Simple Nuclear Structure in $^{111-129}$Cd from Atomic Isomer Shifts

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Isomer shifts have been determined in $^{111-129}$Cd by high-resolution laser spectroscopy at CERN-ISOLDE. The corresponding mean square charge-radii changes, from the $1/2^+$ and the $3/2^+$ ground states to the $11/2^-$ isomers, have been found to follow a distinct parabolic dependence as a function of the atomic mass number. Since the isomers have been previously associated with simplicity due to the linear mass dependence of their quadrupole moments, the regularity of the isomer shifts suggests a higher order of symmetry affecting the ground states in addition. A comprehensive description assuming nuclear deformation is found to accurately reproduce the radii differences in conjunction with the known quadrupole moments. This intuitive interpretation is supported by covariant density functional theory.

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Complexity is known to dominate the nuclear state, hence, sustaining the need for theoretical input to nearly every experimental work in order to disentangle the nuclear problem down to a set of basic concepts, e.g., shell structure, pairing, "magic" numbers, deformation, etc. However, in near-closed-shell nuclei, simple structures may occur as a result of the spherical symmetry breaking up. The cadmium isotopic chain studied here appears to provide one such instance of simplicity.

Already in 1949, Brix and Kopfermann [1] pointed out the connection between the anomalous isotope shift separating ^{150}Sm and ^{152}Sm, observed for the first time by Schüler and Schmidt in 1934 [2], and the jump of the quadrupole moment between ^{151}Eu and ^{153}Eu. Since then, the importance of looking at nuclear charge radii was further highlighted in the measurements of the neutron-deficient mercury isotopes [3] where shape staggering and shape coexistence generate sizable changes in the charge distribution between neighboring isotopes [4,5] or states in the same nucleus [6]. Thus far, radial changes of such magnitude appear to be uncommon, the other prominent examples being the halo structures in light nuclei [7–11] and the onset of deformation at $N = 60$ [12–16]. Cadmium and mercury are analogues in terms of their charge distributions since both incorporate an open shell with two protons less than a magic number, respectively, $Z = 50$ and $Z = 82$. Despite the similarities, the cadmium case presented here shows no abrupt changes of the nuclear size. Instead, one observes a small-to-moderate effect on the radii characterized by regularity which may be attributed to the unique-parity $h_{11/2}$ orbital.

In this Letter, we report on the simplicity of high-precision isomer shifts derived from collinear laser spectroscopy on the neutron-rich cadmium isotopes towards the $N = 82$ shell closure. A comprehensive model resting on some of the basic concepts in nuclear physics suggests an apparent link between the $11/2^-$ radii and quadrupole moments. The credibility of such an interpretation is examined quantitatively within a relativistic mean field calculation.

The work was carried out with the collinear laser spectroscopy setup at the CERN-ISOLDE radioactive-beam facility. Of interest were the odd neutron-rich

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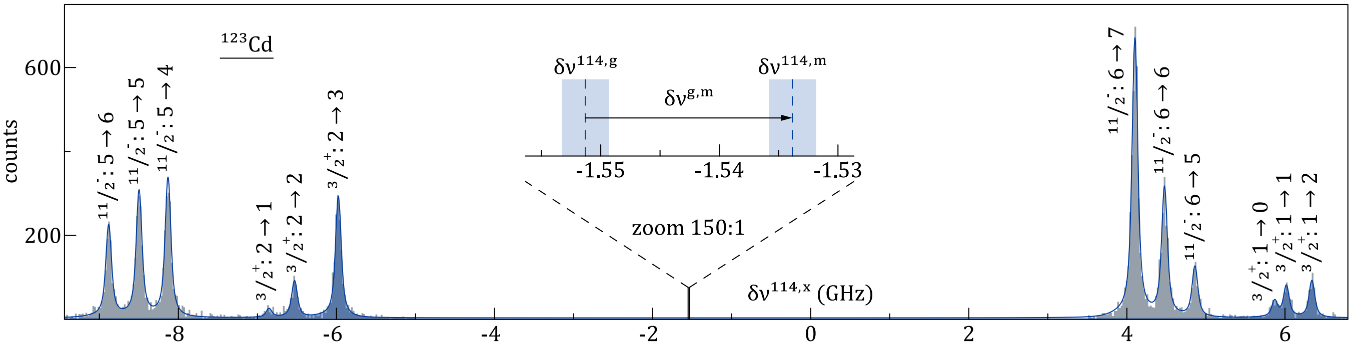

FIG. 1. Spectral lines of ^{123}Cd relative to the excitation frequency of ^{114}Cd in the ionic transition $5s\ ^2S_{1/2} \rightarrow 5p\ ^2P_{3/2}$. The hyperfine structure is comprised of a $3/2^+$ ground state and an $11/2^-$ isomer. The centroids of both states with their statistical uncertainties, as determined by the fit, are shown in the magnified range.

cadmium isotopes whose long-lived isomeric states were readily available as a component in the beam. Most of the measurements were conducted in the cadmium ion in order to benefit from an exceptionally high sensitivity in the $5s\ ^2S_{1/2} \rightarrow 5p\ ^2P_{3/2}$ transition at 214.5 nm [17]. Beam-purity improvement by multiple orders of magnitude was achieved by the use of a neutron converter [18], a quartz transfer line [19], and by resonant laser ionization [20]. Significant background suppression in the fluorescence measurements derived from ion-beam bunching [21,22]. Ground states and isomers were recorded in the same spectrum allowing for a direct measurement of the isomer shift independent of most experimental uncertainties. The work in the ionic system was supported by spectroscopy in the transition $5s5p\ ^3P_2 \rightarrow 5s6s\ ^3S_1$ of the atom at 508.7 nm [17]. The experimental details of both measurements have been previously described in Refs. [23,24] which are reporting on a different aspect of the data.

The isomer shifts in $^{111-129}$Cd are being presented for the first time. The precision of the measurements requires comment, as the studied effects are about 3 orders of magnitude smaller than the hyperfine structure and, in most cases, are appreciably smaller than the natural linewidth. Hence, this study can only be conducted with high resolution. An example spectrum of ^{123}Cd is presented in Fig. 1 where one can acknowledge the relative scale. The following equation relates the nuclear mean square charge-radius change $\delta\langle r^2\rangle^{g,m} = \langle r^2\rangle^m - \langle r^2\rangle^g$ to the isomer shift:

$$\delta\nu^{g,m} = \nu^m - \nu^g = F\delta\langle r^2\rangle^{g,m}. \quad (1)$$

Here, F is the electronic factor related to the change in the total electronic charge density at the site of the nucleus. The term taking into account the difference in mass between the two states is omitted for simplicity since it is negligible with respect to the statistical uncertainty. The atomic masses are still critical for determining the ion-beam velocities since they enter implicitly in the measured Doppler-shifted frequencies. The experimental isomer shifts and thus obtained differential charge-radii changes are presented in Table I. For most of the studied isotopes, the atomic masses of both states are well known [25,26], largely as a result of high-precision Penning-trap measurements [27–29]. In ^{127}Cd and ^{129}Cd, accurate data exist only for one state [30], and therefore, the level ordering is experimentally unknown. With that in mind, we assign the notation "m" to the $11/2^-$ state and "g" to the lower-spin state, either $1/2^+$ or $3/2^+$, coupled to the terms "isomer" and "ground state" even for the latter two cases. Following the systematics in the lighter isotopes, the isomeric excitation energies in ^{127}Cd and ^{129}Cd are assumed to be within a range of ±200 keV. The above considerations contribute an estimated 0.9 MHz to the final uncertainties. The isotope shifts and the known rms charge radii of the stable isotopes [31] have been analyzed by a King-plot procedure establishing the electronic factors in Table I. The relative uncertainties of those are similar at 8% since the measurements in both transitions are referenced to the same set of

TABLE I. Isomer shifts and differential mean square charge radii of $^{111-129}$Cd in the 214.5 nm and 508.7 nm transitions of Cd II and Cd I, respectively. Statistical uncertainties are quoted in parentheses. Square brackets denote systematic uncertainties reflecting the accuracy of the electronic factors.

	$5s\ ^2S_{1/2} \rightarrow 5p\ ^2P_{3/2}$		$5s5p\ ^3P_2 \rightarrow 5s6s\ ^3S_1$	
	$\delta\nu_{\rm II}^{g,m}$	$\delta\langle r^2\rangle^{g,m}$	$\delta\nu_{\rm I}^{g,m}$	$\delta\langle r^2\rangle^{g,m}$
A	(MHz)	(μb)	(MHz)	(μb)
111	−167.5 (9)	272 (2) [21]	31.5 (10)	258 (8) [20]
113	−65.8 (10)	107 (2) [8]	13.8 (8)	113 (6) [9]
115	10.1 (9)	−16 (2) [1]	−2.0 (10)	−16 (8) [1]
117	45.3 (17)	−74 (3) [6]	−7.2 (6)	−59 (5) [5]
119	47.9 (9)	−78 (2) [6]	−9.7 (9)	−80 (7) [6]
121	31.1 (23)	−50 (4) [4]	−6.6 (10)	−54 (8) [4]
123	17.5 (20)	−28 (3) [2]	−5.1 (16)	−42 (13) [3]
125	−13.9 (13)	23 (2) [2]		
127	−119.6 (26)	194 (4) [15]		
129	−235.5 (60)	382 (10) [30]		

$F_{\rm II} = -0.62\,(5)$ MHz/μb $\quad F_{\rm I} = 0.12\,(1)$ MHz/μb

$F_{\rm II}/F_{\rm I} = -5.2\,(2)$

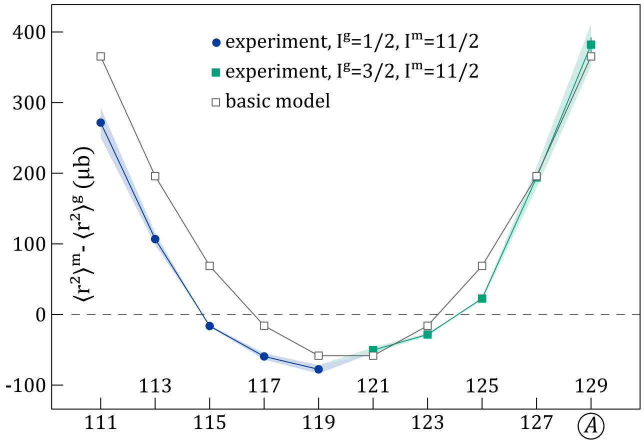

FIG. 2. Mean square charge-radii differences between isomers (m) and ground states (g) in $^{111-129}$Cd compared with calculations discussed in the text. The data correspond to the ionic transition except for ^{117}Cd whose value is taken from the atomic one. In all but one case, the statistical errors are smaller than the dots. The shaded bands represent the systematic uncertainty.

radii. Correspondingly, an 8% systematic uncertainty is associated with all mean square charge-radii changes in the table. The ratio of the electronic factors, being a straightforward ratio of the corresponding isomer shifts, is deduced with a higher precision of 4%. Because of the larger electronic factor, the proportionally larger isomer shifts in the ionic transition dominate the mean square radii changes with smaller uncertainties. The results in the two transitions are statistically consistent. A larger difference of 2.6 standard deviations for ^{117}Cd is potentially caused by random photomultiplier bursts affecting the hyperfine structure of the isomer in the ionic spectrum.

The experimental results are plotted in Fig. 2, where a strikingly simple parabolic trend appears to characterize the differential mean square charge radii as a function of the atomic mass number. To our knowledge, no such clear dependence has ever been observed or discussed before. Furthermore, there is no apparent resemblance of this effect to be found in the mercury analogues [3]. It is reasonable to assume that the present feature is not unique to the cadmium species, but it is likely a general phenomenon that is relevant under common circumstances. In support of this assertion, we consider below a simple model which reproduces the data with remarkable accuracy, and we tie this model to complementary experimental data on the same nuclides. The high experimental precision may be a key factor for making this discovery in the cadmium isotopes and not in the lower-resolution data of tin [32] and lead [33] where, in principle, the unique-parity orbitals $h_{11/2}$ and $i_{13/2}$ should play a similar role.

The question examined here is whether there is a connection between the parabolic mass dependence of the isomer shifts and the linear increase [23] of the $11/2^-$ quadrupole moments in $^{111-129}$Cd. There is a natural link between charge radii and quadrupole moments, since both depend on the nuclear shape and, more specifically, on the deformation parameter β of the charge distribution, in the case of an ellipsoidal nucleus. Below, we propose an interpretation that rests on a number of basic concepts. The mean square charge radius may be decomposed into a spherical and a deformed component [3]. In the case of isomer shifts, i.e., states in the same nucleus, it is approximately true that the spherical component will cancel in the differential mean square charge radii, and therefore, the following relation will hold:

$$\delta\langle r^2\rangle^{\mathrm{g,m}} = \frac{5}{4\pi}\langle r_0^2\rangle(\beta_{\mathrm{m}}^2 - \beta_{\mathrm{g}}^2). \qquad (2)$$

Here, $\langle r_0^2\rangle$ is the mean square charge radius of the nucleus assuming a spherical distribution. A quick analysis of the above equation implies that the parabolic trend in Fig. 2 may be simply a result of a linearly increasing deformation parameter of the isomeric state while the ground-state deformation is constant or relatively small. The above is a compelling argument to make a connection with the quadrupole moments of the $11/2^-$ states which have been found to obey the simple relation [23]

$$Q = \underbrace{\frac{120 - A}{9}Q_{\mathrm{sp}}}_{2Q'} + \underbrace{Q_{\mathrm{core}}}_{2Q''}. \qquad (3)$$

The above single-particle and core parameters take the values $Q_{\mathrm{sp}} = -667(31)$ mb and $Q_{\mathrm{core}} = -85(8)$ mb, and A is the atomic mass number. It was concluded, in the original work, that Q_{sp} is about a factor of 2 larger than the expected contribution from a single neutron on the $h_{11/2}$ orbital, suggesting a sizable polarization of the proton distribution. In other words, the polarized, or deformed, proton core must be responsible for half of each measured quadrupole moment. These findings have been later supported by a dedicated theoretical study [34]. Thus, Eq. (3) is, in effect, a definition of two quantities: Q', representing the quadrupole-moment contribution from polarization of the proton core by the addition of $h_{11/2}$ neutrons, and Q'', representing a constant deformation of the ground state regardless of any $h_{11/2}$ occupation.

One of the basic equations resulting from the collective model of the nucleus [35] is the connection between the observable and the intrinsic quadrupole moment: $Q = \langle IK20|IK\rangle\langle II20|II\rangle Q_0$, where K is the projection of the nuclear spin $\vec{I}$ onto the symmetry axis. With the standard assumption of $K = I$, and by expressing the intrinsic quadrupole moment as a function of the deformation, one can formally write

$$Q = \frac{3}{\sqrt{5\pi}}\langle r_0^2\rangle Z\beta\frac{I(2I-1)}{(I+1)(2I+3)}. \qquad (4)$$

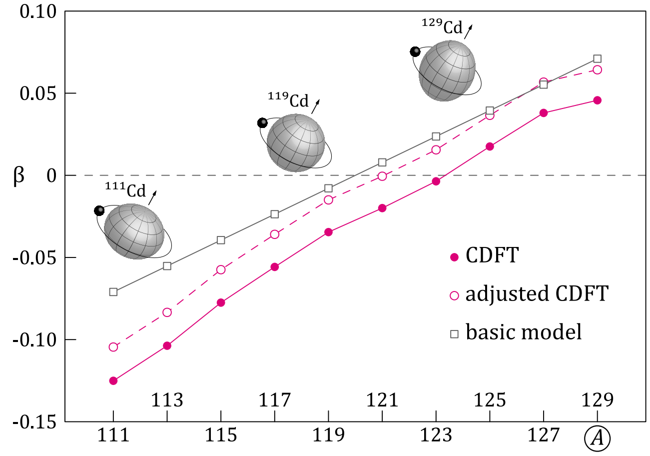

FIG. 3. Quadrupole deformation of the proton distribution in the $11/2^-$ states of $^{111-129}$Cd from covariant density functional theory (CDFT) compared with the basic model discussed in the text. For the sake of clarity, in the visualized examples, the deformation parameters are exaggerated with respect to the model by a factor of 3.

An inclusion of higher powers of β is not necessary because the deformations are relatively small. Indeed, with the aid of Eq. (4) and by approximating the mean square charge radius with a constant value equal to the one of ^{116}Cd [31], one arrives at the deformation parameters plotted with the squares in Fig. 3. For that purpose, the quadrupole moment is substituted by $Q'(A)$ from Eq. (3), and therefore, the obtained deformation parameters correspond to the isomer. Evidently, the parabola in Fig. 2 crosses the zero line. This is a solid evidence that a moderate constant deformation is retained by the cadmium isotopes in their ground states. Clearly, the spin change between ^{119}Cd and ^{121}Cd does not produce any significant effect. We estimate the common deformation of the $1/2^+$ and the $3/2^+$ ground states, on the basis of Q'', to be -0.03. Finally, the obtained β_g and β_m are substituted in Eq. (2) to produce the modeled differential square radii plotted in Fig. 2. The agreement with experiment is remarkable, which in turn strongly supports the formalism and the assumptions along the way. Moreover, this is a manifestation of one of the long-standing concepts in nuclear physics, the deformed nucleus. Yet, one needs to point out that the cadmium species are very close to spherical with deformation parameters never exceeding ±0.07 along the discussed range. It is, perhaps, in this regime that one truly observes “simple structure in complex nuclei” [36].

Having clearly established its relevance, one still needs to examine any apparent difficulties with the proposed picture of weakly deformed nuclei. In our basic model, we postulate a constant ground-state charge deformation. On the other hand, the measured $3/2^+$ quadrupole moments in $^{121-129}$Cd have been found to spread over a wide range of 500 mb [23]. Arguably, these quadrupole moments are, therefore, of noncollective origin. Neglecting the second term in Eq. (3) for the calculation of the isomer's deformation is somewhat more difficult to justify. In this, we are mostly guided by the experimental data which clearly show a minimum in the differential mean square charge radii at $A = 120$ (Fig. 2). Should the offset in Eq. (3) be taken into account, the modeled parabola will exhibit a minimum at a higher mass number resulting in large discrepancies with the experiment especially at the wings of the parabola. As a matter of fact, the inconsistency does not lie with the isomer shifts but with the quadrupole moments. Since the $h_{11/2}$ shell is being filled between ^{111}Cd and ^{129}Cd, the minimum in quadrupole deformation should, indeed, be at $A = 120$, as argued in Ref. [23]. Instead, a zero quadrupole moment is measured at $A = 121$. So far, the origin of this discrepancy of one mass unit has not been fully understood. Further studies should address the theoretical grounds for our hypotheses and this issue in particular.

The quadrupole deformations of the proton distribution for the $11/2^-$ states obtained with the covariant density functional theory (CDFT) [37,38] are shown in Fig. 3 for comparison. The point-coupling functional PC-PK1 [39] is used for the Lagrangian, and the pairing correlations are taken into account by the Bardeen-Cooper-Schrieffer (BCS) method with a zero-range force. Since we are focusing on the $11/2^-$ states of the odd-A nuclei, the last unpaired neutron will block its occupied level in the BCS calculations; i.e., the Pauli principle excludes this level from the scattering process of nucleon pairs by the pairing correlations. In practical calculations, the single-particle orbital $h_{11/2}$ with the third projection of the total angular momentum $j_z = 11/2$ is always blocked in order to obtain the nuclear states with $K = 11/2$. In fact, in most cases, such $K = I$ states are more favorable in energy than other K states.

It has been found in the previous work [34] that the experimental quadrupole moments are well reproduced by the same calculation. In Fig. 3, we present the corresponding quadrupole deformation values for the proton distribution. In general, it supports the linear behavior of the deformation obtained with the basic model. Note that, here, the CDFT deformations are the average deformation of the intrinsic state associated with the collective part of the quadrupole moment, while other effects from the beyond mean field level, such as the configuration mixing, are not included. This may lead to the differences between the basic model and CDFT results. In fact, as in Ref. [34], the contribution from the noncollective parts of the quadrupole moments are roughly 150 mb. If this amount is accounted for in the CDFT calculation, the deformation parameters would essentially align, as shown with the dashed line in Fig. 3.

In summary, isomer shifts in $^{111-129}$Cd have been found to follow a distinct parabolic mass dependence. This observation is, so far, unique to the cadmium chain, although it may be relevant to other nuclear species under the influence of unique-parity orbitals, namely $h_{11/2}$ or $i_{13/2}$. The corresponding regularity in the radii changes is understood in conjunction with previously reported electric quadrupole moments within a common picture assuming axial deformation. This sets a constraint on the ground states which are found to have an identical nearly spherical shape along the chain regardless of the change in configuration from $1/2^+$ to $3/2^+$ at ^{121}Cd. Self-consistent calculations with the covariant density functional theory corroborate our conclusions. Nonetheless, our approach does not address the magnetic properties of the above nuclides [40,41], or their $3/2^+$ quadrupole moments in particular. Further theoretical work, notably by using the spherical shell model in a large model space, would be highly beneficial for understanding all aspects of the nuclear simplicity in this region of the nuclear chart.

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