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**implications for top mass**

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# Radiative and muonic decays of $K_L$: implications for top mass

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**Abstract**

We examine the implications for $K_L$ phenomenology of the recent experimental determinations of the form factor for the decay $K_L \to e^+e^-\gamma$. In particular, we reexamine the role of the top quark in light of the new measurements, and we show that for a branching ratio of $K_L \to \mu^+\mu^-$ larger than 1.25 times the unitarity bound, as indicated by one experiment, the top quark mass has to be larger than 115 GeV. We predict a ratio between the rates for $K_L \to e^+e^-\gamma$ and $K_L \to \mu^+\mu^-\gamma$ of approximately 24.

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Recently, two measurements [1, 2] of the form factor of $K_L$ Dalitz decays, $K_L \to e^+e^-\gamma$, have been reported. The experimental determination of this form factor is an important achievement for several reasons. First, it probes in a rather direct way the various long-distance contributions in the pseudoscalar and vector meson sectors of the theory; in particular it sheds light on the mechanism of the $\Delta I = 1/2$ rule in the non-leptonic weak kaon decays [3], by measuring for the first time a $\Delta S = 1$ weak nonleptonic transition between (virtual) vector states. Secondly, it has implications also for the decay $K_L \to \mu^+\mu^-$, which has been used [4, 5, 6] to constrain the parameters of the electroweak sector, in particular the top quark mass. (For a review of rare kaon decay phenomenology and its implications, see [7]). In fact, in [5] it was shown for the first time that the relatively long B meson lifetime taken together with $K_L \to \mu^+\mu^-$ and other phenomenology of the kaon sector allows generically high values for the top quark mass, above 100 GeV or so, which was quite unexpected at that time.

When extracting the weak contribution to $K_L \to \mu^+\mu^-$ (dominated by the top quark), one faces the problem of subtracting the electromagnetic contribution from the real part of the amplitude. The (imaginary) part of the amplitude coming from real photons in the intermediate state has been unambiguously determined from the measured rate for $K_L \to \gamma\gamma$. The real part coming from virtual photons has been unknown, although it has been argued to be small based on the comparison with $\eta \to \mu^+\mu^-$ [8]. However, it was shown in [5] that there is a class of diagrams, vanishing for on-shell photons, that is present in the case of $K_L \to \mu^+\mu^-$ but absent for $\eta \to \mu^+\mu^-$ and which therefore spoils the analogy. This extra contribution can be parametrized in terms of an unknown constant $\alpha_K$ [3, 5], and it is precisely this parameter that has been measured in the new experiments [1, 2]. Prompted by these new measurements, we have redone the analysis of $K_L \to \mu^+\mu^-$ in light of the new experimental results.

We start by defining the amplitude for $K_L \to \gamma\gamma^*$. It is given by (any CP-violating effects are safely neglected here)

$$\mathcal{A}(K_L \to \gamma\gamma^*) = iA_{\gamma\gamma^*}(s)\epsilon_{\mu\nu\sigma\tau}\epsilon_1^\mu(k_1)\epsilon_2^\nu(k_2)k_1^\sigma k_2^\tau, \tag{1}$$

where $s = k_1^2$ is the invariant mass of the off-shell photon. The form factor $A_{\gamma\gamma^*}(s)$ has two contributions [3], one which smoothly extrapolates to the measured value for the on-shell two-gamma amplitude $A_{\gamma\gamma}^{exp}$ (this is the part that has a correspondence in the $\eta \to \gamma\gamma^*$ amplitude; ordinary vector dominance is known to work here [9]) and one which is unique to the strangeness changing radiative non-leptonic transition and which vanishes for an on-shell photon [3, 10, 11]:

$$A_{\gamma\gamma^*}(s) = \frac{|A^{exp}_{\gamma\gamma}|}{1 - s/m_\rho^2} + \frac{\alpha_K A_{K^*}(s)}{1 - s/m_{K^*}^2}, \tag{2}$$

where

$$A_{K^*}(s) = \sqrt{2} e G_F f_{K^*K\gamma} \frac{m_\rho^2}{f_{K^*} f_\rho^2} \left[\frac{4}{3} - \frac{1}{1 - s/m_\rho^2} - \frac{1}{9}\left(\frac{1}{1 - s/m_\omega^2} + \frac{2}{1 - s/m_\phi^2}\right)\right]. \tag{3}$$

Here we have adopted a long-distance model, also based on vector meson dominance, by which the extra term corresponds to a $K \to K^*\gamma$ transition followed by a non-leptonic $K^* \to \rho, \omega, \phi$ transition (see [3] for details). Inserting the known values of the various couplings and masses appearing in (3), one finds (cf. [2])

$$A_{\gamma\gamma^*}(s) = |A^{exp}_{\gamma\gamma}| f(s/m_K^2), \tag{4}$$

with

$$f(x) = \frac{1}{1 - 0.42x} + \frac{2.5\alpha_K}{1 - 0.31x}\left[\frac{4}{3} - \frac{1}{1 - 0.42x} - \frac{1}{9(1 - 0.41x)} - \frac{2}{9(1 - 0.24x)}\right]. \tag{5}$$

In [3] predictions for the value of $\alpha_K$ were given for two distinct models. A phenomenological octet dominance model [12], which assumes a universal Cabibbo non-suppression for non-leptonic transitions, would correspond to $|\alpha_K| = 1$, whereas the dynamical model of Shifman, Vainshtein and Zakharov [13] (based on the QCD-amended nonleptonic weak Hamiltonian of the standard model) gives $|\alpha_K| = 1.22 \sin\theta_C \cos\theta_C = 0.26$ . The recent measurements of $\alpha_K$ in the decay $K_L \to e^+e^-\gamma$ give $\alpha_K = -0.28 \pm 0.13$ [1] and $\alpha_K = -0.280 \pm 0.083^{+0.054}_{-0.034}$ [2], respectively. It is interesting to note that this lower value of $|\alpha_K|$, predicted by QCD and now seemingly confirmed by the two experiments, indicates that the $\Delta I = 1/2$ enhancement in non-leptonic pseudoscalar-pseudoscalar transitions is indeed dynamical in nature and has no correspondence in vector-vector transitions, which obey the more moderate enhancement of a factor of about 3, provided by gluonic corrections to the coefficients $c_1 - c_3$ of the effective nonleptonic Hamiltonian [13]. One should also note that an important consistency check on this whole scheme is still to be made, namely to measure also the rate for $K_L \to \mu^+\mu^-\gamma$. A negative value of $\alpha_K$ means a relatively larger rate for this (phase-space suppressed) decay, and our prediction for the ratio between the rates for $K_L \to e^+e^-\gamma$ and $K_L \to \mu^+\mu^-\gamma$ is (*cf* Fig. 2 of [3]) approximately 24.

We now turn to the consequences of the experimental determination of $\alpha_K$ for the decay $K_L \to \mu^+\mu^-$. The branching ratio for $K_L \to \mu^+\mu^-$, normalized to $K_L \to \gamma\gamma$, can be written

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} = \frac{2\beta}{\pi^2}\frac{m_\mu^2}{m_K^2}|A_{\mu\mu}|^2, \tag{6}$$

where $\beta = \sqrt{1 - 4m_\mu^2/m_K^2}$. The model-independent absorptive part is given by

$$Im\, A_{\mu\mu} = -\frac{e^2}{8\beta}\log\left(\frac{1+\beta}{1-\beta}\right) \sim -3.8 \cdot 10^{-2}. \tag{7}$$

The real part can be written

$$Re\, A_{\mu\mu} = A_{e.m.} + A_W, \tag{8}$$

where $A_{e.m.}$ is the real part of the $2\gamma$ amplitude and $A_W$ is the weak contribution to the dispersive part. The unitarity lower bound on the branching ratio (6) is obtained by putting $Re\, A_{\mu\mu} = 0$, which corresponds to a branching ratio normalized to the $2\gamma$ decay of $1.2 \cdot 10^{-5}$ and a total branching ratio $\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to \text{all})$ of around $6.8 \cdot 10^{-9}$, for a $K_L \to \gamma\gamma$ branching ratio of $5.7 \cdot 10^{-4}$ [14]. The most recently published experimental values for $K_L \to \mu^+\mu^-$ are $(8.4 \pm 1.1) \cdot 10^{-9}$ [15] (hereafter referred to as EXP1) and $(5.8 \pm 1.0) \cdot 10^{-9}$ [16] (EXP2), respectively. We note that these values are incompatible with each other within the quoted errors, and that EXP2 indicates a very low value for $\Gamma(K_L \to \mu^+\mu^-)$ (in fact, their central value is $1\sigma$ *below* the unitarity limit).

When relating $K_L \to e^+e^-\gamma$ to $K_L \to \mu^+\mu^-$ one still faces the problem of how to continue $A_{\gamma\gamma^*}$ to $A_{\gamma^*\gamma^*}$, i.e. with both photons off-shell. In [5] two alternatives were considered, namely, to saturate one photon by vector mesons (corresponding to a $PV\gamma$ vertex) or to saturate both (corresponding to $PVV$). All realistic models for the off-shell $2\gamma$ form factor were found to give values somewhere inbetween these cases. (We remark that this question could be settled by measuring the decay $K_L \to e^+e^-e^+e^-$, which should occur with a branching ratio of a few times $10^{-8}$.) Using $\alpha_K = -0.28$, we infer from the expressions given in [5] that $A_{e.m.} = -0.6 \cdot 10^{-2}$ and $0.3 \cdot 10^{-2}$ in these two cases, respectively. [2] In our analysis, we will then choose $A_{e.m.}$ to be in this range. This means that the electromagnetic contribution to the real part of $A_{\mu\mu}$ is indeed very small. In the absence of weak contributions the prediction for the branching ratio of $K_L \to \mu^+\mu^-$ would be $1.02 \pm 0.02$ times the unitarity bound.

The weak contribution $A_W$ to $A_{\mu\mu}$ can be written [4, 5, 6] (neglecting the very small $c$ quark contribution)

$$|A_W| \sim \frac{0.43}{\sqrt{\mathrm{B.R.}(K_L \to \gamma\gamma)}} |Re\,\lambda_t| G(x_t)\eta_t, \tag{9}$$

where $\lambda_t = V_{ts}^* V_{td}$ ($V$ is the usual CKM mixing matrix), $x_t = m_t^2/m_W^2$,

$$G(x) = \frac{3}{4}\frac{x^2 \log(x)}{(1-x)^2} + \frac{1}{4}x + \frac{3}{4}\frac{x}{1-x}, \tag{10}$$

(see [17]), and $\eta_t$ is a QCD correction factor of order unity [18]. The unknown quantities entering are thus $m_t$ and $Re\,\lambda_t$. A recent analysis of the parameters entering the kaon phenomenology can be found in [19, 20]. Unfortunately, as is well known many of the parameters that enter the full analysis are still plagued by theoretical and experimental uncertainties. Anyway, we will tentatively use the results of the most recent work [20]. Using this analysis for the range of presently accepted values of $m_t$, namely between 90 and 250 GeV, we take

$$3.2 \cdot 10^{-4} < |Re\,\lambda_t| < 6.7 \cdot 10^{-4}. \tag{11}$$

This can now be used to correlate the $K_L$ branching ratio to its short- distance contribution, thus obtaining new bounds on the top quark mass. Varying $|Re\,\lambda_t|$ over its allowed range and taking the weak and the electromagnetic contributions to the real part of the amplitude to interfere either positively or negatively, we get the allowed range for the top quark mass shown in Fig. 1 displayed as a function of the branching ratio for $K_L \to \mu^+\mu^-$ in units of

[2] We use updated values for the experimental parameters appearing in Eqs. (8) of [5]: $f_\rho = 4.99, f_{K^*} = 5.78$, and $\Gamma(K^{*0} \to K^0\gamma) = 118.0$ keV [2, 14].

the unitarity limit. As can be seen, the upper bound is not very stringent unless the branching ratio is very small. Taking the $1.5\sigma$ upper limit of EXP2 as a limit for the branching ratio one would, e.g., obtain an upper bound for the top quark mass of around 170 GeV (*cf* [21]). However, there are still large uncertainties involved, both theoretical and experimental. For example, taking the central value of EXP1 instead, one gets the allowed range $115\,\mathrm{GeV} < m_t < 280\,\mathrm{GeV}$, which is, interestingly enough, more restrictive at the lower end than the present experimental bound of 90 GeV. Given the uncertainties involved, the most interesting thing is perhaps that one gets a quite reasonable range for $m_t$ also from this method. The importance of settling the discrepancy between the two experiments has to be stressed.

To conclude, we have reanalyzed the decay $K_L \to \mu^+\mu^-$ in light of the new measurement of the form factor in the $K_L \to e^+e^-\gamma$ decay. We have shown that the electromagnetic contribution to the dispersive part of this decay is small, meaning that a large experimental value has to be driven by a relatively heavy top quark. It is also important to emphasize the consistency of our model with the negative value of $\alpha_K$, now established experimentally [1, 2], which leads to a branching ratio for $K_L \to \mu^+\mu^-$ close to the unitarity bound. A positive value of $\alpha_K$ would have led to a much larger rate of twice the unitarity value. The limits on the top quark mass that follow from the experimentally measured branching ratio are not good enough to improve on current knowledge based on other methods, due to the uncertainty in several experimental and theoretical parameters. Our conclusions agree with those of previous work (e.g. [6]), but we now have additional information from the $K_L \to e^+e^-\gamma$ decay to support it. We want once again to point out to our experimental colleagues that there are a couple of processes that should be within reach of being measurable, notably $K_L \to e^+e^-e^+e^-$ and $K_L \to \mu^+\mu^-\gamma$, that would shed additional light on these questions. Given the experimental value of $\alpha_K = -0.28$, our prediction for the branching ratio of $K_L \to \mu^+\mu^-\gamma$ is $4.0 \cdot 10^{-7}$, some 30% larger than without the $K^*\gamma$ contribution.

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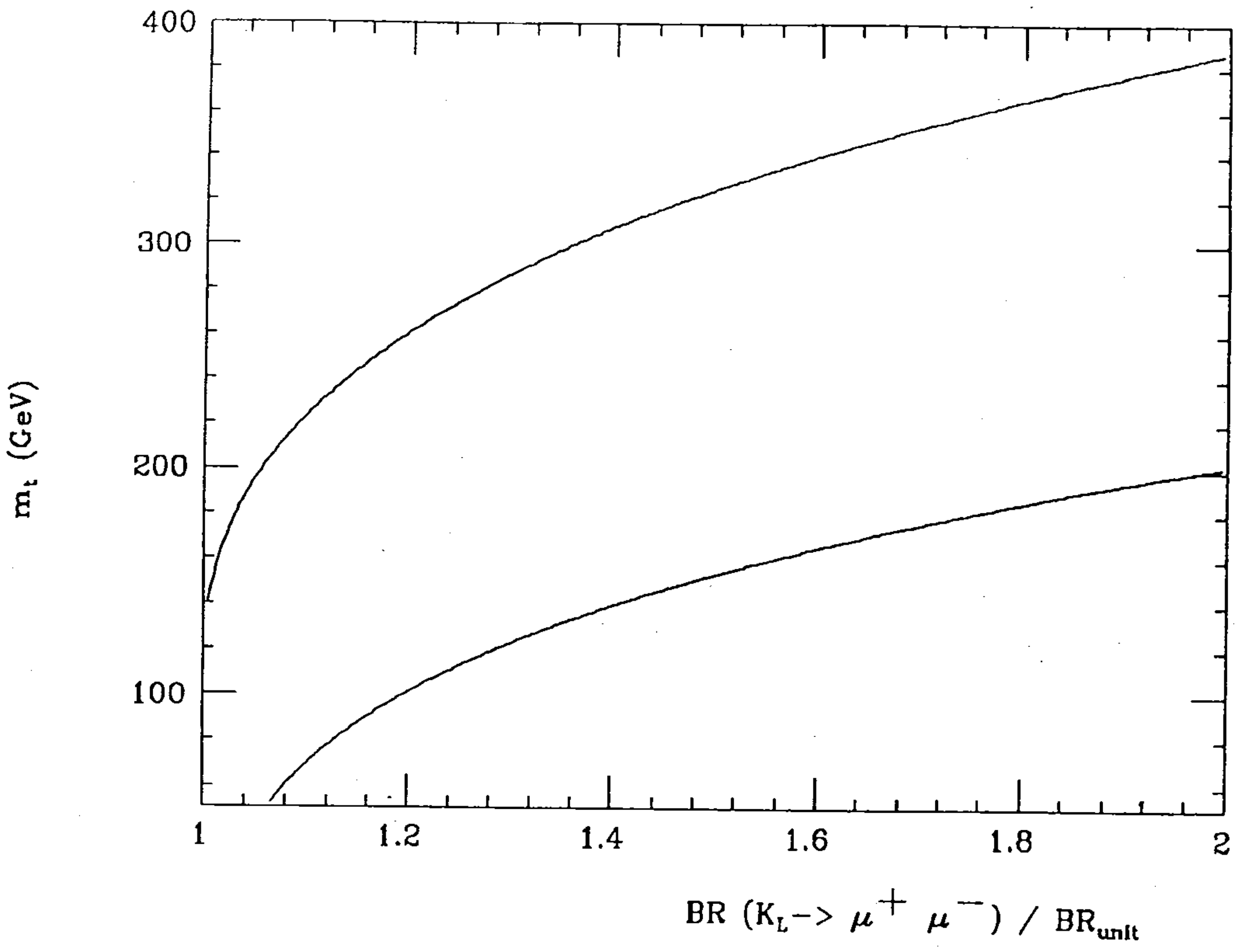
## Figure Caption

Fig. 1. Allowed range for the top quark mass as a function of the branching ratio for the decay $K_L \to \mu^+\mu^-$ normalized to the unitarity lower bound for the branching ratio of this decay, which is $6.8 \cdot 10^{-9}$.


400
300
200
100
$m_t$ (GeV)
1
1.2
1.4
1.6
1.8
2
BR ($K_L \to \mu^+ \mu^-$) / $BR_{unit}$