One-nucleon transfers to resonance states

I.J. Thompson[1]*, J.E. Escher*[1]*, G. Arbanas*[2]*, Ch. Elster*[3]*, V. Eremenko*[3,4]*, L. Hlophe*[3]*, and F.M. Nunes*[5]*, TORUS Collaboration* [1] Lawrence Livermore National Laboratory L-414, Livermore, CA 94551, USA,
[2] Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA,
[3] Ohio University, Athens, OH 45701, USA,
[4] M.V. Lomonosov Moscow State University, Moscow, 119991, Russia
[5] Michigan State University, East Lansing, MI 48824, USA

Abstract
We examine the contributions from interior, surface and exterior parts of the matrix elements for (d,p) neutron-transfer matrix elements, and show how their sum may be written as interior-post, exterior-prior terms along with a surface term. If we locate our surface according to the distance of the transferred neutron to the target, then the three terms depend on the neutron wave function at specific radii, and the surface and exterior terms depend only on that part of the neutron wave function determined by R-matrix parameters for neutron-target scattering.

1. Deuteron stripping

Deuteron-induced reactions, in particular (d,p) one-neutron transfer reactions, have been used for decades to investigate the neutron single-particle structure of bound states of nuclei. The reaction typically involves a large momentum transfer of the neutron from a high-energy beam to a bound state, so the shape of the exit proton angular distribution depend on the transferred angular momentum ℓ. The analyzing powers for polarized deuterons depends on the neutron j value, and the magnitude of that cross sections is used to extract a spectroscopic factor $S_{\ell j}$ for each discrete bound state. In first order the cross section scales exactly with the spectroscopic factor, while for some cases higher-order corrections introduce non-linearities. These high-order corrections arise from collective excitations in the entrance or exit channels, leading to Coupled-Channels Born Approximation (CCBA) methods. They may also arise for weakly-bound neutron bound states, when multi-step transfers become significant, leading to Coupled-Reaction Channels (CRC) methods.

At excitation energies above the weakly-bound states, we see of course resonances. Specific resonances are very often of importance in astrophysics and other applications because their existence may change low-energy cross sections by many orders of magnitude. It is therefore of great importance to measure their structure by whatever means are possible. Resonances with open neutron channels, therefore, can and should be probed by (d,p) transfer reactions. Resonances, however, are not characterized by spectroscopic factors, but by their resonance energies E_r and widths Γ. For a resonance which can be populated (and hence decay) by multiple channels c, each channel has a partial width Γ_c such that the total width of the resonance is the sum $\Gamma = \sum_c \Gamma_c$. These channels, for example, could be inelastic excitations of the target, (n,γ) capture, or proton or α-particle emissions. Ideally each partial width Γ_c should be measured. These energies and widths are directly related to the parameters (pole energies and reduced-width amplitudes) of R-matrix theory [1, 2], either by fitting to observed cross sections, or by derivation from some microscopic model. It is therefore very desirable to have predictions of (d,p) cross sections that are based on R-matrix parameters for the neutron-target interaction. These are exactly the parameters that would describe neutron scattering on the target, so, in ideal cases, observables from (d,p) experiments could be used to predict neutron scattering and the multichannel cross sections for γ, proton or α exit channels. If the target is radioactive and therefore unable to be a target, then results from inverse kinematic experiments with deuteron targets may be used to predict neutron cross

sections.

But is it *possible* to formulate of (d,p) resonant cross sections in terms of R-matrix parameters for the neutron? These parameters describe surface and external properties of the wave function of neutron-target scattering, so we have to determine the dependence of (d,p) cross sections also on the neutron wave function *inside* the surface of the target.

One possible obstacle is that the ℓ-dependence of the (d,p) cross section involving resonances is much less than for reactions to bound states, because the momentum transfer is reduced. Since resonant (d,p) reactions involve the continuum, they represent one form of deuteron breakup. In the limit of 'transfer' to neutron states at the energies of about half the beam energy, the energy and momentum transfers pass through a minimum, and we must then expect almost *no* dependence on the transferred angular momentum ℓ. This means that (d,p) transfers to resonances may well require higher beam energies for the deuteron, even though this leads to lower cross sections.

2. Post-prior transformations

To examine the dependence of (d,p) cross sections on the interior, surface and exterior regions of the final neutron state, we use the new formalism [3] devised by Akram Mukhamedzhanov of our TORUS collaboration. This is to define a *surface operator* by means of transforming from post to prior matrix elements not over all space as usual, but at a specific surface radius ρ measured for the neutron-target distance r_n. Let us see how this comes about.

In first-order DWBA for A(d,p)B reactions, the post and prior matrix elements for transfer from the bound state $\Phi_d(r)$ in a deuteron to a final state $\Phi_n(r_n)$ around a target are

$$M_{dp}^{\rm post} = \langle \Phi_n(r_n) u_p(r_p) | \mathscr{V}_{\rm post} | \Phi_d(r) u_d(R) \rangle \quad (1)$$

$$\text{and} \quad M_{dp}^{\rm prior} = \langle \Phi_n(r_n) u_p(r_p) | \mathscr{V}_{\rm prior} | \Phi_d(r) u_d(R) \rangle, \quad (2)$$

where $u_d(R)$ and $u_p(r_p)$ are the incoming deuteron and outgoing proton optical-model wave functions with potentials U_{dA} and U_{pB} respectively. The transfer operators are $\mathscr{V}_{\rm post} = V_{np} + V_{pA} - U_{pB}$ and $\mathscr{V}_{\rm prior} = V_{nA} + V_{pA} - U_{dA}$.

It can be proven in this first-order case that the post and prior expressions are exactly equivalent. This proof uses the Hermiticity of the total kinetic energy operator T, which has equivalent representations for post $\overleftarrow{T} = T_{nA} + T_{pB}$ and prior $\overrightarrow{T} = T_{np} + T_{dA}$ based on the two equivalent sets of Jacobi coordinates $\{r_n, r_p\}$ and $\{r, R\}$ for this three-body problem consisting of the proton, neutron and target A.

The kinetic energy operator is Hermitian in this case because at least one of the bound states $\Phi_d(r)$ and $\Phi_n(r_n)$ goes to zero at large distances. Since the deuteron bound state wave function $\Phi_d(r)$ decays to zero for large r, the post-prior equivalence of the DWBA matrix element holds not only for bound final states, but also for unbound scattering states of the neutron on the target.

We can also choose to interchange the two forms of the kinetic energy, $\overleftarrow{T}$ and $\overrightarrow{T}$ for *limited* regions of the integral over the neutron coordinate r_n. Let us define $M_{dp}^{\rm post}(a,b)$ and $M_{dp}^{\rm prior}(a,b)$ as the respective integrals over the range $a < r_n < b$ (the integral over the full range of the r_p coordinate is implied). These two matrix element differ by

$$M_{dp}^{\rm post}(a,b) = M_{\rm surf}(a) + M_{dp}^{\rm prior}(a,b) - M_{\rm surf}(b), \quad (3)$$

$$\text{where we define} \quad M_{\rm surf}(\rho) = \langle \Phi_n(r_n) u_p(r_p) | \overleftarrow{T} - \overrightarrow{T} | \Phi_d(r) u_d(R) \rangle_{r_n > \rho}. \quad (4)$$

The result in the previous paragraphs follows from $M_{\rm surf}(0) = M_{\rm surf}(\infty) = 0$.

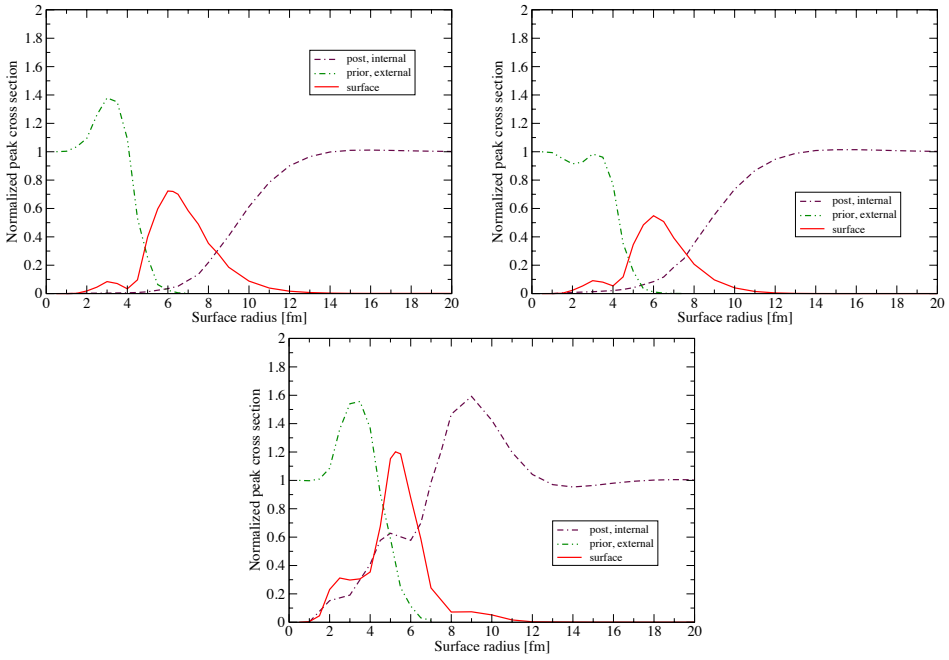

Fig. 1: Interior, surface, and exterior contributions for ^{48}Ca(d,p)^{49}Ca stripping to the $3/2^-$ ground state, at E_d = 13 (top left), 19.3 (top right), and 56 MeV (bottom). Shown are the peak cross sections of the individual contributions (which are proportional to $|M_{\rm int}^{(post)}(0,\rho)|^2$, $|M_{\rm surf}(\rho)|^2$, $|M_{\rm ext}^{(prior)}(\rho,\infty)|^2$, respectively), as a function of surface radius selected. The results are normalized to the peak cross section of the full calculation.

If M_{dp} is the matrix element over the whole space (prior or post, since they are equal), then [3]

$$M_{dp} = M_{dp}^{\rm prior}(0,\rho) + M_{dp}^{\rm prior}(\rho,\infty), \tag{5}$$

$$\text{hence}\quad M_{dp} = M_{dp}^{\rm post}(0,\rho) + M_{\rm surf}(\rho) + M_{dp}^{\rm prior}(\rho,\infty). \tag{6}$$

This final equation (6) is of considerable practical value, as it shows a way to separate interior, surface and exterior contributions of different kinds. The split can be examined for a range of surface radii ρ. The interior post (first) term is model-dependent, while the exterior prior and surface (second and third) terms are related to the asymptotic properties of the wave function.

The term $M_{\rm surf}(\rho)$ is called a *surface* term, although given in equation (4) as a volume integral, because the Green's theorem allows us to convert a volume integral into the surface integral

$$\int\limits_{r\geq\rho} {\rm d}\mathbf{r}\, f(\mathbf{r}) \left[\overleftarrow{T} - \overrightarrow{T}\right] g(\mathbf{r}) = \frac{\hbar^2}{2\mu} \oint\limits_{r=\rho} {\rm d}\mathbf{S}\, \left[g(\mathbf{r})\nabla_{\mathbf{r}} f(\mathbf{r}) - f(\mathbf{r})\nabla_{\mathbf{r}} g(\mathbf{r})\right]$$

$$= \frac{\hbar^2}{2\mu}\,\rho^2 \int {\rm d}\Omega_{\mathbf{r}} \left[g(\mathbf{r})\,\frac{\partial f(\mathbf{r})}{\partial r} - f(\mathbf{r})\,\frac{\partial g(\mathbf{r})}{\partial r}\right]_{r=\rho}. \tag{7}$$

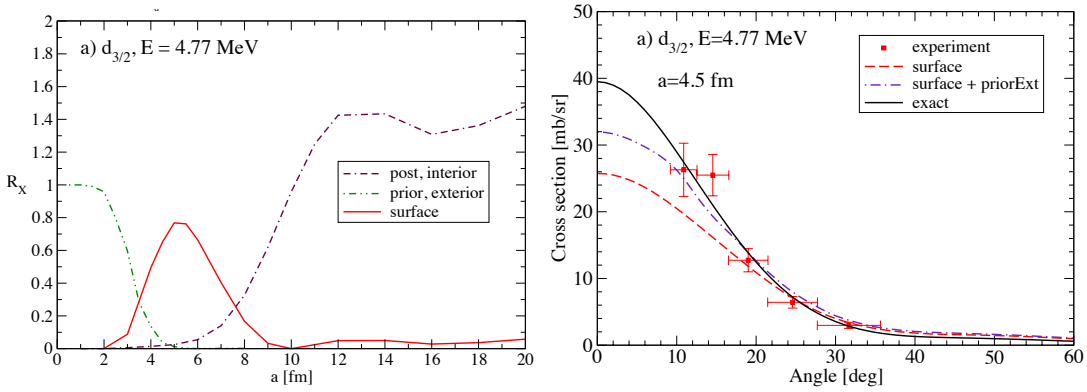

Fig. 2: Examination of interior, surface, and exterior contributions for transfers to resonance states in ^{21}O, for the $3/2^+$ resonance at 4.77 MeV. The left panel shows the interior (post) term, the surface term, and the exterior (prior) term, as a function of the surface radius. The right panel shows that improvements to the surface-term-only approximation can be achieved by including contributions from the prior-exterior term and selecting a small surface radius. The potential V_{nA}, which binds the neutron to the ^{20}O nucleus, has a radius of 3.39 fm and a diffuseness of 0.65 fm.

3. Surface transfer operator in first-order models

The matrix elements of a surface transfer operator $M_{\rm surf}(\rho)$ can be very simply calculated in first-order DWBA. We may simply take the difference of equations (5) and (6) above, giving

$$M_{\rm surf}(\rho) = M_{dp}^{\rm post}(0,\rho) - M_{dp}^{\rm prior}(0,\rho)\ . \tag{8}$$

This can be easily accomplished using any code for first-order finite-range transfer calculations, such as FRESCO [4]. Using this equation we first examined [5, 6] the sizes of the three terms in equation (6) for a range of bound and resonant states for the final neutron in a (d,p) reaction.

Fig. 1 shows, within the DWBA formalism, that the surface contribution is dominant for bound states at around 5-7 fm for the ^{48}Ca(d,p) at three different beam energies. But still there are non-negligible contributions from both the interior (post) and exterior (prior) terms.

We have also carried out calculations that test the dominance of the surface term for transfer reactions that populate resonances, such as the $3/2^+$ resonance in ^{21}O at 4.77 MeV measured recently in a ^{20}O(d,p) experiment [7]. We see in the left panel of Fig. 2 that at around $\rho = 5$ fm the surface term appears to largely dominate. However, the other terms contribute not incoherently as cross sections, but coherently as amplitudes. The red-dashed curve in the right panel shows the angular distribution arising solely from the surface term, and this is clearly short of the exact result. Adding in an exterior-prior contribution does improve the accuracy considerably, but still not entirely.

4. Surface transfer operator in coupled-channel models

The surface contributions extracted in the previous section are within the context of first-order theory, as then post and prior matrix elements give identical results and differences can be taken. If breakup in the entrance channel, say, is important, then it is necessary to go beyond first order. In that case, only the post matrix element uses the coupled-channels wave function in the entrance channel from the CDCC methods discussed earlier. This means that the surface operator has to be calculated explicitly in terms of the multi-channel CDCC wave functions $\psi_{\rm CDCC}(\mathbf{R},\mathbf{r})$. In coupled-channel models for transfers, we use a source term $S_\beta(R')$ that depends on $\psi_{\rm CDCC}(\mathbf{R},\mathbf{r})$ to calculate the matrix elements M_β as the

asymptotic amplitude of the outgoing wave solutions of $[E_\beta - H_\beta]u_\beta(R') = S_\beta(R')$, where $R' \equiv r_p$ is the coordinate of the exiting proton.

In order to calculate transfer cross sections with the surface operator at some final neutron radius $r' = \rho$, we now have to implement the general surface operator of eq. (7), not just its value from the 'prior–post' difference as used in eq. (8) above.

The source term $S_\beta(R')$ for the transfer channel using the surface operator at radius $r' = \rho$ is

$$S^{\rm surf}_\beta(R') = \langle Y_\beta(\hat{R}',\hat{r}')\Phi_\beta(r')|\overleftarrow{T} - \overrightarrow{T}|\psi_{\rm CDCC}(\mathbf{R},\mathbf{r})\rangle_{r'>\rho} \tag{9}$$

$$= \langle Y_\beta(\hat{R}',\hat{r}')\Phi_\beta(r')|\overleftarrow{T_{vA}} - \overrightarrow{T_{vA}}|\psi_{\rm CDCC}(\mathbf{R},\mathbf{r})\rangle_{r'>\rho} \tag{10}$$

where $r' \equiv r_n$ is the coordinate of the neutron in the final state. Transforming this matrix element into a surface integral with Green's theorem, we have

$$S^{\rm surf}_\beta(R') = \frac{\hbar^2}{2\mu_n}\int_0^\infty dr' \left\langle Y_\beta(\hat{R}',\hat{r}')\Big|\delta(r'-\rho)\left[\psi_{\rm CDCC}(\mathbf{R},\mathbf{r})\frac{\partial\Phi_\beta(r')}{\partial r'} - \Phi_\beta(r')\frac{\partial\psi_{\rm CDCC}(\mathbf{R},\mathbf{r})}{\partial r'}\right]\right\rangle$$

$$= \frac{\hbar^2}{2\mu_n}\int_0^\infty dr' \left\langle Y_\beta(\hat{R}',\hat{r}')\Big|\delta(r'-\rho)\left[\frac{\partial\Phi_\beta(r')}{\partial r'} - \Phi_\beta(r')\frac{\partial}{\partial r'}\right]\Big|\psi_{\rm CDCC}(\mathbf{R},\mathbf{r})\right\rangle \tag{11}$$

Since the derivative operator $\partial/\partial r'$ acts on both the radial and angular components of the vectors $(\mathbf{R},\mathbf{r})$ in the entrance channel, a large number of terms and derivatives need to be evaluated:

We will need the following derivatives for the linear combination $\mathbf{r} = p\mathbf{r}' + q\mathbf{R}'$:

$$\frac{\partial}{\partial r'}Y^m_\ell(\hat{\mathbf{r}})\frac{\varphi_\alpha(r)}{r} = \frac{\varphi_\alpha(r)}{r}\frac{\partial}{\partial r'}Y^m_\ell(\hat{\mathbf{r}}) + Y^m_\ell(\hat{\mathbf{r}})\frac{\partial}{\partial r'}\frac{\varphi_\alpha(r)}{r}$$

$$= \frac{p}{r}\left\{\sqrt{\frac{4\pi\ell(2\ell+1)}{3}}\frac{\varphi_\alpha(r)}{r}\sum_{\lambda=-1}^{1}\langle\ell{-}1\ m{-}\lambda, 1\lambda|\ell m\rangle Y^{m-\lambda}_{\ell-1}(\hat{\mathbf{r}})Y^\lambda_1(\hat{\mathbf{r}}') + Y^m_\ell(\hat{\mathbf{r}})\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}'\left[\varphi'_\alpha(r) - \frac{\ell+1}{r}\varphi_\alpha(r)\right]\right\} \tag{12}$$

The needed source term $S^{\rm surf}_{\beta\alpha}(R')$ in a final (proton) channel β from the initial (deuteron) channel α is, with the surface operator,

$$S^{\rm surf}_{\beta\alpha}(R') = \langle Y_\beta(\hat{R}',\hat{r}')\Phi_\beta(r')|\overleftarrow{T_{nA}} - \overrightarrow{T_{nA}}|\Phi_\alpha(r)\,Y_L(\hat{R})u_\alpha(R)\rangle_{r'>\rho}$$

$$= \frac{-\hbar^2}{2\mu_n}\int_0^\infty dr' \left\langle Y_\beta(\hat{R}',\hat{r}')\Big|\delta(r'-\rho)\left[\frac{\partial\Phi_\beta(r')}{\partial r'} - \Phi_\beta(r')\frac{\partial}{\partial r'}\right]\Big|\Phi_\alpha(r)Y_L(\hat{R})\right\rangle u_\alpha(R), \tag{13}$$

where Φ_β is the final state of the neutron, whether bound or unbound. This coupling is non-local as $R' \neq R$, and depends on the derivatives of the deuteron incoming wave function $u_\alpha(R)$, so we need to calculate the two non-local kernel functions $\mathsf{X}_{\beta\alpha}(R',R)$ and $\mathsf{Y}_{\beta\alpha}(R',R)$ to give the source term as

$$S^{\rm surf}_{\beta\alpha}(R') = \int_0^\infty dR\,\mathsf{X}_{\beta\alpha}(R',R)u_\alpha(R) + \int_0^\infty dR\,\mathsf{Y}_{\beta\alpha}(R',R)\left[u'_\alpha(R) - \frac{L_\alpha+1}{R}u_\alpha(R)\right]. \tag{14}$$

The derivative operators in Eq. (13) operate on all of the radii r,R and their angles $\hat{r},\hat{R}$, so X has four terms. Using the $\Phi_\alpha(r)$ and $\hat{\Phi}_\alpha(r) = \frac{1}{r}(\varphi'_\alpha(r) - \frac{\ell+1}{r}\varphi_\alpha(r))$ variables for both entrance and exit

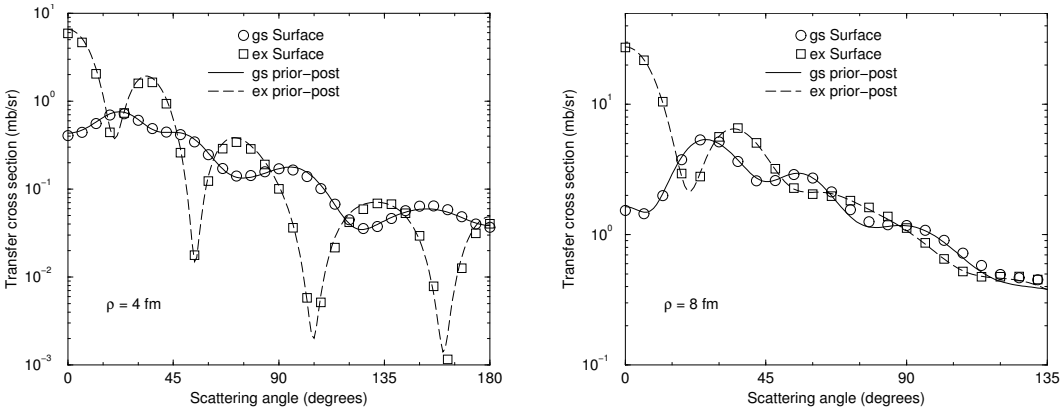

Fig. 3: Comparison of methods to calculate the surface transfer contribution, for the ^{90}Zr(d,p) reaction at deuteron energy of 11 MeV. On the left the surface radius is $a = 4$ fm, and on the right 8 fm. The gs is a $d_{5/2}$ neutron state, and the excited state is a $s_{1/2}$ state.

channels, kinematical coefficients a', b', p, P, J, and Clebsch-Gordan products $G^{\alpha'\alpha}_{m'_\ell M_L m_\ell}$, we have derived

$$
\begin{aligned}
X_{\alpha'\alpha}(R',R) = J\frac{\hbar^2}{2\mu_n}\frac{\rho}{a'b'}\sum_{m'_\ell m_\ell}\sum_{M_L=-1}^{1} G^{\alpha'\alpha}_{m'_\ell M_L m_\ell}\, P_{\ell'}^{|m'_\ell|}(\cos\theta_{r'})P_{L'}^{|M_L+m_\ell-m'_\ell|}(\cos\theta_{R'})
\\
\Bigg[\ \Phi'_\beta(\rho)\ Y_L^{M_L}(\hat{\mathbf{R}})\ Y_\ell^{m_\ell}(\hat{\mathbf{r}})\Phi_\alpha(r)
\\
-\Phi_\beta(\rho)\ Y_L^{M_L}(\hat{\mathbf{R}})\frac{p}{r}\sqrt{\frac{4\pi\ell(2\ell+1)}{3}}\sum_{\lambda=-1}^{1}\langle \ell{-}1\ m_\ell{-}\lambda, 1\lambda\,|\ell m_\ell\rangle Y_{\ell-1}^{m_\ell-\lambda}(\hat{\mathbf{r}})Y_1^\lambda(\hat{\mathbf{r}}')\Phi_\alpha(r)
\\
-\Phi_\beta(\rho)\ Y_L^{M_L}(\hat{\mathbf{R}})p\ \hat{\mathbf{r}}\cdot\hat{\mathbf{r}}'\ Y_\ell^{m_\ell}(\hat{\mathbf{r}})\hat{\Phi}_\alpha(r)
\\
-\Phi_\beta(\rho)\ Y_\ell^{m_\ell}(\hat{\mathbf{r}})\Phi_\alpha(r)\ \frac{P}{R}\sqrt{\frac{4\pi L(2L+1)}{3}}\sum_{\Lambda=-1}^{1}\langle L{-}1\ M_L{-}\Lambda, 1\Lambda|LM_L\rangle Y_{L-1}^{M_L-\Lambda}(\hat{\mathbf{R}})Y_1^\Lambda(\hat{\mathbf{r}}')\Bigg],
\end{aligned}
\tag{15}
$$

and the derivative term (with $M_L = 0$):

$$
\begin{aligned}
Y_{\alpha'\alpha}(R',R) = -J\frac{\hbar^2}{2\mu_n}\frac{\rho}{a'b'}P\Phi_\beta(\rho)\Phi_\alpha(r)\ \ \hat{\mathbf{R}}\cdot\hat{\mathbf{r}}'
\\
\times\sum_{m'_\ell m_\ell} G^{\alpha'\alpha}_{m'_\ell 0 m_\ell}P_{\ell'}^{|m'_\ell|}(\cos\theta_{r'})P_{L'}^{|m_\ell-m'_\ell|}(\cos\theta_{R'})\ Y_\ell^{m_\ell}(\hat{\mathbf{r}})Y_L^0(\hat{\mathbf{R}}).
\end{aligned}
\tag{16}
$$

We use the rotated coordinate frame that has the z-axis parallel to $\mathbf{R}$ and the x-axis in the plane of $\mathbf{R}$ and $\mathbf{R}'$. The resulting operator is still non-local like other finite-range transfer operators, but does not require any internal quadrature over angles. The integral operators, when $\mathbf{r} = a\mathbf{R} + b\mathbf{R}'$, are now

$$
\int d\hat{\mathbf{R}}'\int d\hat{\mathbf{R}}'\ \ \delta(r'-\rho) = \frac{8\pi^2\rho}{a'b'RR'}\Big|_{u=(\rho^2-a'^2R^2+b'^2R'^2)/(2a'b'RR')}.
\tag{17}
$$

These expressions (15) and (16) have been directly implemented in a LLNL version of our coupled-channels code FRESCO [4]. In this way we can go beyond first order for neutron transfers both to bound

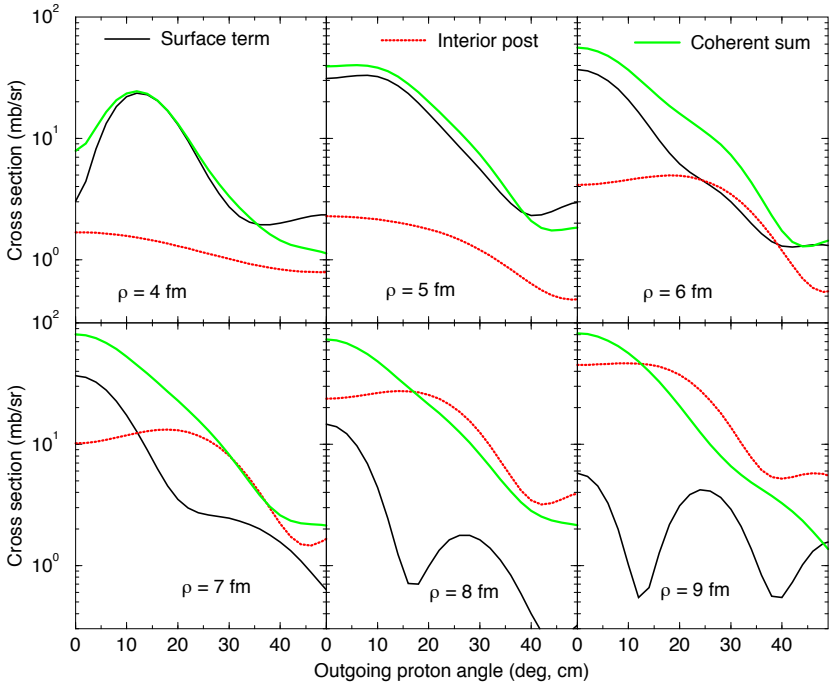

Fig. 4: The cross sections from separate surface and interior-post terms (black and red respectively) along with their coherent sum (green line). The different panels show the variations from using different radii ρ of the surface.

and resonance states. The new calculations for the surface term have been validated by comparison with the angular cross sections obtained in the work described in the previous section 3.. The comparisons are shown in Figure 3.

Furthermore, their values depends only on the wave function $\Phi_\beta(\rho)$ and derivative $\Phi'_\beta(\rho)$ of the final neutron wave function at the surface, and these are precisely the numbers that can be uniquely obtained from standard R-matrix fits of pole positions and reduced-width amplitudes.

When using the surface term derived from CDCC wave functions, the exterior-prior term in eq. (6) does not appear. That is because the prior term is the coupling derived from $\mathcal{V}_{\rm prior} = V_{nA} + V_{pA} - U_{dA}$, and this interaction potential is expected to be that which is diagonalized by the CDCC solution.

5. Results

Nevertheless, as shown in the previous section, the interior-post terms are still significant and need to be added coherently to the surface contributions. The exterior-prior term is not expected to be present if we use sufficiently accurate CDCC wave functions in the source term.

Figure 4 shows the relative sizes of these terms for a ^{20}O(d,p)^{21}O reaction that populates a d-wave neutron resonance at 0.9 MeV. The different panels show the effects of different radii of the surface, where the black curve shows the surface term, the red line the interior post by itself, and the green curve shows their coherent sum. Except for the smallest surface radius, the coherent sums are nearly constant, but there are large variations in the relative sizes of the surface and interior-post terms. This will provide an essential tool for probing how much these transfer cross sections measure the surface properties described by R-matrix theory, compared with measuring in part the interior part of the resonance wave functions.

6. Discussion

Our aim is to fit neutron pole energies and partial widths to (d,p) cross sections across a resonance. There have been experiments with many wide and narrow resonances, often overlapping, such as the early ^{15}N(d,p)^{16}N* experiments of [8, 9]. Our results can be generalized to multichannel exit wave functions, so that, for example, if experiments measure ^{12}N(d,p)^{16}N$^* \to \alpha+^{12}$B, then the results of our analyses can be used to predict the (n,α) cross section for a ^{12}N target.

Experimentally, resonance structures are most often studied in elastic and inelastic scattering reactions. For those reactions, the phenomenological R-matrix approach has been extremely useful for the interpretation of experiments and for extracting resonance energies and widths from measured cross sections. The surface integral formalism [3] is essentially an extension of the R-matrix approach to (d,p) transfer reactions. It holds the potential to overcome present difficulties in describing transfers to resonance states and to become a practical and sound way for extracting structure information from transfer experiments, since: a) It reduces the dependence of the cross section calculations on the model used for the nuclear interior; b) it reduce (in DWBA) or eliminates (in CDCC) the impact of the slow convergence of calculations of the exterior term when resonances are considered; and c) it establishes a useful link between resonance properties and transfer observables, since the surface term $M_{\rm surf}(\rho)$ can be parameterized in terms of quantities that are familiar from traditional R-matrix approaches. When resonance studies were carried out in the DBWA formalism, convergence was found to be difficult to achieve, but the results obtained so far show trends similar to what was found for bound states, with reduced contributions from the nuclear interior.

In conclusion, we note that the surface formalism for studying resonances with (d,p) uses successful R-matrix ideas to emphasize asymptotic properties of the wave function. It is based on a separation into interior and exterior, and leads to a surface term which can be expressed in terms of familiar R-matrix parameters, thus providing spectroscopic information. Our DWBA and CDCC studies show that the surface term is dominant both in first-order and higher-order calculations, and that the dependence on a model for nuclear interior is reduced. The surface term alone is not sufficient, however, to entirely describe the cross sections for transfer reactions, since we find that corrections are required. We expect that an accurate CDCC implementation (which includes breakup effects) should already include copings in the exterior, so that there will be no need to include the exterior-prior term when using the formalism laid out here.

Acknowledgements

This work is supported through the US DOE's Topical Collaboration TORUS (www.reactiontheory.org). It is performed under the auspices of the DOE by LLNL under Contract No. DE-AC52-07NA27344, by Ohio University under Contract No. DE-SC0004087, under the U.T. Battelle LLC Contract No. DE-AC0500OR22725 and by MSU Contract No. DE-SC0004084.

References

[1] A. M. Lane, R. G. Thomas *Rev. Mod. Phys.* **30**, 257 (1958).
[2] P. Descouvemont, D. Baye *Reports on Progress in Physics* **73**(3), 036301 (2010).
[3] A. M. Mukhamedzhanov *Phys. Rev. C* **84**, 044616 Oct (2011).
[4] I.J. Thompson *Computer Physics Reports* **7**(4), 167 – 212 (1988).
[5] J. E. Escher, A. M. Mukhamedzhanov, I. J. Thompson, *J. Phys.: Conf. Series* **403**(1), 012026 (2012).
[6] J. E. Escher, *et al. Phys. Rev. C* **89**, 054605 May (2014).
[7] B. Fernández-Domínguez *et al. Phys. Rev. C* **84**, 011301 Jul (2011).
[8] P. Hewka, C. Holbrow and R. Middleton, *Nucl. Phys.* **88**(3), 561 – 575 (1966).
[9] H. Fuchs *et al.*, *Nucl. Phys. A* **196**(2), 286 – 302 (1972).
[10] Carlson, B. V., Escher, J. E., and Hussein, M. S. *J.Phys.* **G41**, 094003 (2014).