EuCARD-2

Enhanced European Coordination for Accelerator Research & Development

Journal Publication

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14 March 2016

Optimized stability of a modulated driver in a plasma wakefield accelerator

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(Dated: 15 March 2016)

We analyze the transverse stability for a configuration of multiple gaussian bunches subject to the self-generated plasma wakefield. Through a semi-analytical approach we first study the equilibrium configuration for the modulated beam and then we investigate the evolution of the equilibrium configuration due to the emittance-driven expansion of the beam front that results in a rigid backward shift. The rear-directed shift brings the modulated beam out of the equilibrium, with the possibility for some of the bunch particles to be lost with a consequent deterioration of the driver. We look therefore for the proper position of the single bunches that maximize the stability without severely affecting the accelerating field behind the driver. We then compare the results with 3D PIC simulations.

I. INTRODUCTION

Plasma wakefield acceleration, both laser driven[1–4] and particle driven[5,6], has proven to be a possible alternative to conventional accelerators for a wide range of applications, due to the intense electric fields that the plasma can sustain.

Among the possible configurations feasible for beam driven plasma wakefield, a renewed interest is devoted to the use of modulated drivers in view of the forthcoming AWAKE experiment at CERN[7]. The goal of the project is to prove the possibility to employ the 400 GeV proton bunches produced at the Super Proton Synchrotron(SPS) as drivers for the plasma wakefield.

The success of the experiment relies on the capability to produce short bunches whose length is approximately $L = \lambda_p/2$ (being $\lambda_p = 2\pi c/\omega_p$ the plasma wavelength and $\omega_p^2 = 4\pi n e^2/m$ the plasma frequency) from the much longer proton bunches provided at the SPS. Such achievement can be fulfilled through the self-modulation instability[8], in which the interaction of a long bunch with the wave itself leads to a modulation of the driver resulting at the final stage in a train of bunches with the required length and periodicity approximately λ_p.

One of the serious issues of such a framework, and more in general for a configuration employing a pre-modulated beam, is the stability of the driver. A periodicity of λ_p, although provides an intense accelerating field, does not guarantee as well a long lasting driver, which can be deteriorated by the transverse wake field. Dealing with such a problem requires an appropriate analysis of the evolution of a modulated train of bunches interacting with a plasma.

In this paper we study the proper periodicity of the modulated driver in order to improve its stability without severely affecting the resulting accelerating field. We show that by properly placing the single bunches it is possible to reduce the deterioration of the modulated configuration, providing a larger number of bunch particles contributing to the wakefield for a longer time. This will end up eventually in a more intense accelerating field as compared to the case of a modulation of λ_p, after few meters of propagation.

Such result suggests the necessity to rethink the proper shape of the modulated beam. While in the case of a pre-modulated driver it is enough to properly configure the spacing between the bunches, for a self-modulated driver like in the AWAKE experiment, a different approach is necessary. A possible solution is the use of two plasma cells[9], the first intended to modulate the proton bunch through the self-modulation instability, the second to employ the modulated structure to excite the field suitable for the acceleration of the witness bunch.

The paper is structured as follows: in section II we trace the model used for description of the interaction of Gaussian bunches with a plasma, assuming a linear response and the quasi-static approximation; in section III we describe the evolution of the equilibrium configuration for the bunches emerging from the model, comparing the results with the case of flat-top bunches; in section IV we look for the proper position of the bunches in order to increase their stability without seriously affecting the longitudinal electric field; in section V we present our conclusions.

II. ANALYTICAL MODEL

The model describing the interaction of a particle beam with plasma relies on the work of Kenigs and Jones[10].

The authors consider an axi-symmetric bunch linearly interacting with an overdense plasma with immobile ions. Since the plasma is overdense the beam is regarded as an external perturbation. The analysis is developed in the co-moving frame defined by the variables $\xi = \beta ct - z$ and $\tau = t$ with $\beta = v_b/c \simeq 1$, v_b being the beam velocity and c the speed of light. Lastly the quasi-static approximation is assumed, providing $\partial_\tau \simeq 0$.

The two-dimensional transverse and longitudinal fields

arising from the interaction are then:

$$W(r,\xi) = (E_r - \beta B_\theta) = 4\pi k_p \int_0^\infty \int_0^\xi \frac{\partial \rho(r',\xi')}{\partial r'}$$
$$r'\, \mathrm{I}_1(k_p r_<)\, \mathrm{K}_1(k_p r_>) \sin[k_p(\xi - \xi')]d\xi' dr' \quad (1)$$

$$E_z(r,\xi) = -4\pi k_p^2 \int_0^\infty \int_0^\xi \rho(\xi',r')r'$$
$$\mathrm{I}_0(k_p r_<)\, \mathrm{K}_0(k_p r_>) \cos[k_p(\xi - \xi')]d\xi' dr' \quad (2)$$

where $k_p = \omega_p/v_b$ is the plasma wavenumber, $\rho(r,\xi)$ is the bunch charge density, $\mathrm{I}_{1/0}$ and $\mathrm{K}_{1/0}$ are the modified Bessel functions and $r_{</>} = \min/\max(r,r')$.

We study the fields excited by a bunch with a flat-top profile in the transverse direction:

$$\rho(r,\xi) = n_b q_b \left(\frac{r_0}{r_b(\xi)}\right)^2 \mathrm{H}(r_b(\xi) - r)f(\xi) \quad (3)$$

with n_b being the peak bunch density, q_b the bunch charge, r_0 the initial bunch radius, $r_b(\xi)$ the radius of the beam-envelope, H the Heaviside function and $f(\xi)$ the longitudinal bunch profile. The wakefields generated by such a distribution are therefore:

$$W(r,\xi) = -4\pi k_p n_b q_b r_0^2$$
$$\begin{cases} \mathrm{I}_1(k_p r) \int_0^\xi f(\xi') \frac{\mathrm{K}_1(k_p r_b(\xi'))}{r_b(\xi')} \sin[k_p(\xi - \xi')]d\xi' \text{ for } r < r_b \\ \mathrm{K}_1(k_p r) \int_0^\xi f(\xi') \frac{\mathrm{I}_1(k_p r_b(\xi'))}{r_b(\xi')} \sin[k_p(\xi - \xi')]d\xi' \text{ for } r > r_b. \end{cases}$$
$$(4)$$

$$E_z(r,\xi) = -4\pi k_p n_b q_b r_0^2$$
$$\begin{cases} \int_0^\xi f(\xi') \frac{1 - k_p r_b(\xi')\, \mathrm{I}_0(k_p r)\, \mathrm{K}_1(k_p r_b(\xi'))}{k_p r_b^2(\xi')} \cos[k_p(\xi - \xi')]d\xi' \text{ for } r < r_b \\ \mathrm{K}_0(k_p r) \int_0^\xi f(\xi') \frac{\mathrm{I}_1(k_p r_b(\xi'))}{r_b(\xi')} \cos[k_p(\xi - \xi')]d\xi' \text{ for } r > r_b. \end{cases}$$
$$(5)$$

In order to self-consistently include the dynamics of the beam, we couple the fields equation with the beam-envelope equation for the beam radius assuming that the transverse motion of the beam can be described by only the motion of its boundary like in a water-bag model.

Denoting with $r_b = r_b(\xi,\tau)$ and with $r'_b = r_b(\xi',\tau)$, the resulting equation is:

$$\frac{\partial^2 r_b}{\partial \tau^2} = \frac{\epsilon^2 c^2}{\gamma^2 r_b^3} - \frac{4\pi k_p n_b q_b^2 r_0^2}{m_b \gamma}$$
$$\begin{cases} \mathrm{I}_1(k_p r_b) \int_0^\xi f(\xi') \frac{\mathrm{K}_1(k_p r'_b)}{r'_b} \sin[k_p(\xi - \xi')]d\xi' \text{ for } r_b < r'_b \\ \mathrm{K}_1(k_p r_b) \int_0^\xi f(\xi') \frac{\mathrm{I}_1(k_p r'_b)}{r'_b} \sin[k_p(\xi - \xi')]d\xi' \text{ for } r_b > r'_b \end{cases}$$
$$(6)$$

with ϵ being the normalized beam emittance, $\gamma = 1/\sqrt{1 - \beta^2}$ the beam relativistic Lorentz factor and m_b the mass of the beam particles.

From Eq.6 results that the front of the beam is not subject to the wakefield and its dynamics is governed by the emittance-driven expansion according to the equation:

$$r_b(\xi = 0, \tau) = r_{b0}(\tau) = r_0 \sqrt{1 + \frac{\epsilon^2 c^2 \tau^2}{r_0^4 \gamma^2}}. \quad (7)$$

TABLE I: Simulation parameters for plasma and bunch.

Parameter	Value
Plasma density (n_p)	7×10^{14} cm^{-3}
Bunch length (σ)	0.02 cm
Initial bunch radius (r_0)	0.02 cm
Periodicity (Δ)	0.12 cm
Peak bunch density (n_b)	4×10^{12} cm^{-3}
Bunch relativistic Lorentz factor (γ)	400
Normalized bunch emittance (ϵ)	3.5 mm mrad

The purely diverging front of the bunch provides the absence of a global stable configuration for the transverse beam profile. Moreover, since the dynamics at every point along the bunch depends on the upstream part, the evolution of the front of the bunch leads to a change of the whole equilibrium configuration for the beam while propagating in the plasma.

III. EVOLUTION OF THE EQUILIBRIUM CONFIGURATION FOR GAUSSIAN BUNCHES

A previous work[11] has analyzed the equilibrium configuration for a modulated bunch with longitudinal flat-top density profile. The authors have shown that the dynamics of the front of the bunch leads to a backward drift of the whole equilibrium configuration for the modulated beam.

We perform the same analysis for the case of bunches with a longitudinal gaussian density profile. Since the defocusing force driven by the emittance does not depend on the explicit shape of the bunch, we can expect a similar behavior as well for gaussian bunches.

First we consider the case of a modulated beam composed by identical equidistant bunches. The bunch densities are described by Eq.(3) with longitudinal profiles:

$$f(\xi) = \sum_{j=1}^N e^{-\frac{(\xi - \xi_j)^2}{2\sigma_j^2}} = \sum_{j=1}^N e^{-\frac{(\xi - \xi_0 - j\Delta)^2}{2\sigma^2}} \quad (8)$$

with N being the number of bunches, ξ_j the center, σ_j the length and Δ the periodicity. The complete set of the parameters characterizing both the plasma and the beam, with the exclusion of the beam length, is based on the baseline of the AWAKE project and is presented in Table I.

Although this analysis refers explicitly to the AWAKE experiment, the mechanism that leads to the modulation can be general. Therefore the same results hold for both a pre-modulated and a self-modulated beam.

The equilibrium radius of the bunches is defined as the global minimum, at every position in the bunch, of the potential provided by the force in Eq.6, with the prescription that $r_b(\xi,\tau) \leq r_b(\xi = 0,\tau)$.
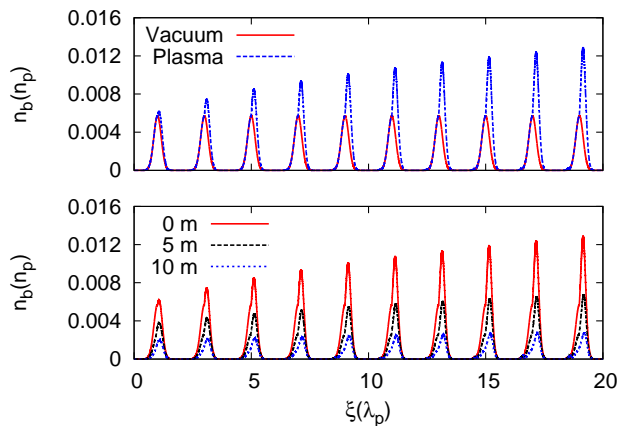
FIG. 1: Quasi-equilibrium configuration for a modulated gaussian bunch with periodicity $2\lambda_p$ in vacuum and in plasma (top) and for different propagation distances in the plasma (bottom).
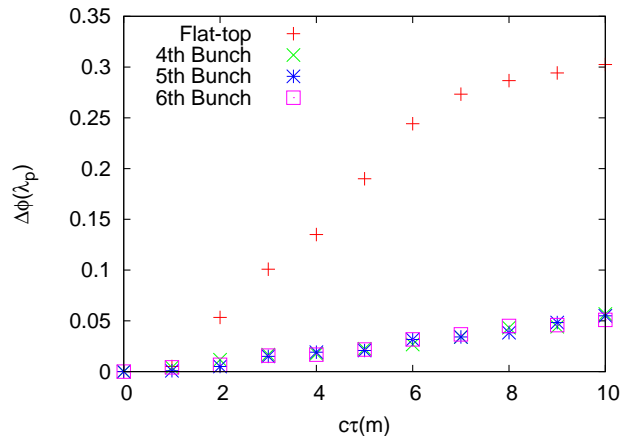

FIG. 2: Shift of the 4th, 5th and 6th bunch respect to its initial position varying the propagation distance in the plasma. The result is compared with the shift experienced by flat-top bunches.

As expected (Fig.1), the interaction with the plasma leads to the focusing of the modulated beam, increasing the peak density of the bunches as compared to the case of pure vacuum. The focusing force increases towards the tail of the configuration due to the interference among the transverse fields generated by the single bunches.

In order to avoid effects due to the overlapping of the single bunches, the periodicity has been set to $\Delta = 2\lambda_p$. Studying the evolution of the equilibrium configuration for gaussian bunches while propagating in the plasma, shows a behavior analogous to the case of flat-top bunches. The modulated beam tends to experience a backward shift in its equilibrium configuration, with the displacement that is increasing with the propagation distance. Nevertheless, as shown in Fig.2, the shift is much smaller for gaussian bunches than that for flat-top ones. Although the backward shift is quantitatively different for the two cases, the reason is the same. The expansion of the front of the bunch corresponds to a decreasing bunch density and therefore a larger amount of bunch is required to obtain the same charge for different propagation distances. This process delays the onset of the focusing force(Fig.3). The inhomogeneity of the gaussian bunches tends anyway to a suppression of the fields, therefore the backward shift is less strong respect to flat-top bunches.

The evolution of the equilibrium configuration implies serious consequences for the stability and the duration of the modulated beam.

A bunch particle initially in an equilibrium position, will find itself in an unstable one while propagating in the plasma as a consequence of the rearrangement of the trapping potential caused by the expansion of the front.

The displacement experienced by the initial bunch radius with respect to the new equilibrium configuration results in a gain of transverse momenta. If the displacement is large enough, the bunch particles can even escape
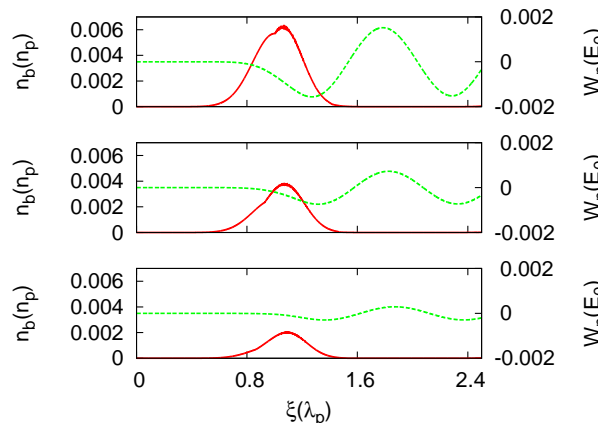

FIG. 3: Transverse field excited by a gaussian bunch for 0, 5 and 10 m of propagation in the plasma.

the trapping potential.

The more driver particles are depleted, the lower is the wakefield amplitude, resulting in the lower efficiency of the process.

IV. OPTIMIZATION OF THE BEAM CONFIGURATION

The framework depicted previously demands a further analysis on the optimal configuration for a modulated beam in order to reduce its deterioration as much as possible, while propagating in the plasma.

A train of bunches with periodicity $\Delta = \lambda_p$ provides an intense accelerating field behind the driver, but the simple expansion of the front of the bunch causes its degradation. The more particles the bunches lose while propagating in the plasma, the weaker is the final accelerating field.
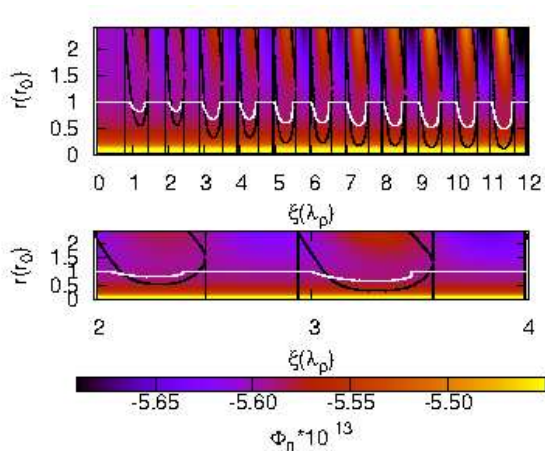
FIG. 4: Potential surface after 10 m of propagation in the plasma. The white line corresponds to the equilibrium beam radius at 0 m, while the black lines are the boundaries for which the particle stay trapped after 10 m of plasma.

We look therefore to the proper position of the single bunches in order to maximize the number of particles keeping trapped while propagating in the plasma.

The trapping is studied between the initial and final position in the plasma channel. Since the backward shift increases with the propagation distance, improving the trapping respect to the final stage guarantees as well an improvement for the whole evolution.

To establish which sections of the bunches are unstable, we compare the initial configuration with that at the end of the propagation. The bunch particles are lost if the difference between the two is large enough to provide the necessary transverse momentum to escape the trapping potential. In Fig.4 we can see an example of the method applied: some sections of the initial equilibrium configuration are, at the end of the propagation distance, out of the trapping region, meaning that those slices of the bunches are lost.

According to this criterion we look for the matching periodicity of the configuration that guarantees the highest number of particles trapped between the initial and final step. The procedure is performed for one bunch at the time, fixing the configuration upstream to the already evaluated matching positions.

In Fig.5 we can see the relative number of particle trapped changing the position of the last bunch of the configuration over a plasma wavelength.

As expected the maximum number of particle trapped is not achieved with a periodicity $\Delta = \lambda_p$. This is a consequence of several aspects: the maximum of the Green function for the transverse field in Eq(2) lies at $\Delta = \lambda_p/2$; the gaussian profile induces a nonlinear shift of the maximum; the backward drift driven by the expansion of the front of the bunch causes an additional shift of the optimal position.
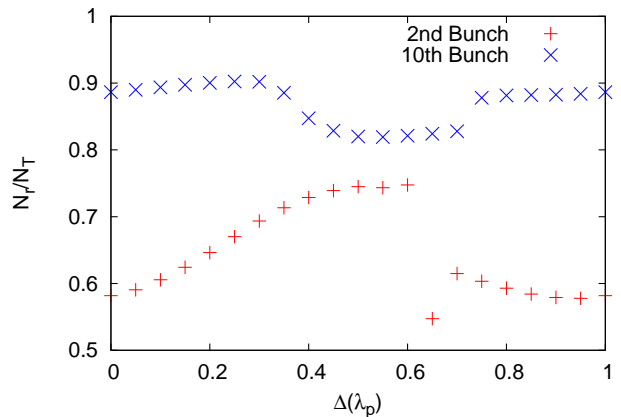

FIG. 5: Relative number of particle trapped after 10 m of propagation varying the position of the last bunch of the configuration.
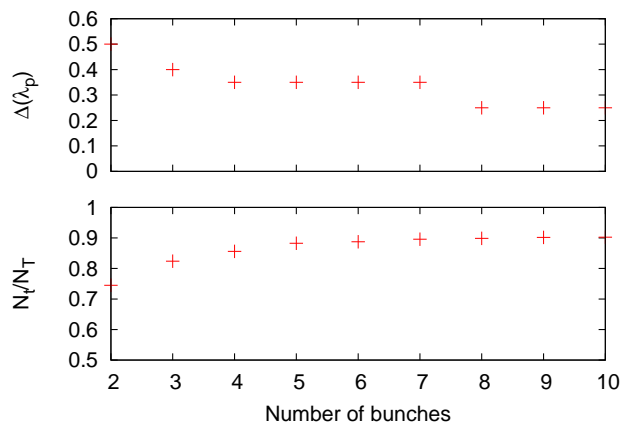

FIG. 6: Matching position of every bunch to improve the trapping (top) and relative number of particle trapped (bottom).

Moreover, as appears from Fig.6, the displacement is not constant for every bunch, meaning that it is not enough to rigidly move the configuration backward, but an analysis for every bunch is required. On the other hand the stability of the configuration, defined as the relative number of particle trapped, increases with the number of bunches involved. This is a consequence of the increasing total transverse field due to the superposition of the single ones generated by all the upstream bunches. Although this optimization provides an increasing final total charge of the beam, it does not necessarily implies as well an intense accelerating field behind the driver. The matching position improving the total charge can coincide with that providing a destructive interference of the accelerating field generated by the single bunches.

This can be understood also by looking at Eq.(2), since the Green function for the longitudinal and transverse fields are $\pi/2$ out of phase, meaning that the maximum
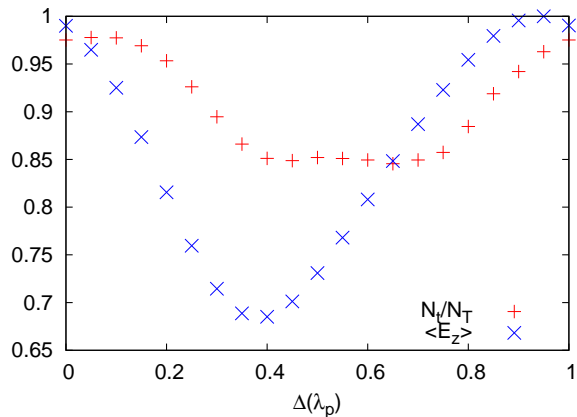
FIG. 7: Relative number of trapped particle with average accelerating field behind the driver for a configuration with ten bunches.
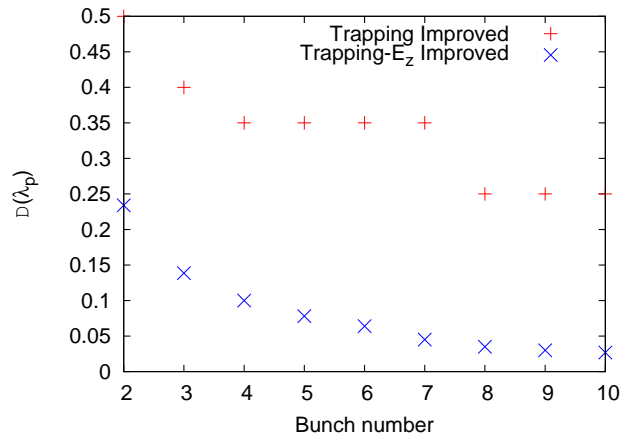

FIG. 8: Matching position of every bunch in order to improve both the stability of the driver and the final accelerating field. The result is compared with the previous one in which only the stability was guaranteed.

of the first will be close to the minimum of the second.

Therefore in order to obtain a configuration suitable for particle acceleration, it is necessary to improve the stability of the train of bunches without severely affecting the resulting longitudinal field behind the driver.

We perform an analogous analysis as the previous one, taking into account this time also the average of the accelerating field behind the driver over the propagation distance. The analysis is performed again for one bunch at the time, fixing the configuration upstream to the matching one.

Fig.7 displays clearly the behavior mentioned previously. The position that guarantees the maximum number of trapped particles does not correspond to that improving also the longitudinal field behind the driver. In order to obtain the best of the two behaviors, the matching condition is set at the crossing between the lines representing the relative number of trapped particle and the average accelerating field behind the driver.

In order to confirm the results, we have performed a three dimensional PIC simulation using the quasi-static VLPL code[12]. We compare both the bunch densities and the longitudinal electric field for a modulation of λ_p and the newly obtained modulation. We can see from Fig.9 that the optimized configuration preserves the bunch densities, obtaining a final charge higher than that in the periodic case.

The longitudinal electric field as well provides the behavior expected by the improved configuration(Fig10). The modulation of λ_p guarantees a stronger electric field initially. On the other hand, the optimized configuration provides an improved focusing field acting on the single bunches. This leads to an increasing peak density and therefore an increasing accelerating field. After about 3 meters of propagation in the plasma, the accelerating field behind the driver becomes stronger in the new configuration(Fig.11) and the improved stability shows

guarantees a slower decrease.

V. CONCLUSION

In this work we have analyzed the stability of a train of gaussian bunches interacting with a plasma, a configuration occurring in plasma wakefield acceleration employing pre-modulated or self-modulated beams.

Through a semi-analytical model we have study the equilibrium configuration for a series of periodic gaussian bunches and the effects of the expansion of the front of the bunch in its evolution. The result shows an analogy with the backward shift occurring for flat-top bunches. To excite focusing fields of the same intensity for increasing propagation distances, it is required a longer section of the bunch, due to the decreasing density of the front. This provides a phase shift of the focusing field and therefore of the entire equilibrium configuration.

We pointed out as this backward shift leads to an instability of the driver and to its deterioration while propagating in the plasma.

Through a numerical analysis we have found first the proper position of the bunches in order to improve the relative number of particle trapped during the propagation showing that is not provided by a periodicity of λ_p. We finally extended the analysis checking as well the resulting accelerating field behind the driver generated by the new configuration. The final result provides a configuration for a train of gaussian bunches with an improved stability as well with a strong accelerating field behind the driver.

The validity of the analysis have been finally tested performing a 3D PIC simulation using the quasi-static VLPL code, providing the effects already observed in the simplified model. The accelerating field arising from the modified train of bunches is initially less intense than
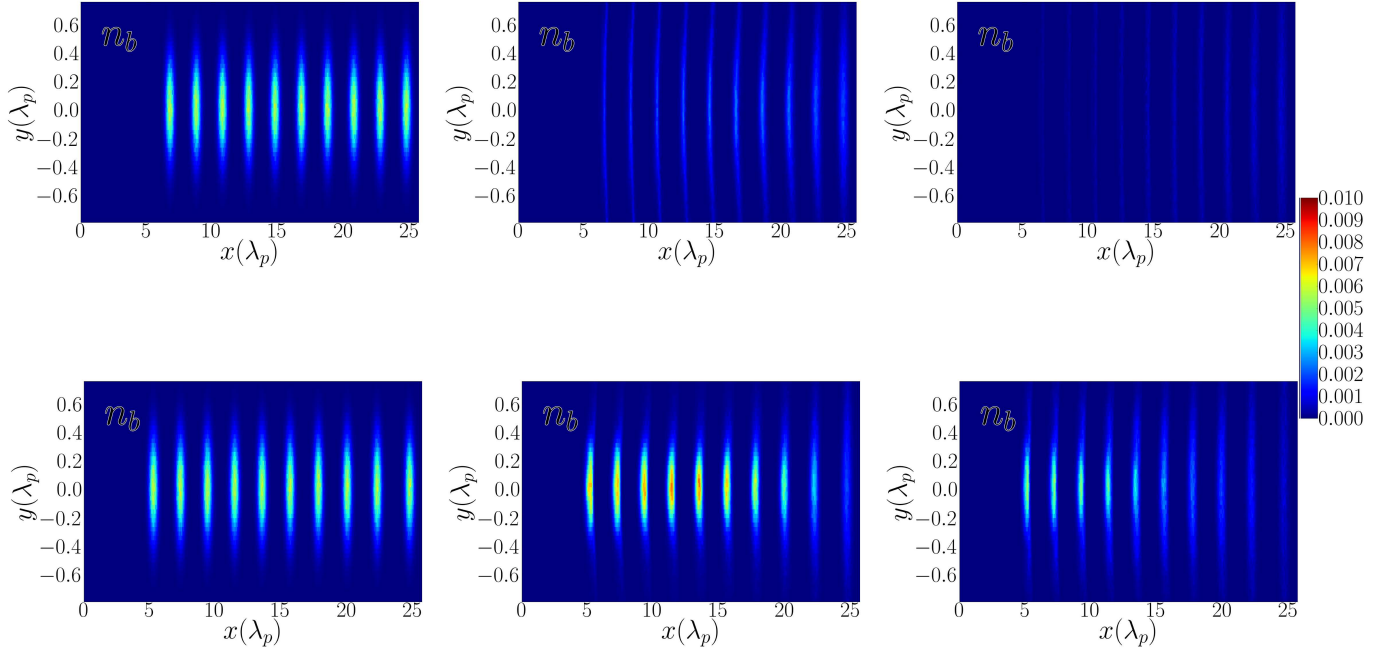
FIG. 9: Density of the modulated beam for a periodicity $\Delta = \lambda_p$ (upper line) and for the matching periodicity (lower line) at 0, 5 and 10 m in the plasma.
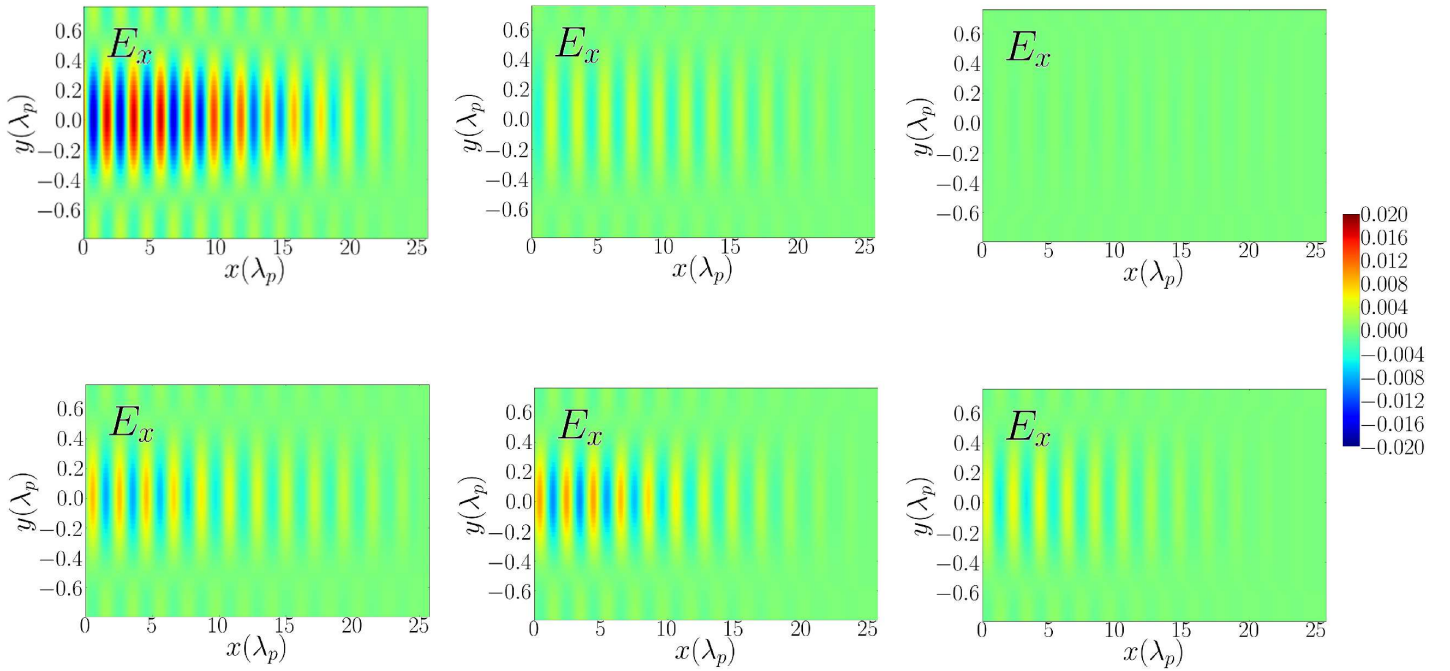

FIG. 10: Longitudinal electric field generated by the modulated beam for a periodicity $\Delta = \lambda_p$ (upper line) and for the matching periodicity (lower line) at 0, 5 and 10 m in the plasma.
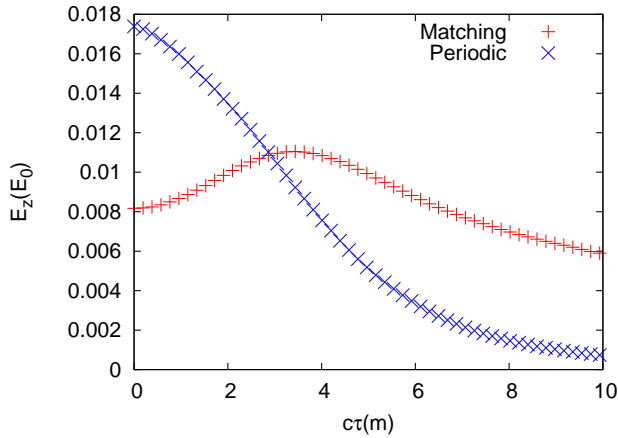
FIG. 11: Comparison of the accelerating field behind the driver for the periodic and optimized case as a function of the propagation distance.

that emerging from a modulation of λ_p, but after about 3 meters of propagation it becomes stronger, the difference between the two increasing further for the rest of the propagation distance.

ACKNOWLEDGMENTS

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