

A simple method to detect spontaneous CP violation in multi-Higgs models

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ABSTRACT: For models with several Higgs doublets we present an alternative method to the one proposed by Branco, Gerard and Grimus, in 1984, to check whether or not CP is spontaneously violated in the Higgs potential. The previous method is powerful and rigorous. It requires the identification of a matrix U corresponding to a symmetry of the Lagrangian and verifying a simple relation involving the vacuum expectation values. The nonexistence of such a matrix signals spontaneous CP violation. This approach may be far from trivial as complexity grows with the number of Higgs doublets. In such cases it may turn out to be easier to analyse the potential by going to the so-called Higgs basis. The transformation to the Higgs basis is straightforward once the vacuum expectation values are known. The method proposed in this work is also powerful and rigorous and can be particularly useful to analyse models with more than two Higgs doublets and with continuous symmetries.

KEYWORDS: Beyond Standard Model, CP violation, Higgs Physics

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1 Spontaneous CP violation

Models with more than one Higgs doublet allow for the possibility of having spontaneous CP violation. The idea of spontaneous T (hence CP) violation was first proposed by T.D. Lee [1] in the context of two-Higgs-doublet models. CP can only be spontaneously violated if the Lagrangian is invariant under CP and if at the same time there is no transformation that can be identified with a CP transformation leaving both the Lagrangian and the vacuum invariant. In the Standard Model there is only one Higgs doublet and the scalar potential necessarily conserves CP.

Under a CP transformation a single Higgs doublet, Φ , transforms into its complex conjugate. In the presence of more than one doublet the most general CP transformation [2] allows for mixing of the scalar doublets under an arbitrary unitary matrix, U :

$$\Phi_i \xrightarrow{\text{CP}} U_{ij} \Phi_j^*. \quad (1.1)$$

This transformation combines the CP transformation of each Higgs doublet with a Higgs basis transformation. Higgs basis transformations do not change the physical content of the model. If the potential is invariant under such a transformation there is explicit CP conservation. At this stage U applied to the Φ_j fields is not required to be a symmetry of the Lagrangian. It is trivial to see that when all the coefficients of the potential are real the above condition is verified by a matrix U equal to the identity and CP is not violated explicitly.

In multi-Higgs models it may not be trivial to check whether CP is violated explicitly or not in the scalar sector due to the freedom one has to make Higgs basis transformations. These transformations change the quadratic and quartic couplings and in particular couplings that are complex in one basis may become real in another, and vice versa. This fact has motivated the study of conditions for CP invariance at the Lagrangian level expressed in terms of CP-odd Higgs-basis invariants [3, 4].

Once it is established that the potential does not violate CP explicitly the question remains of whether or not there is spontaneous CP violation. It has been shown [5] that the

vacuum is CP invariant if the following relation is verified with a matrix U corresponding to a symmetry of the Lagrangian:

$$U_{ij} \langle 0 | \Phi_j | 0 \rangle^* = \langle 0 | \Phi_i | 0 \rangle. \quad (1.2)$$

This is a very powerful relation. It is stated in ref. [5] that: given a particular set of vacuum expectation values (vevs) the simplest way of proving that they do not break CP is to construct a unitary matrix U which satisfies eq. (1.2) and which corresponds at the same time to a symmetry of the Lagrangian. This prescription is rigorous but in some cases the construction of this matrix may not be obvious.

If such a difficulty arises we propose a simple test which proves useful in identifying CP-conserving cases. Once the set of vevs is determined, we go to the so-called ‘‘Higgs basis’’ defined as a basis where only one of the Higgs doublets acquires a vev different from zero and chosen to be real. It is straightforward to build a transformation that takes the fields to such a special basis [6, 7], by means of the product of an orthogonal matrix by a diagonal matrix with phases equal to those of the original vevs but with opposite sign. It should be pointed out that for more than two Higgs doublets such a basis is defined up to a unitary transformation of the new $(n - 1)$ doublets with zero vev. If the coefficients of the scalar potential in such a basis can be made real by means of the rephasing freedom that is still left for the doublets with zero vevs, we may conclude that CP is not spontaneously broken.¹ Obviously, in this case, once in such a Higgs basis, we may define a CP transformation given by eq. (1.1) that verifies the relation given by eq. (1.2) by simply choosing the matrix U to be diagonal. If on the contrary it proves impossible to make the scalar potential real in such a Higgs basis we must make sure that we are not in one of the special cases of CP conservation with irremovable complex coefficients [8] before concluding that CP is violated. On the other hand, this procedure, complemented with the use of CP-odd invariant conditions [3, 4, 9, 10] may also prove useful to confirm the existence of CP violation since in this case it must be possible to find CP-odd invariants that are non-zero.²

2 Special cases in the framework of three-Higgs-doublet models with S_3 symmetry

CP conserving scalar potentials with irremovable phases are very special and rare. Imposing explicit CP conservation in the S_3 -symmetric three-Higgs-doublet model by taking all parameters of the potential real does not lead to loss of generality [8] and was adopted in ref. [12] where a detailed study of the possible vacua of the S_3 -symmetric three-Higgs-doublet potential is performed with emphasis on the cases in which the CP symmetry can be spontaneously broken. Different vacuum solutions correspond to different regions of parameter space which are identified in ref. [12].

¹We thank H. Haber for pointing out to us that in some cases this unitary $(n - 1) \times (n - 1)$ matrix may play a role in making the potential real.

²The technique involved in deriving CP-odd invariant conditions was introduced for the first time in ref. [11] and subsequently applied in many different contexts.

First, we illustrate some of the features of the Higgs basis by analysing a special complex solution for the vevs of the scalar potential written in terms of the S_3 defining representation, i.e., three Higgs doublets such that the potential is invariant under any permutation of these fields. This representation is known to be reducible. The scalar potential ($V = V_2 + V_4$) acquires the following form [13]:

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \frac{1}{2} \gamma \sum_{i < j} [\phi_i^\dagger \phi_j + \text{h.c.}], \tag{2.1a}$$

$$\begin{aligned} V_4 = & A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i < j} \left\{ C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \bar{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \text{h.c.}] \right\} \\ & + \frac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_i^\dagger \phi_j) + \text{h.c.}] + \sum_{i \neq j \neq k \neq i, j < k} \left\{ \frac{1}{2} E_2 [(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \text{h.c.}] \right. \\ & \left. + \frac{1}{2} E_3 [(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \text{h.c.}] + \frac{1}{2} E_4 [(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \text{h.c.}] \right\}. \end{aligned} \tag{2.1b}$$

It was pointed out long ago [14] that a possible complex vacuum solution is given by $(x, x e^{\pm \frac{2\pi i}{3}}, x e^{\mp \frac{2\pi i}{3}})$. This solution was discussed in [5]. It has the remarkable feature of corresponding to a set of vevs with calculable non-trivial phases assuming geometrical values, i.e., fixed values that are not expressed as functions of the parameters of the potential, and which are entirely determined by the symmetry of the scalar potential. These phases cannot be removed by a simple rephasing of the Higgs fields, while at the same time keeping the coefficients of the Higgs potential real. However they do not lead to spontaneous CP violation [5] since there is a matrix U satisfying the constraint of eq. (1.2), namely:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{2.2}$$

which is at the same time a symmetry of the potential. It looks, in fact, as if we are in the presence of irremovable CP conserving phases. However, there is a nontrivial unitary transformation giving rise to real vevs together with a real potential:

$$\begin{pmatrix} \phi'_1 \\ \phi'_2 \\ \phi'_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}}(1 & 1 & 1) \\ \frac{1}{\sqrt{2}}(1 & -1 & 0) \\ \frac{1}{\sqrt{6}}(1 & 1 & -2) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\mp i \frac{2\pi}{3}} & 0 \\ 0 & 0 & e^{\pm i \frac{2\pi}{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}. \tag{2.3}$$

This transformation takes us to the Higgs basis with only one nonzero real vev. Notice that this unitary transformation is also given by geometrical angles, in the sense defined above. The fields ϕ'_2 and ϕ'_3 acquire zero vevs and can be rephased to remove unwanted phases from the potential so that the potential is real. Obviously, the transformation to the Higgs basis always consists of the product of an orthogonal matrix with a diagonal matrix with phases. These phases are the complex conjugates of the phases of the vevs of each doublet in the initial basis. The orthogonal matrix has one row given by $(1/N)(|v_1|, |v_2|, \dots, |v_n|)$,

where n is the number of doublets and $1/N$ a normalisation factor. Finally, there is still freedom to rephase the new doublets with zero vev.

The scalar potential written in terms of the S_3 irreducible representations singlet and doublet fields, respectively (h_S) and (h_1, h_2) , can be written [15–17]:

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \tag{2.4a}$$

$$\begin{aligned} V_4 = & \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) \\ & + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\ & + \lambda_8 (h_S^\dagger h_S)^2. \end{aligned} \tag{2.4b}$$

The irreducible representations can be related to the reducible-triplet fields by:

$$\begin{pmatrix} h_S \\ h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}}(1 & 1 & 1) \\ \frac{1}{\sqrt{2}}(1 & -1 & 0) \\ \frac{1}{\sqrt{6}}(1 & 1 & -2) \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \tag{2.5}$$

and all coefficients of the potential remain real. The translation of the coefficients of the potential from one framework to the other is given explicitly in ref. [12]. Notice the similarity of this transformation with the one given in eq. (2.3). In this basis the vevs of (h_1, h_2, h_S) corresponding to the previous solution $(x, x e^{\pm \frac{2\pi i}{3}}, x e^{\mp \frac{2\pi i}{3}})$ are of the form [12] $(w_1, w_2, w_S) = (\hat{w}_1, \pm i \hat{w}_1, 0)$ with \hat{w}_1 real and positive by convention. A phase redefinition of (h_1, h_2, h_S) through $\text{diag}(1, i, 1)$ gives rise to real vevs and as a result the term in λ_4 splits into different terms with coefficients $i\lambda_4$ or $-i\lambda_4$, i.e., no longer real. All these coefficients can be made real by multiplying h_S by i , note that h_S has zero vev. At this stage both the vevs and coefficients of the potential are real. To go to the Higgs basis one still needs a rotation that mixes h_1 and h_2 . This rotation is real and therefore it does not introduce any new phase.

Now, we revisit some of the solutions with complex vacua and $\lambda_4 = 0$ discussed in ref. [12]. As pointed out there, in the framework of three-Higgs-doublet models with S_3 symmetry spontaneous CP violation cannot occur if $\lambda_4 = 0$. Furthermore, for $\lambda_4 = 0$ the potential acquires an additional SO(2) symmetry.

A particularly interesting vacuum is the one identified as case C-III-c, which in terms of the irreducible representations is of the form $(\hat{w}_1 e^{i\sigma}, \hat{w}_2, 0)$. This complex vacuum requires that three constraints among the coefficients of the potential be verified, one of them being $\lambda_4 = 0$. At first sight it looks as if it violates CP spontaneously, due to the fact that the moduli of w_1 and of w_2 are different. Clearly, there is no obvious simple form for the matrix U satisfying the constraint of eq. (1.2). Therefore, the easiest and most straightforward way of checking for CP conservation is to look at the potential in the Higgs basis, which can be reached via the simple transformation:

$$\begin{pmatrix} h'_1 \\ h'_2 \\ h'_S \end{pmatrix} = \frac{1}{v} \begin{pmatrix} \hat{w}_1 & \hat{w}_2 & 0 \\ \hat{w}_2 & -\hat{w}_1 & 0 \\ 0 & 0 & v \end{pmatrix} \begin{pmatrix} e^{-i\sigma} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix} \tag{2.6}$$

with $v^2 = (\hat{w}_1^2 + \hat{w}_2^2)$ and confirming by inspection that the coefficients of the potential remain real, while now all vevs are real. Notice that in the Higgs basis there is freedom to rephase h'_2 and h'_S .

The construction of the matrix U satisfying the constraint of eq. (1.2) was presented in ref. [12] and makes use of the additional SO(2) symmetry resulting from having $\lambda_4 = 0$:

$$U = e^{i(\delta_1 + \delta_2)} \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.7)$$

where the angle θ is such that the matrix on the right-hand side of eq. (2.7) rotates $(\hat{w}_1 e^{i\sigma}, \hat{w}_2, 0)$ into vevs of the form $(ae^{i\delta_1}, ae^{i\delta_2}, 0)$, where the two nonzero entries have the same modulus. This is possible due to the additional SO(2) symmetry and requires $\tan 2\theta = (\hat{w}_1^2 - \hat{w}_2^2)/(2\hat{w}_1\hat{w}_2 \cos \sigma)$. An overall rotation by the phase factor $\exp[-i(\delta_1 + \delta_2)/2]$ leads then to vevs of the form $(ae^{i\delta}, ae^{-i\delta}, 0)$. The matrix U also makes use of the symmetry under the interchange $h'_1 \leftrightarrow h'_2$, as can be seen from the matrix in the middle. Equations (2.6) and (2.7) have in common the fact that both rotations depend on the vevs of the Higgs doublets. Once the vevs are known a rotation to the Higgs basis can be easily determined. However building the matrix U requires insight and therefore there is the possibility of missing it in cases where in fact CP is conserved since there is no well-defined prescription to build it. On the other hand, once CP conservation is established it follows that the matrix U of eq. (1.2), corresponding to a symmetry of the Lagrangian, must exist.

As mentioned above, in the case of $\lambda_4 = 0$ the S_3 -symmetric potential acquires an additional SO(2) symmetry. Spontaneous breaking of this symmetry leads to a massless scalar field, which is ruled out by experiment. This problem can be avoided by adding soft breaking terms to the potential. Soft breaking terms of the form $(h_S^\dagger h_i + \text{h.c.})$ are only consistent, once the minimisation conditions are imposed, if their coefficients are proportional to λ_4 , therefore, in this case, we are only left with the possibility of adding terms of the form $\mu_2^2(h_1^\dagger h_1 - h_2^\dagger h_2) + \frac{1}{2}\nu^2(h_2^\dagger h_1 + h_1^\dagger h_2)$. It has been checked that the potential with these additional terms still allows for vevs of the form C-III-c and that the transformation to the Higgs basis together with rephasing of the fields with zero vevs can lead to a new potential with only real coefficients.

The only other complex vacuum with $\lambda_4 = 0$ having non trivial phases, i.e., phases differing from $\pm i$ is case C-IV-e which has the form $(\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}} \hat{w}_2 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S)$. In this case there is no zero vev and all vevs have different moduli. The construction of a matrix U satisfying the constraint of eq. (1.2) follows the same steps as in the case C-III-c. However, in this case there is no freedom to apply an overall phase rotation to transform the relative phase of w_1 and w_2 into two symmetric phases, since this would make w_S complex. It turns out that this vacuum is more constrained than case C-III-c, requiring four relations among the coefficients of the potential to be obeyed. As a result, the SO(2) rotation transforming it into $(be^{i\gamma_1}, be^{i\gamma_2}, \hat{w}_S)$ automatically leads to $\gamma_1 + \gamma_2 = 0$. Once again building the matrix U requires special insight. The necessary SO(2) rotation will be a function of the \hat{w}_i and σ_i , for $i = 1, 2$ and is similar to the one of case C-III-c [12] being given by $\tan 2\theta = (\hat{w}_1^2 - \hat{w}_2^2)/(2\hat{w}_1\hat{w}_2 \cos(\sigma_1 - \sigma_2))$. The alternative procedure of going

directly to the Higgs basis is also, in this case, the easiest and most straightforward way of checking for CP conservation where now a possible rotation is:

$$\begin{pmatrix} h'_1 \\ h'_2 \\ h'_S \end{pmatrix} = \begin{pmatrix} \frac{1}{N_1}(\hat{w}_1 & \hat{w}_2 & \hat{w}_S) \\ \frac{1}{N_2}(\hat{w}_2 & -\hat{w}_1 & 0) \\ \frac{1}{N_3}(\hat{w}_1 & \hat{w}_2 & X) \end{pmatrix} \begin{pmatrix} e^{-i\sigma_1} & 0 & 0 \\ 0 & e^{-i\sigma_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix} \quad (2.8)$$

where $1/N_i$ are normalisation factors and the X is chosen in such a way that rows 1 and 3 are also orthogonal. With this transformation only h'_1 acquires a non-zero vev and the coefficients of the potential can all be made real using the freedom to rephase the fields with zero vevs.

Examples C-III-c and C-IV-e show that searching for a matrix U satisfying the constraint of eq. (1.2) may not always be the easiest path to check for CP conservation. In particular, as the complexity grows, it may be more convenient to inspect the potential directly by going to the Higgs basis.

3 The T.D. Lee model

So far we have shown how to use the Higgs basis to prove that CP is not spontaneously broken. In T.D. Lee's two-Higgs-doublet model [1] the potential has the most general form with real coefficients:

$$\begin{aligned} V(\phi) = & -\lambda_1 \phi_1^\dagger \phi_1 - \lambda_2 \phi_2^\dagger \phi_2 \\ & + A(\phi_1^\dagger \phi_1)^2 + B(\phi_2^\dagger \phi_2)^2 + C(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \bar{C}(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\ & + \frac{1}{2}[(\phi_1^\dagger \phi_2)(D\phi_1^\dagger \phi_2 + E\phi_1^\dagger \phi_1 + F\phi_2^\dagger \phi_2) + \text{h.c.}]. \end{aligned} \quad (3.1)$$

CP is violated spontaneously by vevs of the form $(\rho_1 e^{i\theta}, \rho_2)$, in the region of parameters of the potential where ρ_1 and ρ_2 are different from zero and $e^{i\theta} \neq 1$. The transformation to the Higgs basis is given by

$$\begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix} = \frac{1}{v} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\chi} \end{pmatrix} \begin{pmatrix} \rho_1 & \rho_2 \\ -\rho_2 & \rho_1 \end{pmatrix} \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad (3.2)$$

with $v^2 = \rho_1^2 + \rho_2^2$. The potential acquires a new form under this transformation. Even if the model is initially defined with a bilinear part that is real, this change of basis will generate bilinear terms where the two doublets mix. The coefficients of the bilinear terms $\phi_1^\dagger \phi_2$ and $\phi_2^\dagger \phi_1$ are found to be real only if $\lambda_1 = \lambda_2$ or $\sin \chi = 0$ or $\sin 2\theta = 0$ after performing the change of basis given by eq. (3.2). In each case requiring also the quartic part of the potential to be real leads to special conditions on the parameters of the potential and therefore does not hold in general.

In the general two-Higgs-doublet model CP can also be violated explicitly. This requires the potential to have complex coefficients. However, the presence of complex coefficients does not always signal CP violation. Again in this case, the Higgs basis transformation can be used to check for CP conservation [18].

4 Concluding remarks

Both methods presented in this work are powerful and rigorous. In both cases there is the need to determine the vevs of the set of Higgs doublets. The prescription of ref. [5] is especially convenient when working in a basis where there is a symmetry such that the matrix U can be easily identified. If that is the case this procedure is the most direct one of the two. However, this may not always be the case, especially with an increasing number of Higgs doublets. Then the procedure described in this paper, which can be applied in a straightforward manner, is a more adequate alternative method to check whether or not CP is spontaneously violated. In cases where spontaneous CP violation is confirmed, the use of weak-basis CP-odd invariants that signal CP violation, involving vacuum expectation values, can provide insight into physical processes that see such effects. This type of invariants was used in the case of two Higgs doublets to study Feynman rules and CP violation [2] and also to help distinguish experimentally explicit from spontaneous CP violation in the case of two Higgs doublets, with no reference to Yukawa couplings [19, 20].

It should be pointed out that the usefulness of the transformation to the Higgs basis goes beyond the study of the possibility of having spontaneous CP violation. In fact, the method proposed in this paper can also be applied to potentials with complex coefficients. As pointed out in the introduction couplings that are complex in one basis may become real in another, and vice versa, and CP may still be a good symmetry. Once the vevs are known, the same procedure applies to check for CP conservation. If CP happens to be violated, in this case, its origin can either be explicit or spontaneous.

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