2.12 Impedance-Induced Beam Instabilities and Damping Mechanisms in Circular Machines – Longitudinal - Simulations

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2.12.1 Introduction

When studying the dynamics of high intensity beams, in addition to the external guiding fields, it is necessary to take into account, in a self-consistent way, the effects of the self-induced electromagnetic fields, for which in time domain we use the word wakefields, and in frequency domain coupling impedances [1]. In this article we discuss the principal issues and solutions which emerge from simulation codes dealing with longitudinal beam instabilities induced by coupling impedances in circular machines.

To simplify the beam dynamics study, it is generally convenient to distinguish between short-range wakefields, which influence the single turn beam dynamics (often also called single bunch beam dynamics), and long-range wakefields, which last for many turns, are generated by resonant electromagnetic modes with high quality factors, are excited by one or a train of bunches, and produce, under some conditions, coupled-bunch instabilities (multi-turn or multi-bunch beam dynamics). In both cases a linear perturbation theory is generally used to study analytically these instabilities.

However, in order to analyze the behavior of the beam under the influence of wakefields also in the non-linear regime, and for more reliable results, very often simulation codes are used. In these codes, which take into account collective effects related to impedance-induced instabilities, the used equations of motion of a single charge in a bunch are quite simple. However, the inclusion of the effects of wakefield, which is also called beam induced voltage, and which couples the motion of different particles, can be very tricky due to the possible introduction of numerical noise and non-physical phenomena.

The basic idea behind the numerical calculation of the beam induced voltage in longitudinal beam dynamics codes has not changed since the 1980s [2]. The first official release in 1984 of the well-known FermiLab code ESME [3] has been an important reference for many years to calculate the beam induced voltage and its effect on the beam dynamics. Over a period of more than thirty years, several codes using short-range wakefields for longitudinal [4,5,6] and transverse [7,8] beam dynamics, long-range wakefields [9,10], and both short- and long-range wakefields [11,12,13] in circular accelerators have been developed and have been proved to be reliable tools in the comprehension of the collective effects. Many laboratories prefer to rely on their own codes, such as, for example, Fermilab (United States) [14], CERN (Switzerland) [7,15], J-PARC (Japan) [16], CSNS (China) [17]. All these codes use the same techniques or close variants. Certainly the code evolution cannot be compared with the exponential increase in computational power during the past forty years [18].

In order to include the wakefields in a simulation, a convolution sum, which takes into account the electromagnetic fields acting on a charge and produced by all the others, is needed. This issue is discussed in the following section. The evaluation of the effects of short- and long-range wakefields generally requires different approaches which are reviewed in sections 2.12.3 and 2.12.4 respectively, while in section 2.12.5 we briefly

mention some damping mechanisms, both natural and induced, that can be included in simulations, and, finally, other methods of simulation for impedance-induced instabilities, different from tracking codes, are presented in section 2.12.6.

2.12.2 **Common Approach in Wakefield Simulations**

In order to write the longitudinal equations of motion of a single particle in a circular accelerator to be used in a simulation code, we assume, for simplicity, that the energy exchange between a charge and the surrounding accelerator environment is localized in a single place of the machine. In addition, we suppose that the minimum time interval to be integrated is the revolution period T_0, and the variation of the charge longitudinal position and energy with respect to the synchronous particle is evaluated every T_0. These assumptions are generally satisfied in many circular machines. However, in some particular cases, they are not valid. For example, the FCC accelerator [19] has a design circumference of about 100 km, and the previous approximations are too rough. In such cases it is possible to divide the circumference in several sectors and apply the procedure here described to each sector. Also the inclusion of the space charge generally requires to split the circumference, since the local effect of this force varies over one turn because of different environment conditions along the ring, such as beam pipe cross section shapes and dimensions.

If we call ε the energy difference of a charge q with respect to the synchronous particle divided by the nominal energy, which we consider equal to the synchronous particle energy E_s, its variation in one turn, $\Delta\varepsilon$, depends on the energy gain due to the RF system, on the beam induced voltage, and on other sources of energy exchange, such as the synchrotron radiation, possible higher order RF cavities for bunch length control, electron cooler, betatron acceleration due to changing magnetic field in synchrotrons [20], etc.

Without loss of generality, let us consider here only the energy exchange due to the main RF system, the wakefield and the synchrotron radiation, so that $\Delta\varepsilon$ can be written as:

$$\Delta\varepsilon = \frac{qV_{RF}\,(\sin\phi - \sin\phi_s) - qV_{WF}(\phi) + R(T_0)}{E_s} - 2\frac{T_0}{\tau_s}\varepsilon \tag{1}$$

where V_{RF} is the RF peak voltage, $V_{WF}(\phi)$ the induced wakefield voltage, $R(T_0)$ a stochastic variable changing each turn and taking into account the fact that the electromagnetic radiation occurs in quanta of discrete energy, and τ_s the longitudinal damping time, these last two terms related to synchrotron radiation effects and important, in particular, in electron machines [21]. In the above equation, ϕ and ϕ_s are the phases, with respect to the RF voltage, of the charge q and of the synchronous particle, respectively. The value of ϕ_s depends on the acceleration in synchrotrons and on the energy loss per turn due to synchrotron radiation. In the above equation, if necessary, it is possible to add terms due to other sources of energy exchange.

The second quantity necessary to describe 1-D motion in simulations is related to the longitudinal position of the charge with respect to the synchronous particle. Its variation in one turn is given by the relation

$$\Delta\,(\phi - \phi_s) = -\frac{2\pi h\eta}{\beta^2}\varepsilon \tag{2}$$

with h the RF harmonic number, η the slippage factor equal to $1/\gamma^2 - \alpha_c$ with γ the relativistic factor and α_c the momentum compaction, and β the ratio between the particle velocity and the speed of light. In the above equation we have assumed $\phi - \phi_s > 0$ for a particle behind the synchronous one, that is with positive time delay.

The common approach used in longitudinal simulation codes models each bunch as an ensemble of particles, each one governed by the above two coupled equations, and tracks these particles turn after turn. Since in a bunch the number of charges is in the range $10^8 - 10^{12}$ and, sometimes, even more, it would be necessary a very high computing power, with the help of parallel clusters, to track all the particles. For this reason, generally, macro-particles, which gather together the behavior of a given number of charges, are used. The maximum possible number of simulated macro-particles depends on the available computing power, but, nowadays, codes with 10^7 charges can be run on a personal PC.

Without the presence of wakefield, the two equations can be easily solved turn by turn, and they are independent from one macro-particle to another. The term which couples the equations of different particles, making the tracking more complicated, is the induced wakefield voltage $V_{WF}(\phi)$. This is the voltage acting on a charge in a position ϕ, and induced by all the others. This voltage depends on the normalized longitudinal bunch distribution $\lambda(\phi)$ according to the relation

$$V_{WF}(\phi) = Q_{tot} \int_{bunch} d\phi' w_{||}(\phi - \phi')\lambda(\phi') \tag{3}$$

where the integration is performed over the bunch length, Q_{tot} is the total charge of a bunch, and $w_{||}$ is the wake function of a point charge or Green function. The convolution integral is also called wake potential. It represents the energy gained or lost by a unity charge due to the entire bunch. If a charge is travelling with the speed of light, due to the causality property, the upper bound of the integral can be stopped at ϕ because $w_{||}(x) = 0$ for $x < 0$. The opposite happens for simulations having coherent synchrotron radiation effect, for which the wake function is different from zero only ahead of the charge that created it. The above equation is valid only if we take into account the short-range wakefields. In order to include also the long-range wakefields for coupled bunch simulations, we have to add a sum over previous bunches and turns in the above convolution integral, as we will discuss in section 2.12.4.

In writing Eq. (3) we have described the behavior of the particle ensemble, representing a bunch, with a continuous distribution function, as it is generally done with theoretical methods, even if the real structure of a bunch is discrete. With a simulation code we have the opposite approach, which uses a number of macro-particles reduced with respect to the real number of charges in a bunch. In this case Eq. (3), or the expanded version with the long-range wakefields, can be transformed into

$$V_{WF}(\phi_i) = \frac{Q_{tot}}{N_m} \sum_{j=1}^{N_m} w_{||}(\phi_i - \phi_j) \tag{4}$$

with ϕ_i the longitudinal position of the i^{th} macro-particle and N_m the total number of macro-particles. If the bunch is travelling with the speed of light, due to the fundamental theorem of beam loading [22], in the above summation we have to use $w_{||}(0)/2$ instead of $w_{||}(0)$ when $\phi_i = \phi_j$. Equation (4) has to be evaluated at each turn and for each macro-

particle. This means that, at each turn, the calculation of wakefields in simulations requires in general N_m^2 operations, and $(N_m - 1)N_m/2$ for the ultra-relativistic case. In order to track $10^6 - 10^7$ macro-particles, at each turn more than 10^{11} operations are needed, and this task can be accomplished, at least for the moment, only on parallel computing clusters. In order to reduce the computing time and only in the evaluation of the wakefield effects, the bunch is generally divided into N_s slices, or bins, of width Δ and center $\phi_{i\Delta}$, each one containing $n_i(\Delta)$ macro-particles. By supposing that slices act as point charges, the induced voltage at the center of each slice is then evaluated by using the relation

$$V_{WF}(\phi_{i\Delta}) = \frac{Q_{tot}}{N_m} \sum_{j=1}^{N_s} n_j(\Delta) w_{||}(\phi_{i\Delta} - \phi_{j\Delta}) \tag{5}$$

Once the induced voltage is known in the positions $\phi_{i\Delta}$, a linear interpolation (or higher order ones) permits to evaluate the wake potential acting on any macro-particle of the bunch. Since in general the number of slices is between few hundreds to some thousands, this greatly reduces the number of operations.

This kind of approach has been widely used in simulations, and in the years it has demonstrated to give reliable results. However, a particular care has to be taken when deciding the size and the number of the slices (and, of course, of macro-particles). A low number of slices reduces the computing time, but it could suppress some physical microstructures in the bunch leading to instabilities. On the other side, slices can introduce numerical noise additional to that of macro-particles, making necessary, in some cases, a parametric study to investigate any possible dependence of the results on the number of slices and of macro-particles inside the slices.

One possible approach to determine the slice size is to plot the absolute value of the product of the bunch spectrum and the impedance. This allows to identify a certain $\mathrm{f_{max}}$ above which the product can be considered negligible. This frequency defines the length of the slices since $\Delta \mathrm{t_{slice}} = 1/(2\mathrm{f_{max}})$. Once the slices are fixed, the number of macro-particles is increased by steps until a convergence is reached.

2.12.3 **Simulations With Short-Range Wakefields**

Single-bunch simulations can be performed according to the previous equations once the short-range wake function $w_{||}$ of a circular accelerator, or the corresponding coupling impedance, is known.

The coupling impedance model of a machine is generally obtained with the help of dedicated electromagnetic computer codes, such as CST Particle Studio® [23], GdfidL [24], or ACE3P [25]. From the impedance, with the inverse Discrete Fourier Transform (DFT), the wakefield $w_{||}$ in Eq. (5) can be obtained. Another method to calculate directly the induced voltage in circular machines without the use of Eq. (5) is to multiply the impedance by the spectrum of the longitudinal distribution and take the inverse DFT of the result. Often these two methods in time and frequency domain give the same result, even if different numbers of slices and macro-particles have to be chosen to obtain the same accuracy. Of course particular attention has to be paid to the inverse DFT in order to avoid non-physical results.

One important problem that sometimes arises in simulations is the necessity to use a too high number of slices such that the code becomes too cumbersome and other solutions have

to be found. As an example, let us take the wake field of a broad-band resonator, which sometimes is used as a simplified impedance model of an accelerator. Depending on its resonant frequency and on the bunch length, it may happen that a high number of slices is necessary to properly reconstruct the correct induced voltage.

In Fig. 1 we show, for example, the induced voltage for a broad-band resonator with unit quality factor, produced by a Gaussian bunch with RMS bunch length 2.4 times higher than the resonant wavelength in the relativistic case. If we consider that, generally, for single bunch simulations, a longitudinal interval of $\pm 5\sigma$, with σ the RMS bunch length, is used, in order to have about 20 slices for wavelength, about 480 slices are needed. Indeed, from the figure, we can see that, with 500 slices, Eq. (5) for the Gaussian bunch gives a slightly different result (green line) with respect to the theoretical wake potential, and only with 1e3 slices (red curve) the induced voltage is very close to the theoretical one, represented with the black curve.

With such a high number of slices, also a very high number of macro-particles has to be used, because there is the need to have a reasonable number of particles in each slice for proper beam simulations. If the number is not sufficiently high, the strong fluctuations of macro-particles from one slice to another could produce non-physical effects.

There is however the possibility to bypass this problem by using, in Eq. (5), instead of the Green function, the wake potential (induced voltage) of a very short Gaussian bunch. Indeed, in the same figure, with the cyan dashed line, we have also represented the induced voltage as given directly by a simulation with only 100 slices, for which we have used in Eq. (5) in place of $w_{||}$, the wake potential of a Gaussian bunch 10 times shorter than the simulated one. As can be seen from the figure, with 100 slices we obtain a result similar to that obtained with 1000 slices and the Green function. Also with the method in frequency domain described at the beginning of this section, it is possible to obtain the correct induced voltage with only about 150 slices.

2.12.4 **Simulations With Long-Range Wakefields**

For the multi-bunch simulations, in addition to the slice problem, the main issue is the necessity to know the wakefield as a function of time until it becomes zero. Since the long range wakefields are generated by resonant modes with a high quality factor, depending on its value and on the resonant frequency, these modes can last for several hundreds of nanoseconds up to microseconds, influencing many bunches for many turns, and then requiring the calculation of a very long interaction of the wakefield with the beam.

Different simulation codes have different approaches to tackle this problem.

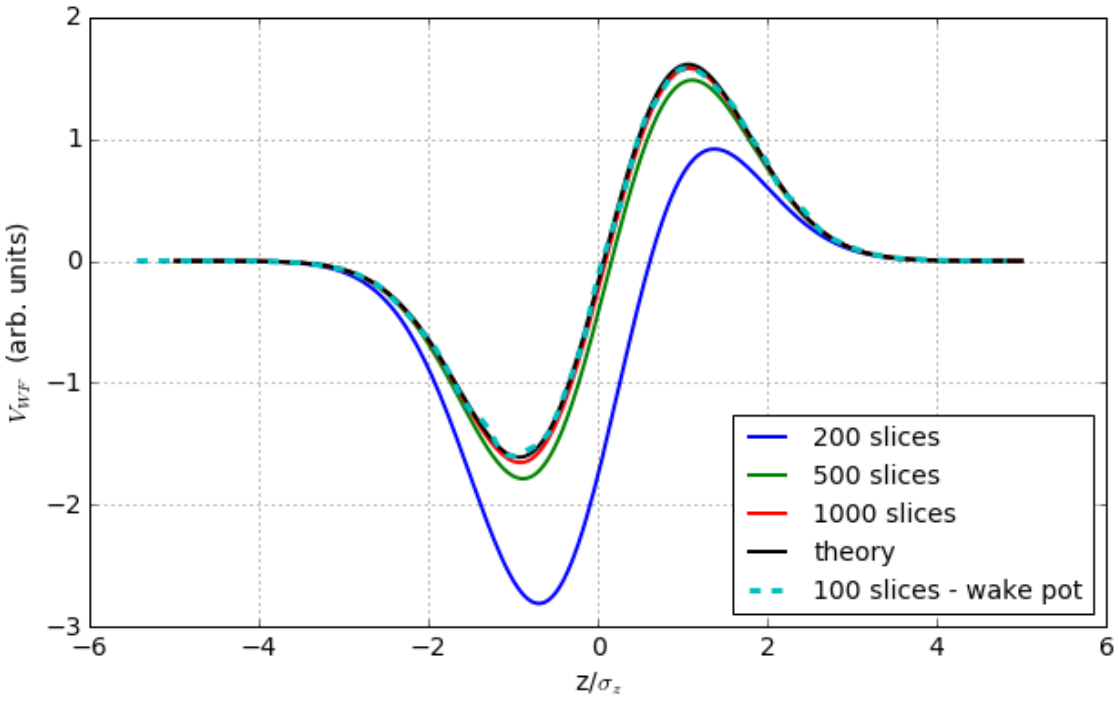

Figure 1: Induced beam voltage for a broad-band resonator produced by a Gaussian bunch with RMS length a factor 2.4 higher than the resonant wavelength. The voltage has been obtained by using 200 slices (blue), 500 slices (green), 1000 slices (red). The cyan dashed line represents the induced voltage obtained with only 100 slices by using, as Green function, the wake potential of a Gaussian bunch 10 times shorter. The black line is the theoretical induced voltage.

For example, in the BLonD code [13], different routines can be selected depending on the length of the wakefield. In particular, two different approaches are used to memorize the long range wakefields. The first one stores into memory, at each turn, the sum of the present induced voltage extended to the future and the induced voltage derived from the past after appropriate time shift of one revolution period. A complication arises in presence of acceleration: the time frame, which length is the revolution period, shrinks turn after turn and, as a consequence, an interpolation is needed each turn when the present voltage is summed to the voltage from the past.

To avoid potentially expensive interpolations in computing time, another approach, operating in frequency domain, has been introduced considering that a multiplication of a Fourier-transformed function by a complex exponential in frequency domain is equivalent to a shift of the same function in time domain. The algorithm uses a $M \times N$ matrix with M the number of turns to be stored, and N the number of slices (this time of the whole accelerator and not of the single bunch) times M. At each turn the beam spectrum is multiplied by the impedance and the result saved into a matrix row. All the remaining rows are multiplied by a complex exponential to shift the past induced voltage by one turn, all the rows are summed element by element, and the result, after an inverse DFT, gives the induced voltage in the present time frame. After the last row of the matrix is filled in, at the turn $M + 1$, the first row of the matrix is overwritten, since the known wakefield has a length corresponding to M turns. The downside of this second approach is that the size of M influences heavily the computation time and the user has to choose wisely which method to use depending on the simulation parameters.

Another method, used in the tracking parallel code SPACE [11], expands the long range wake force in Taylor series and stores the moments of the longitudinal distribution of all the bunches in previous turns. This method requires a slowly varying wake function and, in addition, the number of terms N_{TL} for the Taylor expansion has to be wisely chosen together with the order of the method to calculate the N_{TL} derivatives of the wake. However, the strength of the algorithm derives from the fact that the induced voltage acting on a certain bunch can be calculated in parallel via master-to-slave processor communications. This means that at each turn, after having applied the single particle equations of motion, the various moments of the present longitudinal bunch profiles are calculated independently by different processors. These independent computations are then communicated to the master processor which can sum them to the other calculated moments from previous turns which have been stored into memory.

In some cases, mostly when the distance between the source and the test charges is big, it is reasonable supposing that the wake function doesn't change significantly. This idea has been implemented in [10]. Whenever the wake amplitude does not change more than 0.1% in a certain longitudinal frame, then all the slices contained in that window are replaced by a single slice characterised by a wake that is just an average of the wake values of all the concerned slices. The frame length can be even of the order of the bunch length or the revolution period. Using this approximation, the convolution sum to calculate the long range wakefields can be significantly simplified.

An alternative approach, which has been developed and used in the simulation code MuSiC [12], exploits a matrix formalism to transport the wakefield of resonators, both broadband and narrow band, from one macro-particle to the following one, removing the necessity to resort to the convolution sum, avoiding problems related to bunch slices, and eliminating the necessity to store long range wakefields. The code allows to simulate, simultaneously, the effects of short and long range wakefields without the necessity to distinguish between the single and multi-bunch beam dynamics and including intra-bunch motion. It also contains a frequency domain feedback system to damp coupled bunch instabilities. The drawback of the MuSiC approach is that it requires to fit the machine coupling impedance with a sum of resonators, which are used as input parameters in place of the wakefield.

2.12.5 **Damping Mechanisms**

Intrinsic damping mechanisms, such as Landau damping, are naturally included in simulation codes if the number of macro-particles is high enough. Cases of Landau damping effects in simulations can be found in Refs. [12, 15, 26]. Also the radiation damping is simulated by means of the last term in Eq. (1). A particular mention regards feedback systems. For example, a coupled bunch feedback can be simulated in a given point of the machine exchanging energy with the macro-particles. The correct amount of energy to be exchanged has to be evaluated according to the kind of feedback. In case of time domain, for example, each bunch is treated separately by the others, while in frequency domain the motion of all the bunches has to be first decomposed as sum of eigenmodes, turn after turn, and then the feedback energy kick acting on each eigenmode has to be proportional to its oscillation amplitude.

Examples of coupled bunch feedback systems included in simulation codes can be found in Refs. [15, 27] for the time domain, and in Ref. [12] for the frequency domain.

2.12.6 **Vlasov-Fokker-Planck Solvers and Other Methods**

In addition to simulation codes, which track macro-particles turn after turn, a different approach can be used, and it consists in solving numerically the time domain Vlasov-Fokker-Planck equation with the inclusion of wakefields.

By considering the same effects discussed for Eq. (1), that is damping and fluctuations produced by the synchrotron radiation, the Vlasov's nonlinear integro-differential equation, with the inclusion of the Fokker-Plank terms, can be written as [28]

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial H}{\partial q} = 2\frac{T_0}{\tau_s}\frac{\partial}{\partial p}(pf) + D\frac{\partial^2}{\partial p^2}f \tag{6}$$

where f is the phase space longitudinal distribution, (q, p) are the canonical coordinates related to the position ϕ and relative energy ε respectively, H the Hamiltonian accounting for both the external fields and the collective force produced by the wakefields, and D a diffusion constant of the random process related to $R(T_0)$. If the right hand side of Eq. (6) is zero, hypothesis valid, for example, with a good approximation for proton beams, we have the so-called Vlasov equation. In writing the above equation, we made a sort of smooth approximation in which the dissipation and fluctuations are distributed homogeneously in the independent variable t. The nonlinearity in Eq. (6) derives from the collective force contained in the Hamiltonian, which is proportional to the convolution integral between the wakefield and the longitudinal distribution, according to Eq. (3).

As first approach, it would seem that the above equation could be treated by the usual methods for partial differential equations, as the finite differences, to approximate the phase space longitudinal distribution function f on nodes of a finite grid.

However, such a technique fails completely with or without implicit time stepping, and not because of any effect of the nonlinear terms, but because it does not preserve the symplectic form. Different and more appropriate methods must therefore be applied to preserve the symplectic structure of the equation, as, for example, the one of Ref. [29], where the integration of the equation is based on discretization of the local Perron Frobenius operator. In Ref. [30] instead, an algebraic technique of solution, that is suited for general evolution-type equations and that can also be applied to the Vlasov equation, extended to the non-linear case, has been developed, and it is based on the evolution operator technique, widely exploited in the solution of quantum mechanical problems.

Codes solving Vlasov-Fokker-Planck equations have been developed to study single bunch effects [31, 32] as alternative to multi-particle tracking codes, and they generally guarantee a very smooth evolution of the beam distribution function in time that allows to reduce, and in some cases to completely eliminate, the effect of numerical noise. Usually, the computing time for a simulation solving the Vlasov-Fokker-Plank equation is comparable to that of the multi-particle tracking codes because the problem due to the slices previously discussed is avoided, but, in any case, in order to calculate the collective force in the Hamiltonian term, the convolution integral of Eq. (3) has to be performed over a finite phase space grid.

2.12.7 **References**

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