2.8 Beam-Coupling Impedance and Wake Field – Bench Measurements

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2.8.1 Introduction

Bench measurements nowadays represent an important tool to estimate the coupling impedance [1] of any resonant and not resonant device present in modern particle accelerators. A complete review of the most common methods, including as well practical suggestions, is reported in Ref. [2].

For the non resonant components, the most well-known technique based on the coaxial wire method allows to excite in the device under test a field similar to the one generated by an ultra-relativistic point charge. We discuss the basics of the coaxial wire method and review the formulae widely used to convert measured scattering parameters to longitudinal and transverse impedance data.

For resonant devices (such as accelerating and deflecting cavities) the way to estimate the impedance is the well-known bead-pull technique used, as well, in the design, construction and tuning the structures themselves.

Nowadays, numerical simulations are useful tools also in the measurement stage to compare results and to guide the measurements in order to avoid measurement artifacts. The tools have been described in the previous section; here we will briefly comment on the (typical) reliability of such comparisons against bench measurements.

2.8.2 The Coaxial Wire Method

After the introduction of the beam coupling impedance concept by V. Vaccaro it was realized soon that for highly relativistic beams a very close similarity exists between the Transvers Electric Magnetic (TEM) like field of the charged particles and the field of a wire in a coaxial structure. This is the main basis and motivation of the coaxial wire method.

We review the early concepts of this method in order to show the motivation; we show some issues (and advantages) concerning the practical implementation of those concepts with modern instruments; we show some practical cases to compare the different approaches.

Longitudinal impedance measurements are straightforward, but also transverse impedance measurements using two wires carrying currents with opposite polarity were already done on the late 70s. The concept was extended to the evaluation of dipole and quadrupolar transverse impedances [3] by applying a single displaced wire and pair of wires. We discuss the basis of those methods and present some examples relevant for modern accelerator components.

The coaxial wire set-up can also be used to study the properties of the structure when excited by a beam passing through; trapped modes or beam transfer impedance can, for example, be measured in this way [4].

2.8.2.1 *Motivation and Validation*

The field of a relativistic point charge q in the free space (or in a perfectly conducting beam pipe) is a TEM wave, namely it has only components transverse to the propagation direction (z-axis). The amplitude scales inversely with the distance r from the propagation axis and the propagation constant is ω/c. The fundamental mode of a coaxial wave guide is a TEM wave as well, with the same amplitude dependence on 1/r and the same propagation constant.

Therefore the excitation due to a relativistic beam in a given Device Under Test (DUT) can be "simulated" by exciting a TEM field by means of a conductor placed along the axis of the structure. The impedance source on the DUT will scatter some field, i.e. exciting some higher order modes; such modes must not propagate otherwise the propagating field will not be anymore similar to the TEM beam field. In principle, then, simulating the beam field with the TEM mode of a coaxial waveguide is possible only at frequencies below the first higher mode cut-off, namely below the TM_{01} cut-off frequency for circular waveguides. One can also demonstrate that the modes of the coaxial waveguide converges for vanishing wire radius to the analogous mode of the cylindrical waveguide, at least at the beam pipe boundary, where the impedance source is usually located.

To compare the excitation of a given DUT by a coaxial wire and with the beam itself, we review some measurements done in the framework of the investigations of the shielding properties of coated ceramic vacuum chambers [5]. The 500 MeV CERN EPA electron beam was sent through two identical ceramic vacuum chamber sections; the first one was internally coated with a layer of 1.5 μm depth. Magnetic field probes were placed to measure the beam field just outside the two ceramic chambers (the coated and the reference one). In a first experiment, shielding properties of the resistive coating (thinner than the skin depth) were demonstrated, confirming previous indirect measurements and simulations [6]. In a following experiment, among other results, it was proved that the screening properties of the coating can be spoiled by the addition of a second conducting layer placed outside the field probes and electrically connected to the metallic vacuum chamber sections. In this case, in fact, the magnetic field probe was measuring clearly the field of the 1 ns (r.m.s.) bunched beam (see Fig. 1).

The same chamber in the same configuration (i.e. with this additional external conductor) was then measured in the bench set-up: a 0.8 mm diameter wire was stretched on the axis of the structure. One end of the wire was connected to a 50 Ω load while the other end was connected to one port of a Vector Network Analyzer (VNA); matching resistors were used. The other port of the VNA was connected to the field probe. The network analyzer was set to send through the wire a synthetic pulse (using the so called "time domain option") with 300 MHz bandwidth and measured the transmission between the ports, i.e. the signal through the probe.

This particular kind of set-up is not very often used, but it is very similar to the "time domain" measurement originally proposed by Sands and Rees in the 70s [7]; nowadays time domain measurements are often performed with synthetic pulse techniques in many microwaves applications. The measurement with the beam and with the wire should give virtually the same result, apart from a scaling factor due to the difference of the power carried by the beam and by the VNA signal. The results are shown in Fig. 1 where the beam and the bench data have been normalized and time shifted so that the traces coincides in their minimum point.

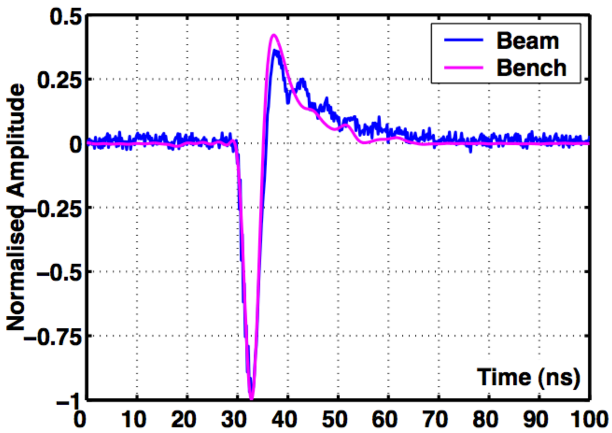

Figure 1: Signal from the field probe after normalization and time shifting in the EPA experiment on coated chamber shielding properties. The field probe is inserted between the coated ceramic and an external conductor connected to the beam pipe.

The external shield, having a DC resistance much smaller than the coating, carries the image currents, the field penetrates the ceramic and the field probe can measure a clear signal. This is only one of the configurations measured both with the beam and in the bench set-up; the agreement with other measurements is similar to the one of Fig. 1.

The results of that comparison confirm the validity of the coaxial wire approach to simulate the beam field effect on a given DUT. Coaxial wire measurements are widely used to estimate impedances of many accelerator devices.

2.8.2.2 *Longitudinal Coupling Impedance*

The wire stretched in the DUT of length L can be modeled, as mentioned before, as a TEM coaxial line of length L. In general, such a line is considered to have distributed parameters but in case of L much smaller than the wavelength λ, the lumped elements approximation is applicable. The DUT beam coupling impedance is then modeled as a series impedance of an ideal reference line (REF). Therefore coupling impedance can be obtained from the REF and DUT characteristic impedances and propagation constants of the lines (see for example Ref. [8]). It is well known that any transmission line can be characterized by measuring its scattering S-parameters, for example with VNA. In principle both reflection (i.e. S_{11}) and transmission measurement (i.e. S_{12}) are possible, but usually transmission measurements are preferred for practical reasons.

In the framework of this transmission line model, the DUT coupling impedance can be exactly computed from measured S-parameters but the procedure is cumbersome and not practically convenient. Therefore a number of approximated formulae are derived in literature and we will report the most used ones, highlighting their approximations.

All the following formulae do not consider the effect of the mismatch at the beginning and at the end of the perturbed transmission line. Therefore matching net- works (resistive networks or cones) are normally used in the actual bench set-ups. Cones are mechanically difficult and act as a frequency dependent distributed transformer, which doesn't work at low frequency; on the contrary resistive networks are affected by parasitic inductances and capacitances affecting their performance at high frequency (depending on the components actually used). Approximated formulas and the "exact" transmission line solution are numerically compared in Ref. [9].

Being Z_c the characteristic impedance of the wire inside the DUT, the beam coupling impedance $Z_{/\!/}$ can be estimated with the "improved log-formula" [8]

$$Z_{LOG} = -Z_c \ln\left(\frac{S_{21}^{DUT}}{S_{21}^{REF}}\right)\left[1 + \frac{\ln\left(S_{21}^{DUT}\right)}{\ln\left(S_{21}^{REF}\right)}\right] \tag{1}$$

Expressing the S_{21}^{REF} in terms of the DUT electrical length L, one can get another equation analogous to Eq. (1) [10]:

$$Z_{LOG} = -Z_c \ln\left(\frac{S_{21}^{DUT}}{S_{21}^{REF}}\right)\left[1 + \frac{jc}{2\omega L}\ln\left(\frac{S_{21}^{DUT}}{S_{21}^{REF}}\right)\right] \tag{2}$$

which can be useful in practice. The improved impedance expression requires the knowledge of the electrical length of the DUT and its accuracy decreases for shorter devices [11]. Reference [9] suggests the use of improved log-formula for DUT longer than the wavelength λ.

For small ratios $Z_{/\!/}/Z_c$, the so-called "standard log-formula" has been proposed for the distributed impedances [12]:

$$Z_{log} = -2Z_c \ln\left(\frac{S_{21}^{DUT}}{S_{21}^{REF}}\right) \tag{3}$$

The log-formula Eq. (3) is generally applicable including lumped components, provided that no strong resonance is present and the perturbation treatment is justified.

For lumped elements, i.e. when the DUT electric length is much smaller than the wavelength, the previous expressions converge to the so called "lumped element formula" [13]:

$$Z_{HP} = -2Z_c \frac{S_{21}^{DUT} - S_{21}^{REF}}{S_{21}^{DUT}} \tag{4}$$

The lumped impedance formula is applicable to single resonances and has the advantage that the scattering coefficient ratio is directly converted into an impedance by the network analyzer [14].

The quantities Z_{LOG}, Z_{log}, Z_{HP} are estimations of the beam coupling impedance $Z_{/\!/}$; the smaller the ratio $Z_{/\!/}/Z_c$, the more accurate the approximated formulae. As an example, a wire in a perfectly conducting cylindrical beam pipe with circular cross section has a characteristic impedance equal to

$$Z_c = \frac{Z_0}{2\pi}\ln\left(\frac{b}{a}\right) \tag{5}$$

where a is the wire radius, b is the (inner) pipe radius and Z_0 is the vacuum impedance. Therefore a smaller wire has an higher Z_c, resulting in a more accurate measurement of the coupling impedance. A detailed discussion of the systematic error done in estimating the beam coupling impedance $Z_{\parallel}$ with Z_{LOG}, Z_{log} or Z_{HP} is reported in Ref. [11].

The difference between the improved log formula of Eqs. (2, 3) and the standard one of Eq. (4) can be shown in measurements performed on the 7 cells module of the MKE kicker [15]. The coupling impedance is much bigger than the characteristic impedance of the wire in the DUT (≈ 300Ω) and therefore the improved log formula must be used:

$$Z_{LOG} = Z_{log}\left[1 + \frac{jc}{2\omega L_f}\ln\left(\frac{S_{21}^{DUT}}{S_{21}^{REF}}\right)\right] \tag{6}$$

Equation (6) differs from Eq. (2) because the length of the ferrite (Lf = 1.66m) is used instead of the length of the whole kicker tank (L = 2.31m), as discussed in Ref. [15]. Figure 2 shows the wire measurement results interpreted with the improved formula Eq. (6) (green line) and the standard one Eq. (3) (blue line). The comparison with theory (black line) shows that, at least for the real part of the impedance, the improved log formula gives a result closer to theoretical expectations for frequencies higher than few hundreds of MHz. At lower frequencies, i.e. where the DUT length is comparable to the wave-length and the impedance is much closer to the characteristic impedance of the wire in the DUT, the standard log formula is a better estimation of the coupling impedance.

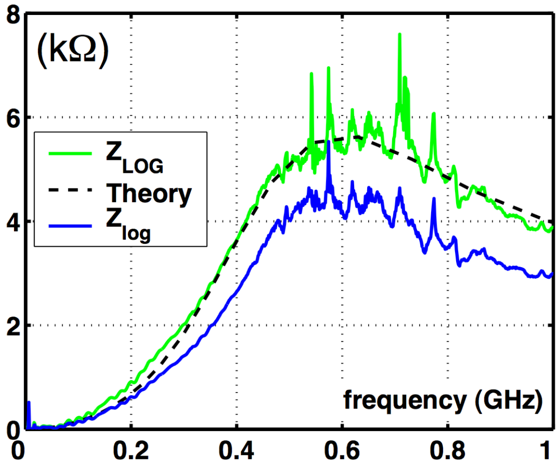

Figure 2: Real part of the longitudinal coupling impedance for the 7 cells MKE kicker module [15]. The measured data are interpreted via the improved log formula (green line) or the standard log formula (blue line) and compared to theoretical expectations (dashed line).

In conclusion, longitudinal coupling impedance bench measurements are reasonably well understood and the technique is well established. Moreover, with modern simulation codes, one can derive directly the coupling impedance or simulate the bench set-up with wire, virtually for any structure. Evaluation of coupling impedance from measured or simulated wire method results require the same cautions; but simulations and RF measurements usually agree well. Moreover comparisons with numerical results are very useful to drive and to interpret the measurements. One should pay attention that simulation may require a simplified DUT model, which will only reproduce the main DUT electromagnetic features.

2.8.2.3 ***Transverse Coupling Impedance***

The transverse impedance is proportional at a given frequency to the change in longitudinal impedance due to the lateral displacement of the beam in the plane under consideration (vertical or horizontal). Therefore the transverse impedance is proportional to the transverse gradient of the longitudinal impedance (Panowsky-Wenzel theorem [1]). Based on this theorem, the most common method to bench measure transverse impedance uses two parallel wires stretched along the DUT [2]. Opposite currents are sent through the wires (odd mode excitation); instead of the wires, a loop can be used to increase the signal to noise ratio [16]. The bench transverse impedance $Z_{\perp,bench}$ is given by [17]

$$Z_{\perp,bench} = \frac{Z_{\|,bench}c}{\omega\Delta^2} \tag{7}$$

where Δ is the wire spacing (usually about 10% of the DUT radius). $Z_{//,bench}$ is the longitudinal coupling impedance measured from the S-parameters as discussed above, e.g. using the improved log-formula

$$Z_{\perp,LOG} = -\frac{Z_c c}{\omega\Delta^2}\ln\left(\frac{S_{21}^{DUT}}{S_{21}^{REF}}\right)\left[1+\frac{\ln\left(S_{21}^{DUT}\right)}{\ln\left(S_{21}^{REF}\right)}\right] \tag{8}$$

where now Z_c is the characteristic impedance of the odd mode of a two wires transmission line. Concerning LHC (and other modern machines as well), low frequency transverse impedance is interesting and therefore the lumped element Eq. (4) must be used in Eq. (8). A practical example of low frequency transverse impedance is reported in Ref. [16] for a simple case; results are compared to theoretical expectations to define a reliable measurement procedure.

In the two wires bench set-up only dipole field components are excited because of the symmetry of the wires/coil; therefore there is no electric field component on the axis. In numerical simulations, this is analogous to putting a metallic image plane between the wires. Nevertheless some accelerator devices may exhibit a strong asymmetry in the image current distribution due to azimuthal variation of conductivity (e.g. ferrite in kickers) or to cross section shape. Two wire techniques can be used with some cautions in these cases because the field in the structure is not TEM-like; in order to get a more complete view of the transverse kick on the beam, it may be useful to characterize the device with a single wire [18].

The transverse impedance itself can be measured with a single wire displaced in various positions, which is measuring the longitudinal coupling impedance as a function of the displacement x_0 of a single wire. From the Panowsky- Wenzel theorem we get

$$Z_{\perp,bench} \simeq \frac{c}{\omega}\frac{Z_{\|,bench}(x_0) - Z_{\|,bench}(x_0=0)}{x_0^2} \tag{9}$$

provided that x_0 is small with respect to the typical variation length of the bench measured coupling impedance $Z_{//,bench}$.

From the practical point of view, transverse impedance measurement techniques are more delicate and require particular attention for asymmetric devices (e.g. traveling wave kickers like). Novel techniques optimized for particular DUTs, are also being proposed, e.g. SNS kicker measurements reported in Ref. [19]. Numerical simulations are necessary to control and validate the measurement procedure. One should pay attention that DUT models feasible for simulations do not introduce non-physical symmetries or approximations; in principle, dealing with transverse problem may require more complex simulations than the longitudinal case.

2.8.3 **Resonant Structures**

An important class of accelerator devices includes cavities, which are now used both for accelerating and deflecting the particle beam. Each cavity is characterized by its resonant frequency f_0, the quality factor of the resonance Q and its shunt impedance R. One can think of measuring all these quantities with the coaxial wire set-up, i.e. measuring strong notches in the transmission scattering coefficient between the ends of the wire. But the wire perturbs longitudinal cavity modes, e.g. lowers the Q and detunes the frequency. Therefore the coaxial wire set-up is not usually recommended for cavity measurements and it is advisable only for special cases, mainly transverse modes [2].

The most used technique to characterize cavities is the “bead pull” measurement [20]. The field in the cavity can be sampled by introducing a perturbing object and measuring the change in resonant frequency: where the field is maximum (minimum) the resonance frequency will be more (less) perturbed. Since it is a perturbation method, the perturbing object must be so small that the field does not vary significantly over its largest linear dimension. Shaped beads are used to enhance perturbation and give directional selectivity among different field components.

Quantitatively, the change of the resonant frequency is related to the perturbed cavity field by the Slater theorem. For the typical case of longitudinal electric field on the axis of accelerating cavities, the variation of the resonance frequency Δf from the unperturbed one is [21]

$$\frac{\Delta f}{f_0} = -\Delta V \varepsilon_0 k_E \frac{E_z^2}{4U} \tag{10}$$

for a conducting bead of volume ΔV; E_z is the field at the perturbing object position and U is the electromagnetic energy stored in the cavity. The form factor k_E of the perturbing object can be exactly calculated for ellipsoids or can be calibrated in a known field (e.g. TM_{0n0} of a pillbox cavity).

The frequency variation can be measured by the variation of the phase φ at the unperturbed resonant frequency, according to [22]

$$\frac{\Delta f}{f_0} = \frac{\tan\phi(f_0)}{2Q_L} \simeq \frac{\phi(f_0)}{2Q_L} \tag{11}$$

where Q_L is the (loaded) quality factor of the resonance. Even if a very precise initial tuning is needed, this method allows easily measuring the field of many points (as many as the points of the instrument trace). The field shape can also be directly visualized on the instrument screen, greatly facilitating the structure tuning procedure, which is typically a very cumbersome procedure in multicell cavities.

An important cavity design parameter is the R/Q, which can be obtained from electric field data using

$$\frac{R/Q}{L_c} = \frac{1}{U\omega L_c}\left|\int_0^{L_c} E_z(z)\exp\left(j\frac{\omega}{c}z\right)dz\right|^2 \qquad (12)$$

where L_c is the length of the structure.

In general measurement on resonant structures are accurate and in very good agreement with simulations. Bead pull measurements are often used to check if the DUT fits the design specifications and they are still required for tuning the multiple cell cavities. R/Q measurements agree always with simulations within the few %.

2.8.4 **Conclusions**

Bench measurements are used for estimating the coupling impedance of accelerator devices, exploiting the very accurate instrumentation used in microwave measurement in the frequency domain (namely vector network analyzer). We have reviewed the standard methods used for non resonant and resonant devices; coaxial wire set-up are used for the first class of devices while the second class of devices are bench measured with bead-pull techniques. The main idea of the coaxial wire technique is the analogy between the field of an ultra-relativistic charged beam and the one of a coaxial waveguide; thus the beam is simulated by an electric pulse travelling on the inner wire. Bead pull techniques allow measuring the resonant field in cavities through perturbation of the cavity space, according to Slater theorem; from the measured field on the beam path, one can compute the resonance parameters (e.g. R/Q). In this paper we have reviewed the most common concepts used in bench measurement, focusing on the motivation, their limit of validity and trying to compare their different results.

2.8.5 **References**

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