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**A Determination of Electroweak Parameters
from Z^0 Decays into Charged Leptons**



The L3 Collaboration

ABSTRACT

We have measured the partial widths for the three reactions $e^+e^- \rightarrow Z^0 \rightarrow e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$. The results are $\Gamma_{ee} = 84.3 \pm 1.3$ MeV, $\sqrt{\Gamma_{ee}\Gamma_{\mu\mu}} = 83.3 \pm 1.3$ MeV, and $\sqrt{\Gamma_{ee}\Gamma_{\tau\tau}} = 83.9 \pm 1.4$ MeV, where the errors are statistical. The systematic errors are estimated to be 1.0 MeV, 0.9 MeV, and 1.4 MeV, respectively. We perform a simultaneous fit to the cross sections for the $e^+e^- \rightarrow e^+e^-$, $\mu^+\mu^-$, and $\tau^+\tau^-$ data, the differential cross section as a function of polar angle for the electron data, and the forward-backward asymmetry for the muon data. We obtain the leptonic partial width $\Gamma_{\ell\ell} = 84.0 \pm 0.9(stat)$ MeV. The systematic error is estimated to be 0.8 MeV. Also, we obtain the axial-vector and vector weak coupling constants of charged leptons, $g_A = -0.500 \pm 0.003$ and $g_V = -0.064^{+0.017}_{-0.013}$.

Introduction

Purely leptonic reactions have been used extensively to study electroweak effects and to test the Standard Model [1]. Reactions involving leptonic channels can be calculated precisely in higher order perturbation theory, and experimentally leptons can be clearly identified and measured with high precision. Thus, the measurement of $\Gamma_{\ell\ell}$, the partial width for the reaction $Z^0 \rightarrow \ell^+\ell^-$, is an important test of the Standard Model. In this paper we present a new measurement of the cross section and our first measurement of the angular distribution of the process $e^+e^- \rightarrow e^+e^-(\gamma)$ at energies near the Z^0 pole with a four-fold increase in statistics over our earlier measurements [2]. Also, we present a new measurement of the $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ cross section. Our results concerning the cross section and asymmetry for the reaction $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ have already been published [3].

We use our measurements to determine the vector and axial vector couplings of the Z^0 , g_V and g_A , to charged leptons, the effective weak mixing angle $\sin^2\bar{\theta}_w$ and the neutral current coupling strength parameter ρ .

Detector and Data Collection

The L3 detector is described in detail elsewhere [4]. It consists of a central tracking chamber, a BGO electromagnetic calorimeter, a plastic scintillator hodoscope, a uranium and brass proportional chamber hadron calorimeter, and a high precision muon spectrometer inside a 0.5 T solenoidal magnet. Forward BGO arrays on either side of the detector, measure the luminosity by detecting small angle Bhabha events.

The $e^+e^-(\gamma)$ and $\mu^+\mu^-(\gamma)$ data samples consist of 3.2 pb^{-1} of data collected during the periods October - December 1989 and March - June 1990. The 1989 data have been published [2]. We describe in this paper the analysis of the 1990 $e^+e^-(\gamma)$ data. The $\tau^+\tau^-(\gamma)$ data sample consists of the 2.2 pb^{-1} collected during March - June, 1990.

Events of the type $e^+e^- \rightarrow e^+e^-(\gamma)$ are detected in the BGO barrel calorimeter. The primary trigger is a total energy requirement of at least 12 GeV. From the comparison of data from redundant triggers the total efficiency is determined to be larger than 0.998.

The $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ events are triggered by three independent triggers: 1) the energy trigger, which requires at least 12 GeV in the electromagnetic calorimeter or

at least 15 GeV total energy in the hadronic and electromagnetic calorimeters; 2) the track trigger, which requires at least two tracks in the central tracking chamber; 3) the muon trigger, for events containing a muon, which requires at least one track in the muon chambers and one scintillator hit. We find the combined trigger efficiency for events satisfying the selection criteria described below to be more than 0.999.

e^+e^- Event Selection

The selection of $Z^0 \rightarrow e^+e^-(\gamma)$ events is based on information from the electromagnetic calorimeter. The selection criteria are as follows:

- (1) The number of BGO shower peaks is required to be less than 12.
- (2) The energy of the most energetic electron candidate is required to be greater than $0.45\sqrt{s}$ and less than $0.55\sqrt{s}$.
- (3) The energy of the second most energetic electron candidate is required to be at least 2 GeV.
- (4) The acollinearity angle (180° minus the angle between the two electrons) must be less than 25° .

Only the information from the BGO calorimeter (which covers the angular range 42.3° to 137.7°) is used for the event selection.

To determine the acceptance, $e^+e^- \rightarrow e^+e^-(\gamma)$ events were generated using a Monte Carlo program [5]. The response of the L3 detector for these events is simulated with a program [6] which includes energy loss, multiple scattering, and electromagnetic and nuclear interactions in detector components. The simulated events are reconstructed using the same analysis chain as for real data.

Applying these selection criteria to a sample of 13,287 simulated events generated by the Monte Carlo program [5] for which both the e^+ and e^- are contained within the angular acceptance of the BGO barrel, we calculate an efficiency of 0.952 ± 0.002 (statistical error only). The efficiency is independent of \sqrt{s} .

We correct the number of detected $e^+e^-(\gamma)$ candidates for backgrounds ($\gamma\gamma$ and $\tau\tau$) and efficiency. The $\gamma\gamma$ background is estimated using Monte Carlo simulation [7] for each \sqrt{s} bin. This background falls as $1/s$ and has a cross section of 18 pb at the Z^0 peak for the angular range covered by the electromagnetic calorimeter. The $\tau\tau$ background is estimated to be $(0.6 \pm 0.1)\%$ at the Z^0 peak using Monte Carlo simulation [8]. By varying the cuts on energy and acollinearity, as well as reducing

the fiducial volume, we estimate the uncertainty of the Monte Carlo efficiency to be 0.7%.

Higher order radiative corrections account for significant deviations from first order predictions in the region of the Z^0 pole. For example the charge asymmetry at $\sqrt{s} = M_Z + 3$ GeV is reduced by $\sim 50\%$ due to hard, initial and final state, photon bremsstrahlung. Thus, a good understanding of photon radiation is essential for precise measurements of electroweak parameters.

We have studied these radiative processes directly using $e^+e^-\gamma$ events. Figures 1a and 1b demonstrate the good agreement between Monte Carlo and data for the observed radiative events.

$\tau^+\tau^-$ Event Selection

The selection of $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ events is based on information from the electromagnetic and hadronic calorimeters, the muon chambers, the scintillation counters and the central tracking chamber. The selection criteria are as follows:

- (1) The energy deposited in the electromagnetic calorimeter has to be greater than 2 GeV and less than 60 GeV, in order to remove $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow e^+e^-(\gamma)$ events.
- (2) The event has to be well contained in the electromagnetic calorimeter region: the polar angle θ of the event thrust axis has to satisfy $|\cos\theta| < 0.7$.
- (3) The hadronic events are removed from the sample by requiring:
 - a) The number of shower peaks in the electromagnetic calorimeter to be less than 13.
 - b) The number of charged tracks in the central tracking chamber to be less than 9.
- (4) The acollinearity angle between the two highest energy clusters has to be less than 14° , in order to remove $e^+e^- \rightarrow e^+e^-\gamma$ and $e^+e^- \rightarrow \mu^+\mu^-\gamma$ events.
- (5) Cosmic rays are removed by requiring that the event has at least one scintillator hit within 6.0 ns relative to the beam crossing.

For events containing an isolated electron, we apply an extra criterion:

- (6) The energy of the electron, deposited in the electromagnetic calorimeter, has to be less than 40 GeV, thus further removing $e^+e^- \rightarrow e^+e^-(\gamma)$ events.

For events containing an isolated muon, we apply the following extra criteria, to reduce the remaining background from $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ and cosmic rays:

- (7) The event should not have more than one isolated muon.
- (8) The muon momentum has to be less than 40 GeV.
- (9) The muon has to satisfy a momentum-dependent vertex cut.

Criterion (7) rejects most events of the type $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \mu^+\mu^-\nu_\mu\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau$, but it also reduces the background from $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$.

To determine the acceptance, $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ events were generated using a Monte Carlo program [8]. The efficiency, including the geometrical acceptance, is 0.467 ± 0.005 (statistical error only). By varying the cuts, we estimate the systematic uncertainty in the event selection to be 2.8 %.

We correct the number of selected $\tau^+\tau^-(\gamma)$ candidates for acceptance and background. For estimating the background from $e^+e^- \rightarrow e^+e^-$, $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow q\bar{q}$ we also use Monte Carlo events. The background from these channels is found to be (1.7 ± 0.2) % of the $\tau^+\tau^-$ sample. By scanning the selected events, we estimate our cosmic ray background to be (0.6 ± 0.4) %. The $\gamma\gamma$ background (e.g. $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$, $e^+e^- \rightarrow e^+e^-e^+e^-$, ...) is negligible.

Adding all systematic errors in quadrature, we find a total systematic error of 3.3 % (including the uncertainty in the luminosity).

Partial Width for $Z^0 \rightarrow e^+e^-$

We determine the cross section for the reaction $e^+e^- \rightarrow e^+e^-(\gamma)$ within the angular range covered by the BGO calorimeter as a function of \sqrt{s} in the energy region of the Z^0 pole. A total of 1991 events, corresponding to an integrated luminosity of 2.3 pb^{-1} , is selected. The results are summarized in Table 1.

Table 1

Number of events and cross sections for $e^+e^- \rightarrow e^+e^-(\gamma)$ (1990)

\sqrt{s} (GeV)	$e^+e^-(\gamma)$ events	Luminosity (nb^{-1})	$\sigma_{e^+e^-}$ (nb)
88.22	48	109.3	0.44 ± 0.06
89.22	128	229.3	0.56 ± 0.05
90.22	214	233.3	0.94 ± 0.06
91.22	1408	1262.2	1.14 ± 0.03
92.22	92	147.8	0.63 ± 0.07
93.22	75	172.7	0.43 ± 0.05
94.22	26	139.9	0.17 ± 0.04
TOTAL	1991	2294.5	

In order to extract the partial width Γ_{ee} we fit the cross section as a function of \sqrt{s} using an improved version of the Caffo-Remiddi [9] program (Method I) based on an earlier [10] and a more recent [11] analytical calculation of the large angle Bhabha cross-section. This analytic expression takes into account both the γ and the Z^0 exchange diagrams in the s and t channels with interference terms. Soft radiation is accounted for by exponentiation, and hard photons are included in the collinear approximation. The improvement with respect to the previous version of the program, used by us in the past analysis [2], mainly consists of the insertion of two-loop QED corrections to the cross section and an improved factorization scheme.

Further cuts are applied to the data in order to reject events containing hard photons (of energy $k > k_{max}$) emitted at large angles ($\delta > \delta_{max}$) with respect to the direction of the electrons (or positrons), since these events are not accounted for by the fitting function. Events with hard acollinear photons in the beam pipe are rejected by an acollinearity cut Δ_{max} on the final state e^+e^- . Choosing Δ_{max} effectively sets the k_{max} used. The choice of δ_{max} and Δ_{max} has to be done bearing in mind that high values of these cuts make the formula less precise while low values make the measurement sensitive to finite resolution of the detector. After a study of the performance of our calorimeter, we chose $\delta_{max} = 5^\circ$, and $\Delta_{max} = 5^\circ$,

corresponding to $k_{max} = 3.7$ GeV. The cut on the acollinearity angle removes a large fraction of events with an undetected radiated photon. From Monte Carlo simulation [5] the ratio between the number of reconstructed events passing all these cuts and the number of events generated within δ_{max} and k_{max} is estimated to be 0.968 ± 0.007 . This includes the correction we apply for the number of events which should be rejected because the photon both has an energy greater than k_{max} and is separated by more than 5° degrees from the e^\pm , but which are retained because the photon is outside of our acceptance and Δ is less than 5° .

Using these measurements and the measurements from 1989 [2], we fit M_Z and Γ_Z using the program described above [9]. We obtain $M_Z = 91.06 \pm 0.05$ GeV and $\Gamma_Z = 2.36 \pm 0.17$ GeV, in agreement with our measurements derived from the hadronic channel [12,13], $M_Z = 91.164 \pm 0.033$ GeV and $\Gamma_Z = 2.494 \pm 0.025$ GeV.

In order to fit Γ_{ee} , M_Z and Γ_Z were set to the values based on our hadron data. In this way we determine: $\Gamma_{ee} = 84.3 \pm 1.3$ MeV, where the statistical error includes the error on the determination of Γ_Z and M_Z . The χ^2 of the fit minimization was 7.6 for 9 degrees of freedom. Fig. 2a shows the measured cross section for the data of 1990 and 1989 and the result of the fit as a function of \sqrt{s} .

As an alternative method (Method II), we use a new calculation of the cross section for large angle Bhabha scattering [14]. It includes second order QED corrections in the leading log approximation and soft photon effects taken into account at all orders.

This calculation integrates the cross section for events with:

- (a) a scattering angle larger than ϑ
- (b) an acollinearity of the final state fermions less than Δ_{max} .

We have used $\vartheta = 42.3^\circ$, $\Delta_{max} = 5^\circ$. We performed a fit to the data sample leaving the partial width into electrons as the only free parameter. From the fit we obtained $\Gamma_{ee} = 84.3 \pm 1.3(stat)$ MeV in agreement with the previous determination.

In our previous analysis [2] we also evaluated Γ_{ee} subtracting the contribution of the t-channel and interference terms. Repeating the same analysis with the new data, using the point $\sqrt{s} = 91.22$ GeV, we get $\Gamma_{ee} = 84.5 \pm 1.4(stat)$ MeV in agreement with the two previous methods. The subtracted contributions are estimated using the program [14].

For this paper a new determination of the 1989 luminosity was performed [13]. Taking into account the error on the luminosity measurement (1.7% and 1.3% for 1989 and 1990 data respectively [2,13]), the uncertainty in the Monte Carlo efficiency and the numerical precision estimated by the authors of the fitting programs [14], we assign a global systematic error of 1.0 MeV to the Γ_{ee} determination. Our final result is therefore:

$$\Gamma_{ee} = 84.3 \pm 1.3(stat) \text{ MeV.}$$

The systematic error is estimated to be 1.0 MeV. The visible cross sections obtained under the cuts required by Methods I and II are listed in Table 2.

Table 2
Visible Cross Sections (1989 and 1990)

\sqrt{s} (GeV)	Method I $\sigma_{e^+e^-}$ (nb)	Method II $\sigma_{e^+e^-}$ (nb)
88.22	0.37 ± 0.04	0.38 ± 0.04
89.22	0.55 ± 0.04	0.57 ± 0.04
90.22	0.87 ± 0.05	0.89 ± 0.06
91.03	1.05 ± 0.09	1.09 ± 0.09
91.22	1.07 ± 0.03	1.10 ± 0.03
91.30	0.98 ± 0.08	1.02 ± 0.08
91.53	0.95 ± 0.08	0.99 ± 0.08
92.22	0.60 ± 0.05	0.61 ± 0.05
93.22	0.35 ± 0.04	0.36 ± 0.04
94.22	0.14 ± 0.04	0.14 ± 0.04

Partial Width for $Z^0 \rightarrow \tau^+\tau^-$ and $Z^0 \rightarrow \mu^+\mu^-$

We determine the cross section for the reaction $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ as a function of \sqrt{s} in the energy region of the Z^0 resonance. A total of 1169 events, corresponding to an integrated luminosity of 2.2 pb^{-1} , are used. The results are summarized in

Table 3 and shown in Fig. 2c. (For comparison, Fig. 2b shows the $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ cross section from Ref. [3].)

Table 3

Number of events and cross sections for $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$

\sqrt{s} (GeV)	$\tau^+\tau^-(\gamma)$ events	Luminosity (nb^{-1})	$\sigma_{\tau^+\tau^-}$ (nb)
88.22	13	108.7	0.25 ± 0.07
89.22	35	198.5	0.36 ± 0.06
90.22	101	231.4	0.91 ± 0.09
91.22	872	1215.7	1.50 ± 0.05
92.22	76	146.9	1.08 ± 0.12
93.22	41	165.0	0.52 ± 0.08
94.22	31	130.5	0.49 ± 0.09
TOTAL	1169	2196.7	

To determine the partial width for $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$, we constrain the mass and width of the Z^0 to the values obtained from the hadron data. We obtain a fit with $\chi^2 = 4.9$ for 6 degrees of freedom. We find:

$$\sqrt{\Gamma_{ee}\Gamma_{\tau\tau}} = 83.9 \pm 1.4(\text{stat}) \text{ MeV.}$$

The systematic error is estimated to be 1.4 MeV.

In Ref. [3], we obtain:

$$\sqrt{\Gamma_{ee}\Gamma_{\mu\mu}} = 83.3 \pm 1.3(\text{stat}) \text{ MeV.}$$

The systematic error is estimated to be 0.9 MeV. Using our measured value of Γ_{ee} , we obtain: $\Gamma_{\tau\tau} = 83.5 \pm 2.7(\text{stat})$ MeV and $\Gamma_{\mu\mu} = 82.3 \pm 2.3(\text{stat})$ MeV. The systematic errors are estimated to be 2.5 and 1.7 MeV, respectively.

e^+e^- Differential Cross Section

The distribution of the electron pairs as a function of the polar angle θ can be expressed at the tree-level through the simple form

$$f(x) = A(1 + x^2) + Bx + C \frac{(1 + x)^2 + 4}{(1 - x)^2} + D \frac{(1 + x)^2}{1 - x}.$$

where x is the cosine of the angle between the incoming e^- and the outgoing e^- .

The coefficients A and B represent the contribution from the s-channel. The coefficient C represents the contribution from the t-channel and the coefficient D represents the contribution from the s- and t-channel interference. The parameters $A, B, C,$ and D can all be expressed as simple functions of s , g_v , and g_A . When comparing experimental data to theory, the presence of radiative corrections necessitates the use of a more complete description of the process, like the one found in [9].

In order to measure the angular distribution $d\sigma/d\cos\theta$ we must determine the sign of the particles using our central chamber (TEC). Events surviving Method I selection criteria are required to have two reconstructed tracks. A total of 1417 events satisfy these criteria. The sign of the particles is determined by fitting of a common circle through the two TEC tracks. The efficiency of TEC track pattern recognition, track reconstruction, and circle fitting as determined using e^+e^- and $\mu^+\mu^-$ data is independent of θ over the angular range of interest.

We use muon pairs measured in the muon spectrometer which can be matched to well-measured TEC tracks in the same event to determine the level of charge-confusion resulting from the circle fit for high-momentum tracks. The circle fit provides the correct charge assignment $(90 \pm 1)\%$ of the time over the range $|\cos\theta| < 0.74$. The charge-confusion is also independent of θ over this angular range. Fig. 4 shows the measured angular distribution $d\sigma/d\cos\theta$ (at $\sqrt{s} = 91.22$ GeV) after unfolding the effect of charge-confusion.

Using the relation:

$$\Gamma_{ee} = \frac{G_\mu M_Z^3}{6\sqrt{2}\pi} (g_A^2 + g_V^2)$$

we find:

$$g_A^2 + g_V^2 = 0.254 \pm 0.005.$$

Fitting the angular distribution for $|\cos\theta| < 0.7$ with the analytic expression [9] we obtain:

$$g_A = -0.499 \pm 0.006$$

$$g_V = -0.073^{+0.031}_{-0.023}$$

where the errors include systematics. The signs of g_V and g_A are inferred from results of other experiments [15-18]. Systematic errors include the uncertainty in the charge determination and the uncertainty in the influence of the t-channel in the fit. The latter is determined by varying the upper bound in the fit over the range between $0.4 \leq \cos\theta \leq 0.7$.

Simultaneous fit to leptonic data

Assuming lepton universality, we make a simultaneous fit to all the leptonic data: e^+e^- , $\mu^+\mu^-$, $\tau^+\tau^-$ cross sections, as well as the forward-backward asymmetry for muons and the electron angular distribution. We fit the data to the effective vector and axial-vector couplings of Z^0 , g_A and g_V . We use the analytical forms given in [19] for the $e^+e^- \rightarrow \mu^+\mu^-$, $e^+e^- \rightarrow \tau^+\tau^-$ cross sections and forward-backward asymmetry (A_{FB}) for muons. The analytical calculation [9] is used in fitting the $e^+e^- \rightarrow e^+e^-(\gamma)$ cross section as well as the angular distribution at $\sqrt{s} = 91.22$ GeV. Only data from the peak is used for fitting the angular distribution in order to minimize the contribution from the t-channel. We take the mass and width of Z^0 as obtained from a fit [13] to hadron data. Systematic errors due to event selection for the three sets of leptonic data are treated separately in the fit program. In addition we use a common systematic error from the luminosity measurement. In this fit, we assume a top quark mass of 150 GeV and a Higgs mass of 100 GeV.

From the simultaneous fit, we obtain

$$g_A = -0.500 \pm 0.003$$

$$g_V = -0.064^{+0.017}_{-0.013}$$

with a χ^2 per degree of freedom of 88/71. The signs of g_A and g_V are again inferred from results of other experiments [15-18]. Fitting the lepton cross sections again

assuming lepton universality, we obtain:

$$\Gamma_{\mu} = 84.0 \pm 0.9 \text{ (stat) MeV.}$$

The systematic error is estimated to be 0.8 MeV. Using the minimal Standard Model relation [20] between ρ and $\sin^2\bar{\theta}_w$ and our measured values for M_Z and Γ_Z , we obtain:

$$\rho = 1.005 \pm 0.012$$

$$\sin^2\bar{\theta}_w = 0.230 \pm 0.004.$$

Our results are consistent with Standard Model predictions as well as measurements reported by other experiments at LEP [12,21].

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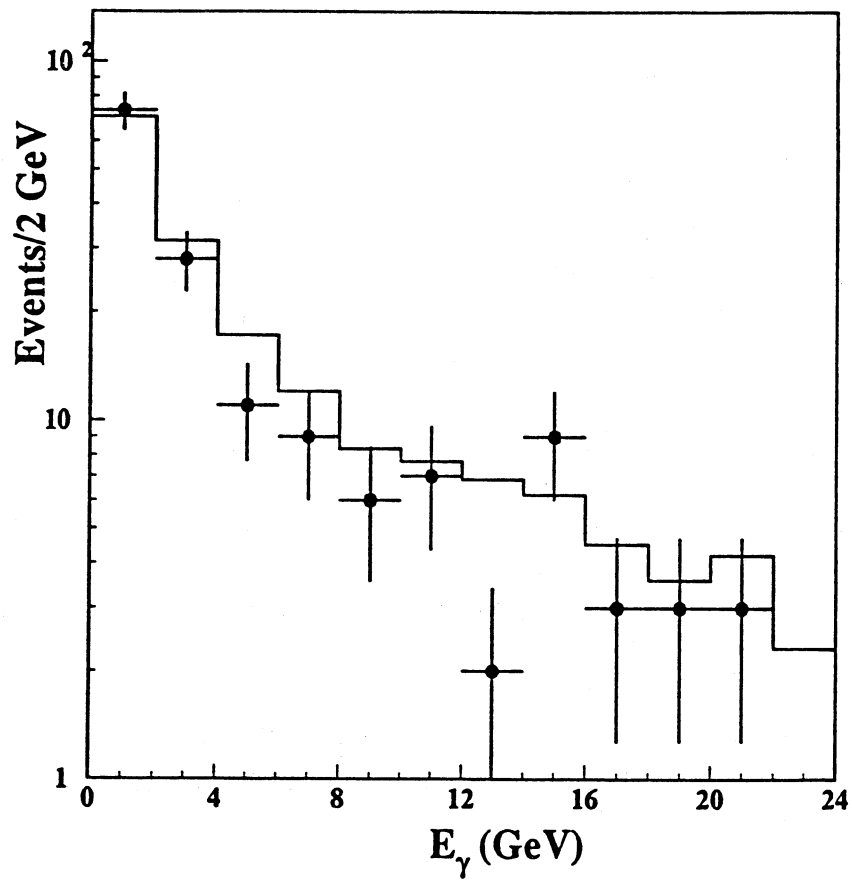
FIGURE CAPTIONS:

Figure 1 (a) Photon energy distribution for $e^+e^-\gamma$ candidates for data (points) and Monte Carlo (histogram) for photon energies above 0.5 GeV. The third most energetic BGO shower peak is defined as a visible photon if its energy is greater than 0.5 GeV and its angle with respect to the nearest particle is greater than 5° . (b) Distribution of the angle between the photon and the closest e^+ or e^- for data (points) and Monte Carlo (histogram) for photons with energy greater than 1 GeV.

Figure 2 (a) The measured cross section for $e^+e^-(\gamma)$ within the angular acceptance of the electromagnetic calorimeter for data (points) and the standard model prediction of the Greco-Caffo-Remiddi analytical formula (highest curve). The values of the contributions of the s-channel, t-channel, and their interference are shown as well. Neighboring energy points are combined. (b) The measured cross section for $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ from [3]; (c) The measured cross section for $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$.

Figure 3 $d\sigma/d\cos\theta$ for $e^+e^- \rightarrow e^+e^-(\gamma)$ for $|\cos\theta| < 0.7$ at $\sqrt{s} = 91.22$ GeV. The curve is the fit to the data using an analytic formula [9] which includes the t-channel contribution.

(a)



(b)

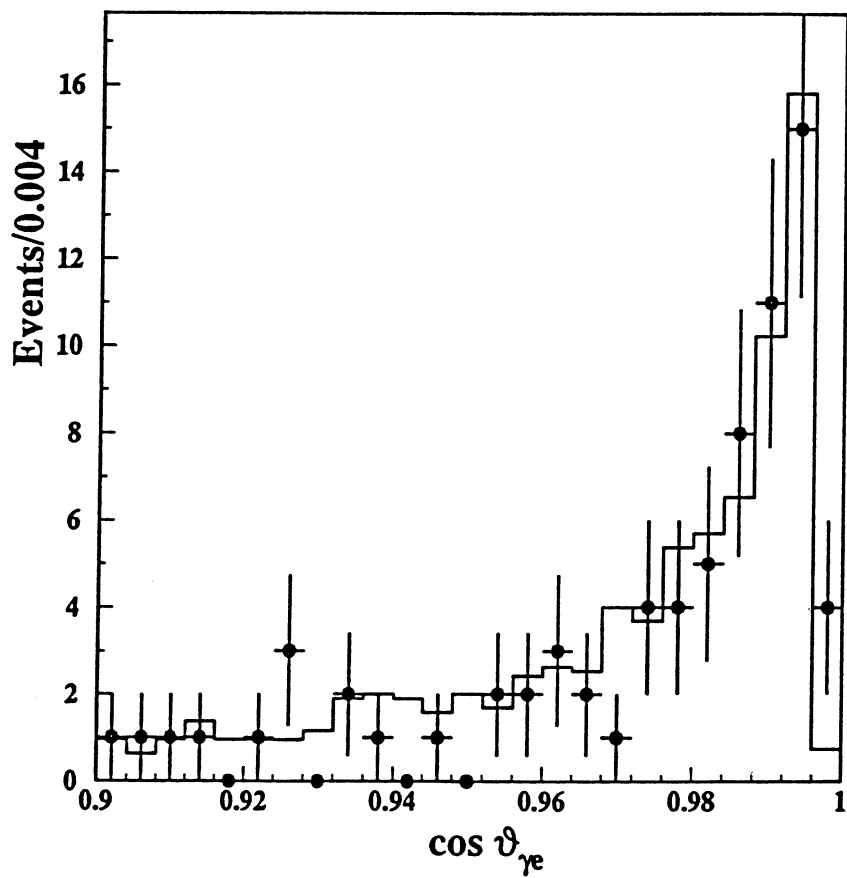
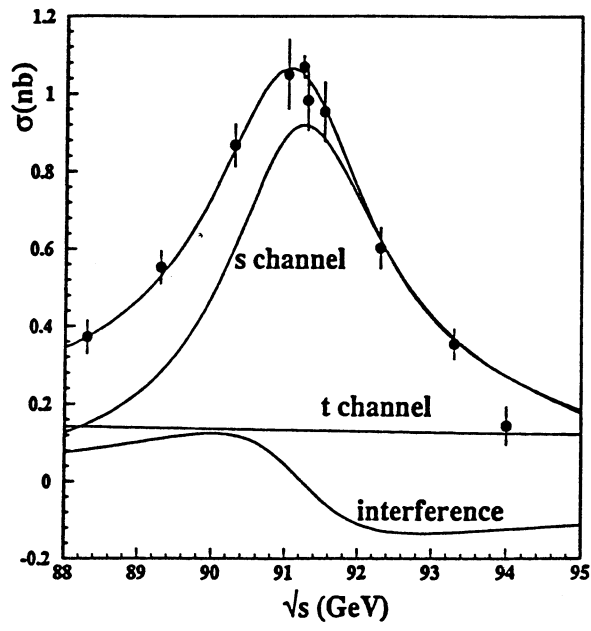
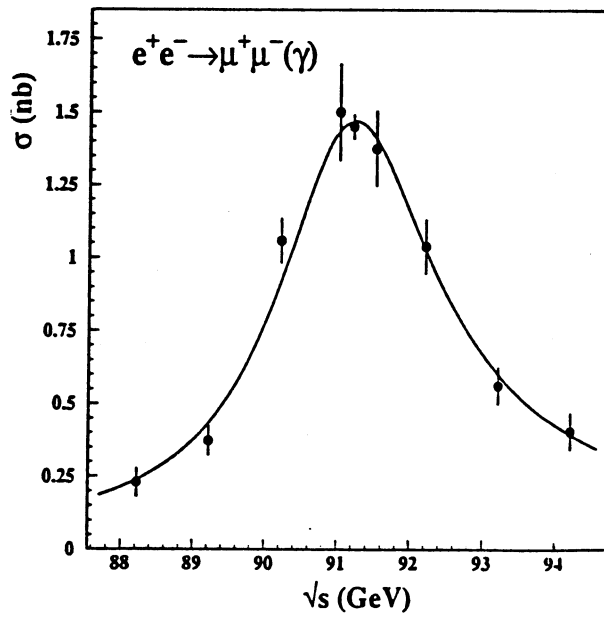


Figure 1

(a)



(b)



(c)

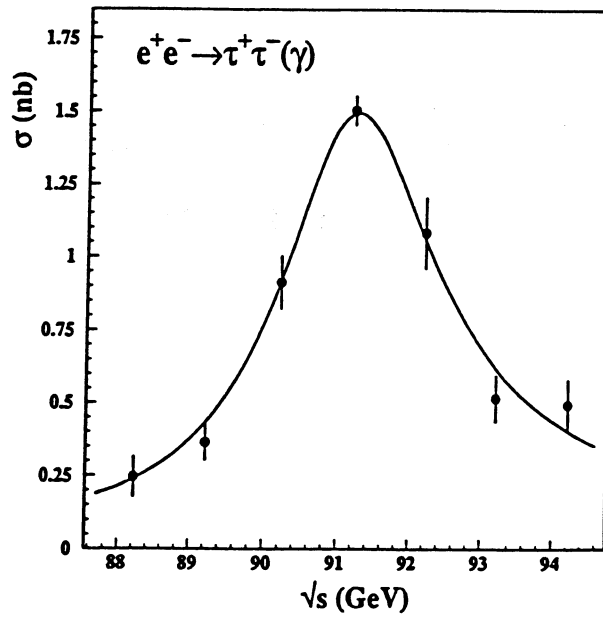


Figure 2

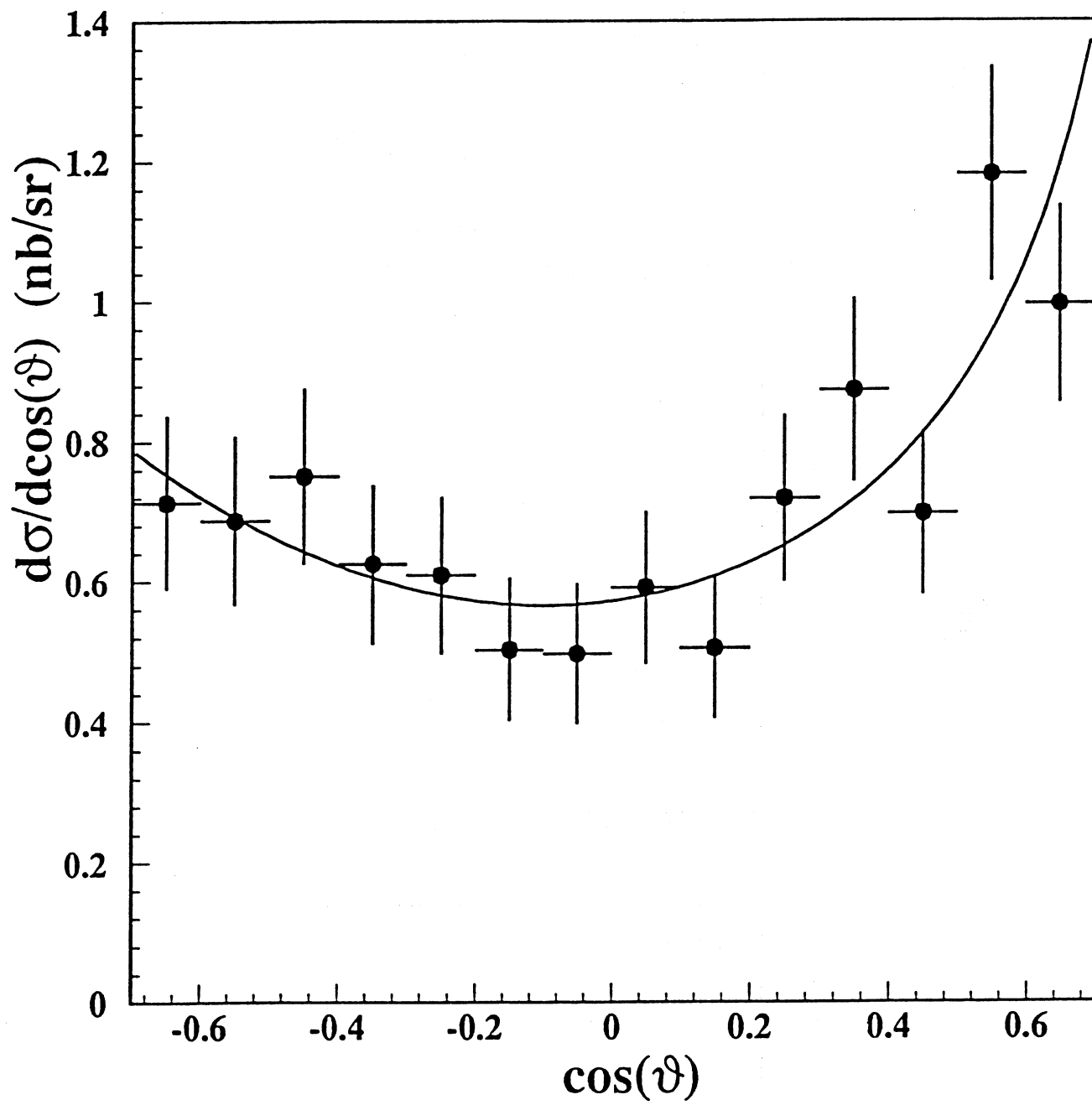


Figure 3