



## Multiplicities of charged kaons from deep-inelastic muon scattering off an isoscalar target



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## ABSTRACT

Precise measurements of charged-kaon multiplicities in deep inelastic scattering were performed. The results are presented in three-dimensional bins of the Bjorken scaling variable  $x$ , the relative virtual-photon energy  $y$ , and the fraction  $z$  of the virtual-photon energy carried by the produced hadron. The data were obtained by the COMPASS Collaboration by scattering 160 GeV muons off an isoscalar <sup>6</sup>LiD target. They cover the kinematic domain  $1 \text{ (GeV}/c)^2 < Q^2 < 60 \text{ (GeV}/c)^2$  in the photon virtuality,  $0.004 < x < 0.4$ ,  $0.1 < y < 0.7$ ,  $0.20 < z < 0.85$ , and  $W > 5 \text{ GeV}/c^2$  in the invariant mass of the hadronic system. The results from the sum of the  $z$ -integrated  $K^+$  and  $K^-$  multiplicities at high  $x$  point to a value of the non-strange quark fragmentation function larger than obtained by the earlier DSS fit.

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## 1. Introduction

The study of strangeness in the nucleon has been arousing constant interest over the last decades, but the data sensitive to strangeness are still rather scarce. The parton distribution functions (PDFs) of strange quarks  $s(x)$  in the nucleon are not yet well constrained. The evaluation of the  $s$ -quark helicity distribution function  $\Delta s(x)$ , where  $x$  denotes the fraction of longitudinal nucleon momentum carried by the quark, is particularly difficult. Different values are obtained for its first moment  $\Delta s$  when using two different methods. On the one hand, a substantially negative value is obtained from QCD fits of measurements of deep-inelastic scattering (DIS) of a polarised beam on a polarised target, when assuming SU(3) flavour symmetry in the hyperon sector [1–4]. On the other hand, the values of  $\Delta s(x)$  extracted using only semi-inclusive DIS measurements (hereafter referred to as SIDIS) of identified kaons using polarised beam and target are found to be compatible with zero in the measured kinematic range [5–7]. The latter evaluations require knowledge of the collinear quark fragmentation functions (FFs), in particular knowledge of the quark-to-kaon FF. The values of the strange quark-to-kaon FF presently published differ by factors of up to three [8,9] and thus can lead to very different values of  $\Delta s(x)$  [4,6]. Note that the authors of Ref. [3] reconciled the two results by using a more flexible shape for  $\Delta s(x)$ , which turned out to be negative at low  $x$  where no data exist and slightly positive at high  $x$  to accommodate SIDIS data. Quark FFs are of general interest by themselves, as they are non-perturbative universal functions describing processes leading to the production of hadrons.

The reason for the presently large uncertainties on the values of the strange quark-to-kaon FF is the lack of appropriate kaon data. Existing data on kaon production from  $e^+e^-$  annihilation [10] and pp collision [11] experiments are only weakly sensitive to the quark and anti-quark flavours individually. In contrast, SIDIS data allow for a full flavour decomposition of FFs [12]. Compared to the kinematic range of the only published SIDIS results, which originate from HERMES [13], COMPASS measurements cover a wider kinematic domain. In addition, they better satisfy the Berger criterion [14] that ensures the validity of factorisation in the description of SIDIS. Kaon multiplicities measured by COMPASS are thus expected to play an important role in future next-to-leading order (NLO) pQCD analyses aiming to constrain quark-to-kaon FFs.

In this Letter, we present COMPASS results on differential multiplicities for charged kaons, which are derived from data taken simultaneously to those used for the determination of the differential multiplicities for charged pions [15]. In both cases, similar analysis methods are used. The multiplicity results are derived from SIDIS measurements with polarised beam and target after averaging over the target polarisation, so that they cover the same kinematic range as used for the extraction of  $\Delta s(x)$  [6], as well as a large  $Q^2$  range to probe evolution.

The multiplicity for a hadron of type  $h$  from the SIDIS measurement  $IN \rightarrow lhX$  is defined as the differential hadron production cross section  $\sigma^h$  normalised to the inclusive DIS cross section  $\sigma^{\text{DIS}}$  and thus represents the mean number of hadrons produced per DIS event when integrated over  $z$ :

$$\frac{dM^h(x, z, Q^2)}{dz} = \frac{d^3\sigma^h(x, z, Q^2)/dxdQ^2dz}{d^2\sigma^{\text{DIS}}/dxdQ^2}. \quad (1)$$

Here,  $x = -q^2/(2P \cdot q)$  is the Bjorken variable,  $z = (P \cdot p_h)/(P \cdot q)$  the fraction of the virtual-photon energy that is carried by the final-state hadron and  $Q^2 = -q^2$  the negative square of the lepton four-momentum transfer. The symbols  $q$ ,  $P$  and  $p_h$  denote the four-momenta of the virtual photon, the nucleon  $N$  and the observed hadron  $h$  respectively. Additional variables used are the lepton energy fraction carried by the virtual photon,  $y = (P \cdot q)/(P \cdot k)$  with  $k$  the four-momentum of the incident lepton, and the invariant mass of the final hadronic system,  $W = \sqrt{(P + q)^2}$ . At LO in pQCD the cross sections are not sensitive to the gluon contribution and can be expressed in terms of only quark PDFs and FFs:

$$\frac{d^2\sigma^{\text{DIS}}}{dxdQ^2} = C(x, Q^2) \sum_q e_q^2 q(x, Q^2),$$

$$\frac{d^3\sigma^h}{dxdQ^2dz} = C(x, Q^2) \sum_q e_q^2 q(x, Q^2) D_q^h(z, Q^2), \quad (2)$$

where the sum is over quarks and antiquarks. Here,  $q(x, Q^2)$  is the quark PDF for flavour  $q$ ,  $D_q^h(z, Q^2)$  denotes the FF that describes the fragmentation of a quark of flavour  $q$  to a hadron of type  $h$ ,  $C(x, Q^2) = 2\pi\alpha^2(1 + (1 - y)^2)/Q^4$  and  $\alpha$  is the fine structure constant.

## 2. Experimental setup and data analysis

The data were taken in 2006 using muons from the M2 beam line of the CERN SPS. The beam momentum was 160 GeV/c with a spread of  $\pm 5\%$ . The solid-state  $^6\text{LiD}$  target was effectively isoscalar, with a small excess of neutrons over protons of about 0.2% due to the presence of additional material in the target (H,  $^3\text{He}$  and  $^7\text{Li}$ ). The target was longitudinally polarised but in the present analysis the data are averaged over the target polarisation. The experimental setup used the same two-stage COMPASS spectrometer [16] as described in the recent publication on the pion multiplicity measurement [15]. For pion, kaon and proton separation, the ring imaging Cerenkov counter (RICH) was used. It was filled with  $\text{C}_4\text{F}_{10}$  radiator gas leading to thresholds for pion, kaon and proton detection of about 2.9 GeV/c, 9 GeV/c and 18 GeV/c respectively. The central region from the RICH is excluded in the analysis in order to avoid particles crossing the beam pipe inside the RICH.

The data analysis includes event selection, particle identification and acceptance correction, as well as corrections for QED radiative effects and diffractive vector-meson production. The kaon multiplicities  $M^K(x, y, z)$  are determined from the kaon yields  $N^K$  normalised by the number of DIS events,  $N^{\text{DIS}}$ , and divided by the acceptance correction  $A$ :

$$\frac{dM^K(x, y, z)}{dz} = \frac{1}{N^{\text{DIS}}(x, y)} \frac{dN^K(x, y, z)}{dz} \frac{1}{A(x, y, z)}. \quad (3)$$

Here,  $y$  is used as a third variable instead of  $Q^2$  because of the strong correlation between  $x$  and  $Q^2$  in the COMPASS fixed-target kinematics.

### 2.1. Event and hadron selection

The present analysis is based on events with inclusive triggers that only use information on the scattered muon. A total of  $13 \times 10^6$  DIS events were collected for an integrated luminosity of  $0.54 \text{ fb}^{-1}$ . The data cover a wide kinematic range of  $1 \text{ (GeV/c)}^2 < Q^2 < 60 \text{ (GeV/c)}^2$ ,  $0.004 < x < 0.4$  and  $W > 5 \text{ GeV/c}^2$ . The relative virtual-photon energy  $y$  is constrained to the range  $0.1 < y < 0.7$  in order to exclude kinematic regions where the momentum

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resolution degrades and radiative effects are most pronounced. Further constraints on  $y$  are discussed in Sect. 2.3. For the selection of kaons in the SIDIS final state,  $z$  is constrained to  $0.2 \leq z \leq 0.85$ . The lower limit avoids the contamination from target remnant fragmentation, while the upper limit is set to keep the same phase space as was used for pions [15]. Further constraints on momentum and polar angle of hadrons are discussed in Sect. 2.2.

The corrections for higher-order QED effects are calculated on an event-by-event basis and depend on  $x$  and  $y$  at a given beam energy. The correction factors are calculated separately for denominator and numerator of Eq. (3), i.e. for the number of DIS events and the number of kaons, respectively. The Dubna TERAD code [17] is used for the DIS correction factor. For the kaon yields, the TERAD correction is calculated excluding the elastic and quasi-elastic tails. As TERAD cannot account for a  $z$  dependence, which leads to an overestimate of the correction,<sup>23</sup> a conservative approach is adopted here. The correction is calculated for the two extreme cases, no correction and full correction to the number of kaons entering the numerator; half of the correction is applied to the multiplicities. This approach leads to an overall correction between 1% and 7% depending on kinematics.

## 2.2. Kaon identification using the RICH detector

The hadron identification using the RICH detector [20] and the unfolding procedure for identified hadrons of different types follow the method described in Ref. [15]. Only the main features are recalled here for completeness. A maximum likelihood method is used, based on the pattern of photons detected in the RICH detector. The likelihood values are calculated by comparing the measured photo-electron pattern with those expected for different mass hypotheses ( $\pi$ , K, p), taking the distribution of background photons into account.

The yields of identified hadrons,  $N_{\text{true}}$ , are obtained by applying an unfolding algorithm to the yields of measured hadrons,  $N_{\text{meas}}$ , in order to correct for the identification and misidentification probabilities:

$$N_{\text{true}}^i = \sum_j (P^{-1})_{ij} \cdot N_{\text{meas}}^j. \quad (4)$$

These probabilities are calculated in a matrix  $P_{ij}$  with  $i, j \in \{\pi, K, p\}$ , which contains as diagonal elements the efficiencies and as off-diagonal elements the misidentification probabilities. The elements of this  $3 \times 3$  matrix are constrained by  $\sum_j P_{ij} \leq 1$ . They are determined from data using samples of  $\pi$ , K and p that originate from the decay of  $K_S^0$ ,  $\phi$ , and  $\Lambda$  respectively, into two charged particles. The probabilities depend mostly on particle momentum  $p_h$  and polar angle  $\theta$  at the entrance of the RICH detector. The  $p_h$  dependence accounts for effects arising from momentum thresholds (see the beginning of Sect. 2) and from saturation at high momentum ( $\beta \rightarrow 1$ ). The  $\theta$  dependence that accounts for varying occupancy and background levels in the RICH photon detectors is weak, so that it is sufficient to divide in two the full polar angle region where the efficiencies are high and precisely measured ( $10 \text{ mrad} < \theta < 40 \text{ mrad}$  and  $40 \text{ mrad} < \theta < 120 \text{ mrad}$ ). In order to achieve excellent kaon-pion separation and high particle-identification probabilities for kaons, only particles with momenta between 12 GeV/c and 40 GeV/c are selected. Fig. 1 shows the

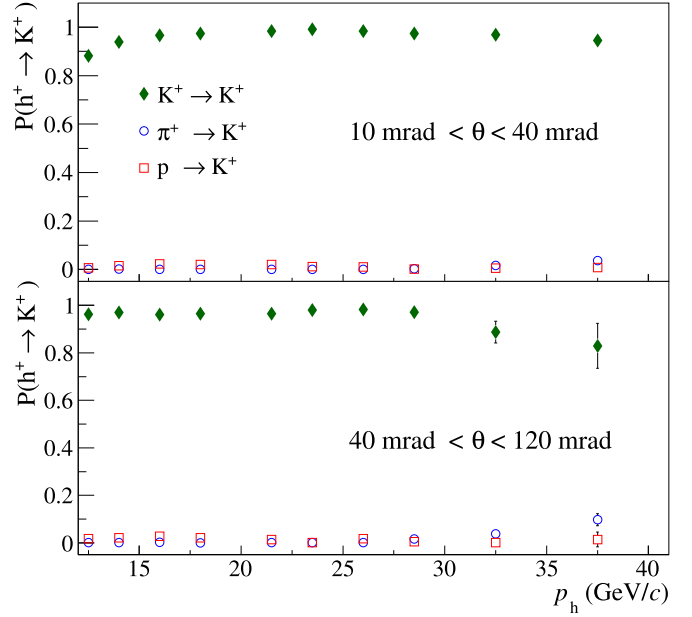


Fig. 1. Probabilities of RICH identification of  $\pi^+$ ,  $K^+$  and p as  $K^+$ , shown versus momentum for the two  $\theta$  regions.

probabilities of positive hadrons to be identified as  $K^+$  vs momentum  $p_h$  for the two  $\theta$  regions. Kaons are identified with a 95% efficiency over most of the momentum range with probabilities to misidentify pions and protons as kaons being smaller than 3%. Similar values are obtained in the case of negative charge. The number  $N_{\text{true}}^{K^+}$  of identified kaons available for the analysis after all particle identification cuts is  $2.8 \times 10^6$ .

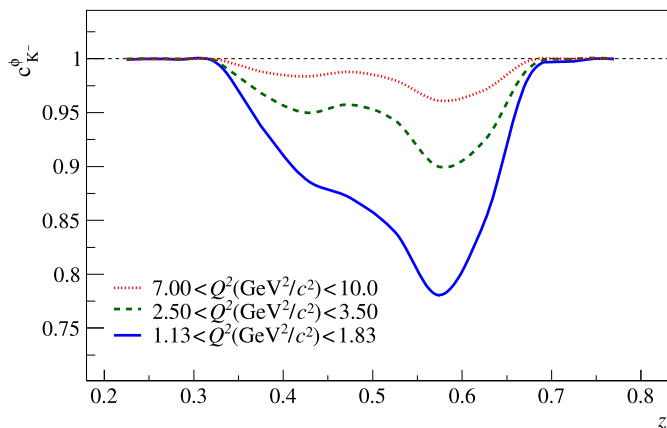
## 2.3. Acceptance correction

The overall correction for the geometric and kinematic acceptance of the COMPASS apparatus and for detector inefficiencies, resolutions and bin migration is evaluated using a Monte Carlo (MC) simulation. A good description of the kinematic distributions of experimental data is obtained using the following tools for the MC simulation: LEPTO [21] for the generation of DIS events, JETSET [22] for hadronisation (tuning from Ref. [23]), GEANT3 [24] for the description of the spectrometer and FLUKA [25] for secondary hadron interactions. The acceptance correction is calculated in narrow regions of  $(x, y, z)$ , thus reducing the dependence on the choice of the generator used in the simulation. It is defined as ratio of reconstructed and generated multiplicities:

$$A(x, y, z) = \frac{dN_{\text{rec}}^K(x, y, z)/N_{\text{rec}}^{\text{DIS}}(x, y)}{dN_{\text{gen}}^K(x, y, z)/N_{\text{gen}}^{\text{DIS}}(x, y)}. \quad (5)$$

The generated kinematic variables are used for the generated events, while the reconstructed kinematic variables are used for the reconstructed events. The reconstructed MC hadrons are subject to the same kinematic and geometric selection criteria as the data, while the generated hadrons are subject to kinematic requirements only. Events generated outside the acceptance are taken into consideration. The average value of the acceptance correction is about 65%. The acceptance correction is almost flat in  $z$ ,  $x$  and  $y$ , except at high  $x$  and low  $y$ , but it is always larger than 35%. For each bin in  $z$ , the  $y$  range for DIS events is restricted to the region accessible with kaon momenta within the range from 12 GeV/c to 40 GeV/c.

<sup>23</sup> We also estimated the radiative corrections using the RADGEN code [18] together with the LEPTO generator, which effectively incorporates a  $z$  dependence. However, by including RADGEN and LEPTO in the COMPASS full MC chain we were not able to correctly describe the data. Note also that an author of RADGEN advises caution when using the program to correct differential cross section distributions for radiative effects beyond the  $(x, y)$  dependence [19].



**Fig. 2.** Correction factor for negative kaon multiplicities due to diffractive production of vector meson  $\phi$  shown versus  $z$  for three  $Q^2$  regions.

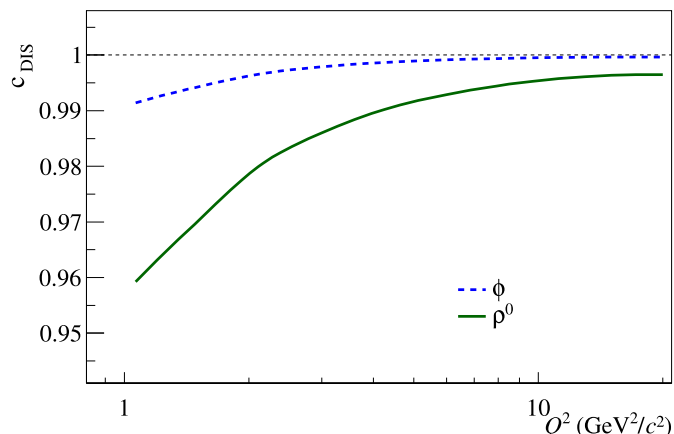
#### 2.4. Vector meson correction

For both the DIS and the kaon SIDIS samples, it is necessary to estimate the fraction of events, in which diffractively produced vector mesons have decayed into lighter hadrons. This fraction can be considered as a higher-twist contribution to the SIDIS cross section [26]. It cannot be described by the pQCD parton model with the independent-fragmentation mechanism that is encoded in the FFs. Hence FFs extracted from data including this fraction would be biased and in particular the universality principle of the model would be violated. The correction for diffractive vector meson decay is applied separately in the DIS and kaon samples. Monte Carlo SIDIS events that are free of diffractive contributions are simulated using LEPTO. Diffractive production of exclusive  $\rho^0$  and  $\phi$  events is simulated using HEPGEN [27], while further exclusive-meson production channels characterised by smaller cross sections and more particles in the final state are not taken into account. Vector meson production with diffractive dissociation of the target nucleon is also simulated and represents about 20% of the cross section given by HEPGEN.

An example of the correction factor calculated for the diffractive contribution of  $\phi$ -meson decay into kaons,  $c_{K^-}^{\phi}$  vs  $z$ , is shown in Fig. 2 for three different  $Q^2$  ranges. It varies between 0.1% and 22% with the largest contribution appearing at small  $Q^2$  and  $z$  close to 0.6. The correction to DIS yields,  $c_{\text{DIS}}$  vs  $Q^2$ , is shown in Fig. 3. It is of the order of a few percent.

#### 2.5. Systematic uncertainties

For the evaluation of systematic uncertainties the same studies are repeated as they were performed for the analysis of pions [15] and summarised here. A maximum uncertainty of 4% on the acceptance correction is derived from the following studies: varying the PDF set used, varying the JETSET parameters, using a four-dimensional space  $(x, y, z, p_T)$  with the hadron transverse momentum  $p_T$  as fourth variable, varying the detector efficiency applied in the MC simulation used for the acceptance calculation, as well as comparing multiplicity results from upstream and downstream target regions. The RICH identification and unfolding procedure leads to uncertainties typically below 1.5%. The uncertainty on the diffractive meson correction factor is on average below 1.5%; it is estimated assuming 30% uncertainty on the cross section for exclusive production of  $\phi$  mesons [28], which is used to normalise the HEPGEN MC calculation. Nuclear effects may be caused by the presence of  $^3\text{He}/^4\text{He}$  and  $^6\text{Li}$  in the target. A detailed study of such effects was previously performed by the EMC [29] in a



**Fig. 3.** Correction factor for DIS events due to diffractive production of vector mesons  $\phi$  and  $\rho^0$  shown versus  $Q^2$ .

kinematic range similar to COMPASS one, for carbon, copper and tin. A decrease of the multiplicities of 5% was observed for copper compared to deuterium. While the effect was larger for tin, no such effect was found for carbon within statistical accuracy, so that possible nuclear effects in the present experiment are expected to be very small and hence neglected. The dependence of the acceptance on the azimuthal angle is negligible [30]. No time dependence is observed in the results obtained in six weeks of data taking. Also, the present results are found in agreement with an independent set of preliminary data taken in 2004 before the upgrade of the spectrometer. We chose to publish only the present set of data, which is of better quality and benefits from a larger spectrometer acceptance and an upgraded RICH detector.

In addition, a conservative estimate of 100% systematic uncertainty is assumed for the corrections for higher-order QED effects. This corresponds to a 1% to 7% systematic uncertainty on the multiplicities.

All these contributions are added in quadrature and yield a total systematic uncertainty,  $\sigma_{\text{sys}}$ , which varies between 5% and 10% depending on kinematics. It is generally larger than the statistical one at low  $z$ , whereas at higher  $z$  the statistical uncertainty dominates.

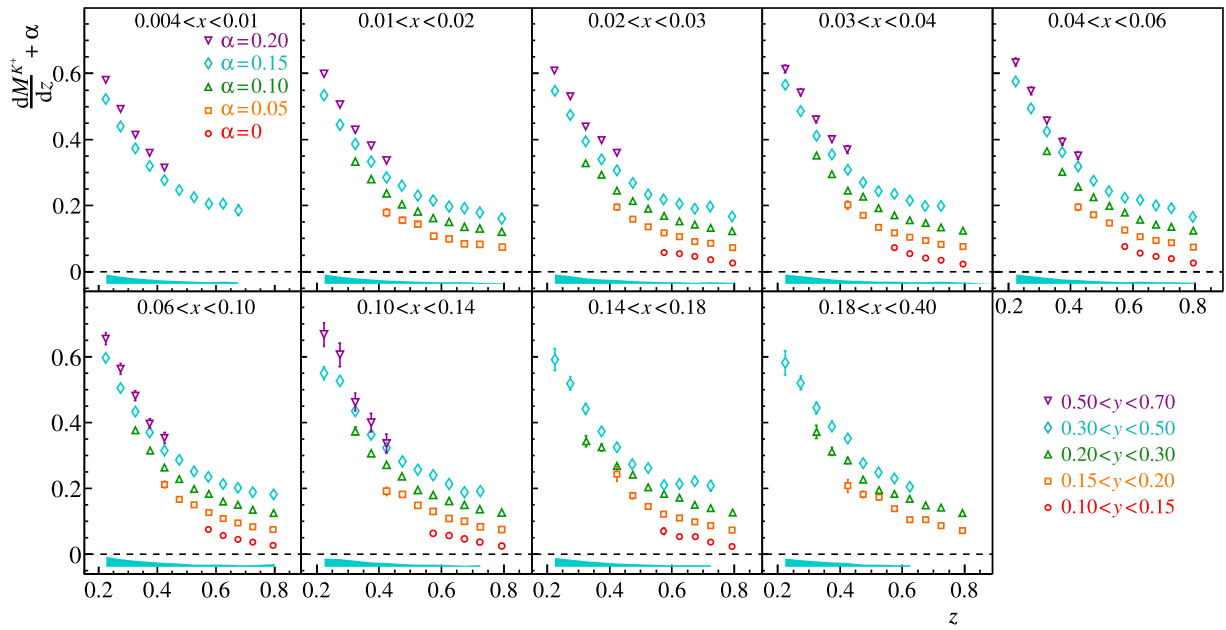
From the total systematic uncertainty a large fraction,  $0.8\sigma_{\text{sys}}$ , is estimated to be correlated from bin to bin, and the remaining fraction,  $\sqrt{(1-0.8^2)\sigma_{\text{sys}}}$ , is uncorrelated.

### 3. Results for charged-kaon multiplicities

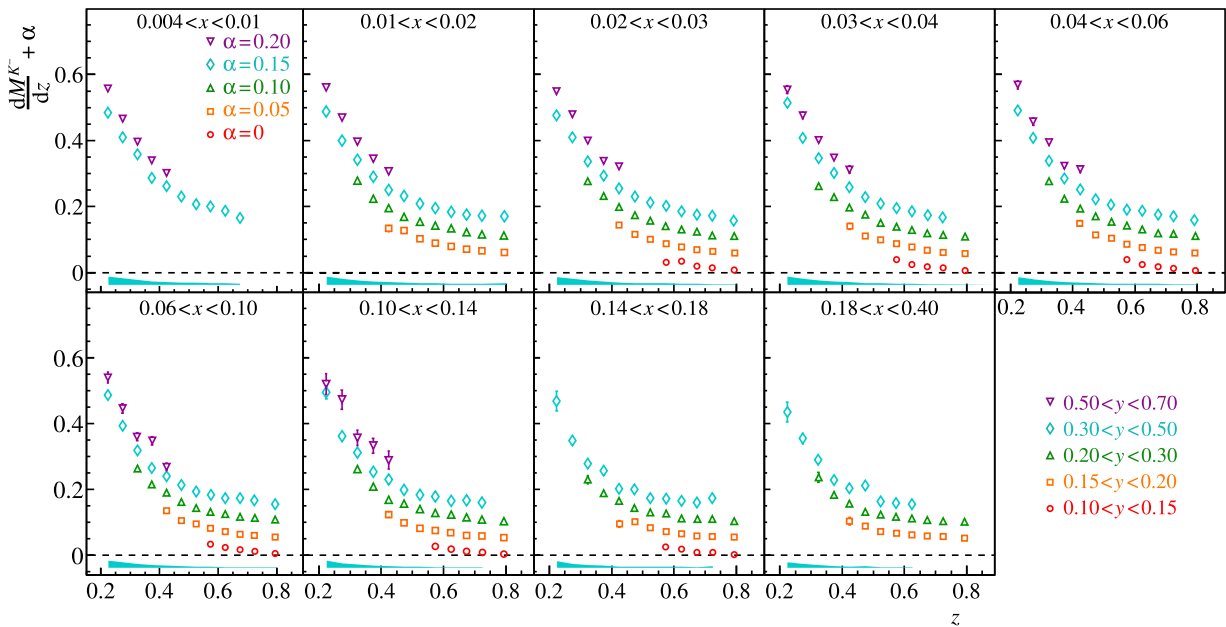
The results for  $K^+$  and  $K^-$  multiplicities represent a total of 618 experimental data points. In this section, results are shown first in the initial three-dimensional  $(x, y, z)$  binning, then averaged in  $y$ , and eventually also integrated over the  $z$  range from 0.2 to 0.85 in order to calculate the sum and ratio of  $K^+$  and  $K^-$  multiplicities vs  $x$ . The data presented in the following figures are all corrected for the diffractive vector-meson contribution as described in Sect. 2.4. For completeness, the numerical values are available on HepData [31] for multiplicities with and without this correction. The separate correction factors for DIS and kaon yields are also provided, as are the factors used for the correction of radiative effects. The bin limits in  $x$ ,  $y$ , and  $z$  are given in Table 1. The  $Q^2$  values range from 1  $(\text{GeV}/c)^2$  at the smallest  $x$  to about 60  $(\text{GeV}/c)^2$  at the largest  $x$ , with  $\langle Q^2 \rangle = 3 (\text{GeV}/c)^2$ . In Figs. 4 and 5, the results for the  $z$  and  $y$  dependences of the  $K^+$  and  $K^-$  multiplicities are presented separately for the nine ranges in  $x$ . Statistical uncertainties are shown for all points, while the band

**Table 1**  
Lower and upper bin limits for the (x, y, z) binning.

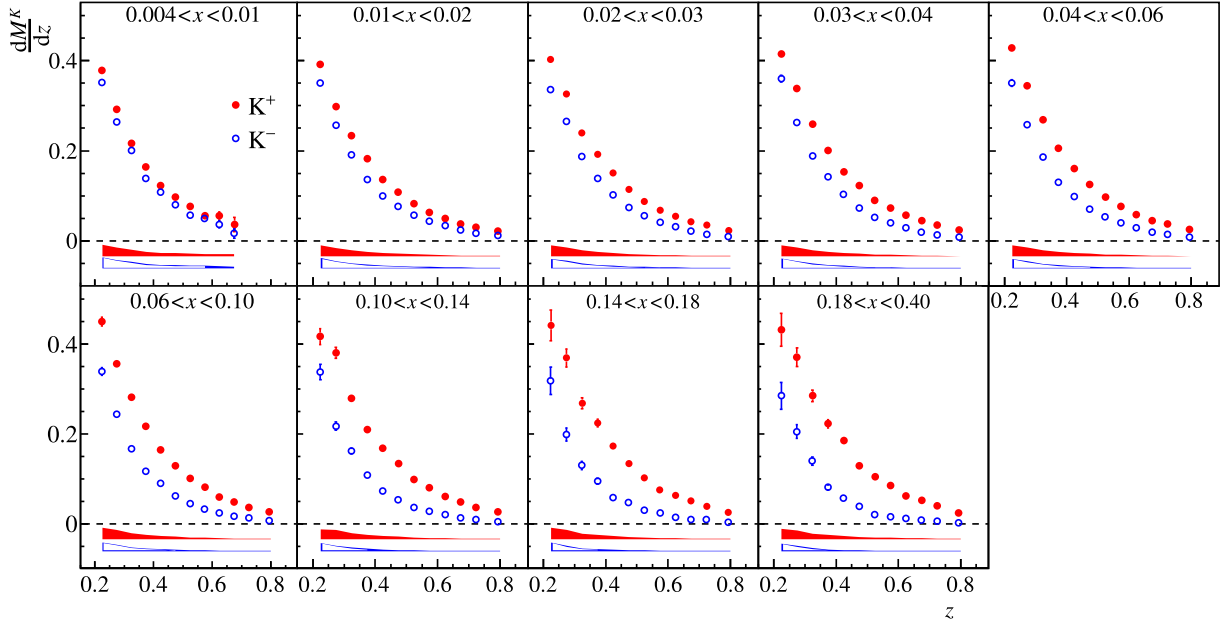
Bin limits													
x	0.004	0.01	0.02	0.03	0.04	0.06	0.1	0.14	0.18	0.4			
y	0.1	0.15	0.2	0.3	0.5	0.7							
z	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.85



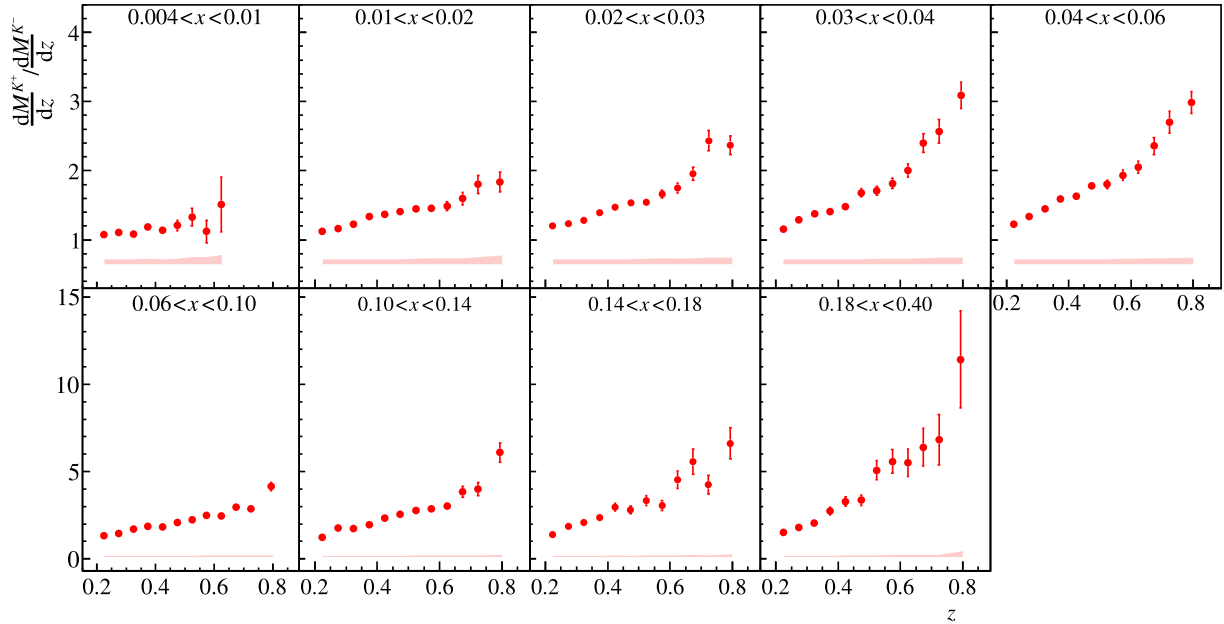
**Fig. 4.** Multiplicities for  $K^+$  shown vs  $z$  for the nine  $x$  ranges and the five  $y$  ranges, adding a constant  $\alpha$  for better visibility. The band corresponds to the total systematic uncertainty for the range  $0.30 < y < 0.50$ .



**Fig. 5.** Same as Fig. 4 but for negative kaons.



**Fig. 6.** Positive (closed) and negative (open) kaon multiplicities vs  $z$  for the nine  $x$  ranges. The bands correspond to the total systematic uncertainties.



**Fig. 7.** Ratio of multiplicities  $(dM^{K^+}/dz)/(dM^{K^-}/dz)$  vs  $z$  for the nine  $x$  ranges. The bands correspond to the total systematic uncertainties.

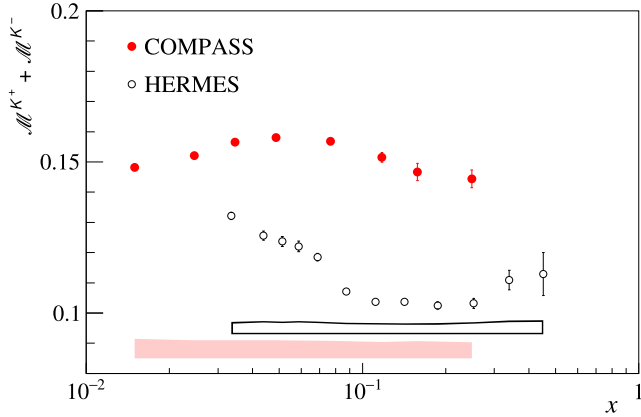
on the bottom of the plots illustrates the size of the total systematic uncertainty for a single  $y$  range (0.30–0.50). For the other  $y$  ranges, the bands are similar or smaller in size. In Fig. 6, multiplicities of positive (closed circles) and negative (open circles) charged kaons are shown vs  $z$ , separately for the nine  $x$  ranges but averaged over  $y$ . The error bars correspond to the statistical uncertainties and the bands to the total systematic ones. A strong dependence on  $z$  is observed as in previous measurements [26] and a weak one on  $x$ . The multiplicities are systematically higher for  $K^+$  as compared to  $K^-$  because of  $u$ -quark dominance and since  $K^-$  do not contain valence quarks from the nucleon.

In Fig. 7, we present the ratio  $R^{K^+/K^-} = (dM^{K^+}/dz)/(dM^{K^-}/dz)$  vs  $z$  in the nine  $x$  ranges. In this ratio, most of the systematic uncertainties cancel, including the one on radiative corrections, so

that it provides a benchmark for future NLO fits. A steep rise in  $z$  is observed.

Since an important goal of the measurement of kaon multiplicities is the determination of quark-to-kaon FFs, the sum of positive and negative charged-kaon multiplicities is of special interest. It was used by HERMES [13] for a LO pQCD extraction of the product of the strange quark PDF and the strange quark-to-kaon FF. For the COMPASS kinematics, the applicability of the LO pQCD formalism to pion multiplicities was already demonstrated [15]. For an isoscalar target and assuming isospin and charge symmetry, the sum of  $K^+$  and  $K^-$  multiplicities can be written in LO in a very simple way [32].

For multiplicities integrated over  $z$ ,  $\mathcal{M}^{K^\pm} = \int \langle M^{K^\pm}(x, y, z) \rangle_y dz$ , the equation reads:



**Fig. 8.** Sum of  $z$ -integrated multiplicities,  $\mathcal{M}^{K^+} + \mathcal{M}^{K^-}$ . COMPASS data (160 GeV, full points) are compared to HERMES data [13] (27.5 GeV, open points) (see text). The bands show the total systematic uncertainties.

$$\mathcal{M}^{K^+} + \mathcal{M}^{K^-} = \frac{U\mathcal{D}_U^K + S\mathcal{D}_S^K}{5U + 2S}, \quad (6)$$

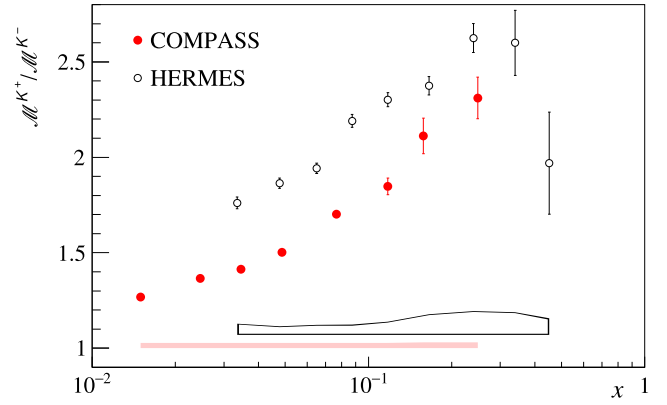
with  $U = u + \bar{u} + d + \bar{d}$ , and  $S = s + \bar{s}$ . The  $z$ -integrated FFs,  $\mathcal{D}^K(Q^2) = \int D^K(z, Q^2) dz$ , depend on  $Q^2$  only. The symbols  $\mathcal{D}_U^K$  and  $\mathcal{D}_S^K$  denote the combinations of FFs  $\mathcal{D}_U^K = 4\mathcal{D}_u^{K^+} + 4\mathcal{D}_u^{K^-} + \mathcal{D}_d^{K^+} + \mathcal{D}_d^{K^-}$  and  $\mathcal{D}_S^K = 2\mathcal{D}_s^{K^+} + 2\mathcal{D}_s^{K^-}$ . At high values of  $x$ , the strange content of the nucleon can be neglected, and in a good approximation the sum of  $K^+$  and  $K^-$  multiplicities is related to  $\mathcal{D}_U^K/5$ . This value is expected to have a rather weak  $Q^2$  dependence when integrated over  $z$ , so that it can be used at smaller values of  $x$  to determine the  $S\mathcal{D}_S^K$  value. The result for  $\mathcal{M}^{K^+} + \mathcal{M}^{K^-}$  is presented in Fig. 8 as a function of  $x$  at the measured values of  $Q^2$ . The data are integrated over  $z$  in the range 0.20 to 0.85 and averaged over  $y$  in the range 0.1 to 0.7. Only those eight  $x$  bins which have a sufficient  $z$  coverage are shown. A weak  $x$  dependence is observed. Fig. 8 also shows the HERMES results [13] that were taken at 27.5 GeV beam energy and correspond to different kinematics in particular accepting lower  $W$  values. They lie well below the COMPASS points and exhibit a different  $x$  behaviour. The  $x$  dependence of the HERMES results for pion and kaon multiplicities gave rise to some dispute [33–35]. Note that COMPASS multiplicities are computed in bins of  $(x, y, z)$  before integration over  $z$  and averaging over  $y$  while in the case of HERMES, the hadron and DIS yields are obtained and integrated over separately, and finally combined in a ratio depending on  $x$  only [35].

From the COMPASS result on  $\mathcal{M}^{K^+} + \mathcal{M}^{K^-}$  at high  $x$  ( $x = 0.25$ ) we extract  $\mathcal{D}_U^K \approx 0.65$ – $0.70$ , depending upon assumptions on strange quark PDFs and FFs. This differs from the earlier DSS fit result at  $Q^2 = 3$  (GeV/ $c$ )<sup>2</sup>,  $\mathcal{D}_U^K = 0.43 \pm 0.04$  [9], which was mainly based on preliminary HERMES results. The latter differed significantly from the published ones [13]. Towards low  $x$ , COMPASS data show a flat behaviour, unlike the rise that is suggested by the HERMES data and the DSS FF parametrisation [9].

Another quantity of interest is the  $x$  dependence of the multiplicity ratio  $\mathcal{M}^{K^+}/\mathcal{M}^{K^-}$ , in which most experimental systematic effects cancel. The results are shown in Fig. 9 as a function of  $x$ . In the region of overlap, COMPASS results are found to be systematically lower than those of HERMES. In contrast, for the case of pions COMPASS and HERMES multiplicity ratios are found in good agreement [15].

#### 4. Summary and conclusions

Precise results for charged-kaon multiplicities are obtained from kaon SIDIS measurements using a muon beam scattering off an



**Fig. 9.** Ratio of  $z$ -integrated multiplicities,  $\mathcal{M}^{K^+}/\mathcal{M}^{K^-}$ . COMPASS data (160 GeV, full points) are compared to HERMES data [26] (27.5 GeV, open points). The bands show the total systematic uncertainties.

isoscalar target. The data are given in a three-dimensional  $x$ ,  $y$ , and  $z$  binning and cover a wide kinematic range:  $1$  (GeV/ $c$ )<sup>2</sup>  $< Q^2 < 60$  (GeV/ $c$ )<sup>2</sup>,  $10^{-3} < x < 0.4$ ,  $0.1 < y < 0.7$ , and  $0.20 < z < 0.85$  with  $W > 5$  GeV/ $c$ . They constitute an important input for future world-data analyses in order to constrain strange quark PDFs and FFs. The sum of  $K^+$  and  $K^-$  multiplicities integrated over  $z$  shows a flat distribution in  $x$ , with values significantly higher than those measured by HERMES at lower energy. By covering lower  $x$  values, the multiplicity sum data will significantly improve the constraint on the product of the strange quark PDF and FF,  $S\mathcal{D}_S^K$ . The present result points to a larger value of  $\mathcal{D}_U^K$  than obtained from the earlier DSS fit.

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