MSSM Higgs sector with dimension-five and dimension-six operators

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An effective approach to understanding the nature of new physics beyond the SM or the MSSM models is presented. We then apply this method to discuss the extension of the MSSM Higgs sector with effective operators of dimension $d = 5$ and $d = 6$, that may be generated by "new physics" beyond this sector. The implications for the value of the mass of the MSSM CP even/odd Higgses are also discussed, to order $1/M_*^2$ where M_* is the scale of new physics that generated these operators. The results show that one can classically increase the Higgs mass m_h above the current LEP bound, to reduce the fine tuning of the MSSM at large m_h. The origin of the effective operators with the largest corrections to m_h is briefly discussed. We extend the discussion beyond the Higgs sector, to analyse briefly all operators of $d = 5$ that respect R-parity, B and L numbers conservations and identify the minimal, irreducible set of such operators. We then point out the existence, at the tree level, of new "wrong"-Higgs Yukawa couplings, similar to those generated in the MSSM at one-loop.

1 Introduction

Although extremely successful, the Standard Model or its supersymmetric version (MSSM) is not a fundamental theory, and this motivated the theoretical efforts to understand the nature of new physics beyond it. This search can be done using an effective field theory approach, in which the "new physics" is parametrised by effective operators. The power of this approach resides in arranging these operators in powers of $1/M_*$ where M_* is the scale of new physics that generated them. To improve the predictive power, one considers additional organising principles, such as: **(i)** symmetry constraints that these operators should respect, often inspired by phenomenology (for example: R-parity, lepton or baryon number conservation, etc). **(ii)** a truncation of the series of operators to a given order in the power of the inverse scale $1/M_*$. The effective low-energy Lagrangian then takes the form

$$\mathcal{L} = \mathcal{L}_0 + \sum_{i,n} \frac{c_{n,i}}{M_*^n} \mathcal{O}_{n,i} \tag{1}$$

where $\mathcal{L}_0$ is the SM or the MSSM Lagrangian; $\mathcal{O}_{n,i}$ is an operator of dimension $d = n + 4$ with the index i running over the set of operators of a given dimension; $c_{n,i}$ are some coefficients of order $\mathcal{O}(1)$. This description is appropriate for scales E which satisfy $E \ll M_*$. Constraints from phenomenology can then be used to set bounds on the scale of new physics M_*.

Regarding the origin of operators $\mathcal{O}_{n,i}$, they can be generated classically or at the quantum level. At the classical level, this can happen by integration of some new massive states, via the equations of motion and one then generates an infinite series as in (1). This can happen even in 4D renormalisable theories; indeed, even though the low energy interaction looks nonrenormalisable, it may actually point to a renormalisable theory valid up to a much higher scale (a familiar example is the Fermi interaction). Such effective operators are also generated at the quantum level, for example following compactification of a higher dimensional theory, by the radiative corrections associated with momentum and winding modes of the compactification [1,2,3,4,5].

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The effects of these operators on the low energy observables can be comparable to the radiative effects of light states in the SM/MSSM[6] and this shows the importance of their study. In the following we shall provide some interesting details regarding the structure and treatment of these operators. We then apply these to more detailed examples, such as the case of the MSSM Higgs sector with additional operators of dimensions $d = 5$ and $d = 6$. In particular we show that the mass of lightest SM-like Higgs can easily be increased above the LEP bound by new physics in the region of few TeV. We then discuss the nature of the "new physics" behind the effective operators.

2 General considerations on models with higher dimensional operators

Consider a supersymmetric Lagrangian, with a Kähler potential K, the superpotential W and the gauge kinetic function f, all functions of chiral superfields Φ_i:

$$\mathcal{L} = \int d^4\theta\, K(\Phi_i^\dagger\, e^V, \Phi^i) + \int d^2\theta \Big[W(\Phi_i) + f_{ab}(\Phi_i)\ \mathcal{W}^a\, \mathcal{W}^b \Big] + h.c. \tag{2}$$

$\mathcal{W}^a$ is the supersymmetric gauge field strength associated to the vector superfield V^a. The presence of effective operators is hidden in the series expansion (in fields) of these functions:

$$\begin{aligned} K &= \Phi_i^\dagger\, e^V\, \Phi^i + \left[\frac{c^i_{jk}}{M_*}\, \Phi_i^\dagger e^V \Phi^j \Phi^k + h.c. \right] + \cdots \\ W &= \lambda_{ijk}\ \Phi^i\Phi^j\Phi^k + \frac{c_{ijkl}}{M_*}\ \Phi^i\Phi^j\Phi^k\Phi^l + \cdots \\ f_{ab} &= \delta_{ab} + \frac{f_{abi}}{M_*}\ \Phi^i + \cdots \end{aligned} \tag{3}$$

The first terms in the rhs would lead to a renormalisable theory. These functions can contain not only operators which are polynomial in fields, but also operators that can involve more than two (one for fermions) space-time derivatives acting on physical fields. For example one can have derivative operators in the superpotential (a) and Kähler potential (b):

$$\begin{aligned} (a)\quad & \frac{\lambda_{ij}}{M_*} \int d^2\theta\, \Phi_i\, \Box\, \Phi_j\ \sim \frac{\lambda_{ij}}{M_*} \int d^4\theta\, \Phi_i\, D^2\, \Phi_j \\ (b)\quad & \frac{k_{ij}}{M_*^2} \int d^4\theta\ \Phi_i^\dagger\, \Box\, \Phi_j,\ \ \frac{k_{ij}}{M_*^2} \int d^4\theta\ \Phi_i^\dagger\, \Phi_j\, D^2\, \Phi_k, ... \end{aligned} \tag{4}$$

where D is the chiral supercovariant derivative. In (a), terms like $\psi\Box\psi$ and $F\Box\phi$ are generated, where $\Phi = \phi + \sqrt{2}\theta\, \psi + \theta^2\, F$. In (b) one finds terms like $|\Box\phi|^2$, $\overline{\psi}\partial_\mu\Box\psi$, $F^\dagger\Box F$. In this case the auxiliary fields become dynamical degrees of freedom. Such operators can be generated even in a 4D supersymmetric *renormalisable* theory. To see this, consider the Lagrangian

$$\mathcal{L} = \int d^4\theta \left[\Phi^\dagger\Phi + \chi^\dagger\chi \right] + \int d^2\theta \Big[m\, \Phi\chi + \frac{M_*^2}{2}\, \chi^2 + \frac{\lambda}{3}\Phi^3 \Big] + h.c. \tag{5}$$

Integrating out the massive field χ by the eqs of motion, one obtains

$$\mathcal{L} = \int d^4\theta \left[\Big(1 + \frac{m^2}{M_*^2} \Big) \Phi^\dagger\, \Phi + \frac{m^2}{M_*^4}\, \Phi^\dagger\Box\Phi + \cdots \right] + \int d^2\theta \left[\frac{-m^2}{2\, M_*}\, \Phi^2 + \frac{\lambda}{3}\Phi^3 + \frac{m^2}{2\, M_*^3} \Phi\Box\Phi \right] + h.c. \tag{6}$$

If one keeps all the terms in the series expansion above, the theory is ghost-free, because the initial theory was so; the effective field theory (6) is valid only below the scale M_*.

From this discussion one natural question emerges: can one reformulate a supersymmetric theory of higher order (with effective operators with extra derivatives), in terms of a standard, second order theory (with at most two derivatives (one for fermions))? As we shall see shortly, the answer is in many cases affirmative. Let us see how this works.

Effective operators in the Kähler function.

Consider the following general Lagrangian, where W is polynomial in fields, but otherwise arbitrary (and not necessarily renormalisable):

$$\mathcal{L} = \int d^4\theta \left[\Phi^\dagger \left(1 + \Box/M_*^2\right)\Phi + \chi^\dagger \chi\right] + \left\{\int d^2\theta\; W[\Phi;\chi] + h.c.\right\} + \mathcal{O}(1/M_*^3) \tag{7}$$

where one can replace $\Phi^\dagger \Box \Phi \rightarrow (-1/16)\overline{D}^2 \Phi^\dagger D^2 \Phi$. This model can be "unfolded" into a Lagrangian without extra derivatives [7] (see also the appendix in [8]). For this, consider a change of basis to $\Phi_{1,2}$:

$$\Phi = s_1 \Phi_1 + s_2 \Phi_2, \qquad (1/m)\,\overline{D}^2 \Phi^\dagger = r_1 \Phi_1 + r_2 \Phi_2 \tag{8}$$

where $s_{1,2}$, $r_{1,2}$ form an unitary matrix, so that the eigenvalue problem is not changed; m is a very small, non-zero mass scale of the theory that can be taken to zero at the end of calculation. Since Φ and $\overline{D}^2\Phi$ are not independent, such transformation must be accompanied by a Lagrangian constraint, which must vanish in the limit $M_* \rightarrow \infty$. This constraint has the form:

$$\delta\mathcal{L} = \int d^2\theta \left[(1/m)\,\overline{D}^2 (s_1\,\Phi_1 + s_2\,\Phi_2)^\dagger - (r_1 \Phi_1 + r_2\,\Phi_2)\right] \Phi_3 \left(\frac{m^2}{4\,M_*}\right) \tag{9}$$

Φ_3 is a chiral superfield (Lagrange multiplier). After we bring $\mathcal{L}' \equiv \mathcal{L} + \delta\mathcal{L}$ to a diagonal form

$$\mathcal{L}' = \int d^4\theta \left[\tilde{\Phi}_1^\dagger \tilde{\Phi}_1 - \tilde{\Phi}_2^\dagger \tilde{\Phi}_2 - \tilde{\Phi}_3^\dagger \tilde{\Phi}_3 + \chi^\dagger \chi\right] + \left\{\int d^2\theta \left[W[\Phi(\tilde{\Phi}_{1,2});\chi] - M_* \tilde{\Phi}_2 \tilde{\Phi}_3\right] + h.c.\right\} + \mathcal{O}(1/M_*^3) \tag{10}$$

where $\Phi = \tilde{\Phi}_2 - \tilde{\Phi}_1$ and where we took the limit $m \rightarrow 0$.

This Lagrangian is that of a second order theory and classically equivalent to the initial one in (7). It shows that two massive ghosts ($\tilde{\Phi}_{2,3}$) are present, of mass of $\mathcal{O}(M_*)$; we observe that the χ field was "spectator" throughout this analysis and did not affect it; in fact the χ-dependence can be replaced by an arbitrary polynomial function. This reformulation of the initial theory into a second-order one has interesting advantages and applications. In particular it would be interesting to apply this technique to the case of higher derivative gravity. Particular attention should be paid to the analytical continuation (pole prescription) of the theory, when going to the "unfolded" form.

This is not the whole story; since the ghost degrees of freedom are massive (of order M_*), one can integrate them out, by the equations of motion. After a careful calculation and consistent Taylor expansion, the result is (see also the appendix in [8])

$$\mathcal{L}' = \int d^4\theta \left[\tilde{\Phi}_1^\dagger \tilde{\Phi}_1 - \frac{1}{M_*^2} W'^\dagger[\tilde{\Phi}_1;\chi] W'[\tilde{\Phi}_1;\chi] + \chi^\dagger \chi\right] + \left\{\int d^2\theta\, W[\tilde{\Phi}_1;\chi] + h.c.\right\} + \mathcal{O}(1/M_*^3) \tag{11}$$

where the derivatives of W are taken wrt its first argument. This Lagrangian contains only polynomial interactions (renormalisable or not) and standard kinetic terms, and is equivalent to the original one, eq.(7). As a side-remark, let us mention that this result agrees with that obtained by using the equations of motion in the derivative term in (7). This is not true in general, as we shall see shortly.

Effective operators in the superpotential.

We extend the previous discussion to similar effective operators in the superpotential, which have now dimension $d=5$. Consider for example the following Lagrangian

$$\begin{aligned}\mathcal{L} &= \int d^4\theta \Big[\Phi^\dagger \Phi + \chi^\dagger \chi\Big] + \Big\{ \int d^2\theta \Big[\frac{\sigma}{M_*} \Phi \,\Box\, \Phi + W[\Phi;\chi]\Big] + h.c.\Big\} \\ &= \int d^4\theta \Big[\Phi^\dagger \Phi + \frac{\sigma}{4M_*}(\Phi D^2 \Phi + h.c.)\Big] + \Big\{ \int d^2\theta \; W[\Phi;\chi] + h.c.\Big\} + \mathcal{O}(1/M_*^3) \qquad (12)\end{aligned}$$

with $\sigma = \pm 1$ and where W can contain additional higher dimensional (polynomial) interactions. The method presented earlier works identically, to find the "unfolded" Lagrangian [7]

$$\mathcal{L}' = \int d^4\theta \left[\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2 + \chi^\dagger \chi\right] + \left\{ \int d^2\theta \left[\frac{\sigma M_*}{4}\Phi_2^2 + W[\Phi_2 - \Phi_1;\chi]\right] + h.c.\right\} + \mathcal{O}(1/M_*^3) \quad (13)$$

There is only one ghost superfield here, because unlike earlier, the auxiliary field is not dynamical. We integrate out the massive ghost superfield which has a mass $\mathcal{O}(M_*)$, to find

$$\mathcal{L}' = \int d^4\theta \left[\Phi_1^\dagger \Phi_1 - \frac{4}{M_*^2} W' \, W'^\dagger + \chi^\dagger \chi\right] + \left\{ \int d^2\theta \left[W - \frac{\sigma W'^2}{M_*} + \frac{2}{M_*^2} W'' W'^2\right] + h.c.\right\} + \mathcal{O}(1/M_*^3) \quad (14)$$

where the argument of W, W', W'' above is $[-\Phi_1;\chi]$ and derivatives are taken wrt the first argument. This is equivalent to the starting Lagrangian (12) and has new interactions, but only polynomial in fields. A similar result is obtained by using non-linear field redefinitions [8]. Note that the use of equations of motion in original $\mathcal{L}$ to remove the higher derivatives is not leading to an identical result at $\mathcal{O}(1/M_*^2)$, but gives a result where in (14) the coefficients 4 (2) are replaced by 3 (3/2). The reason for this discrepancy is that this last method does not take into account that Euler-Lagrange equations are changed in the higher order theory (see also [9,10,11]), so in our case it gave correct results valid in $\mathcal{O}(1/M_*^2)$ order only.

To conclude, the presence of effective operators with extra space-time derivatives can be replaced, in the low energy effective theory, by additional polynomial interactions and wavefunction renormalisations. The examples discussed were exactly supersymmetric; when supersymmetry is broken, new effects are present, such as soft terms and μ term renormalisation [6].

3 MSSM with d=5 operators.

As an application, consider the MSSM extended by all possible $d=5$ operators that respect R-parity, baryon and lepton number symmetry. The Lagrangian is $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}^{(5)}$ where

$$\begin{aligned}\mathcal{L}_0 &= \int d^4\theta \sum_{i=1,2} Z_i(S,S^\dagger)\, H_i^\dagger \, e^{V_i}\, H_i + \Big\{ \int d^2\theta \; \mu_0\,(1 + B_0\, S)\, H_1.H_2 + h.c.\Big\} + \cdots \\ &+ \int d^2\theta \Big[Q\,\lambda_U(S)\, U^c\, H_2 - Q\,\lambda_D(S)\, D^c\, H_1 - L\,\lambda_E(S)\, E^c\, H_1\Big] + h.c. \qquad (15)\end{aligned}$$

The dots stand for Higgs-independent terms and $Z_i(S,S^\dagger) = 1 - c_i\, S^\dagger S$, $c_i \sim \mathcal{O}(1)$. Further

$$\begin{aligned}\mathcal{L}^{(5)} &= \frac{1}{M_*}\int d^4\theta \Big[H_1^\dagger\, e^{V_1} Q\, Y_U U^c + H_2^\dagger\, e^{V_2} Q Y_D D^c + H_2{}^\dagger\, e^{V_2} L\, Y_E E^c \qquad (16) \\ &+ a\, D^\alpha\big[b\, H_2 e^{-V_1}\big] D_\alpha\big[c\, e^{V_1} H_1\big] + \delta(\overline{\theta}^2)\big[QU^c\, T_Q\, QD^c + QU^c\, T_L\, LE^c + \lambda_H (H_1 H_2)^2\big] + h.c.\Big]\end{aligned}$$

with a standard notation. We introduced here some spurion dependent functions: $a, b, c, Z_{1,2}, Y_F$, $F = U, D, E$, which are general functions of $(S, S^\dagger)$ while T_Q, T_L, λ_H, are holomorphic functions of S. Here $S = m_0\theta^2$ is the spurion superfield and m_0 the supersymmetry breaking scale, with $m_0 = \langle F_{hidden}\rangle/M_P$, so supersymmetry breaking is transmitted via gravitational interaction. Any supersymmetry breaking associated with the presence of the above interactions is included using the spurion field technique. Not all operators in (16) are independent [6]. To remove the redundant operators we use field re-definitions:

$$
\begin{aligned}
H_1 &\;\rightarrow\; H_1 - \frac{1}{M_*}\,\overline{D}^2\left[\Delta_1\, H_2^\dagger\, e^{V_2}\,(i\,\sigma_2)\right]^T + \frac{1}{M_*}\,Q\,\rho_U\,U^c \\
H_2 &\;\rightarrow\; H_2 + \frac{1}{M_*}\,\overline{D}^2\left[\Delta_2\, H_1^\dagger\, e^{V_1}\,(i\sigma_2)\right]^T + \frac{1}{M_*}\,Q\,\rho_D\,D^c + \frac{1}{M_*}\,L\,\rho_E\,E^c
\end{aligned}
\tag{17}
$$

where $\rho_F = \rho_F(S)$, $F = U, D, E$, $\Delta_i = \Delta_i(S, S^\dagger)$, $i = 1, 2$, can be chosen arbitrarily. To avoid the presence of flavour changing neutral currents, the following simple ansatz can be made:

$$
\begin{aligned}
T_Q(S) &= c_Q(S)\,\lambda_U(0)\otimes\lambda_D(0), \qquad T_L(S) = c_L(S)\,\lambda_U(0)\otimes\lambda_E(0), \\
\rho_F(S) &= c_F(S)\,\lambda_F(0), \qquad\qquad Y_F(S,S^\dagger) = y_F(S,S^\dagger)\,\lambda_F(0), \quad F = U, D, E.
\end{aligned}
\tag{18}
$$

and, as usual $\lambda_F(S) = \lambda_F(0)\,(1 + A_F\,S)$. Using a suitable choice for the (otherwise arbitrary) coefficients of the spurion entering in $\Delta_{1,2}$, one can set $T_Q = T_L = 0$ also $a = b = c = 0$ and $Y_F \rightarrow y_F(S^\dagger)\,\lambda_F(0)$. Then one finds [6]

$$
\begin{aligned}
\mathcal{L}^{(5)} &= \frac{1}{M_*}\int d^4\theta\left[H_1^\dagger\,e^{V_1}\,Q\,Y_U'(S^\dagger)\,U^c + H_2^\dagger\,e^{V_2} Q\,Y_D'(S^\dagger)\,D^c + H_2^\dagger\,e^{V_2}\,L\,Y_E'(S^\dagger)\,E^c + h.c.\right] \\
&+ \frac{1}{M_*}\int d^2\theta\;\lambda_H'(S)\,(H_1\,H_2)^2 + h.c.
\end{aligned}
\tag{19}
$$

Detailed calculations show[6] that the new Yukawa couplings $Y_F'(S^\dagger)$ now depend on $S^\dagger$ only, $Y_F'(S^\dagger) = \lambda_F(0)\,(x_0^F + x_2^F\,S^\dagger)$. After (17), the couplings of $\mathcal{L}_0$ also acquired, at tree level, threshold corrections which depend on M_* [6]. The new form of $\mathcal{L}^{(5)}$ in (19) gives the minimal irreducible set of R-parity, B, L conserving $d{=}5$ operators that can be present beyond MSSM.

A consequence of this analysis is the generation of new couplings, beyond those in the MSSM at the tree level. For example there is a "wrong"-Higgs Yukawa coupling, that exchanges the holomorphic dependence on one Higgs by that on the hermitian conjugate of the other [12,13]. Such couplings do arise in the MSSM at one-loop, after integrating out massive squarks and are suppressed by $m_0^2/M_*^2\times$(loop-factor). Here they are suppressed by m_0/M_* only, as seen below:

$$
\frac{M_s}{M_*}\left[x_2^U\,[\lambda_U(0)]_{ij}\,(h_1^\dagger\,q_{L\,i})\;u_{R\,j}^c + x_2^D\,[\lambda_D(0)]_{ij}\,(h_2^\dagger\,q_{L\,i})\;d_{R\,j}^c + x_2^E\,[\lambda_E(0)]_{ij}\,(h_2^\dagger\,l_{L\,i})\;e_{R\,j}^c + h.c.\right] \tag{20}
$$

These couplings bring a $\tan\beta$ enhancement of a prediction for a physical observable, such as the bottom quark mass, relative to bottom quark Yukawa coupling:

$$
m_b = (1/\sqrt{2})\,v\cos\beta\,\left(\lambda_b + \delta\lambda_b + \Delta\lambda_b\tan\beta\right) \tag{21}
$$

Here λ_b is the usual bottom quark Yukawa coupling, $\delta\lambda_b$ is its one-loop correction in MSSM and $\Delta\lambda_b$ is a "wrong"-Higgs coupling' corrections, obtained after integrating out at one-loop massive squarks in MSSM; in our case $\Delta\lambda_b$ receives an extra correction from (20), which can actually be larger than its one-loop-generated MSSM counterpart [12,14,15,16]. This can bring a $\tan\beta$ enhancement of the Higgs decay rate into bottom quarks pairs (for further details see [6]).

4 MSSM Higgs sector with d=5 and d=6 operators

We can extend the previous discussion by including all effective operators of both dimension $d = 5$ and $d = 6$ that can exist beyond the MSSM Higgs sector. This can be motivated in various ways. The MSSM Higgs sector is a minimal construction and extension of that of the SM. It does not take into account possible non-perturbative effects [17] or additional massive states that can couple to the Higgs sector and generate, when integrated out, new contributions. The fine tuning [18,19] needed to have the SM-like Higgs mass well above the LEP bound [20] can also be a problem and it may indicate the existence of new physics beyond the Higgs sector. Such problems may be addressed by using a model-independent approach, using the effective operators. In the leading order, new physics beyond the MSSM Higgs sector can manifest itself as operators of either $d = 5$ [6,21,22,23] or $d = 6$ [8,24] or both. If generated by the same new physics, by comparing $\mathcal{O}(1/M^*)$ and $\mathcal{O}(1/M_*^2)$ terms one can estimate when the series expansion in $1/M_*$ breaks down. The operators in the Higgs sector of dimension $d = 5$ were:

$$\begin{aligned}
\mathcal{L}_1 &= \frac{1}{M_*}\int d^2\theta\ \lambda'_H(S)\,(H_2.H_1)^2 + h.c. = 2\,\zeta_{10}\,(h_2.h_1)(h_2.F_1 + F_2.h_1) + \zeta_{11}\,m_0\,(h_2.h_1)^2 + h.c, \\
\mathcal{L}_2 &= \frac{1}{M_*}\int d^4\theta\ \Big\{ a(S,S^\dagger) D^\alpha \Big[b(S,S^\dagger)\,H_2\,e^{-V_1}\Big] D_\alpha \Big[c(S,S^\dagger)\,e^{V_1}\,H_1\Big] + h.c.\Big\}
\end{aligned} \tag{22}$$

where

$$\lambda'_H(S)/M_* = \zeta_{10} + \zeta_{11}\,m_0\,\theta\theta, \qquad \zeta_{10},\,\zeta_{11} \sim 1/M_*, \tag{23}$$

$\mathcal{L}_1$ can be generated by integrating out a massive gauge singlet or $SU(2)$ triplet. Indeed, in the MSSM with a massive gauge singlet, with an F-term of type $M_*\Sigma^2 + \Sigma\, H_1.H_2$, when integrating out Σ generates $\mathcal{L}_1$. $\mathcal{L}_2$ can be generated in various ways (see Appendix A, B in [6]) but perhaps the simplest way is via an additional pair of massive Higgs doublets of mass of order M_*. As already discussed, $\mathcal{L}_2$ can be removed by general spurion-dependent field redefinitions, up to soft terms and μ term renormalisation and $\mathcal{O}(1/M_*^2)$ corrections [6].

We assume that $m_0 \ll M_*$, so that the effective approach is reliable. If this is not respected and the "new physics" is represented by "light" states (like the MSSM states), the $1/M_*$ expansion is not reliable and one should work in a setup where these are not integrated out.

The list of $d = 6$ operators is longer [25]:

$$\begin{aligned}
\mathcal{O}_1 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_1\ (H_1^\dagger\, e^{V_1}\, H_1)^2, &
\mathcal{O}_5 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_5\ (H_1^\dagger\, e^{V_1}\, H_1)\ H_2.\,H_1 + h.c. \\
\mathcal{O}_2 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_2\ (H_2^\dagger\, e^{V_2}\, H_2)^2, &
\mathcal{O}_6 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_6\ (H_2^\dagger\, e^{V_2}\, H_2)\ H_2.\,H_1 + h.c. \\
\mathcal{O}_3 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_3\ (H_1^\dagger\, e^{V_1}\, H_1)\,(H_2^\dagger\, e^{V_2}\, H_2), &
\mathcal{O}_7 &= \frac{1}{M_*^2}\int d^2\theta\, \mathcal{Z}_7\, \mathrm{Tr}\, W^\alpha\, W_\alpha\,(H_2\, H_1) + h.c. \\
\mathcal{O}_4 &= \frac{1}{M_*^2}\int d^4\theta\, \mathcal{Z}_4\ (H_2.\,H_1)\,(H_2.\,H_1)^\dagger, &
\mathcal{O}_8 &= \frac{1}{M_*^2}\int d^4\theta\, \mathcal{Z}_8\ (H_2\, H_1)^2 + h.c.
\end{aligned} \tag{24}$$

where $W^\alpha = (-1/4)\,\overline{D}^2 e^{-V} D^\alpha\, e^V$ is the chiral field strength of $SU(2)_L$ or $U(1)_Y$ vector superfields V_w and V_Y respectively. Also $V_{1,2} = V_w^a(\sigma^a/2) + (\mp 1/2)\,V_Y$ with the upper (minus) sign for V_1. The remaining $d = 6$ operators involve extra space-time derivatives:

$$\mathcal{O}_9 = \frac{1}{M_*^2} \int d^4\theta \; \mathcal{Z}_9 \, H_1^\dagger \, \overline{\nabla}^2 \, e^{V_1} \, \nabla^2 \, H_1 \qquad \mathcal{O}_{12} = \frac{1}{M_*^2} \int d^4\theta \; \mathcal{Z}_{12} \, H_2^\dagger \, e^{V_2} \, \nabla^\alpha \, W_\alpha^{(2)} \, H_2$$
$$\mathcal{O}_{10} = \frac{1}{M_*^2} \int d^4\theta \; \mathcal{Z}_{10} \, H_2^\dagger \, \overline{\nabla}^2 \, e^{V_2} \, \nabla^2 \, H_2 \qquad \mathcal{O}_{13} = \frac{1}{M_*^2} \int d^4\theta \; \mathcal{Z}_{13} \, H_1^\dagger \, e^{V_1} \; W_\alpha^{(1)} \, \nabla^\alpha \, H_1$$
$$\mathcal{O}_{11} = \frac{1}{M_*^2} \int d^4\theta \; \mathcal{Z}_{11} \, H_1^\dagger \, e^{V_1} \, \nabla^\alpha \, W_\alpha^{(1)} \, H_1 \qquad \mathcal{O}_{14} = \frac{1}{M_*^2} \int d^4\theta \; \mathcal{Z}_{14} \, H_2^\dagger \, e^{V_2} \; W_\alpha^{(2)} \, \nabla^\alpha \, H_2 \quad (25)$$

Also $\nabla_\alpha H_i = e^{-V_i} D_\alpha e^{V_i} H_i$ and W_α^i is the field strength of V_i. To be general, in the above operators one should include spurion (S) dependence under any ∇_α, of arbitrary coefficients, to account for supersymmetry breaking effects associated to them. Finally, the wavefunction coefficients are spurion dependent and have the structure

$$(1/M_*^2)\, \mathcal{Z}_i(S, S^\dagger) = \alpha_{i0} + \alpha_{i1}\, m_0\, \theta\theta + \alpha_{i1}^*\, m_0\, \overline{\theta\theta} + \alpha_{i2}\, m_0^2\, \theta\theta\overline{\theta\theta}, \qquad \alpha_{ij} \sim 1/M_*^2. \tag{26}$$

Regarding the origin of these operators: $\mathcal{O}_{1,2,3}$ can be generated in the MSSM by an additional, massive $U(1)'$ gauge boson or $SU(2)$ triplets, when integrated out [21]. $\mathcal{O}_4$ can be generated by a massive gauge singlet or $SU(2)$ triplet, while $\mathcal{O}_{5,6}$ can be generated by a combination of $SU(2)$ doublets and massive gauge singlet. $\mathcal{O}_7$ is essentially a threshold correction to the gauge coupling, with a moduli field replaced by the Higgs. $\mathcal{O}_8$ exists only in non-susy case, but is generated when removing the $d = 5$ derivative operator $\mathcal{L}_2$ by field redefinitions [6], so we keep it.

It can be shown that operators $\mathcal{O}_{9,\ldots,14}$, can be eliminated along the lines discussed in the previous sections. For example, in the absence of gauge interactions, $\mathcal{O}_9$ is similar to the operator in eq.(7) and only brings a wavefunction renormalisation, $\mathcal{O}_9 \sim |\mu|^2/M_*^2 \int d^4\theta H_1^\dagger H_1$, and similar for $\mathcal{O}_{10}$. Regarding $\mathcal{O}_{11,12}$, in the supersymmetric case they vanish, following the definition of ∇^α and an integration by parts. Further, $\mathcal{O}_{13,14}$ are similar to $\mathcal{O}_{9,10}$, which can be seen by using the definition of $W_\alpha^{(i)}$ and the relation between ∇^2, $(\overline{\nabla}^2)$ and D^2, $(\overline{D}^2)$. In the presence of supersymmetry breaking, elimination of these operators and their supersymmetry breaking contribution is still possible, up to a renormalisation of the soft terms and μ term [6].

5 Higgs mass corrections from $d = 5$ and $d = 6$ operators.

With the remaining set of independent, effective operators $\mathcal{L}_1$, $\mathcal{O}_{1,\ldots,8}$ of dimensions $d = 5$ and $d = 6$, one finds the scalar potential V and its EW minimum; this is perturbed by $\mathcal{O}(1/M_*^2)$ corrections from that of the MSSM. The expression of V is long and it is not given here (see [8] for its form). From V one computes the mass of CP-odd/even Higgs fields. One has:

$$m_A^2 = (m_A^2)_{\rm MSSM} - \frac{2\,\zeta_{10}\,\mu_0\, v^2}{\sin 2\beta} + 2\, m_0\, \zeta_{11}\, v^2 + \delta m_A^2, \quad \delta m_A^2 = \mathcal{O}(1/M_*^2) \tag{27}$$

for the pseudoscalar Higgs, with $(m_A^2)_{\rm MSSM}$ the MSSM value, with δm_A^2 due to $\mathcal{O}(1/M_*^2)$ corrections from $d = 5$ and $d = 6$ operators. For the CP-even Higgs one has [6,21,23]

$$\begin{aligned} m_{h,H}^2 &= (m_{h,H}^2)_{\rm MSSM} \\ &+ (2\,\zeta_{10}\,\mu_0)\, v^2\, \sin 2\beta \left[1 \pm \frac{m_A^2 + m_Z^2}{\sqrt{\tilde{w}}}\right] + \frac{(-2\,\zeta_{11}\, m_0)\, v^2}{2} \left[1 \mp \frac{(m_A^2 - m_Z^2)\, \cos^2 2\beta}{\sqrt{\tilde{w}}}\right] \\ &+ \delta m_{h,H}^2, \qquad \text{where} \qquad \delta m_{h,H}^2 = \mathcal{O}(1/M_*^2) \end{aligned} \tag{28}$$

The upper (lower) signs correspond to h (H), and $\tilde{w} \equiv (m_A^2 + m_Z^2)^2 - 4\, m_A^2\, m_Z^2\, \cos^2 2\beta$. With this result one can show that the mass m_h can be increased above the LEP bound, also with the help of quantum corrections [6,21,22,23].

Regarding the $\mathcal{O}(1/M_*^2)$ corrections of $\delta m_{h,H}^2, \delta m_A^2$ and $\delta m_{h,H}^2$ of (27), (28), in the general case of including all operators and their associated supersymmetry breaking, they have a complicated form. Exact expressions can be found in [8,24]. For most purposes, an expansion of these in $1/\tan\beta$ is accurate enough. At large $\tan\beta$, $d=6$ operators bring corrections comparable to those of $d=5$ operators. The relative $\tan\beta$ enhancement of $\mathcal{O}(1/M_*^2)$ corrections compensates for the extra suppression that these have relative to $\mathcal{O}(1/M_*)$ operators (which involve both h_1, h_2 and are not enhanced in this limit). Note however that in some models only $d=6$ operators may be present, depending on the details of the "new physics" generating the effective operators.

Let us present the correction $\mathcal{O}(1/M^2)$ to $m_{h,H}^2$ for the case m_A is kept fixed to an appropriate value. The result is, assuming $m_A > m_Z$, (otherwise δm_h^2 and δm_H^2 are exchanged):

$$\begin{aligned} \delta m_h^2 \;=\;& -2\, v^2 \Big[\alpha_{22}\, m_0^2 + (\alpha_{30}+\alpha_{40})\mu_0^2 + 2\alpha_{61}\, m_0\, \mu_0 - \alpha_{20}\, m_Z^2\Big] - (2\,\zeta_{10}\,\mu_0)^2\, v^4\, (m_A^2 - m_Z^2)^{-1} \\ & + v^2 \cot\beta \Big[(m_A^2 - m_Z^2)^{-1}\Big(4\, m_A^2\, \big((2\alpha_{21}+\alpha_{31}+\alpha_{41}+2\alpha_{81})\, m_0\, \mu_0 + (2\alpha_{50}+\alpha_{60})\,\mu_0^2 + \alpha_{62}\, m_0^2\big) \\ & - \;(2\alpha_{60}-3\alpha_{70})\, m_A^2\, m_Z^2 - (2\alpha_{60}+\alpha_{70})\, m_Z^4\Big) + 8\,(m_A^2+m_Z^2)\,(\mu_0\, m_0\, \zeta_{10}\, \zeta_{11})\, v^2/(m_A^2-m_Z^2)^2\Big] \\ & + \;\mathcal{O}(1/\tan^2\beta) \end{aligned} \tag{29}$$

A similar formula exists for the correction to m_H:

$$\begin{aligned} \delta m_H^2 \;=\;& \Big[-2\,\big(m_0\mu_0\,(\alpha_{51}+\alpha_{61}) + \alpha_{82}\big), m_0^2\big)\, v^2 + (2\,\zeta_{10}\,\mu_0)^2\, v^4 (m_A^2 - m_Z^2)^{-1}\Big] \\ & + v^2 \cot\beta \Big[(m_A^2-m_Z^2)^{-1}\Big(2 m_A^2 \big(2(\alpha_{11}-\alpha_{21})\, m_0\mu_0 + (\alpha_{60}-\alpha_{50})\,\mu_0^2 + (\alpha_{52}-\alpha_{62}) m_0^2 - \alpha_{60} m_A^2\big) \\ & - \;\big[4\,(\alpha_{11}+\alpha_{21}+\alpha_{31}+\alpha_{41}+2\alpha_{81})\, m_0\mu_0 + 6(\alpha_{50}+\alpha_{60})\,\mu_0^2 + 2(\alpha_{52}+\alpha_{62})\, m_0^2 \\ & - \;(\alpha_{50}+5\alpha_{60}-2\alpha_{70})\, m_A^2\big] m_Z^2 - (\alpha_{50}-\alpha_{60}) m_Z^4\Big) - 8\,(m_A^2+m_Z^2)(\mu_0 m_0 \zeta_{10}\zeta_{11})\, v^2/(m_A^2-m_Z^2)^2\Big] \\ & + \;\mathcal{O}(1/\tan^2\beta) \end{aligned} \tag{30}$$

The mass corrections in (29), (30) must be added to the rhs of eqs.(28) to obtain the full value of $m_{h,H}^2$. Together with (24), (26), these corrections identify the operators of $d=6$ with the largest contributions, which is important for model building beyond the MSSM Higgs sector. These operators are $\mathcal{O}_{2,3,4}$ in the absence of supersymmetry breaking and $\mathcal{O}_{2,6}$ when this is broken. It is preferable, however, to increase m_h^2 by supersymmetric rather than supersymmetry-breaking effects of the effective operators, because the latter are less under control in the effective approach; also, one would favour a supersymmetric solution to the fine-tuning problem associated with increasing the MSSM Higgs mass above the LEP bound. Therefore $\mathcal{O}_{2,3,4}$ are the leading operators, with the remark that $\mathcal{O}_2$ has a smaller effect, of order $(m_Z/\mu_0)^2$ relative to $\mathcal{O}_{3,4}$ (for similar α_{j0}, $j=2,3,4$). At smaller $\tan\beta$, $\mathcal{O}_{5,6}$ can also give significant contributions, while $\mathcal{O}_7$ has a relative suppression factor $(m_Z/\mu_0)^2$. Note that we kept all operators $\mathcal{O}_i$ independent. By doing so, one can easily single out the individual contribution of each operator, which helps in model building, since not all operators are present in a specific model.

One limit to consider is that where the operators of $d=6$ have coefficients such that their contributions add up to maximise δm_h^2. Since α_{ij} are not known, one can choose:

$$-\alpha_{22} = -\alpha_{61} = -\alpha_{30} = -\alpha_{40} = \alpha_{20} > 0 \tag{31}$$

In this case, at large $\tan\beta$:

$$\delta m_h^2 \approx 2\,v^2 \alpha_{20}\big[m_0^2 + 2\,m_0\mu_0 + 2\,\mu_0^2 + m_Z^2\big] \tag{32}$$

A simple numerical example is illustrative. For $m_0 = 1$ TeV, $\mu_0 = 350$ GeV, and with $v \approx 246$ GeV, one has $\delta m_h^2 \approx 2.36\,\alpha_{20} \times 10^{11}$ (GeV)^2. Assuming $M_* = 10$ TeV and ignoring $d = 5$ operators, with $\alpha_{20} \sim 1/M_*^2$ and the MSSM value of m_h taken to be its upper classical limit m_Z (reached for large $\tan\beta$), we obtain an increase of m_h from $d = 6$ operators alone of about $\Delta m_h = 12.15$ GeV to $m_h \approx 103$ GeV. An increase of α_{20} by a factor of 2.5 to $\alpha_{20} \sim 2.5/M_*^2$ would give $\Delta m_h \approx 28$ GeV to $m_h \approx 119.2$ GeV, which is already above the LEP bound. Note that this increase is realised even for a scale M_* of "new physics" beyond the LHC reach.

The above choice of $M_* = 10$ TeV was partly motivated by the fine-tuning results [22] (for $d = 5$ operators) and on convergence grounds: the expansion parameter of our effective analysis is m_q/M_* where m_q is any scale of the theory, in particular it can be m_0. For a susy breaking scale $m_0 \sim \mathcal{O}(1)$ TeV (say $m_0 = 3$ TeV) and $c_{1,2}$ or α_{ij} of $\mathcal{Z}_i(S, S^\dagger)$ of order unity (say $c_{1,2} = 2.5$) one has for $M_* = 10$ TeV that $c_{1,2}\,m_0/M_* = 0.75$ which is already close to unity, and at the limit of validity of the effective expansion in powers of $1/M_*$. To conclude, even for a scale of "new physics" above the LHC reach, one can still classically increase m_h to the LEP bound.

6 Final remarks

The final step is to identify the nature of "new physics" that generated the operators with the largest correction to m_h, ideally from a renormalisable model. At the level of dimension $d = 5$ operators, this is clear from previous discussion: a massive gauge singlet can generate operator $\mathcal{L}_1$ of (22) and the needed increase of m_h, for a scale $M_* \sim 5 - 10$ TeV [22]; this can provide a solution to the little hierarchy problem, provided that one can fix dynamically the scale M_*.

For dimension-six operators, from the above discussion one finds that to increase m_h it is needed that one or more of the following conditions are satisfied:

$$\alpha_{20} > 0,\; \alpha_{30} < 0,\; \alpha_{40} < 0 \tag{33}$$

First recall that $\mathcal{O}_{1,2,3}$ can be most easily generated by integrating out a massive gauge boson $U(1)'$ or $SU(2)$ triplets [21], while $\mathcal{O}_4$ can be generated by a massive gauge singlet or $SU(2)$ triplets. Let us discuss the signs of the operators when they are generated as above:
(a): Integrating out a massive vector superfield $U(1)'$ under which Higgs fields have opposite charges (to avoid a Fayet-Iliopoulos term), one finds $\alpha_{20}<0$ and $\alpha_{30}>0$ (also $\alpha_{10}<0$) [21], which is opposite to what we need. This can be changed, if for example there are additional pairs of massive Higgs doublets also charged under new $U(1)'$; then $\mathcal{O}_3$ could be generated with $\alpha_{30} < 0$.
(b): Integrating massive $SU(2)$ triplets that couple to the MSSM Higgs sector would bring $\alpha_{20}>0$, $\alpha_{40}<0$, $\alpha_{30} > 0$, so the first two relations agree with what we need.
(c): Integrating a massive gauge singlet would bring $\alpha_{40} > 0$, which would instead decrease m_h.

Finally, at large $\tan\beta$, due to additional corrections that effective operators bring to the ρ parameter [26], it turns out that α_{40} and α_{30} can have the largest correction to m_h^2, while avoiding ρ-parameter constraints. The case of a massive gauge singlet or additional $U(1)'$ vector superfield (giving $\mathcal{O}_{3,4}$) have the advantage of preserving gauge couplings unification at one-loop. Following the above information, one can proceed to construct explicit models with additional states that can generate these effective operators.

Let us mention that the method provided here to reduce the fine tuning in the MSSM for m_h larger than the LEP bound, relies on introducing an additional scale in the visible sector, due to “new physics” in this sector. Other solutions to this problem may exist, which essentially rely on a low scale in the hidden sector of supersymmetry breaking [27]. In this case the quartic coupling and the mass of the SM-like Higgs are increased by a factor proportional to $(\mu^2/f)^2$, where f is the hidden sector supersymmetry breaking scale. While not without problems, the advantage of this latter method is that it does not pay the “cost” of an additional parameter (scale) in the visible sector, as models with effective operators do.

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References

1. D. M. Ghilencea and H. M. Lee, JHEP **0509** (2005) 024.
2. D. M. Ghilencea, Phys. Rev. D **70** (2004) 045018.
3. S. Groot Nibbelink, M. Hillenbach, Phys. Lett. B **616** (2005) 125.
4. S. Groot Nibbelink, M. Hillenbach, Nucl.Phys. B **748** (2006) 60.
5. J. F. Oliver, J. Papavassiliou and A. Santamaria, Phys. Rev. D **67** (2003) 125004.
6. I. Antoniadis, E. Dudas, D. Ghilencea, P. Tziveloglou, Nucl. Phys. B **808** (2009) 155.
7. I. Antoniadis, E. Dudas, D. Ghilencea, JHEP **0803** (2008) 045.
8. I. Antoniadis, E. Dudas, D. Ghilencea, P. Tziveloglou, Nucl. Phys. B **831** (2010) 133.
9. H. Georgi, Nucl. Phys. B **361** (1991) 339.
10. H. D. Politzer, Nucl. Phys. B **172** (1980) 349.
11. C. Arzt, Phys. Lett. B **342** (1995) 189.
12. H. E. Haber, J. D. Mason, Phys. Rev. D **77** 115011 (2008).
13. S. P. Martin, Phys. Rev. D **61** 035004 (2000).
14. M. S. Carena, H. E. Haber, H. E. Logan, S. Mrenna, Phys. Rev. D **65** 055005 (2002).
15. D. M. Pierce, J. A. Bagger, K. T. Matchev, R. J. Zhang, Nucl. Phys. B **491** 3-67 (1997).
16. L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D **50** 7048-7065 (1994).
17. G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP **0706** (2007) 045.
18. S. Cassel, D. M. Ghilencea, G. G. Ross, Nucl. Phys. B **835** (2010) 110.
19. S. Cassel, D. M. Ghilencea and G. G. Ross, Phys. Lett. B **687** (2010) 214.
20. R. Barate *et al.* [LEP Working Group for Higgs boson searches and ALEPH, DELPHI, L3, OPAL Collaborations, Phys. Lett. B **565** (2003) 61; S. Schael *et al.* ALEPH, DELPHI, L3, OPAL and LEP Working Group for Higgs Boson Searches, Eur. Phys. J. C**47**(2006)547
21. M. Dine, N. Seiberg, S. Thomas, Phys. Rev. D **76** (2007) 095004.
22. S. Cassel, D. Ghilencea, G. G. Ross, Nucl. Phys. B **825** (2010) 203.
23. K. Blum, C. Delaunay and Y. Hochberg, arXiv:0905.1701 [hep-ph].
24. M. Carena, K. Kong, E. Ponton and J. Zurita, arXiv:0909.5434 [hep-ph].
    M. Carena, E. Ponton and J. Zurita, arXiv:1005.4887 [hep-ph].
25. D. Piriz and J. Wudka, Phys. Rev. D **56** (1997) 4170.
26. A. Brignole, J. A. Casas, J. R. Espinosa and I. Navarro, Nucl. Phys. B **666** (2003) 105.
27. I. Antoniadis, E. Dudas, D. M. Ghilencea and P. Tziveloglou, arXiv:1006.1662 [hep-ph].

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