadronic matter formed in relativistic nuclear collisions ($\sqrt{s_{\rm NN}}=7.7~{\rm GeV}$ to 2.76 TeV) using experimentally observed quantities. $k_{\rm T}$ is related to the fluctuation in particle multiplicity, temperature, and volume of the system formed in the collisions. Multiplicity fluctuations are obtained from the event-by-event distributions of charged particle multiplicities in narrow centrality bins. The dynamical components of the fluctuations are extracted by removing the contributions to the fluctuations from the number of participating nucleons. From the available experimental data, a constant value of $k_{\rm T}$ has been observed as a function of collision energy. The results are compared with calculations from UrQMD, AMPT, and EPOS event generators, and estimations of $k_{\rm T}$ are made for Pb-Pb collisions at the CERN Large Hadron Collider. A hadron resonance gas (HRG) model has been used to calculate $k_{\rm T}$ as a function of collision energy. Our results show a decrease in $k_{\rm T}$ at low collision energies to $\sqrt{s_{\rm NN}}\sim 20~{\rm GeV}$, beyond which the $k_{\rm T}$ values remain almost constant.

Keywords: Quark-gluon plasma, compressibility, multiplicity fluctuation, hadron resonance gas.

1. Introduction

The determination of the thermodynamic state of matter formed in high-energy nuclear collisions is of great importance in understanding the behaviour of the matter formed at high temperature and/or energy density. A set of basic macroscopic quantities, such as temperature, pressure, volume, entropy, and energy density, as well as a set of response functions, including specific heat, compressibility and different susceptibilities define the thermodynamic properties of the system. These quantities are related by the equation of state (EOS), which on the other hand, governs the evolution of the system. One of the basic goals of calculating the thermodynamic quantities, such as the specific heat (c_v) and isothermal compressibility (k_T) is to obtain the EOS of the matter [1, 2, 3, 4, 5, 6, 7]. The c_v is the amount of energy per unit change in temperature and is related to

the fluctuation in the temperature of the system [8, 9].

Heavy-ion collisions at ultra-relativistic energies produce matter at extreme conditions of energy density and temperature, where a phase transition from normal hadronic matter to a deconfined state of quark-gluon plasma (QGP) takes place. Lattice QCD calculations have affirmed a crossover transition at zero baryonic chemical potential ($\mu_{\rm B}$) [10, 11]. On the other hand, QCD inspired phenomenological models [12, 13, 14, 15] predict a first order phase transition at high $\mu_{\rm B}$. This suggests the possible existence of a QCD critical point where the first order transition terminates. The current focus of theoretical and experimental programs is to un-

The $k_{\rm T}$ describes the relative variation of the volume of a system due to a change in the pressure at constant temperature. Thus $k_{\rm T}$ is linked to density fluctuations and can be expressed in terms of the second derivative of the free energy with respect to the pressure. In a second order phase transition $k_{\rm T}$ is expected to show a singularity. The determination of $k_{\rm T}$ as well as c_v can elucidate the existence of a phase transition and its nature.

^{*}maitreyee.mukherjee@cern.ch; Presently at CCNU, Wuhan, 430079, PR China

^{**}sumit.basu@cern.ch

derstand the nature of the phase transition and to locate the critical point by exploring multiple signatures. Since $k_{\rm T}$ is sensitive to the phase transition, its dependence on the $\mu_{\rm B}$ or the collision energy provides one of the basic measurements on this subject.

Recently, collision energy dependence of c_v has been reported by analysing the event-by-event mean transverse momentum ($\langle p_T \rangle$) distributions [18]. In this approach, the $\langle p_T \rangle$ distributions in finite p_T ranges are converted to distributions of effective temperatures. The dynamical fluctuations in temperature are extracted by subtracting widths of the corresponding mixed event distributions.

In the present work, we have calculated the isothermal compressibility of matter formed in high energy collisions using experimentally observed quantities, as prescribed in Ref. [1]. This method uses the fluctuations of particle multiplicities produced in the central rapidity region. It may be noted that enhanced fluctuation of particle multiplicity had earlier been proposed as signatures of phase transition and critical point [19, 20, 21, 22, 23]. Thus the study of event-by-event multiplicity fluctuations and estimation of $k_{\rm T}$ are important for understanding the nature of matter at extreme conditions. The experimental data of event-by-event multiplicity fluctuations at the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) and Super Protron Synchrotron (SPS) of CERN have been used in combination with temperatures and volumes of the system at the chemical freeze-out to extract the values of $k_{\rm T}$. These results are compared to that of three event generators and the hadron resonance gas (HRG) model. Our results provide important measures for the beam energy scan program of RHIC and the experiments at the CERN Large Hadron Collider (LHC), and gives guidance for experiments at the Facility for Antiproton and Ion Research (FAIR) at GSI and the Nuclotron-based Ion Collider facility (NICA) at JINR, Dubna.

2. Methodology

Isothermal compressibility is the measure of the relative change in volume with respect to change in pressure [1].

$$k_T|_{T,\langle N\rangle} = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)|_{T,\langle N\rangle}$$
 (1)

where V, T, P represent volume, temperature, and pressure of the system, respectively, and $\langle N \rangle$ stands for the mean yield of the particles. In the Grand Canonical Ensemble (GCE) framework, the variance (σ^2) of the

number of particles (N) is directly related to isothermal compressibility [1, 24], i.e,

$$\sigma^2 = \frac{k_{\rm B}T\langle N\rangle^2}{V}k_{\rm T},\tag{2}$$

where $k_{\rm B}$ is the Boltzmann constant. Charged particle multiplicity fluctuations have been characterised by the scaled variances of the multiplicity distributions, defined as.

$$\omega_{\rm ch} = \frac{\langle N_{\rm ch}^2 \rangle - \langle N_{\rm ch} \rangle^2}{\langle N_{\rm ch} \rangle} = \frac{\sigma^2}{\mu}$$
 (3)

where $N_{\rm ch}$ is the charged particle multiplicity per event, and $\mu = \langle N_{\rm ch} \rangle$. Following the above two equations, we obtain,

$$\omega_{\rm ch} = \frac{k_{\rm B}T\mu}{V}k_{\rm T},\tag{4}$$

which makes a connection between multiplicity fluctuation and $k_{\rm T}$. This formalism, using GCE properties, may be applied to experimental measurements at midrapidity, as energy and conserved quantum numbers are exchanged with the rest of the system [25]. At the chemical freeze-out surface, the inelastic collisions cease, and thus the hadronmultiplicities get frozen. While the ensemble average thermodynamic properties like the temperature and volume can be extracted from the mean hadron yields, $k_{\rm T}$ can be accessed through the measurements of the event-by-event multiplicity fluctuations.

3. Multiplicity fluctuations: experimental data

The multiplicity fluctuations have been measured for a range of collision energies by the E802 collaboration [26] at BNL-AGS, WA98 [27], NA49 [28, 29], NA61 [30, 31] and CERES [32] experiments at CERN-SPS, and PHENIX experiment [24] at RHIC. The results of these measurements could not be compared directly because of differences in the kinematic acceptances and detection efficiencies. The experimental results are normally reported after correcting for detector efficiencies. But the acceptances in pseudorapidity (η) need not be the same for these experiments. The results from the experiments have been scaled to midrapidity so that these can be presented in the same footing [24, 33]. Fig. 1 shows the values of $\omega_{\rm ch}$ for $|\eta| < 0.5$ in central (0-5%) collisions as a function of the collision energy [33]. The solid circles represent experimental measurements. An increase in the scaled variances with the increase in collision energy has been observed from these data.

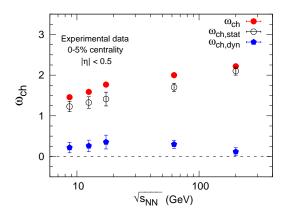


Figure 1: Beam-energy dependence of scaled variances of multiplicity distributions ($\omega_{\rm ch}$) for central (0-5%) Au-Au (Pb-Pb) collisions from the available experimental data [24, 26, 28, 29, 30, 31, 32]. The statistical components of fluctuations ($\omega_{\rm ch,stat}$) using the participant model calculations have been shown. The dynamical components of the fluctuations ($\omega_{\rm ch,dyn}$) are obtained by subtracting the statistical components from the measured values.

It is to be noted that the widths of the charged particle distributions and $\omega_{\rm ch}$ get their contributions from several sources, some of which are of statistical in nature and the rest have dynamical origins. The dynamical components are connected to thermodynamics and have been used in the present work to extract $k_{\rm T}$ [1]. Thus an estimation of the statistical part is necessary to infer about the dynamical component of multiplicity fluctuations

One of the major contributions to statistical fluctuations comes from the geometry of the collision, which includes variations in the impact parameter or the number of participating nucleons. In a participant model [20], the nucleus-nucleus collisions are treated as superposition of nucleon-nucleon interactions. Thus the fluctuation in multiplicity arises because of the fluctuation in the number of participants ($N_{\rm part}$) and the fluctuation in the number of particles produced per participant. In this formalism, based on Glauber type of initial conditions, $\omega_{\rm ch}$ can be expressed as,

$$\omega_{\rm ch} = \omega_{\rm n} + \langle n \rangle \omega_{N_{\rm part}}, \tag{5}$$

where n is the number of charged particles produced per participant, ω_n denotes fluctuations in n, and $\omega_{N_{part}}$ is the fluctuation in N_{part} . The value of ω_n has a strong dependence on acceptance. The fluctuations in the number of accepted particles (n) out of the total number of produced particles (m) can be calculated by assuming that the distribution of n follows a binomial distribution.

This is given as [20, 27],

$$\omega_{\rm n} = 1 - f + f\omega_{\rm m},\tag{6}$$

where f is the fraction of accepted particles. The values of f and $\omega_{\rm m}$ are obtained from proton-proton collision data of the number of charged particles within the midrapidity range and the total number of charged particles produced in the collision [34, 35, 36, 37, 27]. Using these, we obtain the values of $\omega_{\rm n}$ as a function of collision energy.

The values of ω_n vary within 0.98 to 2.0 corresponding to $\sqrt{s_{\rm NN}}$ =7.7 GeV to 2.76 TeV, and are in agreement with those reported for SPS energies [27]. The distribution of $N_{\rm part}$ for narrow centrality bins yields the value of $\omega_{N_{\rm part}}$. With the choice of narrow bins in centrality selection, $\omega_{N_{\rm part}}$ values remain close to unity from peripheral to central collisions. With the knowledge of ω_n , $\langle n \rangle$ and $\omega_{N_{\rm part}}$, the statistical components of $\omega_{\rm ch}$ from the participant model have been extracted. The values of $\omega_{\rm ch,stat}$ are presented as open symbols in Fig. 1 as a function of collision energy. The uncertainties in $\omega_{\rm ch,stat}$ are derived from the statistical and systematic uncertainties in n and $\omega_{\rm n}$.

The dynamical fluctuations of ω_{ch} (denoted as $\omega_{ch,dyn}$) are extracted by subtracting the statistical fluctuations from the measured ones. In Fig. 1, the values of $\omega_{ch,dyn}$ are plotted (as diamond symbols) as a function of collision energy. Within the quoted errors, $\omega_{ch,dyn}$ is seen to remain constant as a function of collision energy. However, a decreasing trend may be seen for $\sqrt{s_{\rm NN}} > 20$ GeV. More experimental data at low and intermediate collision energies are needed to conclude the nature of the fluctuations as a function of the collision energy.

4. Multiplicity fluctuations from event generators

In order to validate the results from experimental data, we have analysed the results from three different event generators, which are: AMPT (A Multi Phase Transport) [38, 39, 40], UrQMD (Ultrarelativistic Quantum Molecular Dynamics) [41, 42], and EPOS [43, 44, 45]. Multiplicity fluctuations using the AMPT model have been studied for the default (DEF) and string melting (SM) modes [33]. In the default mode, hadronization takes place via the string fragmentation, whereas in the SM mode, hadronization takes place via quark coalescence. The UrQMD is a microscopic transport model, where the hadron-hadron interactions and the space-time system evolution are studied based on the covariant propagation of all hadrons

in combination with stochastic binary scatterings, color string formation, and resonance decay. UrQMD has been previously used to simulate production of different particles and analysis of their event-by-event fluctuations [47, 48, 49, 50, 51, 52].

The EPOS(3+1) viscous hydrodynamical model incorporates multiple scattering approach based upon the Gribov-Regge (GR) theory and perturbative QCD [44]. The hydrodynamical evolution starts from flux tube (or relativistic strings) initial conditions, generated by the GR framework. The string formation occurs due to initial scatterings, which later breaks into segments identified as hadrons. One of the salient features of the model is the classification of two regions of physical interest on the basis of density, such as core (high density) and corona (low density) [45]. For the centrality dependence of observables, the corona plays a major role at large rapidity and low multiplicity events and contributes to hadronization. However, for most central collisions, a core with collective hadronization is created from corona because of a large number of nucleons suffering inelastic collisions. Results from EPOS match experimental data at RHIC and LHC for particle multiplicities, transverse momenta and correlation patterns [43, 44, 45, 46].

For the present study, a large number of events are generated using the event generators for Au-Au collisions between $\sqrt{s_{\rm NN}}$ = 7.7 to 200 GeV, corresponding to the RHIC energies, and for for Pb-Pb collisions at $\sqrt{s_{\rm NN}}$ = 2.76 TeV. In all cases, the centrality of the collision has been selected using minimum bias distributions of charged particle multiplicities in the range, $0.5 < |\eta| < 1.0$. The multiplicities and multiplicity fluctuations have been obtained within the kinematic range, $|\eta| < 0.5$ and $0.2 < p_T < 2.0$ GeV/c. The η -range used for the centrality selection is different from the one for the fluctuation study, and thus poses almost no bias on the fluctuation analysis. To minimise the geometrical fluctuations, calculations are first done for narrow (1%) centrality bins. These results are then combined to make wider bins by using centrality bin width correction method which takes care of the impact parameter variations [33].

Fig. 2 shows the collision energy dependence of ω_{ch} for central (0-5%) collisions from the event generators. Statistical errors are calculated using the Delta theorem [53]. It is observed that the fluctuations remain somewhat constant over the energy range considered, except for the AMPT events, where a small rise is seen at higher energies. The statistical components of the fluctuations have been calculated from the participant model calculations, using the same procedure as dis-

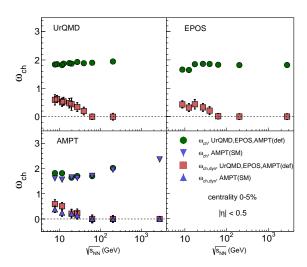


Figure 2: Collision energy dependence of scaled variances of charged particle multiplicity distributions for central (0-5%) Au-Au (Pb-Pb) collisions from event generators, UrQMD, EPOS and AMPT. The dynamical multiplicity fluctuations ($\omega_{ch,dyn}$) are obtained after subtracting the statistical fluctuations from participant model.

cussed in the previous section. The dynamical components, $\omega_{ch,dyn}$, are obtained after subtracting the statistical fluctuations, and are also shown in the Fig. 2. In all cases, the dynamical multiplicity fluctuations decrease with the increase of high collision energy to $\sqrt{s_{\rm NN}} > 62.4$ GeV, beyond which the fluctuations are close to zero.

5. $k_{\rm T}$ from HRG model

The values of $k_{\rm T}$ can be obtained by employing a hadron resonance gas model, which is based on a list of majority of the hadrons and their resonances as per the Particle Data Book [54]. It works within the framework of a multiple species non-interacting ideal gas in complete thermal and chemical equilibrium [55, 56, 57]. The HRG model has been found to provide a good description of the mean hadron yields using a few thermodynamic parameters at freeze-out (for a recent compilation of the freeze-out parameters, see Ref. [58]). The goal in the HRG model calculation is to obtain $k_{\rm T}$ directly from eqn. 1, where instead of total number of charged particles, the attempt has been to calculate in terms of species dependence (i) of the hadrons. The differential for the pressure $P(T, \{\mu_i\})$ can be written as,

$$dP = \left(\frac{\partial P}{\partial T}\right) dT + \sum_{i} \left(\frac{\partial P}{\partial \mu_{i}}\right) d\mu_{i},\tag{7}$$

and so:

$$\left. \left(\frac{\partial P}{\partial V} \right) \right|_{T, \{\langle N_i \rangle\}} = \sum_{i} \left(\frac{\partial P}{\partial \mu_i} \right) \left(\frac{\partial \mu_i}{\partial V} \right) \right|_{T, \{\langle N_i \rangle\}}.$$
 (8)

While the first factor is straightforward to compute from the expression for P, the second factor $\left(\frac{\partial \mu_i}{\partial V}\right)\Big|_{T,\{\langle N_i\rangle\}}$ is obtained from the condition of constancy of N_i as follows,

$$dN_i = \left(\frac{\partial N_i}{\partial T}\right) dT + \left(\frac{\partial N_i}{\partial V}\right) dV + \left(\frac{\partial N_i}{\partial \mu_i}\right) d\mu_i. \tag{9}$$

For fixed N_i , the above equation becomes,

$$\left. \left(\frac{\partial \mu_i}{\partial V} \right) \right|_{T, \{\langle N_i \rangle\}} = -\frac{\left(\frac{\partial N_i}{\partial V} \right)}{\left(\frac{\partial N_i}{\partial \mu_i} \right)}. \tag{10}$$

Within HRG, $\frac{\partial N}{\partial V} = \frac{\partial P}{\partial u}$. Thus, Eq. 8 becomes

$$\left. \left(\frac{\partial P}{\partial V} \right) \right|_{T, \left\{ \langle N_i \rangle \right\}} = -\sum_{i} \frac{\left(\frac{\partial P}{\partial \mu_i} \right)^2}{\left(\frac{\partial N_i}{\partial \mu_i} \right)} \tag{11}$$

which is used to get k_T using Eq. 1,

$$k_T|_{T,\{\langle N_i\rangle\}} = \frac{1}{V} \frac{1}{\sum_i \left(\frac{\partial P}{\partial u_i}\right)^2 / \left(\frac{\partial N_i}{\partial u_i}\right)}.$$
 (12)

This prescription of the HRG model has been used to calculate $k_{\rm T}$ for Au-Au collisions as a function of collision energy, which are presented in terms of the solid curve in Fig. 3. With the increase of collision energy, the values of $k_{\rm T}$ decrease up to $\sqrt{s_{\rm NN}}$ =20 GeV. However, at higher energies, $k_{\rm T}$ remains almost constant. This follows primarily from the behaviour of chemical freezeout temperature as a function of collision energy.

6. Compilation of $k_{\rm T}$

Finally, the values of $k_{\rm T}$ are calculated from the available experimental data and event generators using the dynamical fluctuations, $\omega_{\rm ch,dyn}$, which are presented in the figures 1 and 2. The mean charged particle multiplicities are obtained under the same kinematic conditions. The calculation of $k_{\rm T}$ requires temperature and volume, which are obtained from different sets of measurements. The chemical freezeout temperature ($T_{\rm ch}$) and the corresponding volume of the system have been obtained by fitting the measured identified particle yields using thermal model calculations [57, 58, 59, 60, 61, 62]. For the calculation of $k_{\rm T}$, both $T_{\rm ch}$ and V have been obtained from Ref. [58].

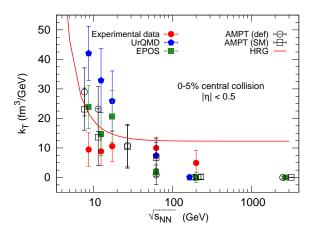


Figure 3: Isothermal compressibility, $k_{\rm T}$, as a function of $\sqrt{s_{\rm NN}}$ for available experimental data for central (0-5%) Au-Au (Pb-Pb) collisions. Results for three event generators are presented. Results from HRG calculations are superimposed.

A compilation of $k_{\rm T}$ as a function of $\sqrt{s_{\rm NN}}$ for central Au-Au (Pb-Pb) collisions is presented in Fig. 3. In the absence of experimental data at the LHC, calculations from AMPT and EPOS have been presented. From the available experimental data, it is observed that, $k_{\rm T}$ remains almost constant within the assigned errors. The results from the event generators are seen to decrease with an increase in the collision energy and remain constant at higher energies. The results from HRG calculations show a sharp decrease in $k_{\rm T}$ at low collision energies. Thus more experimental data points at collision energies below $\sqrt{s_{\rm NN}} \sim 20$ GeV are needed to validate our findings.

The extraction of k_T may be affected by several sources of uncertainty. The evaluation of the statistical component of the fluctuation poses one of the largest uncertainties. We have used a participant model calculation to obtain the $\omega_{ch,stat}$ based on the Glauber type of initial conditions. Another effect which affects the charged particle production is the resonance decay of particles. This is studied for Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV and Pb-Pb collisions at $\sqrt{s_{\rm NN}}$ = 2.76 TeV using AMPT and EPOS event generators by turning off and on the higher order resonances. The differences between the two cases are very small and within the errors, implying that resonance decay effects are negligible for multiplicity fluctuations. Other sources of fluctuations which affect the extraction of $\omega_{ch,dyn}$ include uncertainty in the initial state fluctuations and fluctuations in the amount of stopping. In view of the uncertainties from different sources which could not be considered presently, the extracted values are the upper limits of k_T .

7. Summary

We have studied the isothermal compressibility of the system formed at the time of chemical freeze-out in relativistic nuclear collisions for $\sqrt{s_{\rm NN}}$ from 7.7 GeV to 2.76 TeV. We have shown that $k_{\rm T}$ is related to the fluctuation in particle multiplicity in the central rapidity region. Multiplicity fluctuations have been obtained from available experimental data and event generators. The dynamical fluctuations are extracted from the total fluctuations by subtracting the statistical components using contributions from the number of participating nucleons. For the calculation of $k_{\rm T}$, the temperature and volume were taken from the thermal model fits of the measured particle yields at the chemical freeze-out. Within quoted errors, the values of $k_{\rm T}$ from the experimental data remain almost constant as a function of energy. Using the event generators, we have seen that k_T decreases with an increase of the collision energy. The estimation of $k_{\rm T}$ presented in the present manuscript relies on several assumptions, most importantly on the estimation of dynamical fluctuations. The results of k_T represent the upper limits because of unknown contributions to the statistical components.

We have calculated the values of $k_{\rm T}$ from the HRG model for a wide range of collision energy. With the increase of collision energy, $k_{\rm T}$ values decrease up to $\sqrt{s_{\rm NN}} \sim 20$ GeV, beyond which the $k_{\rm T}$ remain almost constant. The nature of $k_{\rm T}$ as a function of collision energy is similar to what has been observed for c_v [18]. A higher value of $k_{\rm T}$ at low energies compared to higher energies indicates that the collision system is more compressible at the lower energies. This study gives a strong impetus for the second phase of the beam energy scan program of RHIC and planned experiments at FAIR and NICA.

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