

BROKEN SYMMETRIES AT HIGH TEMPERATURES AND THE PROBLEM OF BARYON EXCESS OF THE UNIVERSE

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ABSTRACT

We discuss a class of gauge theories, where spontaneously broken symmetries, instead of being restored, persist as the temperature is increased. Applying these ideas to the specific case of the soft CP-violation in grand unified theories, we discuss a mechanism to generate the baryon to entropy ratio of the universe.

1. INTRODUCTION

Several years ago it was suggested by Kirzhnits and Linde^{1,2)} that the spontaneously broken gauge and global symmetries of nature may be restored if the system is heated to a sufficiently high temperature exactly the same way as heating a superconductor breaks Cooper pairs above the critical temperature and destroys the state of order. Since spontaneously broken gauge theories are excellent candidates for description of weak interactions, above the critical temperature $T_c \sim m_W/g$, the charged and neutral gauge mesons mediating weak interactions would become massless leading to a basically different form of weak interactions at higher temperatures. This may enable us to test (at least in principle) whether the broken gauge symmetry of weak interactions is real or just a mathematical artifact to arrive at an effective Lagrangian. If this wisdom is accepted the same kind of symmetry restoration would take place for discrete symmetries such as P ³⁾ and CP ⁴⁾ as well if they were broken softly at low temperature. On the other hand, the recently suggested mechanism⁵⁾ to understand the matter-antimatter asymmetry of the universe requires that there must be B as well as CP -violating interactions at extremely high temperature ($T \sim 10^{15}$ GeV) in the very early moments ($t \sim 10^{-36}$ sec.) of the universe⁶⁾. Since, according to the prevailing folklore, the characteristic temperature at which soft CP -violation disappears is $\sim 10^3$ GeV, the Lagrangian describing weak interaction must be CP -violating⁷⁾ prior to symmetry breakdown (i.e. hard CP -violation), if we want to tackle the problem of baryon to entropy ratio in these terms. Such a hard CP -violation model, however, would preclude any understanding⁸⁾ of the strong CP -problem⁹⁾ posed by Quantum Chromodynamics. In order to develop theories, which would simultaneously cure the strong CP -problem as well as provide a mechanism for the cosmological production of baryons via the scenario outlined above, we recently began a careful study¹⁰⁾ of the behaviour of broken gauge symmetries at high temperature in the context of various Quantum Flavor Dynamics models. A plausible basis for our discussion can be given in terms of the following intuitive argument. In flavor

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dynamics models with a single order parameter (i.e. one Higgs doublet) the restoration of symmetry at high temperature would appear automatic by the laws of thermodynamics (i.e. entropy must increase!). However, in models with more than one order parameter (more Higgs doublets), the thermodynamic principles simply require that, the entropy of the whole system increase; but it is conceivable that the state of order in one sub-system could be reinforced whereas in the complementary system, the state of order is destroyed as temperature rises. In the context of a large class of gauge models¹⁰⁾ we showed that there exist perfectly acceptable domains of coupling parameters, where one or more Higgs fields $\langle\phi_1\rangle$ remains nonzero while another one $\langle\chi\rangle$ goes to zero above critical temperature as the system is heated. They provide counter examples to the common lore in that in these models, the gauge (or global) symmetry is not restored at high temperature and also in such models, soft CP-violation persists even at ultrahigh temperature. This has important cosmological applications. This review is organized as follows: in sec. II, we summarize the main points of our work in the context of the $O(N)$ gauge model with two vector Higgs multiplets. In sec. III, we discuss the question of cosmological baryon production using these ideas in the context of a grand unified model. There we also summarize our result and conclude our discussion giving other applications.

2. $O(N)$ MODEL WITH BROKEN GAUGE SYMMETRY

Here we discuss a simple $O(N)$ model with spontaneous symmetry breaking occurring at zero temperature. The model consists of two vector representations $\vec{\phi}$ and $\vec{\psi}$. At $T=0$ the Higgs potential allowed by gauge symmetry is given by

$$V(\vec{\phi}, \vec{\psi}) = -\frac{\mu_1^2}{2} \vec{\phi}^2 - \frac{\mu_2^2}{2} \vec{\psi}^2 + \frac{\lambda_1}{4} (\vec{\phi}^2)^2 + \frac{\lambda_2}{4} (\vec{\psi}^2)^2 + \frac{\lambda_3}{2} \vec{\phi}^2 \vec{\psi}^2 + \frac{\lambda_4}{2} (\vec{\phi} \cdot \vec{\psi})^2. \quad (2.1)$$

We look for the minimum of the potential in the form of:

$$\langle\vec{\phi}\rangle^2 = v_1^2, \quad \langle\vec{\psi}\rangle^2 = v_2^2 \quad (v_1, v_2 \neq 0). \quad (2.2)$$

We also chose:

$$\lambda_4 > 0$$

In the case, $\langle\vec{\phi}\rangle \cdot \langle\vec{\psi}\rangle = 0$ at the minimum; so we can choose:

$$\langle\vec{\phi}\rangle = \begin{pmatrix} 0 \\ \vdots \\ \vdots \\ v_1 \end{pmatrix}, \quad \langle\vec{\psi}\rangle = \begin{pmatrix} 0 \\ \vdots \\ \vdots \\ v_2 \\ 0 \end{pmatrix}. \quad (2.3)$$

The equations for v_1 and v_2 are

$$\begin{aligned} \mu_1^2 &= \lambda_1 v_1^2 + \lambda_3 v_2^2 \\ \mu_2^2 &= \lambda_2 v_2^2 + \lambda_3 v_1^2. \end{aligned} \quad (2.4)$$

The positivity of Higgs scalar masses leads to further constraints

$$\lambda_1 \lambda_2 > \lambda_3^2 \tag{2.5}$$

We now compute the one loop induced temperature dependent terms in the potential $V_1(T)$ using the general form given by Weinberg²⁾

$$V_1(T) = \frac{1}{48} T^2 [6g^2(T_\alpha T_\alpha)_{ij} + f_{ijkl}] \psi_i \psi_j \tag{2.6}$$

where T_α are the generators of the gauge group; ψ_i counts all the scalar fields and $f_{ijkl} \equiv \frac{\partial^4 V}{\partial \psi_i \partial \psi_j \partial \psi_k \partial \psi_l}$. In our case, we get

$$V_1(T) = \frac{1}{24} T^2 [b_1 \vec{\phi}^2 + b_2 \vec{\psi}^2] \tag{2.7}$$

with

$$b_i = (N+2)\lambda_i + N\lambda_3 + \lambda_4 + \frac{3}{4} (N-1)g^2 \tag{2.8}$$

$i = 1, 2$. It is now clear that if we chose $\lambda_3 < 0$ and

$$(N+2)\lambda_1 + \lambda_4 + \frac{3}{4} (N-1)g^2 < N|\lambda_3| \tag{2.9}$$

then $b_1 < 0$. Notice that $b_2 > 0$, since λ_2 has to be taken sufficiently large (still of order g^2) to satisfy (2.5). Therefore, above critical temperature T_C given by

$$12\mu_2^2 = [(N+2)\lambda_2 + N\lambda_3 + \lambda_4 + \frac{3}{4} (N-1)g^2]T_C^2 \tag{2.10}$$

$\langle \vec{\psi} \rangle$ vanishes, but $\langle \vec{\phi} \rangle$ remains broken at all temperatures. In conclusion, the symmetry remains broken at high temperatures.

An important point to note is that for this to happen, several Higgs couplings (such as λ_2, λ_3) must be bigger than g^2 , the square of the gauge coupling constant. Furthermore, at zero temperature, we must have $v_1 \gg v_2$. Thus, it is the smaller of the two vacuum expectation values, that vanishes at high temperatures. It is also obvious from eqs. (2.5) and (2.8) that, it is never possible to have both $\langle \vec{\phi} \rangle, \langle \vec{\psi} \rangle \neq 0$.

A few remarks are needed regarding the fermions. Up to now we have ignored them completely. Their contribution to $V_1(T)$ can be easily shown to be of order of h^2 , where h is a typical Yukawa coupling. In most gauge theories, for all the known fermions $h^2 \ll g^2$, which means that their inclusion will not affect our analysis.

One further noteworthy feature of this model is the behaviour of the particle masses with temperature. Namely, at high temperatures ($T > T_C$)

$$\langle \vec{\psi} \rangle = 0$$

and

$$|\langle \vec{\phi} \rangle_T| = \sqrt{\langle \vec{\phi} \rangle_{T=0}^2 + c^2 T^2} \tag{2.11}$$

where c is a constant $c \geq 1$. Approximately then (at $T \gg T_C$)

$$|\langle \vec{\phi} \rangle_T| = cT \tag{2.12}$$

Therefore gauge mesons, and those fermions and Higgs particles that get the

masses from the $\langle \phi \rangle$ expectation value, will have their masses increase with temperatures

$$m(T) \propto T \quad (2.13)$$

at sufficiently high temperatures. This will become relevant when we discuss a realistic grand unified theories in the subsequent section.

3. GRAND UNIFIED THEORIES AND THE PROBLEM OF BARYON EXCESS

In the preceding sections we have learned how a symmetry may remain broken at high temperatures. Of course, one of the basic motivations for studying such effects as we emphasized in the introduction, is the fact that if CP and baryon nonconserving interactions survive at high temperature then they may be responsible for creating matter-antimatter symmetry out of an originally symmetric universe. In this section we therefore apply these ideas to the grandunified theories¹²⁾ which provide a natural basis for such phenomena since they are in general characterized by baryon number violating interactions.

Our analysis is done for the simplest of grandunified theories, the SU(5) model of Georgi and Glashow.¹²⁾ In order to discuss CP-violation, we have to recall the Higgs sector of the theory. It consists of two types of multiplets: a 24 dimensional adjoint representation which provides the strong symmetry breaking down to SU(3) x SU(2) x U(1) and gives the masses to super-heavy gauge mesons which mediate proton decay and a 5 dimensional fundamental representation which is responsible for the breaking of SU(2) x U(1) down to U(1) (it gives the masses to W^\pm , Z and fermions).

Now, in the minimal scheme with one five dimensional multiplet, the CP has to be built in the Lagrangian prior to the symmetry breaking through the complex Yukawa couplings, since the vacuum expectation value of 5 can always be chosen to be real by means of a gauge rotation. It is therefore necessary for our purpose of constructing a soft CP violating theory to increase the number of fundamental representations of SU(5). Since the Higgs self-couplings that mix 24 and 5 are small in order not to affect the light mass scale by the heavy one, we cannot rely on such couplings to dominate over positive contributions of gauge mesons of order g^2 in the coefficients of the temperature dependent terms in the potential (for the sake of our discussion, we will safely ignore such couplings). Now, similarly as in the case of the single O(N) model discussed in section II, it turns out that in the model with two 5's the vacuum expectation values of both the multiplets cannot be nonvanishing at high temperatures. This implies that a necessary condition for soft CP-violation in SU(5) models to persist at high temperatures is that, there must be more than two 5 dimensional Higgs multiplets in the theory, the situation completely analogous to the one in SU(2)_L x U(1) gauge model of weak and electromagnetic interactions where one needs at least three Higgs doublets for the same reason. The SU(2)_L x U(1) model was discussed previously by us in detail.¹⁰⁾

In conclusion, the soft CP-violation at high temperatures requires three ϕ 's denoted by ϕ_1 , ϕ_2 and χ , with the zerotemperature pattern of symmetry breaking

$$\langle \phi_i \rangle_{T=0} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \rho_i e^{i\theta_i} \end{pmatrix}, \quad \langle \chi \rangle_{T=0} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix} \quad (3.1)$$

where $\theta_1=0$, $\theta_2=\theta$. If we write the one loop induced temperature dependent term in the potential as

$$V_1(T) = T^2 [b_i \phi_i^+ \phi_i + b_\chi \chi^+ \chi] \quad (3.2)$$

then, as we have shown,¹⁰⁾ one can choose

$$b_i < 0, b_\chi > 0. \quad (3.3)$$

Therefore, above a critical temperature $\langle \chi \rangle = 0$, but $\langle \phi_i \rangle \neq 0$ and increase with temperature, i.e. $\langle \phi_i \rangle \approx c_i T$ for $T \gg T_c$. The CP phase θ also remains nonzero at the minimum at all temperatures.

To discuss the baryon production in this model, we have to know how the fermion masses grow with temperature and in particular, whether at temperatures of the order of 10^{15} GeV, the fermions are light enough to be produced in the decay of the superheavy bosons, X, Y. It actually turns out that, fermion masses $m_f \approx 10^{-4} T$ (in GeV). Therefore, at 10^{15} GeV, fermion masses are negligible compared to the masses of X and Y bosons. To see this, note that at high T

$$\langle \phi_i \rangle \approx T \quad (3.4)$$

so the fermion mass, $m_f(T)$ is for large T given by

$$m_f(T) \approx h(T)T \quad (3.5)$$

where $h(T)$ refers to Yukawa couplings at temperature T. Now

$$h(0) \approx g(0) \frac{m_f(0)}{m_w(0)} \approx 10^{-4} \text{ where } (0) \text{ denotes the zero temperature value of}$$

relevant couplings. Since Yukawa couplings in gauge theories are asymptotically free, that is they decrease with temperature, we conclude from (3.5) that $m_f(T) \lesssim 10^{-4} T$. This implies that, if $T \sim m_X$, then $m_f/m_X \lesssim 10^{-4}$. The second fact to be considered is that since fermions are still massive at these high temperatures, the interaction of both the superheavy gauge bosons as well as the Higgs mesons, with fermions containing CP-violating pieces, as does the normal W-boson weak interactions of fermions. Now we are ready to discuss the problem of cosmological baryon production.⁵⁾ It has already been emphasized in several papers that, if either the X, and Y bosons or the Higgs boson H_i decay into two channels with final state baryon numbers B_1 and B_2 , with branching ratios γ and $1-\gamma$ then the baryon to entropy ratio n_B/n_γ is given by:⁵⁾

$$\frac{n_B}{n_\gamma} \approx \frac{N_X}{N} \Delta B \quad (3.6)$$

where N_X is the concentration of the bosons that mediate B violating interactions and N is the concentration of photons at 10^{15} GeV,

$$\Delta B = (\gamma - \bar{\gamma}) (B_1 - B_2) , \quad (3.7)$$

$\bar{\gamma}$ is the corresponding branching ratio for antiparticles \bar{X} , \bar{Y} and \bar{H} . N_X/N is of the order of 10^{-2} . To compute $(\gamma - \bar{\gamma})$, we note that, we have to take into account the interference between the Born diagram and the one loop correction to it. Writing the various decay widths

$$\gamma \Gamma_{\text{tot}} = |g + h A(s + i\epsilon)|^2 \quad (3.8)$$

$$\bar{\gamma} \Gamma_{\text{tot}} = |g^* + h^* A(s + i\epsilon)|^2 \quad (3.9)$$

one gets,

$$(\gamma - \bar{\gamma}) \Gamma_{\text{tot}} \approx 2(\text{Im } gh^*) \text{Im} A \quad (3.10)$$

where $\text{Im} A$ stands for the s-channel discontinuity. In our case, the various graphs contributing to eq. (3.10) are shown in fig. 1. It can be shown¹³⁾ that for SU(5) grand unified theories, with an arbitrary number of $\{5\}$ -dim Higgs multiplets, the flavor interaction of the gauge (W,X,Y) and Higgs bosons (H_i^a) can be written in the form that involves Cabibbo rotation U and some Yukawa couplings H_i . It is clear from eq. (3.10) that the contribution of the interference between Fig. 1a and fig. 1b to $(\gamma - \bar{\gamma})$ is of the form:

$$\text{Im Tr } (UU^+UU^+) = 0 . \quad (3.11)$$

However, the interference between Fig. 1a and 1c is of the form $\text{Im Tr } (UU^+H_iH_j^+)$ which is not zero in general and therefore, the contribution to ΔB from such graphs is

$$\Delta B \sim \frac{g^2 \text{Im Tr } (UU^+H_iH_j^+)}{g^2} . \quad (3.12)$$

If $H \sim 10^{-2}$ to 10^{-3} $\Delta B \approx 10^{-4}$ to 10^{-6} leading to $n_B/n_\gamma \sim 10^{-6}$ to 10^{-8} . Our purpose here is not to highlight the prediction for the magnitude of ΔB for that depends on many more details of the model but rather to point out that a mechanism for baryon production capable of yielding reasonable values for n_B/n_γ exists and the potentially dangerous gauge loop graphs (Fig. 1b) do not contribute. A similar kind of contribution also arises from Fig. 2, where again the gauge loop corrections vanish.

We would like now to summarize the main results of this paper and conclude with some additional remarks on the implications of our work. The main point we wish to make is that, contrary to conventional wisdom, broken symmetry may persist at high temperatures if boundary conditions at zero temperature are suitably chosen. Such a model necessarily contains heavy Higgs mesons and leads to fermion and boson masses growing with temperature.

We have also shown how realistic theory of soft CP-violation, constructed within the grand unified theory based on SU(5), retains its CP-violating character at high temperatures.

The phenomenon we have uncovered is similar to the one discovered by Linde¹⁴ earlier. However, Linde's work relies on there being a large neutrino density in the universe which may not necessarily be the case. Furthermore, it is not clear whether a generalization of Linde's work to the case of soft CP-violation is possible.

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FOOTNOTES AND REFERENCES

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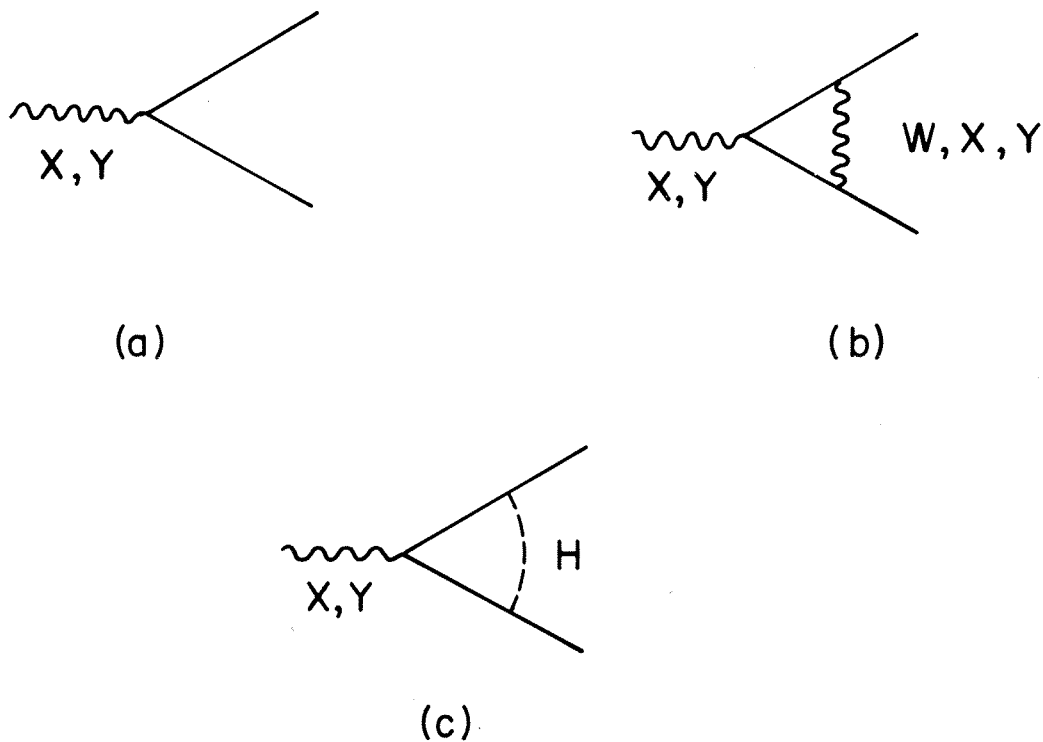


Fig. 1 The Feynman graphs expected to play a role in the generation of baryon excess in the universe, through the decay of heavy gauge mesons. X,Y stand for superheavy gauge mesons and H for superheavy Higgs scalars of SU(5).

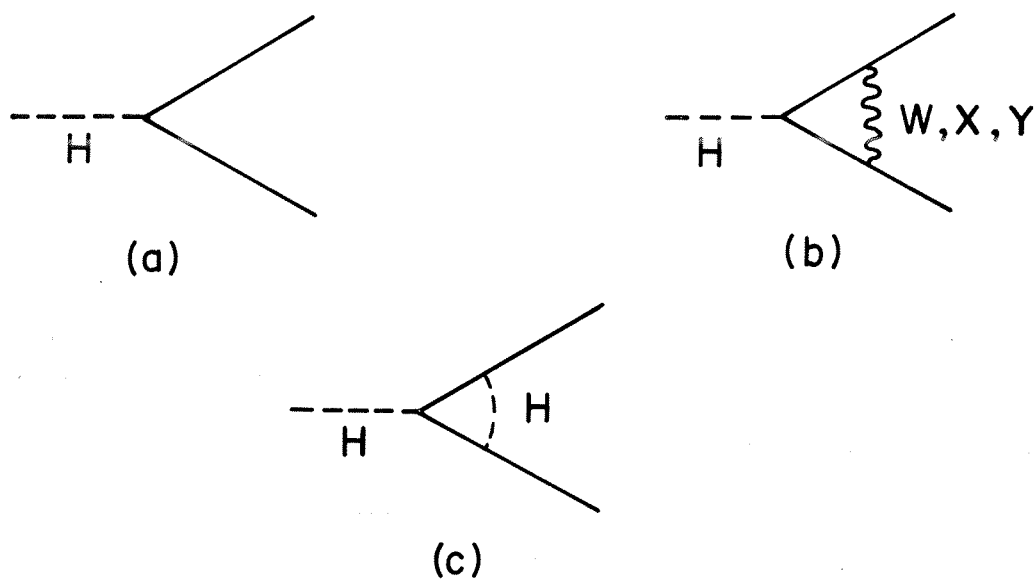


Fig. 2 Baryon number violating decays of superheavy Higgs scalars