

# CHROMATICITY DEPENDENCE OF THE TRANSVERSE EFFECTIVE IMPEDANCE IN THE CERN PROTON SYNCHROTRON

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## Abstract

The current knowledge of the transverse beam coupling impedance of the CERN Proton Synchrotron (PS) has been established with beam-based measurements at different energies. The transverse coherent tune shift as a function of the beam intensity has been measured in order to evaluate the total effective imaginary part of the transverse impedance in the accelerator at the energies of 7, 13 and 25 GeV. Measurements have been performed changing the vertical chromaticity for each vertical tune scan with intensity. The data analysis revealed an increase of impedance with chromaticity for all the considered energies. The transverse impedance can be compared with the previously evaluated theoretical impedance budget taking into account the individual contribution of several machine devices.

## INTRODUCTION

### Coherent Transverse Tune Shift

The number of betatron oscillations per turn of the bunch center of mass is called coherent betatron tune, and it is defined as

$$Q_0 = \frac{\omega_\beta}{\omega_0}, \quad (1)$$

where  $\omega_0$  is the machine angular revolution frequency and  $\omega_\beta$  is the angular betatron frequency. To perform tune measurements, a chirp signal is used to excite the beam, and the variation of the intensity of the bunch allows to observe a tune shift that is linear with the measured intensity. The transverse tunes can be measured in the PS with a Base Band Tune system based on diode detectors, known as the BBQ [1]. The transverse position of the bunch is acquired every turn by a beam position monitor (BPM). High amplitude short pulses measured by the BPM are then sent to a diode detector, which converts the modulation of the BPM pulses, related to beam oscillations, into a signal in the audio frequency range. This signal is then processed in order to deliver the tune content. BBQ measurements are performed here on a single circulating bunch of particles.

Measuring the tune shift with intensity gives information on the total reactive transverse impedance. For a Gaussian bunch of r.m.s. bunch length  $\sigma_z$  traveling with velocity  $v = \beta c$ , the coherent tune shift with intensity  $\Delta Q$  is proportional to the imaginary part of the total (driving plus

detuning) transverse effective impedance  $Z_t^{eff}$  by [2]

$$\Delta Q = -\frac{\beta e I_0}{4\sigma_z \sqrt{\pi} \omega_0^2 \gamma Q_0 m_0} \Im \{ Z_t^{eff} \}, \quad (2)$$

where  $I_0$  is the bunch current,  $Q_0$  is the unperturbed betatron tune,  $\gamma$  is the relativistic factor,  $e$  the particle charge and  $m_0$  the particle mass at rest. The effective transverse impedance is defined as the impedance weighted by the transverse bunch power spectrum centered at the chromatic frequency  $\omega_\xi$ :

$$Z_t^{eff} = \frac{\sum_{p=-\infty}^{\infty} Z_t(\omega') h(\omega' - \omega_\xi)}{\sum_{p=-\infty}^{\infty} h(\omega' - \omega_\xi)}, \quad (3)$$

where  $\omega' = \omega_0(p + Q_0)$  with  $p$  an integer,  $\omega_\xi = \omega_0 Q_0 \xi / \eta$ , with  $\xi$  the chromaticity and  $\eta$  the slippage factor, and the power spectrum of the Gaussian zero azimuthal bunch mode is  $h(\omega) = \exp(-(\omega^2 \sigma_z^2 / c^2))$ . If the bunch length does not change with intensity, Eq. 2 predicts a tune shift linear with bunch intensity, with a slope proportional to the imaginary part of the transverse total effective impedance.

### Chromaticity

The tune variation with the momentum is a machine parameter called chromaticity, defined as

$$\xi = \frac{\Delta Q / Q_0}{\Delta p / p_0}, \quad (4)$$

where  $p_0$  is the particle momentum on the nominal closed orbit and  $\Delta p$  is the momentum deviation. Chromaticities in the PS can be measured by acquiring the tune shift while varying  $\Delta p / p_0$ . Introducing a radial offset, we generate a momentum offset that lead to a variation of the revolution frequency. The tune can be written as a Taylor series of  $\frac{\Delta p}{p_0}$

$$Q \left( \frac{\Delta p}{p_0} \right) = Q_0 + Q' \frac{\Delta p}{p_0} + \frac{Q''}{2!} \left( \frac{\Delta p}{p_0} \right)^2 + \dots + \frac{Q^n}{n!} \left( \frac{\Delta p}{p_0} \right)^n, \quad (5)$$

where

$$Q' = \frac{\Delta Q}{\Delta f / f_0} \quad (6)$$

and  $Q^n$  are the higher order terms. The chromaticity is computed applying a polynomial fit on the measured data: from the linear term we can calculate the linear chromaticity as

$$\xi = \frac{Q'}{Q_0}. \quad (7)$$

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Control of the Working Point

In the PS, bending and focusing of the beam is provided by the main magnet units; the working point, in absence of any auxiliary magnet or coils, is determined by the momentum of the beam. For example, the bare machine at the energy of 2 GeV is working in linear condition (natural working point) with measured tunes of  $Q_x = 6.253$  and  $Q_y = 6.285$ , chromaticities of  $\xi_x = -0.83$  and  $\xi_y = -1.12$ , corresponding to chromatic angular frequencies of 213 MHz and 280 MHz, respectively. To correct the effects of linear and second order chromaticity in the PS, the magnetic field higher order components are needed and kept under control with auxiliary windings, called Pole Face Windings (PFW) [3]. PFW consist of four auxiliary coils mounted on the iron pole of each magnet (two coils for the focusing and two for the defocusing yoke) and they are used to control the working point and chromaticity. In addition, the figure-of-eight loop creates opposite fields in the two yokes. In this way, the horizontal and vertical tunes, horizontal and vertical chromaticities and non-linear chromaticity, can be controlled by five parameters.

TUNE SHIFT MEASUREMENTS AT ZERO CHROMATICITY

Extraction Energy

The measurements at the extraction kinetic energy of 25.48 GeV have been performed during several Machine Development (MD) sessions [4] on a dedicated cycle, provided by a long extraction energy plateau, as shown in Fig. 1. The measured vertical tune shift as a function of the beam intensity is shown in Fig. 2.

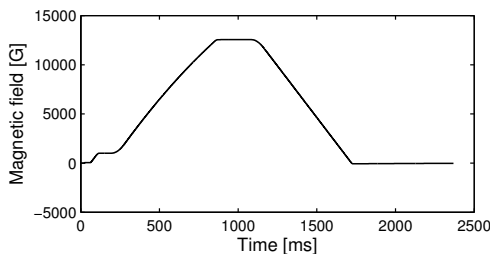


Figure 1: Magnetic field applied along the cycle used for tune shift measurements at kinetic energy of 25.48 GeV.

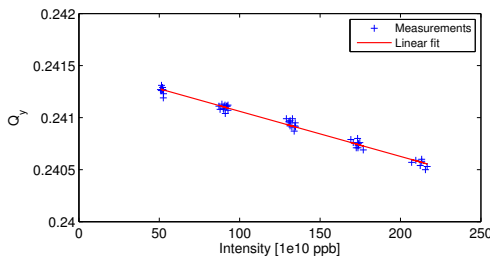


Figure 2: Vertical tune as a function of beam intensity measured at the kinetic energy of 25.48 GeV.

Intermediate Energies

To better understand the transverse effective beam coupling impedance and the indirect space charge contribution, two intermediate kinetic energies of 7.25 and 13.09 GeV have been considered for tune shift measurements. The measurements have been performed in several MD sessions on a dedicated MD cycle with several long plateaux at different energies. The measured vertical tune shift as a function of the beam intensity is shown in Figs. 3 and 4. In Table 1

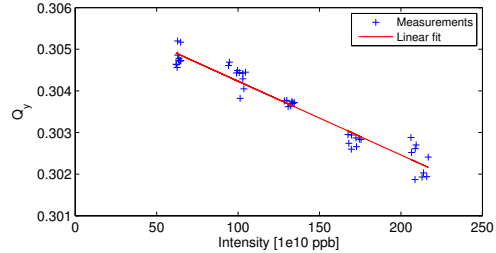


Figure 3: Vertical tune as a function of beam intensity measured at the kinetic energy of 7.25 GeV.

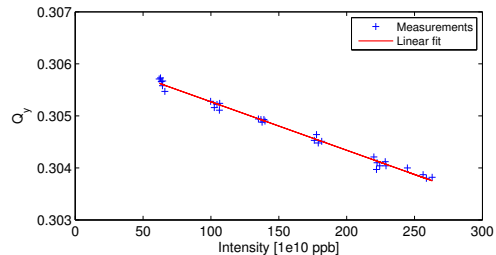


Figure 4: Vertical tune as a function of beam intensity measured at the kinetic energy of 13.09 GeV.

the imaginary part of the total vertical effective impedance measured at different energies and zero chromaticity is summarized.

Table 1: Imaginary part of the total vertical effective impedance measured at different energies and zero chromaticity

$E_{kin}$ [GeV]	$\Im \{Z_t^{eff}\}$ [MΩ/m]	$4\sigma_z$ [ns]	$\xi_y$
7.25	$3.51 \pm 0.13$	55	-0.02
13.09	$3.06 \pm 0.12$	55	-0.02
25.48	$2.23 \pm 0.05$	45	0.02

TUNE SHIFT MEASUREMENTS WITH CHROMATICITY SCAN

Several MD sessions have been dedicated to investigate the change of the imaginary part of the effective vertical impedance with the chromaticity. To perform this measurement, a vertical chromaticity value was initially set using the PS working point application. Vertical and horizontal tunes and horizontal chromaticity were kept to fixed value,

far from possible resonances. The setting was then remotely sent through the working point application in the CERN control center to the PFW in the ring. The PS figure-of-eight loop was kept to zero. A measurement of the vertical chromaticity was performed after each change in the working point to assess the effective value. Three sets of measurements were performed at the kinetic energies of 7.25, 13.09 and 25.48 GeV. For each energy, about ten values of vertical chromaticity have been set and measured with a dedicated application. Above transition the PS operates with zero or slightly positive chromaticity. The possibility of pushing the chromaticity to values far from zero, as well setting a negative chromaticity, was limited by the stability of the beam in the specific magnetic cycle and by the beam losses in the machine. After each chromaticity measurement, a tune scan with intensity was performed in order to calculate the effective vertical impedance for the given chromaticity and for the given bunch length. The three sets of tune shift

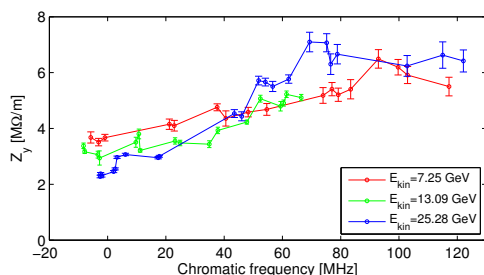


Figure 5: Imaginary part of the effective vertical impedance scan with chromatic frequency at different energies.

measurements show the same increasing trend of the vertical effective impedance with the chromatic frequency, as shown in Fig. 5.

## TRANSVERSE IMPEDANCE BUDGET FROM SIMULATIONS

Theoretical estimations based on numerical codes and analytical formulas have been performed in order to confirm the measured impedance budget. The sum of transverse impedances of each machine element, calculated analytically and with electromagnetic simulations, allows to compute the frequency dependence of the total machine impedance. The computed impedance model takes into account the contributions of indirect space charge, resistive wall, kickers, vacuum equipment (pumps, bellows, flanges valves and step transition) and RF cavities. In Table 2, the vertical effective impedances computed at different energies for the main machine elements groups, are summarized. Kicker magnets, followed by vacuum equipment, are predicted to be the most important source of transverse impedance in the PS at high energies, while RF cavities introduce a negligible contribution [5] [6]. The indirect space charge contribution becomes dominant only at injection energy. In Table 3, a comparison between the impedance calculation of the ma-

chine elements and impedance measurements at different energies, is shown.

Table 2: Imaginary part of the vertical effective impedance expressed in  $M\Omega/m$  calculated at different energies for the main machine elements groups

$E_{kin}$ [GeV]	Kickers	Vacuum	SC+RW
7.25	1.4	0.55	0.58
13.09	1.4	0.55	0.34
25.48	1.4	0.55	0.25

Table 3: Comparison between the measured zero chromaticity vertical effective impedance and the computed vertical effective impedance, expressed in  $M\Omega/m$

$E_{kin}$ [GeV]	Measured $\Im\{Z_y^{eff}\}$	Computed $\Im\{Z_y^{eff}\}$
7.25	$3.51 \pm 0.13$	2.54
13.09	$3.06 \pm 0.12$	2.29
25.48	$2.23 \pm 0.05$	2.20

## CONCLUSIONS

The vertical effective impedance at zero chromaticity of the PS has been measured at different energies. The measured impedance revealed a good agreement with the total impedance of the machine computed with simulations and numerical codes at 25 GeV. The computed impedance slightly underestimates the measurements at intermediate energies. This observation suggests that improvements are needed in the indirect space charge impedance computation. A scan of the imaginary part of the vertical impedance with chromaticity has been performed, revealing an increasing trend for all the measurements at different energies. This effect, not yet predicted by the computed impedance model, is in the process of being further confirmed with other measurements.

## ACKNOWLEDGEMENTS

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## REFERENCES

- [1] M. Gasior, CERN-LHC Project Report 853 (2005).
- [2] A.W. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators", JohnWiley&Sons, (1993).
- [3] S. Gilardoni (ed.), "Fifty Years of the CERN Proton Synchrotron", CERN, 2011.
- [4] S. Persichelli, "The Beam Coupling Impedance Model of the CERN Proton Synchrotron", PhD thesis, 2015.
- [5] S. Persichelli, M. Migliorati, N. Biancacci, S. Gilardoni, E. Métral, B. Salvant, "The Proton Synchrotron Transverse Impedance Model", Proc. of IPAC14, 2014, p. 4096.

- [6] M. Migliorati, S. Persichelli, H. Damerau, S. Gilardoni, S. Hancock, and L. Palumbo, "Beam-wall Interaction in the CERN Proton Synchrotron for the LHC Upgrade", Phys. Rev. ST Accel. Beams, 16 3 (2013).