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PERTURBATIVE CHERN-SIMONS THEORY IN THE LIGHT-CONE GAUGE. THE ONE-LOOP VACUUM POLARIZATION TENSOR IN A GAUGE-INVARIANT FORMALISM

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ABSTRACT

Perturbative Chern-Simons (CS) theory, with gauge group SU(N), is studied in the physical light-cone gauge $n^\sigma A^a_\sigma = 0,\ n^2 = 0$. Using a gauge-invariant regularization procedure, we compute the vacuum polarization tensor to one-loop order. The regularized theory is consistent with BRS invariance. It is shown that the vacuum polarization tensor is UV-finite and transverse, but contains one non-local term which is strictly gauge-dependent. A finite, non-multiplicative (gauge-dependent) wave function renormalization enables us to express the one-loop effective action as

$$\Gamma_{1-\mathrm{loop}}(A^{(\mathrm{ren})}_\mu, k) = \Big(\frac{k + \mathrm{sign(k)} c_v}{4\pi}\Big) S_{CS}(A^{(\mathrm{ren})}_\mu),$$

where S_{CS} is the classical action and k is the bare Chern-Simons parameter.

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1 Introduction

Two years ago Witten [1] gave a three-dimensional field theoretical derivation of the Jones polynomial and its generalizations in the context of Chern-Simons field theory. The classical action of this theory is the integral over an oriented three-dimensional manifold $\mathcal{M}$ of the Chern-Simons three-form and, therefore, independent of the choice of metric for the manifold. This property is a key issue in showing the connection between Chern-Simons theory and knot theory. Moreover, we know that the Wilson loop for the gauge field is likewise independent of the metric of the manifold. Given a collection of oriented, non-intersecting curves on the manifold, one may, therefore, define the vacuum expectation value of the product of Wilson loops along each curve and then demonstrate that this topological invariant yields (for $\mathcal{M} = S^3$) the Jones polynomial and its generalizations. In addition, there exists a natural and most interesting connection between Chern-Simons theory and two-dimensional conformal field theory that has been studied extensively [2].

During the past three years, a great deal of attention has been devoted to the analysis of perturbative Chern-Simons theory [3]-[11]. There are two solid reasons for this interest. (1) Since canonical quantization methods involving path integrals are known to be not rigorous from a mathematical viewpoint, it is desirable to have an independent check of the numerical results at one's disposal [12]. The perturbative approach offers an alternative way of verifying these results. (2) Perturbative computations of vacuum expectation values of Wilson loops provide us with integral representations for the invariant coefficients in the Jones polynomial [10],[13].

The majority of perturbative computations [3]-[8], [10], [13] has been carried out in a Lorentz covariant gauge and with various regularization methods. It turns out that all gauge-invariance preserving techniques produce the famous one-loop shift of the bare Chern-Simons parameter $k, k \rightarrow k + \mathrm{sign}(k)N$ [4]-[7]. If, however, the regularized theory is not gauge invariant, the aforementioned shift does not materialize [3], [7]. Of course, by exploiting the regulator-independent analysis of Ref. [11] , we may always pass from the shifted result ($k \rightarrow k + \mathrm{sign}(k)N$) to the unshifted one ($k \rightarrow k$), and vice versa, by a finite renormalization of k. The authors of Ref. [11] proved among other things that both the beta function and the anomalous dimensions of the elementary fields vanish at any order of perturbation theory.

The principal purpose of this paper is to extend the analysis of the perturbative Chern-Simons model to non-covariant gauges, specifically to the physical light-cone gauge. A secondary purpose is to test the prescription currently in vogue for the light-cone gauge [14],[15] and indirectly develop new computational techniques for perturbative Chern-Simons field theory. We recall that the light-cone gauge prescription introduced in Ref. [14] for the unphysical poles of $(p \cdot n)^{-\alpha}, \alpha = 1, 2, 3, ...$ has been tested extensively in four-dimensional QED and QCD (see Ref. [16] for a review). The prescription permits a Wick rotation to Euclidean space and is consistent with power counting. By contrast, the prescription (or its generalization to the axial and temporal gauges) has not been checked explicitly in three-dimensional Chern-Simons theory.

Efforts to exploit the non-covariant axial-type gauges are not new. For instance, in Ref. [9], the Gauss linking number was expressed in terms of the propagator in an axial-type gauge, and in Ref. [17] a connection was established between the WZW model, knot theory and a collection of Green functions, the latter having been obtained heuristically from the Chern-Simons action in the light-cone gauge. Furthermore, the perturbative computation of the vacuum expectation

value of Wilson loops in an axial gauge ought to lead to analytic expressions for the HOMFLY [13] coefficients when expressed in terms of the axial propagator.

Finally, it is important to distinguish clearly between regularizations which respect gauge invariance and those that break it. In Ref. [9], perturbative Chern-Simons theory was defined in an axial-type gauge, but the regularizations employed were not explicitly gauge invariant. Even so, the complete effective action of the theory turned out to be the tree-level action. (A similar conclusion in covariant gauges is still lacking, even for simple regularization methods [3],[7].) The computation in this paper, on the other hand, is based on an explicitly gauge – invariant regularization, a hybrid regularization: it utilizes the higher-derivative term, $F^2_{\mu\nu}$, and an algebraically consistent, and BRS invariant, dimensional regularization. The latter regularizes those diagrams which remain unregularized even after inclusion of the $F^2_{\mu\nu}$-term.

The plan of the paper is as follows. In Section 2 we implement our gauge invariant regularization method in the light-cone gauge. In Section 3 we analyze those one-loop contributions from the D-dimensional propagator that vanish at the tree level as $D \rightarrow 3$. The vacuum polarization tensor is computed in Section 4, and the one-loop effective action discussed in Section 5. The main technical tools and principal results are summarized in Section 6.

2 Feynman Rules and Regularization

2.1 The complete action

Let $A_\mu = A^a_\mu T^a$ be the SU(N) gauge connection over R_3 endowed with a Minkowski metric. Let T^a be the SU(N) generators in the fundamental representation with the normalization $Tr(T^aT^b) = (1/2)\delta^{ab}$. The BRS invariant Chern-Simons action in the homogeneous light-cone gauge $n \cdot A^a = 0,\ n^2 = 0$, is given by

$$\mathcal{S}_{CS} = \int d^3x \varepsilon^{\mu\rho\nu}(\frac{1}{2}A^a_\mu \partial_\rho A^a_\nu + \frac{g}{3!} f^{abc} A^a_\mu A^b_\rho A^c_\nu) + \int d^3x [-\frac{1}{2\alpha}(n \cdot A^a)^2 + \bar{c}^a n^\mu D^{ab}_\mu c^b], \alpha \rightarrow 0, \quad (2.1)$$

where $D^{ab}_\mu = \partial_\mu \delta^{ab} + g f^{abc} A^c_\mu$ is the covariant derivative and f^{abc} are the structure constants of SU(N). The limit $\alpha \rightarrow 0$ has to be taken in the Feynman rules obtained from the action (2.1). Here $g^2 = \frac{4\pi}{k}$, $k > 0$ being the bare Chern-Simons parameter.

In defining the formal series expansion in powers of g of the Green functions for (2.1), one meets two main difficulties. The first one occurs already at the tree-level and has to do with a suitable definition of the free propagators as distributions. The second difficulty comes from the UV behaviour of the Feynman diagrams.

The three-dimensional gauge propagator turns out to be

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle = \frac{i\delta^{ab}}{p^2}\{\varepsilon_{\mu\rho\nu} p^\rho + \frac{p_\mu \varepsilon_{\nu\rho\sigma} p^\rho n^\sigma - p_\nu \varepsilon_{\mu\rho\sigma} p^\rho n^\sigma}{p \cdot n}\}, \quad (2.2)$$

and the ghost propagator reads

$$\langle c^a(p) \bar{c}^b(p) \rangle = -\frac{i\delta^{ab}}{p \cdot n}. \quad (2.3)$$

Apart from the $p^2 = 0$ pole in (2.2) that is handled by the usual Feynman prescription, the propagators in (2.2) and (2.3) are not well defined because of the $p \cdot n = 0$ singularity. This extra singularity, a characteristic feature of axial-type gauges (see Ref. [16] for a review), has to be defined consistently. A good prescription, particularly suitable for the light-cone gauge, is given by [14]

$$\frac{1}{[p\cdot n]} \equiv \lim_{\eta\to 0} \frac{p\cdot n^*}{p\cdot n\; p\cdot n^* + i\eta},\ \eta > 0, \tag{2.4}$$

where $n^*_\mu = (n_0, -\vec{n})$ is a second light-like vector. Prescription (2.4) permits a Wick rotation to Euclidean space and naive power counting, and may be implemented in any number of space-time dimensions and in any gauge theory. In fact, one of the purposes of this paper is to verify by means of an explicit computation that prescription (2.4) really works for three-dimensional Chern-Simons gauge theory.

The second difficulty in calculating Feynman diagrams concerns the UV behaviour of some of the integrals. The superficial UV degree of divergence, Ω_{CS}, of a 1PI diagram with E_A external gauge lines and E_C external ghost lines is given by

$$\Omega_{CS} = 3 - E_A - \frac{1}{2}E_C. \tag{2.5}$$

We see that $\Omega_{CS} \geq 0$ for any diagram with fewer than four external legs, so that a regularization method must be implemented. A consistent dimensional regularization procedure does not exist for the theory at hand, since the "three-dimensional" $\varepsilon_{\mu\rho\nu}$-tensor prevents inversion of the kinetic term in n-dimensional space [6]. In order to obtain a regularized Chern-Simons theory consistent with gauge invariance at the regularized level, we shall follow the strategy adopted in Ref. [6]. We simply add to the Chern-Simons action S_{CS} the three-dimensional Yang-Mills term S_{YM}

$$S_{YM} = -\frac{1}{4m}\int d^3x F^a_{\mu\nu}F^a_{\mu\nu},\ m \equiv \text{regulator mass} \tag{2.6}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$. The action of the theory then becomes

$$S_m = S_{CS} + S_{YM}, \tag{2.7}$$

and leads to the following Feynman rules:

Propagators

$$\begin{aligned}\langle A^a_\mu(p)A^b_\nu(-p)\rangle &= \frac{im\,\delta^{ab}}{p^2(p^2-m^2)}\Big[-im\varepsilon_{\mu\rho\nu}p^\rho - im\frac{p_\mu\varepsilon_{\nu\rho\sigma}p^\rho n^\sigma - p_\nu\varepsilon_{\mu\rho\sigma}p^\rho n^\sigma}{p\cdot n} \\ &- p^2 g_{\mu\nu} + \frac{p^2}{p\cdot n}(p_\mu n_\nu + p_\nu n_\mu)\Big] \equiv \Delta^{ab}_{\mu\nu}(p;m); \end{aligned}\tag{2.8a}$$

$$\langle c^a(p)\bar{c}^b(+p)\rangle = -i\delta^{ab}/p\cdot n; \tag{2.8b}$$

Vertices

$$\begin{aligned}V^{abc}_{\mu\nu\rho}(p,q,r) &= -g f^{abc}\Big\{i\varepsilon_{\mu\nu\rho} + \frac{1}{m}[(q-r)_\mu g_{\nu\rho} \\ &+ (r-p)_\nu g_{\rho\mu} + (p-q)_\rho g_{\mu\nu}]\Big\}\,; \end{aligned}\tag{2.9}$$

$$\begin{aligned}V^{abcd}_{\mu\nu\sigma\rho} &= -\frac{ig^2}{m}\Big\{f^{abe}f^{cde}(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}) \\ &+ f^{ace}f^{bde}(g_{\mu\nu}g_{\sigma\rho} - g_{\mu\rho}g_{\nu\sigma}) + f^{ade}f^{cbe}(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\sigma\rho})\Big\}\,; \end{aligned}\tag{2.10}$$

$$V^{abc}_\mu = -ig f^{abc} n_\mu. \tag{2.11}$$

2.2 UV-behaviour

Let us now examine the UV behaviour of the propagator. The behaviour of $\Delta^{ab}_{\mu\nu}(\lambda p; m)$ as $\lambda \to \infty$, is given by λ^{-2}, whereas the corresponding behaviour of the gauge propagator in Eq. (2.2) is λ^{-1}. By adding the Yang-Mills term to the Chern-Simons action, we have improved the UV behaviour of the gauge propagator. Unfortunately, the improvement is insufficient to render the theory finite by power-counting. There are two reasons for this: (i) the presence of the vertex (2.9), and (ii) the fact that ghost loops remain untouched. Indeed, the new superficial UV degree, Ω_m, of a diagram with E_A external gauge lines, E_g external ghost lines, V_3 vertices of type (2.9), V_4 vertices of the form (2.10), and V_g vertices of type (2.11), is given by

$$\Omega_m = 3 - \frac{1}{2}(E_A + 2E_g + V_3 + V_4 + V_g). \tag{2.12}$$

If $E_A + 2E_g + V_3 + V_4 + V_g \geq 7$, then $\Omega_m < 0$, and unlike pure Chern-Simons theory, we now have a model with a finite number of superficially UV divergent Feynman diagrams.

Unfortunately, our ghost and gauge propagators are not Lorentz covariant, so that their large momentum behaviour will depend on the direction in q-space. Even if $\Omega_m < 0$ for a certain diagram and for any of its subdiagrams, this does not necessarily ensure that the diagram is UV finite. Thus, situations where $\ell\cdot n$ remains fixed (ℓ_μ are loop momenta), while the components orthogonal to n_μ go to infinity, have to be analyzed separately [18],[19].

In order to characterize the UV divergences along the direction transverse (T) to n_μ, we follow Becchi [19] and split every loop momentum ℓ_μ into two parts ℓ^T_μ and ℓ^L_μ: $\ell_\mu = \ell^T_\mu + \ell^L_\mu$, where

$$\ell^L_\mu = -(n\cdot n^*)^{-1}(\ell\cdot n^* n_\mu + \ell\cdot n n^*_\mu); \ell^T\cdot n = \ell^T\cdot n^* = 0. \tag{2.13}$$

It is easy to see that $\Delta^{ab}_{\mu\nu}(\lambda\ell^T, \ell^L; m) \sim \lambda^{-1}$, as $\lambda \to \infty$.

Since $\Delta^{ab}_{\mu\nu}(\lambda\ell^T, \lambda\ell^L) \sim \lambda^{-2}$, a given Feynman diagram could be more divergent than is indicated by Ω_m. From power-counting we see that the UV degree along the ℓ^T-direction, Ω_T, is

$$\Omega_T = 1 - V_4 \tag{2.14}$$

for any graph without ghost lines. Hence, a graph may be UV divergent, even though Ω_m is negative for the diagram, or for any of its sub-diagrams.

In order to regularize the integrals obtained from S_m, specifically those involving parity-violating objects such as $\varepsilon_{\mu\nu\rho}$, we shall adopt the dimensional regularization recipe introduced by 't Hooft and Veltman [20], and systematized by Breitenlohner and Maison [21], and Collins [22].

Consider the following action in R_n (with a Minkowski metric)

$$\begin{aligned}S_m(n) &= \int d^n x\, \varepsilon^{\mu\rho\nu}\Big\{\frac{1}{2}A^a_\mu\partial_\rho A^a_\nu + \frac{1}{3!}g f^{abc}A^a_\mu A^b_\rho A^c_\nu\Big\} \\ &- \frac{1}{4m}\int d^n x F^a_{\mu\nu}F^a_{\mu\nu} + \int d^n x\Big[-\frac{1}{2\alpha}(n\cdot A^a)^2 + \bar{c}^a n\cdot D^{ab}c^b\Big], \end{aligned}\tag{2.15}$$

where A^a_μ, n_μ and ∂_μ are n-dimensional vectors. The n-dimensional symbol $\varepsilon_{\mu\rho\nu}$ is defined as a completely antisymmetric object satisfying

$$\varepsilon_{\mu_1\mu_2\mu_3}\varepsilon_{\nu_1\nu_2\nu_3} = \sum_{\pi\in S_3}\text{sign}(\pi)\prod_{i=1}^{3}\tilde{g}_{\mu_i\nu_{\pi(i)}}; \tag{2.16}$$

$\tilde{g}_{\mu\nu}$ is the "3-dimensional" Minkowski metric, i.e., the projection of the n-dimensional Minkowski metric $g_{\mu\nu}$ onto the 3-dimensional subspace R_3 of R_n; $\tilde{g}_{\mu\nu}$ obeys

$$\tilde{g}^{\mu}_{\mu} = 3,\ \tilde{g}_{\mu\rho}g^{\rho\nu} = \tilde{g}_{\mu\rho}\tilde{g}^{\rho\nu} = \tilde{\delta}^{\nu}_{\mu}, \qquad \tilde{g}_{\mu\rho}v^{\rho} = \tilde{v}_{\mu}; \tag{2.17}$$

$\tilde{v}_\rho$ denotes the projection of v_ρ onto R_3, and $\tilde{\delta}^{\nu}_{\mu} = 1$ if $\mu,\nu = 0,1,2$ and zero otherwise. One may, likewise, introduce an "(n-3)-dimensional" metric $\hat{g}_{\mu\nu} = g_{\mu\nu} - \tilde{g}_{\mu\nu}$, which satisfies

$$\begin{aligned} &\hat{g}^{\mu}_{\mu} = n-3,\ \tilde{g}^{\mu\rho}\hat{g}_{\rho\nu} = 0; \qquad g^{\mu\rho}\hat{g}_{\rho\nu} = \hat{\delta}^{\mu}_{\nu}; \qquad \hat{g}_{\mu\rho}v^{\rho} = \hat{v}_{\mu}; \\ &\varepsilon_{\mu\rho\nu}\hat{g}^{\nu\sigma} = 0,\ \text{with } \hat{\delta}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \tilde{\delta}^{\mu}_{\nu},\ \text{and } \hat{v}_{\mu} = v_{\mu} - \tilde{v}_{\mu}. \end{aligned} \tag{2.18}$$

All indices in Eqs.(2.16-2.18) run from 0 to (n-1). In view of (2.16), the n-dimensional action $S_m(n)$ in Eq. (2.15) is not $SO(1,n-1)$ invariant, but rather $SO(2,1)\otimes SO(n-3)$ invariant. This fact complicates computations considerably.

The Feynman rules for the vertices and ghost propagator of S_m, Eq. (2.15), are given by Eqs. (2.9), (2.10), (2.11) and (2.8b), provided we interpret the metric tensor and all vectors as n-dimensional objects, and $\varepsilon_{\mu\rho\nu}$ as in (2.16). The gauge propagator emerging from $S_m(n)$ reads $(\alpha \to 0)$:

$$\langle A^a_\mu(p)A^b_\nu(-p)\rangle = \Delta^{ab}_{\mu\nu}(\tilde{p},\hat{p};m) = i\delta^{ab}\sum_{i=1}^{8}\Delta^{(i)}_{\mu\nu}(p), \tag{2.19}$$

with

$$\begin{aligned} \Delta^{(1)}_{\mu\nu} &= -\frac{mp^2}{Den(p)}g_{\mu\nu}; \qquad \Delta^{(2)}_{\mu\nu}(p) = m^3\frac{\hat{p}^2}{p^2}\frac{\hat{g}_{\mu\nu}}{Den(p)}; \\ \Delta^{(3)}_{\mu\nu}(p) &= m^3\frac{[(\hat{p}\cdot\hat{n})^2 + \hat{p}^2\hat{n}^2]}{p^2(p\cdot n)^2 Den(p)}p_\mu p_\nu; \\ \Delta^{(4)}_{\mu\nu}(p) &= \frac{m^3}{p^2}\frac{\hat{p}_\mu\hat{p}_\nu}{Den(p)}; \quad \Delta^{(5)}_{\mu\nu}(p) = \frac{mp^2}{p\cdot n}\frac{(p_\mu n_\nu + n_\mu p_\nu)}{Den(p)}; \\ \Delta^{(6)}_{\mu\nu}(p) &= \frac{m^3\hat{p}^2}{p^2 p\cdot n}\frac{(p_\mu\hat{n}_\nu + p_\nu\hat{n}_\mu)}{Den(p)}; \quad \Delta^{(7)}_{\mu\nu}(p) = -\frac{im^2\varepsilon_{\mu\rho\nu}p^\rho}{Den(p)}; \\ \Delta^{(8)}_{\mu\nu}(p) &= -im^2\frac{(p_\mu\varepsilon_{\nu\rho\sigma}p^\rho n^\sigma - p_\nu\varepsilon_{\nu\rho\sigma}p^\rho n^\sigma)}{p\cdot n\ Den(p)}, \end{aligned} \tag{2.20}$$

where

$$Den(p) = (p^2)^2 - m^2\tilde{p}^2. \tag{2.21}$$

2.3 Regularization

The integrals are regularized by analytically continuing the integer n to complex dimensions D as discussed in Collins [22]. Notice that the n-dimensional action $S_m(n)$ is invariant under the n-dimensional BRS transformations

$$sA^a_\mu = D^{ab}_\mu c^b; \quad s\bar{c}^a = \frac{1}{\alpha}n\cdot A^a; \quad sc^a = -\frac{g}{2}f^{abc}c^b c^c, \tag{2.22}$$

so that the resulting perturbative Green functions satisfy the appropriate Salvnov-Taylor identities. The properties of the dimensionally regularized integrals [22] then ensure that the regularized theory is BRS-invariant. A key argument is that the n-dimensional free propagators are the inverses of the corresponding kinetic terms of the n-dimensional theory.

We note in passing that prescription (2.4) admits the useful decomposition formula [24]

$$\frac{1}{[q\cdot n][(q+p)\cdot n]} = \frac{1}{[p\cdot n]}\left\{\frac{1}{[q\cdot n]} - \frac{1}{[(q+p)\cdot n]}\right\},\ p\cdot n \neq 0. \tag{2.23}$$

By repeated application of (2.23), any ghost loop may be expressed as a sum of massless tadpole integrals that vanish in dimensional regularization. Hence any diagram containing one or more ghost loops vanishes identically in dimensional regularization. On the other hand, we also know that

$$n^\mu\Delta^{ab}_{\mu\nu}(\tilde{p},\hat{p};m) = 0, \tag{2.24}$$

where $\Delta^{ab}_{\mu\nu}(\tilde{p},\hat{p};m)$ is given in (2.19), so that every 1PI diagram containing at least one internal gauge line attached to a vertex V^{abc}_μ (cf. Eq. (2.11)) vanishes. Thus the 1PI functions involving ghost fields are zero beyond the tree-level.

The 1PI functions $\Gamma^{a_1\cdots a_n}_{\mu_1\ldots\mu_n}(p_1,\ldots,p_n)$ for the gauge fields of an SU(N) Chern-Simons theory in the light-cone gauge are defined as follows:

$$\Gamma^{a_1\cdots a_n}_{\mu_1\ldots\mu_n}(p_1,\ldots,p_n) = \lim_{m\to\infty}\lim_{D\to 3}\Gamma^{a_1\cdots a_n}_{\mu_1\ldots\mu_n}(p_1,\ldots,p_n;m,D), \tag{2.25}$$

where $\Gamma^{a_1\cdots a_n}_{\mu_1\ldots\mu_n}(p_1,\ldots,p_n;m,D)$ are the dimensionally regularized 1PI functions for the gauge fields of the action $S_m(n)$, Eq. (2.15). Definition (2.25) assumes the existence of the indicated sequence of limits. In this article we shall verify by explicit computation that the double limit in Eq. (2.25) does indeed exist for the one-loop vacuum polarization tensor. An important feature of (2.25) is that the functions $\Gamma^{a_1\cdots a_n}_{\mu_1\ldots\mu_n}(p_1,\ldots,p_n)$ satisfy the BRS identities, since the functions $\Gamma^{a_1\cdots a_n}_{\mu_1\ldots\mu_n}(p_1,\ldots,p_n;m,D)$ do. (It is assumed that the external momenta $p_1,\ldots,p_n$ remain bounded as m goes to infinity.) Notice that the reverse order of the limits in (2.25) does not exist, even at the tree-level.

The action $S_m(n)$ may be interpreted as the light-cone gauge action of a topologically massive theory of mass m [23]. To see this we note that the propagator (2.8a) exhibits a pole at $p^2 = m^2$. Definition (2.25) seems to suggest, therefore, that light-cone gauge Chern-Simons theory may be defined as the large mass limit of a topologically massive Yang-Mills theory. A similar interpretation has been studied in the Landau gauge (see second reference in Ref. [6]).

3 Evanescent Objects

3.1 Introduction

The purpose of this section is to show that some of the cumbersome pieces of the gauge propagator (2.19) do not contribute as $D \to 3$, at least to one loop.

According to Eq. (2.25), we are only interested in keeping those contributions which do not vanish as $D \to 3$. The propagator $\Delta^{ab}_{\mu\nu}(\tilde{p},\hat{p};m)$ in Eq. (2.19) contains many "evanescent" operators in the sense of Collins [22], i.e., operators which vanish at the tree-level. These "evanescent" operators, namely $\Delta^{(2)}_{\mu\nu},\Delta^{(3)}_{\mu\nu},\Delta^{(4)}_{\mu\nu}$ and $\Delta^{(6)}_{\mu\nu}$, are needed to ensure that the dimensionally regularized theory is BRS invariant, even though they may not contribute in certain instances. Specifically, if $I_{\mu_1\ldots\mu_n}$,

$$I_{\mu_1\ldots\mu_n} = \int\prod_{j=1}^{L}\frac{d^D q_j}{(2\pi)^D}\prod_{i=1}^{I}\frac{q_{\mu_1}\cdots q_{\mu_n}}{(\ell_i)^{n_1}[(\ell_i^2)^2 + m^2\tilde{\ell}_i^2]^{n_2}(n\cdot\ell_i)^{n_3}}, \tag{3.1}$$

is both UV and IR convergent by power-counting at $D=3$ for non-exceptional momenta (ℓ_i is a linear combination of external momenta and loop momenta), then

$$\lim_{D\to 3} \hat{g}_{\nu_1\mu_1} I_{\mu_1...\mu_n} = 0. \tag{3.2}$$

In this paper we shall compute the vacuum polarization tensor at the one-loop level and also analyze the one-loop contributions from the evanescent operators in $\Delta^{ab}_{\mu\nu}(\tilde{p},\hat{p};m,D)$.

3.2 UV-behaviour characterized by Ω_m and Ω_T

Let us consider the UV behaviour of an arbitrary one-loop diagram. Since diagrams containing ghosts vanish in dimensional regularization, we only need to examine diagrams without ghosts. The UV behaviour of a one-loop 1PI diagram is characterized by Ω_m and Ω_T in Eqs. (2.12) and (2.14), respectively; Ω_m corresponds to the UV behaviour when the loop momentum goes to infinity along any direction. To get Ω_T we allow ℓ^T [cf. Eq. (2.13)] to become large, while ℓ^L remains bounded. In deriving the final values for Ω_m and Ω_T, the evanescent operators in (2.19) are taken to be zero. Of course, there is the potential danger that evanescent operators might give rise to integrals such as (3.1) which have a positive UV degree along some direction, while Ω_m and Ω_T are negative. Fortunately, this possibility does not materialize at the one-loop level.

If we define α_i and α_i^T, $i=1,...,8$, by

$$\begin{aligned}\Delta^{(i)}_{\mu\nu}(\lambda p) &\sim \lambda^{\alpha_i}, \text{ as } \lambda\to\infty,\\ \Delta^{(i)}_{\mu\nu}(\lambda p^T, p^L) &\sim \lambda^{\alpha_i^T}, \text{ as } \lambda\to\infty,\end{aligned} \tag{3.3}$$

we find that

$$\begin{aligned}&\alpha_1=-2,\alpha_2=-4,\alpha_3=-4,\alpha_4=-4,\alpha_5=-2,\alpha_6=-4,\alpha_7=-3,\alpha_8=-3,\\ &\alpha_1^T=-2,\alpha_2^T=-4,\alpha_3^T=-2,\alpha_4^T=-4,\alpha_5^T=-1,\alpha_6^T=-3,\alpha_7^T=-3,\alpha_8^T=-2.\end{aligned} \tag{3.4}$$

Notice that $\alpha_i < \alpha_1 = -2\ \forall i\neq 1,5$ and $\alpha_i^T < \alpha_5^T = -1$, $\forall i \neq 5$. Ω_m and Ω_m^T were obtained by taking the leading behaviour of $\Delta^{ab}_{\mu\nu}(p,m)$, Eq. (2.8a), as $p\to\infty$ and $p^T\to\infty$, respectively. This leading behaviour corresponds to $\alpha_1=-2$ (or $\alpha_5=-2$) and $\alpha_5^T=-1$.

Next we shall define what we mean by a "bit" of a Feynman diagram. A "bit" is that portion of the whole diagram which is derived by replacing every gauge propagator by precisely one of the $\Delta^{(i)}_{\mu\nu}$-components in Eq. (2.20). Thus, in a "bit" every line has attached to it only one $\Delta^{(i)}_{\mu\nu}$, so that $\Omega_m^{(\mathrm{bit})}$ and $\Omega_T^{(\mathrm{bit})}$ are always less than or equal to the corresponding Ω_m and Ω_T of the whole diagram:

$$\begin{aligned}\Omega_m^{(\mathrm{bit})} &= \Omega_m - 2I^{(-4)} - I^{(-3)},\\ \Omega_T^{(\mathrm{bit})} &= \Omega_T - 3I_T^{(-4)} - 2I_T^{(-3)} - I_T^{(-2)}.\end{aligned} \tag{3.5}$$

$I^{(-4)}$ and $I^{(-3)}$ represent the number of $\Delta^{(i)}_{\mu\nu}$-terms in a "bit", having $\alpha_i=-4$ and $\alpha_i=-3$, respectively. Similarly, $I_T^{(-4)}, I_T^{(-3)}$ and $I_T^{(-2)}$ denote the number of $\Delta^{(i)}_{\mu\nu}$ in a "bit" with $\alpha_i^T = -4, \alpha_i^T=-3$, and $\alpha_i^T=-2$, respectively.

The evanescent operators in $\Delta^{ab}_{\mu\nu}(\tilde{p},\hat{p};m)$ have $\alpha_i=-4$, and $\alpha_i^T=-4$, or $\alpha_i^T=-3$ if $i\neq 3$. According to (2.12), $E_A\geq 2$ implies $\Omega_m\leq 1$. Hence, if a "bit" involves at least one

$\Delta^{(i)}_{\mu\nu}, i=2,4,6$, or more than one $\Delta^{(3)}_{\mu\nu}$,

$$\Omega_m^{(\mathrm{bit})} < 0, \text{ and } \Omega_T^{(bit)} < 0, \tag{3.6}$$

and the "bit" turns out to be UV finite by power-counting. Its contribution, according to (3.1) and (3.2), would therefore vanish as $D\to 3$ (the integrals are taken to be IR-finite by power counting). The "bits" involving one $\Delta^{(3)}_{\mu\nu}$ and at least a single $\Delta^{(j)}_{\mu\nu}$, $j=1,7,8$, have both $\Omega_m^{(\mathrm{bit})}<0$ and $\Omega_T^{(\mathrm{bit})}<0$, and should also disappear as $D\to 3$.

The above analysis leads to the following conclusion. In the absence of IR divergences at non-exceptional momenta, the only non-vanishing "bits" in the sense of Eq. (3.2) ($D\to 3$), are those which have one $\Delta^{(3)}_{\mu\nu}$ and the remaining $\Delta^{(i)}$'s are all $\Delta^{(5)}$'s. Of course, "bits" not involving evanescent operators may, in principle, contribute as $D\to 3$.

3.3 Contributions from evanescent operators

Even with these simplifications, most integrals contain the computationally inconvenient denominator $Den(p)=(p^2)^2-m^2\tilde{p}^2$. Fortunately, we exploit the decomposition formula

$$\frac{1}{(p^2)^2-m^2\tilde{p}^2} = \frac{1}{p^2(p^2-m^2)} + \frac{m^2\hat{p}^2}{((p^2)^2-m^2\tilde{p}^2)p^2(p^2-m^2)}, \tag{3.7}$$

in which the second term on the right-hand side is an evanescent operator. Its overall UV degree is two units smaller than the UV degree of $1/Den(p)$. The same holds for the UV degree along the p^T-direction. On the other hand, each term in (3.7) contributes terms of the kind

$$\frac{m^2\hat{p}^2}{[(p^2)^2-m^2\tilde{p}^2]p^2(p^2-m^2)}, \tag{3.8}$$

that have degrees of divergence $\Omega_m^{(\mathrm{bit})}<0$ and $\Omega_T^{(\mathrm{bit})}<0$, for $E_A\geq 2$. Hence, the contributions from these evanescent operators vanish in the limit $D\to 3$. This result is based on the assumption that all integrals are IR convergent. To show that this is indeed the case for any one-loop diagram with non-exceptional momenta, we proceed as follows.

Let q_μ be the loop momentum; the overall IR index of $\Delta^{(i)}_{\mu\nu}(q)$, called β_i, is then defined by

$$\Delta^{(i)}_{\mu\nu}(\lambda q) \sim \lambda^{\beta_i}, \text{ as } \lambda\to 0, \tag{3.9}$$

so that $\beta_i\geq -2\ \forall i:1,...,8$. The propagators of the other lines are $\Delta^{ab}_{\mu\nu}(q+\sum_{i=1}^r p_i)$, where $p_i...p_{E-1}$ are non-exceptional, i.e., $\sum_{i=1}^r p_i\neq 0\ \forall r:1,...,E-1$; E is the number of external legs. These propagators have IR index equal to zero (along the q_μ-direction), as do the vertices. As $q\to 0$, the overall IR degree of the diagrams turns out to be $3+\min\{p_i\}=1$ at the one-loop level. Note that for non-exceptional momenta,

$$\sum_{i=1}^r p_i \neq \sum_{i=1}^{r'} p_i\ \forall r,r'.$$

In summary, none of the evanescent operators $\Delta^{(2)}_{\mu\nu},\Delta^{(3)}_{\mu\nu},\Delta^{(4)}_{\mu\nu},\Delta^{(6)}_{\mu\nu}$ contribute for non-exceptional momenta as $D\to 3$.

Finally, we must analyze the contribution to the vacuum polarization tensor coming from a single $\Delta^{(3)}_{\mu\nu}$, the remaining $\Delta^{(i)}_{\mu\nu}$ in the "bit" being $\Delta^{(5)}_{\mu\nu}$-terms. This contribution can be written as

$$\hat{I}_{\mu\nu} = (\hat{n}_\alpha \hat{n}_\beta + \hat{n}^2 g_{\alpha\beta}) I^{\alpha\beta}_{\mu\nu}, \tag{3.10}$$

$$I^{\alpha\beta}_{\mu\nu} = \int d^D q\, V^{abc}_{\mu\mu_1\mu_2}(p, q, -q-p) \Delta^{(3)\alpha\beta\mu_1\mu_2}(q) \Delta^{(5)\nu_1\nu_2}(q+p)$$

$$V^{dbc}_{\mu\nu_1\nu_2}(-p, -q, q+p), \tag{3.11}$$

where $V^{abc}_{\mu\rho\nu}(p,q,r)$ is defined in Eq. (2.9), and $\Delta^{(5)}_{\mu\nu}(q+p)$ and $\Delta^{(3)}_{\mu\nu}(q) \equiv (\hat{n}_\alpha \hat{n}_\beta + \hat{n}^2 \tilde{g}_{\alpha\beta}) \Delta^{(3)\alpha\beta}_{\mu\nu}(q)$, are given in Eq. (2.20). The overall UV degree of $I^{\alpha\beta}_{\mu\nu}$ is equal to -1, [cf. Eq. (3.5)], whereas the UV degree along the q^T-direction is zero [cf. Eq. (3.5)]. Thus the UV divergent contribution to (3.11) is obtained by setting $p = 0$ in the numerator of the integrand and discarding all epsilon tensors. The non-vanishing contribution to $I^{\alpha\beta}_{\mu\nu}$ now reads ($D \to 3$)

$$\int \frac{d^D q V^{(YM)abc}_{\mu\mu_1\mu_2}(0,q,-q) q^\alpha q^\beta q^{\mu_1} q^{\nu_1} q^{\mu_2} n^{\nu_2} V^{(YM)dbc}_{\nu_1\nu_2\nu}(0,q,-q)}{(q^2)^2 (q^2-m^2)(q\cdot n)^2 [(q+p)\cdot n][(q+p)^2 - m^2]}; \tag{3.12}$$

$V^{(YM)abc}_{\mu\nu\rho}(p,q,r)$ denotes $V^{abc}_{\mu\nu\rho}(p,q,r)$ without the $\varepsilon_{\mu\nu\rho}$. It is easy to check that any term in (3.12) which corresponds to the contraction of n^{ν_2} with an external momentum, has UV degree equal to -1 along the q^T-direction. Such terms do not contribute, therefore, to $\hat{I}_{\mu\nu}$ ($D \to 3$), and one is left with expressions of the type

$$\int \frac{d^D q (q^2)^2 q_{\mu'} n_{\nu'} q^{\alpha'} q^{\beta'}}{(q^2)^2 (q^2-m^2)(q\cdot n)^2 [(q+p)\cdot n][(q+p)^2 - m^2]}, \tag{3.13}$$

where the index set $\{\mu', \nu', \alpha', \beta'\}$ corresponds to any ordering of μ, ν, α, β. The vector n_μ is <u>not</u> to be contracted.

Application of

$$\frac{1}{(q+p)^2 - m^2} = \frac{1}{q^2 - m^2} - \frac{2q\cdot p + p^2}{(q^2-m^2)((q+p)^2 - m^2)} \tag{3.14}$$

permits us to recast (3.13) in the form

$$\int \frac{d^D q\, q_{\mu'} n_{\nu'} q^{\alpha'} q^{\beta'}}{(q^2-m^2)^2 (q\cdot n)^2 (q+p)\cdot n} + \text{UV finite terms.} \tag{3.15}$$

Utilizing (2.23), we may rewrite the above integral as

$$\frac{1}{p\cdot n} \int \frac{d^D q\, q_{\mu'} n_{\nu'} q^{\alpha'} q^{\beta'}}{(q^2-m^2)(q\cdot n)^2} - \frac{1}{(p\cdot n)^2} \int \frac{d^D\, q q^{\alpha'} q^{\beta'} q_{\mu'} n_{\nu'}}{(q^2-m^2)^2 q\cdot n}$$
$$+ \frac{1}{(p\cdot n)^2} \int \frac{d^D q\, q_{\mu'} n_{\nu'} q^{\alpha'} q^{\beta'}}{(q^2-m^2)^2 (q+p)\cdot n}. \tag{3.16}$$

The first term in (3.16) is antisymmetric and so vanishes in dimensional regularization. A .hough the second and third integrals are UV divergent by power-counting, they do not possess poles at $D = 3$ because space-time is <u>*odd*</u>. Accordingly, the integral $I^{\alpha\beta}_{\mu\nu}$ vanishes as $D \to 3$ [cf. (3.2)].

The practical results of this section for the action S_m in Eq. (2.7) may be summarized as follows. In order to compute the non-vanishing contributions to the one-loop vacuum polarization tensor, we use the gauge propagator $\Delta^{ab}_{\mu\nu}(p,m)$ in Eq. (2.8a) in which every vector is interpreted as a D-dimensional object and $\varepsilon_{\mu\rho\nu}$ is defined according to Eq. (2.16).

4 Evaluation of Vacuum Polarization Tensor

4.1 Preliminaries

In this section we summarize the computation of the one-loop Chern-Simons vacuum polarization tensor in the light-cone gauge,

$$n \cdot A^a(x) = 0, \quad n^2 = 0. \tag{4.1}$$

In this gauge, the Lagrangian density, Eq. (2.7), leads to the bare boson propagator $\Delta^{ab}_{\mu\nu}(p;m)$ in Eq. (2.8a), where m is the regulator mass introduced in Eq. (2.6). The form of $\Delta^{ab}_{\mu\nu}$ seems sufficiently complicated to discourage detailed perturbative calculations - even in the light-cone gauge. Fortunately, there exists an identity involving the epsilon-tensors that was discovered by one of the authors [9] and that simplifies the structure of $\Delta^{ab}_{\mu\nu}$ considerably. This identity reads

$$\frac{1}{q\cdot n} n^\rho \varepsilon_{\mu\rho\nu} = \frac{1}{q^2}\left[q^\rho \varepsilon_{\mu\rho\nu} + \frac{1}{q\cdot n}(q_\mu q^\rho n^\sigma \varepsilon_{\nu\rho\sigma} - q_\nu q^\rho n^\sigma \varepsilon_{\mu\rho\sigma})\right], \tag{4.2}$$

and reduces the Chern-Simons propagator (2.8a) to the manageable form:

$$\Delta^{ab}_{\mu\nu}(q) = \frac{i\delta^{ab}}{q^2(q^2-m^2)}\left[-\frac{im^2 q^2}{q\cdot n}\varepsilon_{\mu\rho\nu} n^\rho - mq^2 g_{\mu\nu} + \frac{mq^2}{q\cdot n}(q_\mu n_\nu + q_\nu n_\mu)\right]. \tag{4.3}$$

The three-point and four-point vertices are given, respectively, by Eqs. (2.9) and (2.10).

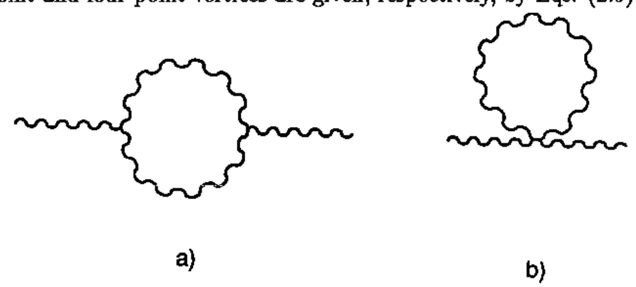

Figure 1: One-loop contributions to the vacuum polarization tensor. The wavy lines represent gauge fields.

4.2 The vacuum polarization tensor

There are two contributions to the Chern-Simons vacuum polarization tensor $\Pi^{tot}_{\mu\nu}$, shown in Fig. 1. The D-dimensional space-time contribution, $\Pi^{(blob)}_{\mu\nu}$, coming from Fig. 1a, follows from the Feynman rules (4.3) and (2.9). Thus,

$$\Pi^{(blob)ab}_{\mu\nu}(p) = \delta^{ab} c_v g^2 \int \frac{d^D q}{(2\pi)^D} V_{\mu\mu_1\mu_2}(p,q,-q-p) \Delta_{\nu_1\mu_1}(q) \Delta_{\mu_2\nu_2}(q+p)$$
$$\cdot\ V_{\nu\nu_1\nu_2}(-p,-q,q+p), \tag{4.4}$$

where $\Delta^{ab}_{\mu\nu}(q) \equiv \delta^{ab}\Delta_{\mu\nu}(q)$, and

$$\begin{aligned} V_{\mu\mu_1\mu_2}(p,q,-q-p) &= -i\varepsilon_{\mu\mu_1\mu_2} - \frac{1}{m}[(2q+p)_\mu g_{\mu_1\mu_2} - (q+2p)_{\mu_1} g_{\mu\mu_2} \\ &+ (p-q)_{\mu_2} g_{\mu\mu_1}], \qquad (4.5a) \\ V_{\nu\nu_1\nu_2}(-p,-q,q+p) &= -i\varepsilon_{\nu\nu_1\nu_2} - \frac{1}{m}[-(2q+p)_\nu g_{\nu_1\nu_2} + (q+2p)_{\nu_1} g_{\nu\nu_2} \\ &- (p-q)_{\nu_2} g_{\nu\nu_1}], \qquad (4.5b) \end{aligned}$$

and $f^{acd}f^{bcd} = c_v\delta^{ab}$.

For convenience, we split the propagator in Eq. (4.3) into light-cone (LC) and non-light-cone (NLC) components,

$$\begin{aligned} \Delta_{\mu\nu}(q) &= \Delta^{NLC}_{\mu\nu}(q) + \frac{1}{q\cdot n}\Delta^{(LC)}_{\mu\nu}(q), \qquad (4.6a) \\ \Delta^{NLC}_{\mu\nu}(q) &= \frac{i}{q^2(q^2-m^2)}(-mq^2 g_{\mu\nu}), \qquad (4.6b) \\ \Delta^{LC}_{\mu\nu}(q) &= \frac{i}{q^2(q^2-m^2)}\left[-im^2q^2\varepsilon_{\mu\rho\nu}n^\rho + mq^2(q_\mu n_\nu + q_\nu n_\mu)\right]. \qquad (4.6c) \end{aligned}$$

With these"improvements", the vacuum polarization tensor separates into four components:

$$\begin{aligned} \Pi^{(blob)ab}_{\mu\nu}(p) &\equiv \delta^{ab}c_v g^2 \Pi^{(blob)}_{\mu\nu}(p), \\ \Pi^{(blob)}_{\mu\nu}(p) &= \Pi^{(1)}_{\mu\nu}(p) + \Pi^{(2)}_{\mu\nu}(p) + \Pi^{(3)}_{\mu\nu}(p) + \Pi^{(4)}_{\mu\nu}(p), \qquad (4.7) \end{aligned}$$

where

$$\Pi^{(1)}_{\mu\nu}(p) = \int \frac{d^Dq}{(2\pi)^D} V_{\mu\mu_1\mu_2}(p,q,-q-p)\Delta^{NLC}_{\nu_1\mu_1}(q)\Delta^{NLC}_{\mu_2\nu_2}(q+p)V_{\nu\nu_1\nu_2}(-p,-q,q+p), \qquad (4.8)$$

$$\Pi^{(2)}_{\mu\nu}(p) = \int \frac{d^Dq}{(2\pi)^D} V_{\mu\mu_1\mu_2}(p,q,-q-p)\frac{\Delta^{LC}_{\nu_1\mu_1}}{q\cdot n}\Delta^{NLC}_{\mu_2\nu_2}(q+p)V_{\nu\nu_1\nu_2}(-p,-q,q+p), \qquad (4.9)$$

$$\Pi^{(3)}_{\mu\nu}(p) = \int \frac{d^Dq}{(2\pi)^D} V_{\mu\mu_1\mu_2}(p,q,-q-p)\Delta^{NLC}_{\nu_1\mu_1}(q)\frac{\Delta^{LC}_{\mu_2\nu_2}(q+p)}{(q+p)\cdot n}V_{\nu\nu_1\nu_2}(-p,-q,q+p), \qquad (4.10)$$

$$\Pi^{(4)}_{\mu\nu}(p) = \int \frac{d^Dq}{(2\pi)^D} V_{\mu\mu_1\mu_2}(p,q,-q-p)\frac{\Delta^{LC}_{\nu_1\mu_1}(q)}{q\cdot n}\frac{\Delta^{LC}_{\mu_2\nu_2}(q+p)}{(q+p)\cdot n}V_{\nu\nu_1\nu_2}(-p,-q,q+p). \qquad (4.11)$$

Due to the presence of $\varepsilon_{\mu\nu\sigma}$-tensors in the propagator and vertex function, the computation of $\Pi^{(blob)}_{\mu\nu}$ turns out to be much longer than in the case of Yang-Mills theory [see second reference in [14]]. Fortunately, both $\Delta_{\mu\nu}$ and $V_{\mu\nu\sigma}$ possess certain symmetries, such as

$$\begin{aligned} \Delta^{NLC}_{\mu\nu}(-q) &= \Delta^{NLC}_{\nu\mu}(q),\ \frac{1}{q\cdot n}\Delta^{LC}_{\mu\nu}(q) = -\frac{1}{q\cdot n}\Delta^{LC}_{\nu\mu}(-q), \qquad (4.12a) \\ V_{\mu\rho\nu}(p,q,r) &= -V_{\mu\nu\rho}(p,r,q), etc. \qquad (4.12b) \end{aligned}$$

which help reduce the amount of algebra by at least 50%. Accordingly, we find that

$$\Pi^{(3)}_{\mu\nu}(p) = \Pi^{(2)}_{\mu\nu}(p). \qquad (4.13)$$

The treatment of $\Pi^{(4)}_{\mu\nu}$ may also be simplified by recalling the separation formula [24]

$$[q\cdot n(q+p)\cdot n]^{-1} = (p\cdot n)^{-1}[(q\cdot n)^{-1} - ((q+p)\cdot n)^{-1}],\ p\cdot n \neq 0, \qquad (4.14)$$

and then convincing oneself that

$$\Pi^{(4)}_{\mu\nu}(p) = (p\cdot n)^{-1}[\Pi^{(41)}_{\mu\nu}(p) - \Pi^{(42)}_{\mu\nu}(p)] = 2(p\cdot n)^{-1}\Pi^{(41)}_{\mu\nu}(p), p\cdot n \neq 0, \qquad (4.15)$$

$$\Pi^{(41)}_{\mu\nu}(p) = \int \frac{d^Dq}{(2\pi)^D} V_{\mu\mu_1\mu_2}(p,q,-q-p)\frac{1}{q\cdot n}\Delta^{LC}_{\nu_1\mu_1}(q)\Delta^{LC}_{\mu_2\nu_2}(q+p)V_{\nu\nu_1\nu_2}(-p,-q,q+p). \qquad (4.16)$$

Clearly

$$\Pi^{(blob)}_{\mu\nu}(p) = \Pi^{(1)}_{\mu\nu}(p) + 2\Pi^{(2)}_{\mu\nu}(p) + 2(p\cdot n)^{-1}\Pi^{(41)}_{\mu\nu}(p). \qquad (4.17)$$

The computation of $\Pi^{(blob)}_{\mu\nu}(p)$ was carried out in three stages:

(1) Reduction of the integrands of $\Pi^{(1)}, \Pi^{(2)}$ and $\Pi^{(4)}$ with the help of the computer program REDUCE.

(2) Integration over d^Dq-space.

(3) Simplification of the integrated expressions, if necessary, by means of certain identities. We highlight the procedure for $\Pi^{(1)}_{\mu\nu}$.

4.3 The component $\Pi^{(1)}_{\mu\nu}$

Substitution of the Feynman rules (2.9) and (4.6b) into Eq. (4.8) and reduction of the integrand yields

$$\begin{aligned} \Pi^{(1)}_{\mu\nu}(p) &= \int \frac{d^Dq}{(2\pi)^D} \frac{1}{(q^2-m^2)((q+p)^2-m^2)}\{[2m^2+4p^2+q^2+(q+p)^2]g_{\mu\nu} \\ &+ 6q_\mu q_\nu + 3(q_\mu p_\nu + q_\nu p_\mu) - 3p_\mu p_\nu - 6imp^\alpha\varepsilon_{\alpha\mu\nu}\}. \qquad (4.18) \end{aligned}$$

The integrals in Eq. (4.18) can be computed with the procedure indicated in the example below. Consider the UV-finite integral:

$$J = m\int \frac{d^Dq}{(2\pi)^D(q^2-m^2)((q+p)^2-m^2)}. \qquad (4.19)$$

Rescaling the momentum variable by $q_\mu \to mq_\mu$, and then operating with $\lim_{m\to\infty}$ $\lim_{D\to 3}$ on both sides of Eq. (4.19), we obtain

$$\lim_{m\to\infty}\lim_{D\to 3} J \equiv J_0 = \lim_{m\to\infty}\lim_{D\to 3} m^{D-3}\int \frac{d^Dq}{(2\pi)^D}\frac{1}{(q^2-1)((q+p/m)^2-1)},$$

$$J_0 = \lim_{m\to\infty}\lim_{D\to 3}\left\{m^{D-3}\int\frac{d^Dq}{(2\pi)^D}\left(\frac{1}{(q^2-1)^2} - \frac{2q\cdot(p/m)+p^2/m^2}{(q^2-1)^2((q+p/m)^2-1)}\right)\right\}, \qquad (4.20)$$

where we used the identity

$$\frac{1}{(q+\tilde{p})^2-1}=\frac{1}{q^2-1}-\frac{2q\cdot\tilde{p}+\tilde{p}^2}{(q^2-1)((q+\tilde{p})^2-1)},\ \tilde{p}\equiv p/m. \tag{4.21}$$

Taking the limit $D\to 3$ in Eq. (4.20), we find that

$$J_0=\lim_{m\to\infty}\int\frac{d^3q}{(2\pi)^3(q^2-1)^2}-\lim_{m\to\infty}\frac{1}{m}\int\frac{d^3q\,(2q\cdot p+p^2/m)}{(2\pi)^3(q^2-1)^2((q+p/m)^2-1)}. \tag{4.22}$$

The first integral is both UV- and IR-finite and has the value $i/8\pi$. (Cf. Appendix B). The second integral is likewise UV-convergent by the Hahn-Zimmermann power-counting theorem [25], enabling us to take $m\to\infty$. Hence

$$J_0=\lim_{m\to\infty}[i/8\pi-m^{-1}(\text{convergent integral})]=\bar{I},\ \bar{I}\equiv\frac{i}{8\pi}. \tag{4.23}$$

The remaining integrals in Eq. (4.18) are derived in similar fashion, so that we finally get

$$\Pi^{(1)}_{\mu\nu}(p)=(14g_{\mu\nu}-6ip^\alpha\varepsilon_{\alpha\mu\nu})\bar{I}. \tag{4.24}$$

4.4 Results for $\Pi^{(2)}_{\mu\nu},\Pi^{(41)}_{\mu\nu}$ and $\Pi^{tot}_{\mu\nu}$

The un-integrated expression for $2\Pi^{(2)}_{\mu\nu}$ has been banished to Appendix C, whereas $\Pi^{(41)}_{\mu\nu}$ has not been documented in this paper, because it is twice as long as $\Pi^{(2)}_{\mu\nu}$. After performing the various integrations and collecting terms, we eventually get these answers:

$$\begin{aligned}2\Pi^{(2)}_{\mu\nu}(p) &= \Big[-24mg_{\mu\nu}+18ip^\alpha\varepsilon_{\alpha\mu\nu}+\frac{4ip\cdot n}{n\cdot n^*}n^*_\alpha\varepsilon_{\alpha\mu\nu}\\ &- \frac{8ip\cdot n^*}{n\cdot n^*}n^\alpha\varepsilon_{\alpha\mu\nu}+\frac{8m}{n\cdot n^*}(n_\mu n^*_\nu+n_\nu n^*_\mu)\Big]\bar{I};\end{aligned} \tag{4.25}$$

$$\begin{aligned}\frac{2}{p\cdot n}\Pi^{(41)}_{\mu\nu}(p) &= \Big[10mg_{\mu\nu}+\frac{2ip\cdot n^*}{n\cdot n^*}n^\alpha\varepsilon_{\alpha\mu\nu}-4ip^\alpha\varepsilon_{\alpha\mu\nu}\\ &- \frac{4ip\cdot n}{n\cdot n^*}n^{*\alpha}\varepsilon_{\alpha\mu\nu}-\frac{4m}{n\cdot n^*}(n_\mu n^*_\nu+n_\nu n^*_\mu)\\ &+ \frac{6ip\cdot n^*}{p\cdot n\ n\cdot n^*}n^\alpha p^\beta(n_\mu\varepsilon_{\alpha\beta\nu}-n_\nu\varepsilon_{\alpha\beta\mu})\Big]\bar{I}.\end{aligned} \tag{4.26}$$

The vacuum polarization tensor also receives the following finite contribution from the massive tadpole in Fig. 1b:

$$\begin{aligned}\Pi^{(tadpole)}_{\mu\nu}g^2c_v\delta^{ab} &= \int\frac{d^Dq}{(2\pi)^D}V^{abcd}_{\mu\nu\sigma\rho}\Delta^{cd}_{\rho\sigma}(q)\\ &= g^2\delta^{ab}c_v\Big[-\frac{4m}{n\cdot n^*}(n_\nu n^*_\mu+n_\mu n^*_\nu)\Big]\bar{I}.\end{aligned} \tag{4.27}$$

Addition of Eq. (4.27) to Eqs. (4.24)-(4.26), and inclusion of the symmetry factor 1/2, lead to the final result

$$\begin{aligned}\Pi^{(tot)ab}_{\mu\nu}(p) &= 2^{-1}\Pi^{(blob)ab}_{\mu\nu}(p)+2^{-1}\Pi^{(tadpole)}_{\mu\nu}(p),\\ \Pi^{(tot)ab}_{\mu\nu}(p) &= ic_vg^2\delta^{ab}\Big[4p^\alpha\varepsilon_{\alpha\mu\nu}-\frac{3p\cdot n^*}{n\cdot n^*}n^\alpha\varepsilon_{\sigma\mu\nu}\\ &+ \frac{3p\cdot n^*}{p\cdot nn\cdot n^*}n^\alpha p^\beta(n_\mu\varepsilon_{\nu\alpha\beta}-n_\nu\varepsilon_{\mu\alpha\beta})\Big]\bar{I},\bar{I}=\frac{i}{8\pi}.\end{aligned} \tag{4.28}$$

The Chern-Simons vacuum polarization tensor (4.28) is UV-finite and transverse, but contains one non-local term which is strictly gauge-dependent, just as in the case of Yang-Mills theory in the light-cone gauge. The simple structure of $\Pi^{(tot)}_{\mu\nu}$ is due in part to the application of certain identities, such as Eq. (4.2) and

$$\frac{1}{n\cdot n^*}p\cdot n\ n^{*\alpha}\varepsilon_{\alpha\mu\nu}=p^\alpha\varepsilon_{\alpha\mu\nu}-\frac{1}{n\cdot n^*}n^{*\alpha}p^\beta(n_\nu\varepsilon_{\mu\alpha\beta}-n_\mu\varepsilon_{\nu\alpha\beta}), \tag{4.29a}$$

$$\frac{1}{(n\cdot n^*)^2}p\cdot nn^\alpha n^{*\beta}(n^*_\nu\varepsilon_{\mu\alpha\beta}-n^*_\mu\varepsilon_{\nu\alpha\beta})=\frac{p\cdot n}{n\cdot n^*}n^{*\alpha}\varepsilon_{\alpha\mu\nu}. \tag{4.29b}$$

Several other identities have been employed in the reduction of $\Pi^{(tot)}_{\mu\nu}$.

5 The Effective Action

The purpose of this section is to analyze our results for the vacuum polarization tensor by using BRS techniques. We begin by introducing external fields K^a and J^a and coupling them, respectively, to the non-linear BRS transformations sc^a and sA^a. In three dimensions, K^a and J^a have the dimension of a mass; they have ghost number -2 and -1, respectively, and are BRS invariant: $sK^a=sJ^a=0$.

Proceeding as in four-dimensional Yang-Mills theory in the homogeneous light-cone gauge, we find that the one-loop effective action, $\Gamma_{1\text{-loop}}$, obeys the BRS equation

$$\Delta\Gamma_{1\text{-loop}}(A,c,\bar{c},K,J)=0, \tag{5.1}$$

where Δ is the Slavnov-Taylor operator:

$$\Delta=\int d^3x\left\{\frac{\delta\bar{\Gamma}_0}{\delta A^a_\mu}\frac{\delta}{\delta J^a_\mu}+\frac{\delta\bar{\Gamma}_0}{\delta J^a_\mu}\frac{\delta}{\delta A^a_\mu}+\frac{\delta\bar{\Gamma}_0}{\delta c^a}\frac{\delta}{\delta K^a}+\frac{\delta\bar{\Gamma}_0}{\delta K^a}\frac{\delta}{\delta c^a}\right\},\ \Delta^2=0, \tag{5.2}$$

with

$$\bar{\Gamma}_0=S_{CS}+\int d^3x\left\{\bar{c}^an^\mu(D_\mu c)^a+J^a_\mu(D_\mu c)^a-\frac{1}{2}gf^{abc}K^ac^bc^c\right\}. \tag{5.3}$$

We also recall that the equation of motion for the field $\bar{c}^a$ implies that $\bar{c}^a$ and J^a always occur in the combination $J^a_\mu+n_\mu\bar{c}^a\equiv\rho^a_\mu$, in the effective action.

The solution of the cohomology problem (5.1) can be expressed in the form

$$\Gamma_{1\text{-loop}}=cS_{CS}+\Delta X, \tag{5.4}$$

where X is an arbitrary, non-local, integrated functional of the fields $A^a,c^a,\bar{c}^a,K^a$ and J^a, with mass dimension 2, and ghost number -1. To motivate the functional structure of X, consider a 1PI function which involves at least one of the fields K^a,J^a,c^a and $\bar{c}^a$. Now, any one-loop diagram, contributing to such a function has at least one vertex of the type $gf^{abc}\bar{c}^an\cdot A^bc^c$, where A^b_μ corresponds to an internal line. Since $n^\mu\Delta_{\mu\nu}=0$, this diagram vanishes. Accordingly, there is no contribution to $\Gamma_{1\text{-loop}}$ from terms involving one or more of the fields K^a,J^a,c^a and $\bar{c}^a$. This conclusion implies that ΔX in Eq. (5.4) should only depend on A^a_μ. However, it remains to convince ourselves that X can indeed be written as

$$X=\int d^3x\rho^a_\mu\Phi^a_\mu(A)+X'(A,J,K,c,\bar{c}), \tag{5.5}$$

where $\Phi^a_\mu(A)$ has mass dimension 1.

Under a continuous gauge transformation $U, T^a\Phi^a_\mu(A)$ transforms according to

$$T^a\Phi^a_\mu(A') = UT^a\Phi^a_\mu(A)U^\dagger. \tag{5.6}$$

As for the functional $X'(A,J,K,c,\bar{c})$, it does not contain terms of the form $\int d^3x \rho^a_\mu \Phi^a_\mu(A)$ and can be shown to satisfy

$$\Delta X'(A,J,K,c,\bar{c}) = 0. \tag{5.7}$$

Equation (5.7) implies that X' does not contribute to $\Gamma_{1\text{-loop}}$. We notice the following point, however. Since the cohomology problem (5.7) is not solved over the space of local functionals of ghost number 1 and mass dimension 2, its solution will not be of the form $X' = 0$. For instance, the functional X',

$$X' = \Delta\left\{\int d^3x f^{abc} K^a(x)\left(\frac{1}{\partial\cdot n}\right) c^b(x)\bar{c}^c(x)\right\},$$

is a solution of Eq. (5.7).

Eqs. (5.4)-(5.5) and (5.7) lead to the following solution of Eq. (5.1):

$$\Gamma_{1\text{-loop}} = cS_{CS} + \int d^3x \frac{\delta S_{CS}}{\delta A^a_\mu(x)}\Phi^a_\mu(A). \tag{5.8}$$

We must compute $\Phi^a_\mu(A)$ and then match the r.h.s. of Eq. (5.8) with $-i\Pi^{(tot)ab}_{\mu\nu}$, in Eq. (4.28). We know that $\Pi^{(tot)ab}_{\mu\nu}$ does not contain any parity preserving terms, i.e. it only involves expressions proportional to the Levi-Civita symbol. Moreover, since the only non-local term in $\Pi^{(tot)}_{\mu\nu}$ is proportional to $(p\cdot n)^{-1}, \Phi^a_\mu$ should only depend on that type of non-locality. A plausible ansatz for $\Phi^a_\mu(A)$ is:

$$\begin{aligned}\Phi^a_\mu(A) &= c_1\frac{n_\mu}{n\cdot n^*}\left(\frac{1}{n\cdot D}n^\sigma F_{\sigma\nu}n^{*\nu}\right)^a \\ &+ c_2\left(\frac{1}{n\cdot D}F_{\mu\nu}n^\nu\right)^a + O\left(\left(\frac{1}{n\cdot D}\right)^2\right),\end{aligned} \tag{5.9}$$

where $n\cdot D^{ab} = n\cdot\partial\delta^{ab} + gf^{abc}n\cdot A^c; \Phi^a_\mu(A)$ has mass dimension +1, and transforms as in Eq. (5.6). Substituting Eq. (5.9) into Eq. (5.8), extracting the part quadratic in A^a_μ and, finally, comparing the result with $-i\Pi^{(tot)ab}_{\mu\nu}$, we find that

$$c_1 = \frac{3c_v g^2}{8\pi}, \quad c - 2c_2 = \frac{c_v g^2}{8\pi}. \tag{5.10}$$

To deduce either c_2, or c, one needs to compute the three-point function for the field A^a_μ, which is beyond the scope of this article. We note, however, that the value of c in Eq. (5.8) ought to coincide for two distinct gauge - fixing terms, provided the same regularization method is used in both gauges. We may therefore "borrow" the coefficient c from computations in other gauges, for instance, from computations in the background field gauge [6]; these yield the value $c = c_v g^2/4\pi$. Hence

$$c_2 = \frac{c_v g^2}{16\pi}, \tag{5.11}$$

and the effective action $\Gamma(A)$ becomes

$$\Gamma(A) = \left(1+\frac{c_v g^2}{4\pi}\right)S_{CS} + \int d^3x \frac{\delta S_{CS}}{\delta A^a_\mu}\Phi^a_\mu + O(\hbar^2), \tag{5.12}$$

with

$$\Phi^a_\mu(A) = \frac{c_v g^2}{8\pi}\frac{1}{n\cdot n^*}\left\{\frac{1}{n\cdot D}\left(3n_\mu n^\sigma F_{\sigma\nu}n^{*\nu} + \frac{1}{2}F_{\mu\nu}n^\nu\right)\right\} + O\left(\left(\frac{1}{n\cdot D}\right)^2\right).$$

The finite, non-multiplicative wave function renormalization,

$$A^a_\mu \longrightarrow A'^a_\mu = A^a_\mu - \Phi^a_\mu(A), \tag{5.13}$$

recasts $\Gamma(A)$ into the form

$$\Gamma(A') = \left(1+\frac{c_v g^2}{4\pi}\right)S_{CS}(A'). \tag{5.14}$$

By defining $A^{(ren)}_\mu = g^{-1}A'_\mu$ and recalling that $g^2 = 4\pi/k$, we may write Eq. (5.14) as

$$\Gamma(A^{(ren)}) = \frac{k+c_v}{4\pi}S_{CS}(A^{(ren)}), \quad k>0. \tag{5.15}$$

It is not difficult to convince oneself that if k is negative, Eq. (5.15) becomes

$$\Gamma(A^{(ren)}) = \frac{k-c_v}{4\pi}S_{CS}(A^{(ren)}), \tag{5.16}$$

and for arbitrary k,

$$\Gamma(A^{(ren)}) = \frac{k+\text{sign}(k)c_v}{4\pi}S_{CS}(A^{(ren)}). \tag{5.17}$$

Eq. (5.17) is compatible with the result obtained in the Landau gauge [4], [7] and the background field gauge [6], where the one-loop effective action can be written as in Eq. (5.17), provided a finite, local, multiplicative wave function renormalization is carried out. Furthermore, if in addition to renormalizing the wave function as in Eq. (5.13), we also renormalize $k, k \rightarrow k_{ren} - \text{sign}(k)c_v$, then the effective action is the same as that computed with the regularization methods of Refs. [3], [9], although the latter regularization does not explicitly preserve gauge invariance.

6 Conclusion

We have studied perturbative Chern-Simons theory in the light-cone gauge $n\cdot A^a = 0, n^2 = 0$, by evaluating the vacuum polarization tensor (4.28) to one-loop order. From a technical point of view, the computation is highlighted by:

(a) The use of new identities such as Eqs. (4.2) and (4.29).

(b) The application of a gauge-invariant regularization procedure, which consists of an algebraically meaningful version of dimensional regularization and the addition to the Chern-Simons action S_{CS} of a higher-derivative term

$$S_{YM} = -(4m^{-1})\int d^3x (F^a_{\mu\nu})^2,$$

where m is the regulator mass [cf. Eq. (2.7): $S_m = S_{CS} + S_{YM}$].

(c) A careful analysis of contributions from evanescent operators (Section 3) and of the UV-behaviour of the propagator (2.19) in Section 2.

(d) An explicit computation of about sixty new integrals, some of which have been listed in Appendix B.

The results of this paper may be summarized as follows:

1) The one-loop vacuum polarization tensor (4.28) is UV-finite as well as transverse, but contains a non-local term which is strictly gauge-dependent, in analogy with massless Yang-Mills theory in the light-cone gauge.

2) The dimensionally regularized theory is BRS invariant.

3) A finite, non-multiplicative (but gauge-dependent) wave-function renormalization enables us to express the one-loop effective action as

$$\Gamma_{1-loop} A(^{(ren)}_{\mu}, k) = \left(\frac{k + \text{sign}(k) c_v}{4\pi}\right) S_{CS}(A_\mu^{(ren)}),$$

with

$$S_{CS}(A_\mu^{(ren)}) = \int d^3x \varepsilon^{\mu\rho\nu} \Big[A_\mu^{(ren)a} \partial_\rho A_\nu^{(ren)b} + \frac{1}{3!} f^{abc} A_\mu^{(ren)a} A_\rho^{(ren)b} A_\nu^{(ren)c} \Big],$$

where k is the bare "coupling constant", and $A_\mu^{(ren)a}$ the non-multiplicatively renormalized gauge field. It appears, therefore, that the effective contribution coming from the one-loop two-point and three-point functions are given by a shift in $k, k \to k + c_v$, in agreement with results obtained in covariant gauges [4]-[5]. We should emphasize that the non-multiplicative renormalization of the wave function arises entirely (for a certain functional X) from the ΔX-term in Eq. (5.4). This non-multiplicative normalization is, therefore, not expected to contribute to the vacuum expectation values of Wilson loops. Of course, as the reader is probably well aware, the occurrence of a non-multiplicative wave function renormalization is not a special feature of the Chern-Simons model in the light-cone gauge: it has already been documented in Yang-Mills theory [16].

4) A finite renormalization of the form $k + \text{sign}(k) c_v \equiv k^{(ren)}$, permits us to write

$$\Gamma_{1-loop}(A_\mu^{(ren)}, k^{(ren)}) = \frac{k^{(ren)}}{4\pi} S_{CS}(A_\mu^{(ren)}).$$

In summary, the radiative corrections can be absorbed into a non-multiplicative wave-function renormalization, and a multiplicative coupling constant renormalization.

5) The physical light-cone gauge in the prescription (2.4) seems to work admirably also in three-dimensional Chern-Simons theory.

Let us finally notice that the situation described in items 3) and 4) is analogous to the one encountered in the Landau gauge, where explicitly gauge invariant regulators yield the same shift of the bare Chern-Simons parameter k [4]-[6], whereas methods that break gauge invariance at the regularized level yield zero radiative corrections [3], [7]. In accordance with Ref. [11], in the Landau gauge, radiative corrections, obtained with an explicitly gauge invariant regulator, can be absorbed into both a wave function and a coupling constant renormalization. We should also mention that axial-type gauges might prove useful in elucidating the connection between perturbative Chern-Simons theory and two-dimensional current algebra. Recent progress in this direction has been made by Emery and Piguet [26].

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Appendix A
Large mass limit of the integrals

In this appendix we compute the large m limit of various integrals which are UV *finite* by power-counting. These integrals have the form

$$I_{\mu_1\ldots\mu_E}(p,m) = \int d^3q \frac{(q_{\mu_1}\ldots q_{\mu_E})m^r}{(q^2-m^2)^{n_1}((q+p)^2-m^2)^{n_2}q\cdot n}, \tag{A.1}$$

where r is real. Suppose s is defined by $s \equiv 2+r+E-2n_1-2n_2, E=\mu_1+\ldots+\mu_E$. Then for $s<0$,

$$\lim_{m\to\infty} I_{\mu_1\ldots\mu_E}(p,m) = 0. \tag{A.2}$$

To prove (A.2) we first Wick-rotate (A.1) to Euclidean space ($q_0 \to iq_0, \vec{q} \to \vec{q}$) and use prescription (2.4), so that the corresponding Euclidean-space integral reads

$$I^{\rm Eucl}_{\mu_1\ldots\mu_E}(p,m) = \int \frac{d^3q\, q_{\mu_1}\ldots q_{\mu_E} m^r(-\vec{q}\cdot\vec{n}+iq_0n_0)}{(q^2+m^2)^{n_1}((q+p)^2+m^2)^{n_2}[(q_0n_0)^2+(\vec{q}\cdot\vec{n})^2]},$$

and then rescale $q\to qm$ to get

$$\begin{aligned} I^{\rm Eucl}_{\mu_1\ldots\mu_E}(p,m) &= m^s \int \frac{d^3q\, q_{\mu_1}\ldots q_{\mu_E}(-\vec{q}\cdot\vec{n}+iq_0n_0)}{(q^2+1)^{n_1}((q+\frac{p}{m})^2+1)^{n_2}[(q_0n_0)^2+(\vec{q}\cdot\vec{n})^2]}, \\ &\equiv m^s I^{\rm Eucl}_{\mu_1\ldots\mu_E}(p/m,1). \end{aligned} \tag{A.3}$$

From the inequalities [25]

$$\begin{aligned} (q+p/m)^2+1 &\le A(p/m)(q^2+1), \\ (q^2+1) &\le A(p/m)((q+p/m)^2+1), \end{aligned} \tag{A.4}$$

where $A(p/m) = 1+|p/m|+|p^2/m^2|$, we see that

$$\begin{aligned} &|I^{\rm Eucl}_{\mu_1\ldots\mu_E}(p/m,1)| \le A(p/m)|I^{\rm Eucl}_{\mu_1\ldots\mu_E}(0,1)|, \\ &|I^{\rm Eucl}_{\mu_1\ldots\mu_E}(0,1)| \le A(p/m)|I^{\rm Eucl}_{\mu_1\ldots\mu_E}(p/m,1)|. \end{aligned} \tag{A.5}$$

Hence

$$\lim_{m\to\infty} I^{\rm Eucl}_{\mu_1\ldots\mu_E}(p/m,1) = \lim_{m\to\infty} I^{\rm Eucl}_{\mu_1\ldots\mu_E}(0,1) < +\infty.$$

Since

$$I^{\rm Eucl}_{\mu_1\ldots\mu_E}(p,m) = m^s I^{\rm Eucl}_{\mu_1\ldots\mu_E}(p/m,1),$$

we conclude that

$$\lim_{m\to\infty} I^{\rm Eucl}_{\mu_1\ldots\mu_E}(p,m) = 0, \tag{A.6}$$

provided $s<0$. This completes the proof of (A.2).

Appendix B

The following non-covariant-gauge Feynman integrals occur in the computation of the vacuum polarization tensor in Eq. (4.4), or (4.17).

$$\begin{aligned}
I_1 &= m^2 \int dq \frac{1}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = 0, \quad \frac{d^Dq}{(2\pi)^D} \equiv dq, \\
I_2 &= \int dq \frac{1}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = 0 \\
I_3 &= \int dq \frac{(q+p)^2}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = 0 \\
I_4 &= \int dq \frac{q_\mu}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = 0 \\
I_5 &= m \int dq \frac{q_\mu}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{\bar{I}}{n\cdot n^*} n^*_\mu, \; \bar{I} = \frac{i}{8\pi} \\
I_6 &= m^3 \int dq \frac{1}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{\bar{I}}{2n\cdot n^*} p\cdot n^* \\
I_7 &= m \int dq \frac{1}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = 0 \\
I_8 &= m \int dq \frac{(q+p)^2}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{\bar{I}}{2n\cdot n^*} p\cdot n^* \\
I_9 &= m \int dq \frac{q^2}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = -\frac{3}{2}\frac{\bar{I}}{n\cdot n^*} p\cdot n^* \\
I_{10} &= m \int dq \frac{1}{((q+p)^2-m^2)q\cdot n} = -\frac{2\bar{I}}{n\cdot n^*} p\cdot n^* \\
I_{11} &= m \int dq \frac{q_\beta q_\mu}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = -\frac{\bar{I}}{2(n\cdot n^*)^2} [n\cdot n^*(g_{\beta\mu} p\cdot n^* \\
&+ p_\mu n^*_\beta + p_\beta n^*_\mu) - n_\mu n^*_\beta p\cdot n^* - n^*_\mu n^*_\beta p\cdot n - n_\beta n^*_\mu p\cdot n^*] \\
I_{12} &= m^2 \int dq \frac{q_\mu}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{m\bar{I}}{n\cdot n^*} n^*_\mu \\
I_{13} &= \int dq \frac{(q+p)^2 q_\mu}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{3m\bar{I}}{n\cdot n^*} n^*_\mu \\
I_{14} &= \int dq \frac{q_\mu}{(q^2-m^2)q\cdot n} = \frac{2m\bar{I}}{n\cdot n^*} n^*_\mu \\
I_{15} &= \int dq \frac{q^2 q_\mu}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{3m\bar{I}}{n\cdot n^*} n^*_\mu \\
I_{16} &= \int dq \frac{q^2}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = 0 \\
I_{17} &= \int dq \frac{1}{((q+p)^2-m^2)q\cdot n} = 0 \\
I_{18} &= m \int dq \frac{(q+p)^2 q_\mu}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{3m^2\bar{I}}{n\cdot n^*} n^*_\mu \\
&+ \frac{p^2\bar{I}}{12n\cdot n^*} n^*_\mu - \frac{p\cdot n^*\bar{I}}{3n\cdot n^*} p_\mu \\
&+ \frac{\bar{I}}{6(n\cdot n^*)^2}(2p\cdot n\, p\cdot n^*\, n^*_\mu + (p\cdot n^*)^2 n_\mu)
\end{aligned}$$

$$
\begin{aligned}
I_{19} &= m\int dq \frac{q^2 q_\mu}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{3m^2\bar{I}}{n\cdot n^*}\, n^*_\mu \\
&+ \frac{p^2\bar{I}}{12n\cdot n^*} n^*_\mu + \frac{5p\cdot n^*\bar{I}}{3n\cdot n^*} p_\mu \\
&- \frac{5\bar{I}}{6(n\cdot n^*)^2}(2p\cdot n\; p\cdot n^*\; n^*_\mu + (p\cdot n^*)^2 n_\mu) \\
I_{20} &= m^4\int dq \frac{1}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{m\bar{I}p\cdot n^*}{2n\cdot n^*} \\
I_{21} &= m^2\int dq \frac{(q+p)^2}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = \frac{m\bar{I}p\cdot n^*}{2n\cdot n^*} \\
I_{22} &= m^2\int dq \frac{q^2}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = -\frac{3m\bar{I}p\cdot n^*}{2n\cdot n^*} \\
I_{23} &= \int dq \frac{q^2(q+p)^2}{(q^2-m^2)((q+p)^2-m^2)q\cdot n} = -\frac{3m\bar{I}p\cdot n^*}{2n\cdot n^*} \\
I_{24} &= m^2\int dq \frac{1}{(q^2-m^2)q\cdot n} = 0 \\
I_{25} &= m^2\int dq \frac{1}{((q+p)^2-m^2)q\cdot n} = -\frac{2m\bar{I}p\cdot n^*}{n\cdot n^*}.
\end{aligned}
$$

Appendix C

The un-integrated component $\Pi^{(2)}_{\mu\nu}(p)$, defined in Eq. (4.9), has the following structure.

$$
\begin{aligned}
2\Pi^{(2)}_{\mu\nu} &= \int \frac{d^Dq}{(2\pi)^D}\frac{1}{(q^2-m^2)((q+p)^2-m^2)q\cdot n}\Big\{4[-2m^2q\cdot n \\
&+ m^2p\cdot n + p^2q\cdot n + 2p^2p\cdot n - (q+p)^2q\cdot n - 2(q+p)^2p\cdot n]g_{\mu\nu} \\
&- 4q\cdot nq_\mu q_\nu + 2(-q\cdot n + 3p\cdot n)(q_\mu p_\nu + q_\nu p_\mu) - 4q\cdot np_\mu p_\nu \\
&+ 4im(q\cdot n + 2p\cdot n)q^\alpha\varepsilon_{\alpha\mu\nu} + 2im[-m^2 - 4p^2 + 3(q+p)^2 - 2q^2]n^\alpha\varepsilon_{\alpha\mu\nu} \\
&- 2imp^\beta n^\alpha(\varepsilon_{\alpha\beta\nu}p_\mu - \varepsilon_{\alpha\beta\mu}p_\nu) \\
&+ 18imp^\beta n^\alpha(-\varepsilon_{\alpha\beta\nu}q_\mu + \varepsilon_{\alpha\beta\mu}q_\nu) \\
&+ 2imq^\beta n^\alpha(\varepsilon_{\alpha\beta\nu}p_\mu - \varepsilon_{\alpha\beta\mu}p_\nu) \\
&- 2imp^\alpha q^\beta(\varepsilon_{\alpha\beta\mu}n_\nu - \varepsilon_{\alpha\beta\nu}n_\mu) \\
&+ 2(q^2 - 2p^2 + (q+p)^2 - 2m^2)(n_\nu q_\mu + n_\mu q_\nu) \\
&+ 3(q^2 - p^2 + (q+p)^2 - 2m^2)(n_\nu p_\mu + n_\mu p_\nu)\Big\}.
\end{aligned}
$$

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