A model of neutron capture and deuteron stripping on deformed nuclei

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Abstract

A coupled channel model of direct neutron capture and deuteron stripping reactions, which consistently accounts for effects of nuclear deformations in both reactions, is constructed by coupling all incoming and outgoing partitions of both reactions to the same set of collective states. This model is demonstrated using the FRESCO coupled-channels code [I. J. Thompson, *Comp. Phys. Rep.* **7**, 167 (1988)], and it is applied to capture and stripping reactions on even-mass calcium isotopes ^{40,42,44,46,48}Ca. All incoming and outgoing partitions in capture and stripping reactions were coupled to 2^+, 4^+, and 3^- collective states using a consistent set of deformation lengths. Coupling to these collective states significantly decreases the direct capture cross section relative to the capture in a spherical model for the nuclides considered. Similarly, deuteron stripping is approximately cut in half for the same nuclides. These results suggest that single-particle spectroscopic factors used in this model of direct capture ought to be refitted by computing deuteron stripping with coupling to the same collective states.

1. Introduction

The neutron radiative capture cross section can be computed as a sum of interfering amplitudes of direct, semi-direct, and compound-resonant capture processes. Direct capture take place by a single electromagnetic transition of the neutron from its incoming state in the continuum to its final bound state. Semidirect capture is a two step process that occurs via excitation of a giant dipole nuclear resonance, for example, that subsequently decays via a γ-ray emission. Compound nuclear resonant capture occurs via narrow compound nuclear resonances conventionally described by R-matrix formalism [2], and it constitutes the dominant component of the total low-energy neutron capture on heavy stable nuclides.

Direct capture cross sections can contribute a significant fraction of the total capture on light nuclides or on neutron-rich doubly closed shell nuclei like Ca^{48} and Sn^{132} [3]. Nucleosynthesis models of nuclear astrophysics have been found to be sensitive to direct capture cross sections [4]. Furthermore, direct capture is combined with compound resonant capture in evaluations of neutron capture data on light and medium mass nuclei [5].

Direct neutron capture cross sections are conventionally computed using single particle potential models where the incoming neutron wave function is computed in a complex optical potential and the final bound state is computed in a real potential.[3] A matrix element of electromagnetic operators between the initial and the final state is then computed to yield a direct capture cross section. Semidirect

[3]Potential depth is fitted to the binding energy of the final neutron state.

capture could be modeled by adding a Lorentzian term to the electromagnetic operator [6] or by a coupled-channel formalism [7, 8].

Previous models of direct neutron capture have accounted for the effects of nonspherical nuclei either in the incoming wave functions only (via nonspherical optical model potentials), *or* in the final bound states only (via nonspherical real potential wells). Since it is known that spherical optical potentials do not yield good agreement with low energy neutron-scattering observables of deformed nuclei, calculations have been performed in which initial and final states are consistently treated in a nonspherical-nucleus picture. This was accomplieshed by introducing coupling to the 2+, 4+, and 3- collective states into incoming and outgoing partitions. A spherically symmetric model of direct capture is restored in the limit of removing all couplings to collective states.

This work focuses on direct capture of thermal neutrons (incident energy 25.3 meV) into even mass calcium isotopes ^{40,42,44,46,48}Ca for which good data exist [9, 11]. Collective strengths and excitation energies for this set of isotopes span a wide range of magnitudes between the two closed shells, as shown in Fig. 1. This leads to a corresponding variety of effects computed by coupling to those collective states.

Thermal neutron capture cross sections are often measured to a relatively high accuracy, including measurements of prompt γ-ray energies and corresponding branching ratios, all of which could be used to test models of neutron capture. Furthermore, relatively large spectroscopic factors of bound states with orbital angular momentum $l = 1$ found in ^{40,42,44,46,48}Ca allow electric dipole (E1) capture of low-energy s-wave neutrons into those bound states. This feature makes E1 capture a large and dominant component of total thermal neutron capture. Specifically, the E1 components of the thermal neutron capture components has been reported to be 82%, 93%, 98%, 96%, and 100% for ^{40,42,44,46,48}Ca, respectively [9]. The compound resonant component of the thermal neutron capture was computed from the resonance parameters [12] fitted to neutron capture data via the R-matrix formalism in [5]. Compound resonant contribution to thermal neutron capture is practically negligible under the assumption that there are no subthreshold resonances.

Single particle models of direct capture and deuteron stripping are related by spectroscopic factors. A spectroscopic factor of a given bound state quantifies the overlap between it and an idealized single-particle wave function. Spectroscopic factors are conventionally inferred by fitting angular distributions computed by a (d,p) model to measured data [10]. A spectroscopic factor of a given bound state is then used to multiply the direct capture cross section into that state, as shown in Sec. 2.. This connection between direct capture and deuteron stripping suggests that a consistent treatment of both reactions should be pursued.

2. Direct capture with couplings to collective states

The expression for direct capture cross section of a neutron in the incoming channel α_i into a bound state b via the electromagnetic transition of multipolarity (LM) coupled to total angular momentum J_{tot} is computed by FRESCO using the partial-wave T-matrix

$$\mathbf{T}^{J_{tot}}_{LM;b\alpha_i} = -i^{L+1}\sqrt{4\pi}\sqrt{2L+1}\sqrt{\frac{8\pi\hbar c(L+1)}{kL}}\frac{q}{k}\frac{k^L}{(2L+1)!!}\sum_{\alpha}\langle\phi_{b\alpha}|r^L Y^M_L|\psi_{\alpha\alpha_i}\rangle \tag{1}$$

used in the usual expression for cross sections

$$\sigma^L_{\gamma;b\alpha_i} = S_b\frac{4\pi}{k_i^2}\frac{1}{(2I_p+1)(2I_t+1)}\frac{c}{v_i}\sum_{MJ_{tot}}(2J_{tot}+1)|\mathbf{T}^{J_{tot}}_{LM;b\alpha_i}|^2\,, \tag{2}$$

where the neutron's incoming velocity is v_i, γ-ray exit velocity is c, and I_p (I_t) is the intrinsic spin of the neutron (target). S_b is the single-particle spectroscopic factor of the bound state obtained by analysis of (d,p) reaction addressed in Sec. 3.. Here the label α indicates a partial wave component of the initial or the bound neutron wave-functions, with the core Φ_α being either in its ground state ($\alpha = 0^+$) or in any one of the excited collective states to which these wave functions may couple, namely $\alpha = 2^+, 4^+, 3^-$, so $\phi_{b\alpha} = [\Phi_\alpha u_{l_\alpha j_\alpha}(r_n)]_{J_b}$ for neutron wave function $u_{l_\alpha j_\alpha}(r_n)$ [13]. At this stage transitions are considered for which the core remains in the same fractional state α as it was prior to the EM transition. This is indicated by the same superscript α labeling for both the initial and the final states in Eq. (1), as is also shown schematically in Fig. 1(a). In future work, electromagnetic transitions between different components of the initial and final states (i.e., $\sum_{\alpha\alpha'} \langle \phi_{b\alpha'} | r^J Y_J^M | \psi_{\alpha\alpha_i} \rangle$) will be accounted for. The energies and deformation strengths of $2^+, 4^+, 3^-$ collective states were taken from RIPL [14] and are plotted in Fig. 1(b) and (c). All computations in this work use Koning-Delaroche optical potential [15] for neutron in the continuum, and its real part is used for single-particle neutron bound states.

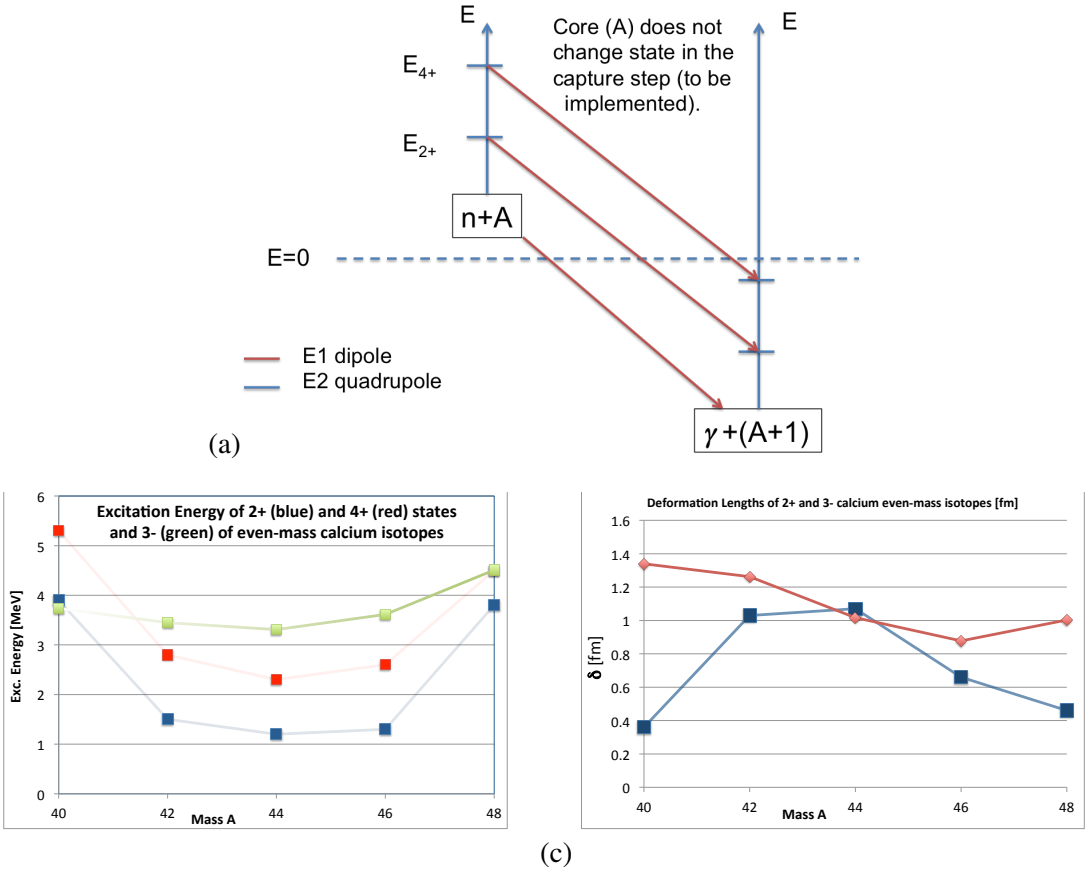

Fig. 1: Couplings between the ground state, 2^+, and 4^+ quadrupole states in the coupled-channel model of neutron capture and deuteron stripping (a), and the energies (b) and deformation lengths (c) of those states for ^{40,42,44,46,48}Ca isotopes used in the coupled-channel computations.

In Eqs. (1) and (2), Siegert's theorem [16] was employed because it conveniently expresses electromagnetic transition matrix elements in terms of effective charge density. Since Siegert's theorem is strictly valid for many-body nuclear wave functions, it should be used with caution in single-particle capture models where its validity cannot be guaranteed. However, it can be shown that Siegert's theorem

remains valid when the binding potential for the neutron in the final state is approximately equal to the real part of the optical potential for the incoming neutron. It has been empirically verified that this condition is reasonably satisfied for the calcium isotopes considered.

A comparison of direct capture computed using the deformed and spherically symmetric models for even-mass calcium isotopes ^{40,42,44,46,48}Ca shows that the deformed method yields a substantially smaller direct capture cross section than the spherically symmetric method in between the two closed shells with a minimum at ^{44}Ca, as seen in Fig. 2(a).

3. Deuteron stripping (d,p) with couplings to collective states

To examine the connection between (d,p) and (n,γ) cross sections in a model that consistently accounts for nuclear deformations in both reactions, coupling is introduced to the same collective states for computation of the (d,p) reactions. The deformation lengths of collective states used for deuteron stripping are the same as those used for direct capture. Daehnick global potential for elastic deuteron scattering [17] is used for the incoming deuteron parition.

We find again a decrease in the magnitude of the deuteron stripping cross section shown in Fig. 2(b), suggesting that spectroscopic factors should be refitted (to experimental data) by using this deformed-potential model of (d,p) reactions. This is suggested because the direct capture cross section computed in Eq. (2) is multiplied by a corresponding single particle spectroscopic factor of the capturing final state. Extant spectroscopic factors are extracted by fitting the computed deuteron stripping cross section to measured (d,p) data. With few notable exceptions [18], spectroscopic factors have generally been extracted using spherically symmetric models of deuteron stripping [19, 20].

If a direct neutron capture model accounts for coupling to collective states, it should use spectroscopic factors that were extracted with an analogous model of deuteron stripping, i.e., a model that accounts for the effects of deformed nuclei by coupling to the same set of collective states. The computation of total deuteron stripping cross section shown in Fig. 2 suggests that a proposed refit of the spectroscopic factor would increase the spectroscopic factors by approximately a factor of 2, and this increase would in turn increase the computed direct neutron capture by the same factor. Such a rescaling process may restore a magnitude of neutron capture that is more consistent with the data.

4. Conclusions

A coupled channel model of direct capture and deuteron stripping that consistently takes nuclear deformation into account in both reactions by coupling to the same set of collective states in both reactions is presented. The results indicate that coupling to collective states decreases direct capture and deuteron stripping by a factor of approximately two for ^{40,42,44,46,48}Ca.

These results suggest that a more thorough application of this model should use spectroscopic factors refitted by using the model of deuteron stripping with coupling to collective states. Such spectroscopic factors could then be consistently used in the model of neutron capture with coupling to the same collective states. Considerations such as these should lead to a more intricate understanding of the spectroscopic factors. Since the core is considered to be inert during the capture step, further improvements of the presented model should allow for core (de-)excitations in the capture step.

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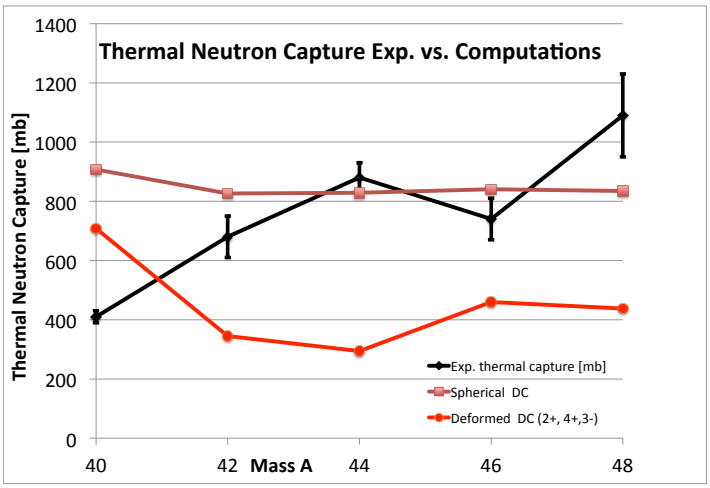

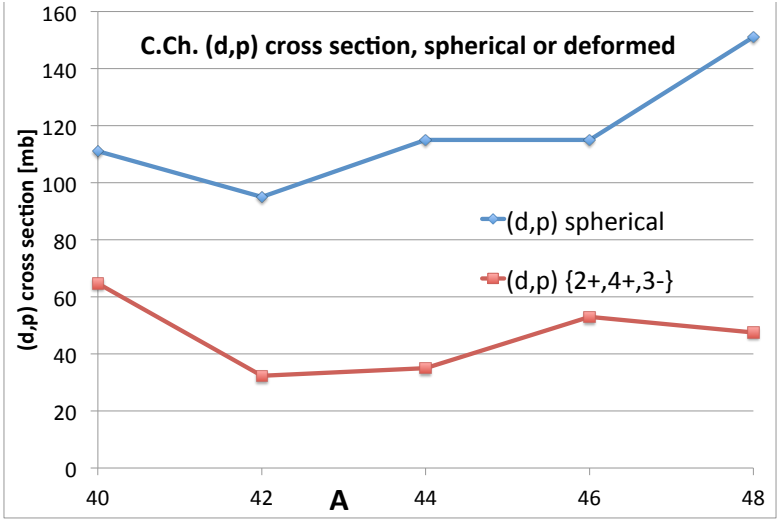

Fig. 2: FRESCO computations of thermal neutron capture (a, upper) and deuteron stripping (b, lower) with coupling to 2^+, 4^+, and 3^- collective states in the coupled-channel model on ^{40,42,44,46,48}Ca isotopes. These results suggest that refitting of spectroscopic factors to the (d,p) data would make (n,γ) computations more consistent with the capture data. The decrease observed for deformed (d,p) calculations suggests that an off-setting increase in spectroscopic factors would increase the computed capture cross section for better agreement with the data.

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References

[1] I. J. Thompson, *Computer Physics Reports* **7**, 167 (1988).
[2] A. M. Lane and R. G. Thomas, *Rev. Mod. Phys.* **30** 257 (1958).

[3] S.-S. Zhang, M. S. Smith, G. Arbanas, and R. L. Kozub, *Phys. Rev.* **C86**, 032802 (2012).
[4] R. Surman, J. Beun, G. C. McLaughlin, and W. R. Hix, *Phys. Rev.* **C79**, 045809 (2009).
[5] N. M. Larson, *Updated Users' Guide for SAMMY: Multilevel R-Matrix Fits to Neutron Data Using Bayes' Equations*, ORNL/TM-9179/R8 (2008).
[6] W. E. Parker, et al., *Phys. Rev.* **C52**, 252 (1995).
[7] I. J. Thompson, J. E. Escher, G. Arbanas, *Nuclear Data Sheets* **118**, 292 (2014).
[8] J. P. Boisson and S. Jang, *Nuclear Physics* **A189**, 334 (1972).
[9] S. Kahane, et al., *Phys. Rev.* **C36**, 533 (1987).
[10] A. M. Mukhamedzhanov and F. M. Nunes em Phys. Rev. **C72**, 017602 (2005).
[11] S. Raman, et al., *Phys. Rev.* **C39**, 1297 (1989).
[12] M. B. Chadwick, et al., *Nuclear Data Sheets* **112**, 2887-2996 (2011); `http://nndc.bnl.gov`.
[13] I. J. Thompson, F. M. Nunes, *Nuclear Reactions for Astrophysics*, (Cambridge Univ. Press, 2009).
[14] R. Capote *et al.*, *Nuclear Data Sheets* **110**, 3107 (2009); *Reference Input Parameter Library* (RIPL) `https://www-nds.iaea.org/`.
[15] A. J. Koning and J. P. Delaroche, *Nucl. Phys.* **A713**, 231 (2003).
[16] A. J. F. Siegert, *Phys. Rev.* **52**, 787 (1937).
[17] W. W. Daehnick, J. D. Childs, and Z. Vrcelj, *Phys. Rev.* **C21**, 2253 (1980).
[18] P. Capel, P. Danielewicz, and F. M. Nunes, *Phys. Rev.* **C82**, 054612 (2010).
[19] K. L. Jones, et al., *Nature* (London) **465**, 454 (2010); *Phys. Rev.* **C84**, 034601 (2011).
[20] R. L. Kozub, et al. *Phys. Rev. Lett.* **109**, 172501 (2012).