TRANSVERSE TOLERANCES OF A MULTI-STAGE PLASMA WAKEFIELD ACCELERATOR

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Abstract

Plasma wakefield acceleration (PWFA) provides GeV/m-scale accelerating fields, ideal for applications such as a future linear collider. However, strong focusing fields imply that a transversely offset beam with an energy spread will experience emittance growth from the energy dependent betatron oscillation. We develop an analytic model for estimating tolerances from this effect, as well as an effective simplified simulation tool in Elegant. Estimations for a proposed 1 TeV PWFA linear collider scheme indicate tight tolerances of order 40 nm and 1 μrad in position and angle respectively.

INTRODUCTION

Plasma wakefield acceleration (PWFA) is an emerging technology promising GeV/m acceleration gradients [1], orders of magnitude higher than conventional technologies. A potential application is a future linear collider of significantly shorter lengths than current designs like ILC [2] and CLIC [3]. Schemes have been proposed where multiple consecutive PWFA cells are staged such that the energy-depleted drive bunch is swapped for a fresh one at regular intervals, accelerating the witness bunch to TeV-scale.

The drive bunch creates a 100 μm-scale wake in the plasma, in which strong electric fields accelerate the witness bunch longitudinally, but also focus the witness bunch transversely. In the non-linear "blowout" regime, these focusing fields are mostly linear in radius r, and hence the transverse emittance is conserved. However, much like in a quadrupolar FODO-lattice, a transverse offset will disperse a beam with an energy spread and increase its projected emittance. In this paper, we study transverse tolerances due to this effect. Plasma-beam interaction may also lead to emittance growth through the hosing instability [4] or the beam break-up instability [5], but this is not considered here.

Note that since the drive beam defines the channel, stage-to-stage alignment of plasma cells is irrelevant, and only relative drive-to-witness alignment matters. Also, static offsets are assumed to be tunable, leaving dynamic offsets (shot-to-shot jitter) as the main source of emittance growth.

MODEL

We adopt a simple analytic model: the witness bunch enters the plasma channel, defined by the drive bunch, with a relative offset position Δx and angle $\Delta x'$ (see Figure 1), and matched to the channel with a Twiss beta-function

$$\beta_m = \frac{\sqrt{2\gamma}}{k_p}, \tag{1}$$

where γ is the Lorentz factor of the witness beam and the k_p is the wavenumber of the plasma. Since inside the channel, the beam x and x' centroids will oscillate, but we don't care about the phase, it is useful to instead work with an *action*

$$J = \frac{\Delta x^2}{2\beta} + \frac{\beta}{2}\Delta x'^2, \tag{2}$$

where β is the beta-function of the incoming beam and we assume $\alpha = 0$. The model neglects the effect of acceleration, synchrotron radiation, beam loading and other collective effects.

For verification, analytic results are compared to a simplified simulation setup. Since we are working in the blowout regime, where fields are mostly linear, conventional particle tracking in Elegant [6] can be used to emulate the effect of the plasma channel. A long radially symmetric "quadrupole" is used for focusing, and thin accelerating cavities are distributed along the channel to include also the effect of acceleration.

EMITTANCE GROWTH

When the witness bunch enters the channel, its centroid starts oscillating in phase space with a conserved action J, with a wavelength $\lambda_\beta = 2\pi\beta_m$ along s. Since λ_β is energy dependent, higher energy particles oscillate more slowly, and

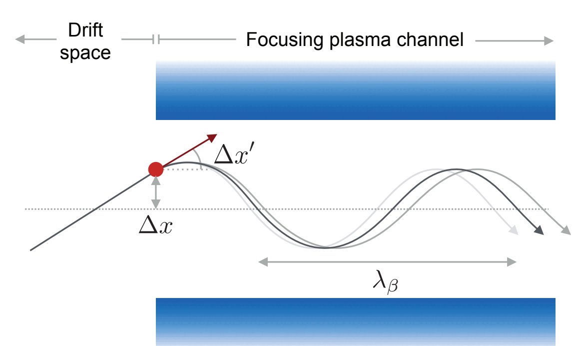

Figure 1: The witness beam enters the plasma channel with a positional offset Δx and angular offset $\Delta x'$ relative to the drive beam (which defines the channel). Each energy slice then oscillates with wavelength $\lambda_\beta(\gamma) = 2\pi\beta_m(\gamma)$, resulting in increased projected emittance.

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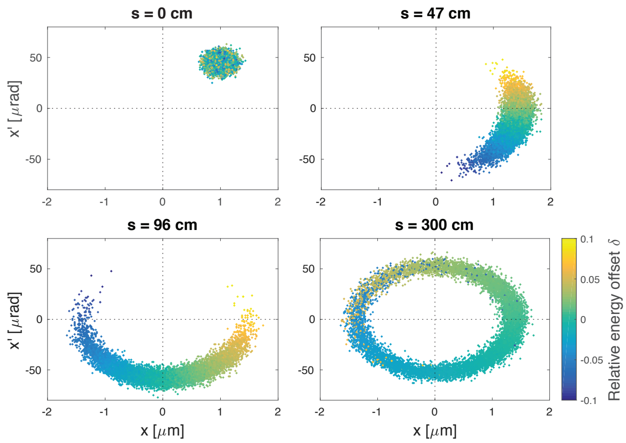

Figure 2: Evolution of the witness bunch in phase space, where colors indicate the relative energy offset. The initial beam, offset by 1 μm and 40 μrad in x and x' respectively, gets progressively stretched while it traverses the circumference of the offset ellipse (unsaturated regime) until ending up as a steady-state ring (saturated regime).

hence the bunch smears out in phase space until it finally forms a ring (see Figure 2). To describe this behavior, we differentiate between the *unsaturated regime* of continuous smearing and the steady-state *saturated regime* where bunch is fully smeared.

Unsaturated Emittance Growth

Consider a beam of transverse rms size σ_x and geometric emittance ϵ. Without loss of generality, we assume the offset is purely positional, $\Delta x = \sqrt{2\beta J}$, giving an rms phase spread

$$\sigma_{\mu 0} = 2\pi \frac{\sigma_x}{2\pi \Delta x} = \frac{\sqrt{\beta\epsilon}}{\sqrt{2\beta J}} = \sqrt{\frac{\epsilon}{2J}}. \qquad (3)$$

Different energies advance their phase by $\mu(s) = 2\pi s/\lambda_\beta = s/\beta_m$, giving an energy dependence (chromaticity) of

$$\xi = \left.\frac{\partial \mu}{\partial \delta}\right|_{\delta=0} = \frac{\partial}{\partial \delta}\left.\left(\frac{s}{\sqrt{1+\delta}\,\beta_m}\right)\right|_{\delta=0} = -\frac{s}{2\beta_m}. \qquad (4)$$

Their new phase spread, assuming Gaussian distributions, can be found by adding sigmas in quadrature

$$\sigma_\mu{}^2 = \sigma_{\mu 0}{}^2 + (\sigma_E \xi)^2. \qquad (5)$$

Since all smearing occurs along the circumference of the offset ellipse, the relative increase in phase spread equals the relative increase in emittance

$$\frac{\epsilon}{\epsilon_0} = \frac{\sigma_\mu}{\sigma_{\mu 0}} = \sqrt{1 + \frac{\sigma_E{}^2}{\sigma_{\mu 0}{}^2}\xi^2} = \sqrt{1 + \frac{\sigma_E{}^2 s^2 J}{2\epsilon_0 \beta_m{}^2}}. \qquad (6)$$

Expanding the square root by assuming the emittance growth is small, we obtain the unsaturated emittance growth

$$\left.\frac{\Delta\epsilon}{\epsilon_0}\right|_{\text{unsat}} = \frac{\sigma_E^2 s^2 J}{4\epsilon_0 \beta_m{}^2} = \frac{\sigma_E^2 s^2 k_p^2 J}{8\epsilon_{N0}}. \qquad (7)$$

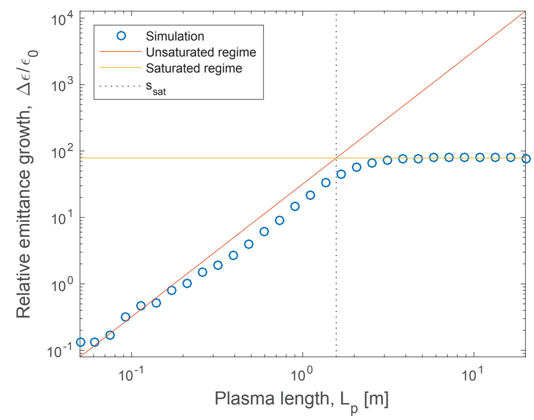

Figure 3: Emittance growth vs. plasma length for a 100 GeV beam with 3 % rms energy spread, normalized emittance 100 nm and an initial offsets of 1 μm and 40 μrad. Emittance growth is seen to transition from the unsaturated regime (red line) to the saturated regime (yellow line) at the predicted saturation length (dotted line).

Saturated Emittance Growth

To find the saturated emittance growth of a plasma channel given an action J, we must find the emittance of the offset ellipse. A ring of radius $j = \sqrt{2J}$ has a 2D rms radius of $j_{rms} = j/\sqrt{2}$, and since the geometric emittance is $\epsilon = j_{rms}{}^2$ we get the saturated emittance growth

$$\left.\frac{\Delta\epsilon}{\epsilon_0}\right|_{\text{sat}} = \frac{J}{\epsilon_0} = \frac{\gamma J}{\epsilon_{N0}}, \qquad (8)$$

where we assume the final emittance is much greater than the initial emittance.

Saturation Length

To determine the length required for saturation, we solve for when the unsaturated emittance growth, Eq. (7), equals the saturated emittance growth, Eq. (8), and find

$$s_{\text{sat}} = \frac{2\beta_m}{\sigma_E}. \qquad (9)$$

This saturation length is energy dependent, growing with higher energy. Using plasma cells of the same length L_p, emittance growth will transit from the saturated to the unsaturated regime when $s_{\text{sat}} = L_p$, at energy

$$\gamma_{\text{swap}} = \frac{(\sigma_E k_p L_p)^2}{8}. \qquad (10)$$

PLASMA DENSITY RAMPS

The effective matched β as seen from outside can be significantly increased while preserving the emittance, as well as the action, by using plasma density ramps [7]. This can be used to trade between angular and positional offset tolerances.

Consider a ramp of demagnification $\beta_0/\beta_m > 1$, where β_0 is the initial beta-function. To minimize emittance growth from offsets, in either regime, we must minimize J with respect to β_0, and since the offsets Δx and $\Delta x'$ are stochastic, we use their rms values $\sigma_{\Delta x}$ and $\sigma_{\Delta x'}$:

$$\frac{\partial \sigma_J}{\partial \beta_0} = \frac{\partial}{\partial \beta_0}\left(\frac{\sigma_{\Delta x}^2}{\beta_0} + \beta_0 \sigma_{\Delta x'}^2\right) = 0 \qquad (11)$$

$$\beta_{0,\mathrm{min}} = \frac{\sigma_{\Delta x}}{\sigma_{\Delta x'}} \qquad (12)$$

The above result indicates that if the plasma ramp demagnification is determined by other considerations (e.g. chromatic effects in staging optics [8]), the drive beam injector should be constructed such as to match the ratio of positional and angular jitter to β_0.

MULTIPLE STAGES

Generalizing to multiple stages necessarily introduces randomness to the system, with an offset jitter in each stage. Since the number of stages N will likely be small (a few to a few dozen), the total emittance growth is subject to significant relative jitter $\sim 1/\sqrt{N}$. Nevertheless, we make an estimate of the tolerances.

The emittance growth starts in the saturated regime, where after every cell the action has been canceled by smearing, and hence the normalized emittances add in quadrature. After this, and more importantly as γ_{swap} is often small, in the unsaturated regime, the beam is not fully smeared and the action is largely preserved. Action jitters σ_J therefore add (i.e. offsets add in quadrature) over n stages to give

$$\sigma_{J,n} = \sum_{i=1}^{n} \frac{i}{n}\sigma_J \approx \frac{n}{2}\sigma_J, \qquad (13)$$

where acceleration damping of the action ($J \sim 1/\gamma$) is accounted for. The normalized emittance added at stage n and in total after N stages are thus given by

$$(\Delta\epsilon_N)_n = \frac{\sigma_E^2 L_p^2 k_p^2 n \sigma_J}{16} \qquad (14)$$

$$(\Delta\epsilon_N)_{\mathrm{tot}} = \sqrt{\sum_{n=1}^{N} (\Delta\epsilon_N)_n^2} = (\Delta\epsilon_N)_1 \sqrt{\sum_{n=1}^{N} n^2} \qquad (15)$$

$$\approx N^{\frac{3}{2}} \frac{\sigma_E^2 L_p^2 k_p^2 \sigma_J}{16\sqrt{3}}. \qquad (16)$$

Equivalently, we can express this as a tolerance for the square root of the action jitter (proportional to the offset jitters)

$$\sqrt{\sigma_{J,\mathrm{max}}} = \frac{\sqrt{16\sqrt{3}\,(\Delta\epsilon_N)_{\mathrm{tot}}}}{\sigma_E L_p k_p N^{\frac{3}{4}}}, \qquad (17)$$

where dependence on plasma density n_p is found by $k_p^2 = n_p e^2/m_e \epsilon_0 c^2$. This can be converted to positional and angular offset tolerances via $\sigma_{\Delta x} = \sqrt{2\beta_1 \sigma_J}$ and $\sigma_{\Delta x'} = \sqrt{2\sigma_J/\beta_N}$ where β_1 and β_N are evaluated at the first and last cell, respectively, as they require the tightest tolerances.

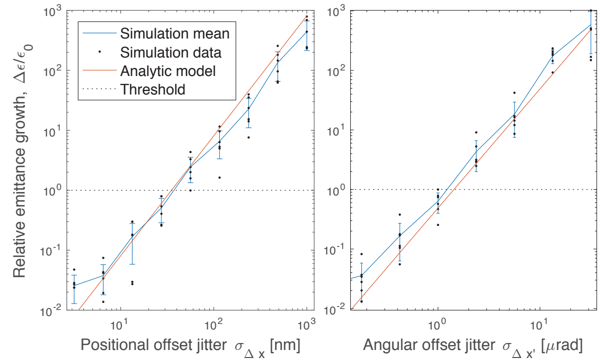

Figure 4: Emittance growth vs. positional and angular offset jitter for the worked example. Tolerances are individually approximately 40 nm and 1 μrad. As predicted by the model, emittance growth scales with the square of the rms offset, and is numerically consistent (35 nm and 1.5 μrad).

WORKED EXAMPLE

The 1 TeV PWFA linear collider scheme in Ref. [9] uses the following parameter set: 20 stages of 25 GeV energy gain, 3 m long, density 2×10^{16} cm^{-3}, and a beam of 1% energy spread and initial normalized emittance of 100 nm. The regime-swap occurs at 40 GeV, which means the unsaturated regime is dominant throughout the accelerator. Assuming a maximum of 100 % emittance growth, Eq. (17) estimates approximately a tolerance in position and angle of 35 nm and 1.5 μrad respectively, in good agreement with simulation (40 nm and 1 μrad) in Figure 4.

CONCLUSIONS

Plasma wakefield accelerators, while providing very large acceleration gradients, will require very tight alignment jitter tolerances. An analytic model was developed to estimate the emittance growth from position and angle offsets in the plasma channel, backed by a simplified simulation setup using conventional particle tracking. Tolerances were estimated for the 1 TeV PWFA linear collider scheme in Ref. [9] to approximately 40 nm and 1 μrad in position and angle rms jitter, respectively.

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